Intensity correlation function for light elastically scattered from a randomly rough metallic grating

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(Received 30 January 1989)

The speckle statistics of light elastically scattered from a randomly rough metal grating are calculated in the limit of partially developed speckle. Specifically, the intensity correlation function of the diffusely scattered light is determined. A multiple-scattering formulation which takes into account the dielectric properties of the metal is used. The calculation proceeds as a diagrammatic expansion in powers of the surface-roughness profile function. Results are presented for several sets of values of the parameters characterizing the Gaussian random surface roughness. For a fixed surface roughness, the dependence of features in the speckle pattern on the complex dielectric constant of the metal is studied by artificially changing the value of the imaginary part of the dielectric constant for a fixed value of the real part. The intensity correlation function of the diffusely scattered light from two beams of different frequencies which are simultaneously incident on the same rough surface is also calculated.

I. INTRODUCTION

Recently there has been renewed interest[1–3] in optical speckle due to its close relationship to the study of universal conductance fluctuations in mesoscopic electron systems[4–6] and backscattering effects associated with Anderson-localization phenomena.[7–13] These phenomena all arise due to the phase coherence of wavelike excitations propagating in a disordered medium. Shapiro,[1] and Stephen and Cwilich[3] have recently used this fact to apply mathematical techniques developed for the study of mesoscopic electron systems and Anderson localization to calculate the intensity correlation function for light interacting with a disordered bulk medium. This correlation function allows from the determination of speckle size, the intensity fluctuations in the speckle pattern, and the frequency dependence of the pattern in terms of the statistical properties of the disordered medium.

In this paper we shall undertake a calculation similar to those of Shapiro,[1] and Stephen and Cwilich,[3] but now for the intensity correlation function of light elastically scattered from a randomly rough metal surface. Specifically, we shall calculate the intensity correlation function for the diffuse component of light scattered from a randomly rough metallic grating. Previous work on speckle, which treats the surface statistics, considers surfaces which are either perfectly conducting[14–17] or take the phase shifts of the scattered waves to be proportional to the surface profile function.[18–22] In this work we shall improve upon these by correctly treating the interaction of light with the metallic surface, described by a complex dielectric constant. The calculation shall employ a unitarity- and reciprocity-preserving formulation for the scattering of light from rough surfaces which has recently been developed by Brown *et al.*,[23,24] which is based on the Rayleigh hypothesis.[25,26] This last restriction will limit us to the consideration of only partially developed speckle patterns.

As the work of Brown *et al.*[23,24] has already been applied by us, within the context of Anderson localization,[11] to the consideration of the scattering of light from randomly rough metal gratings, we shall quote a number of results from our paper, Ref. 11, below. Along with these, a brief discussion of the formulation of the light-scattering problem will be given, but the reader is referred to Ref. 11 for the detailed derivation of the formulas taken over from it.

II. THE INTENSITY CORRELATION FUNCTIONS

We consider a grating whose profile is given by $x_3=\zeta(x_1)$, where the profile function $\zeta(x_1)$ is assumed to be a Gaussianly distributed random variable[27] with the properties

$$\langle \zeta(x_1)\rangle = 0 , \quad \text{(1a)}$$

$$\langle \zeta(x_1)\zeta(x_1')\rangle = \sigma^2 \exp\left[-\frac{|x_1-x_1'|^2}{a^2}\right] , \quad \text{(1b)}$$

where the angular brackets denote an average over the ensemble of realizations of $\zeta(x_1)$. In Eq. (1b) σ is the root-mean-square deviation of the surface from the plane $x_3=0$ and a is the transverse correlation length, which is a measure of the average distance between successive peaks and valleys on the rough surface. The region

$x_3 > \zeta(x_1)$ is a vacuum, while the region $x_3 < \zeta(x_1)$ is filled by an isotropic dielectric medium characterized by the complex dielectric constant $\epsilon(\omega)=\epsilon_1(\omega)+i\epsilon_2(\omega)$, with $\epsilon_1(\omega)<-1$, $\epsilon_2(\omega)>0$, and $\epsilon_2(\omega)\ll|\epsilon_1(\omega)|$ at the frequency ω of the incident light.

The light in our system is taken to be p polarized, and the plane of incidence is the (x_1x_3) plane. The magnetic field vector thus has the form $\mathbf{H}(\mathbf{x},t)=(0, H_2(x_1,x_3|\omega), 0)\exp(-i\omega t)$, and the amplitude $H_2(x_1,x_3|\omega)$ in the region $x_3>\zeta(x_1)$ can be written as

$$H_2^>(x_1,x_3|\omega)=e^{ikx_1-i\alpha_0(k,\omega)x_3}+\int\frac{dp}{2\pi}R(p|k)e^{ipx_1+i\alpha_0(p,\omega)x_3}\ , \tag{2}$$

where $k=(\omega/c)\sin\theta_i$, with θ_i the angle of incidence measured from the normal to the mean surface,

$$\alpha_0(p,\omega)=\begin{cases}(\omega^2/c^2-p^2)^{1/2}, & p^2<\omega^2/c^2 & \text{(3a)}\\ i(p^2-\omega^2/c^2)^{1/2}, & p^2>\omega^2/c^2 & \text{(3b)}\end{cases}$$

and $\alpha_0(k,\omega)$ is always real. Note that Eq. (2) invokes the Rayleigh hypothesis,[25,26] and that the first term on the right-hand side of Eq. (2) describes an incident beam of frequency ω which is scattering from the grating surface.

By an application of Green's theorem and the extinction theorem to the above described grating problem, an integral equation for the scattering amplitude $R(p|k)$ is obtained whose solution can be written in the form[11,23,24]

$$R(p|k)=2\pi\delta(p-k)R_0(k)-2iG_0(p)T(p|k)G_0(k)\alpha_0(k)\ , \tag{4}$$

where

$$R_0(p)=\frac{\epsilon(\omega)\alpha_0(p,\omega)-\alpha(p,\omega)}{\epsilon(\omega)\alpha_0(p,\omega)+\alpha(p,\omega)} \tag{5}$$

is the Fresnel coefficient for the reflection of p-polarized light from a planar dielectric surface, $\alpha(p,\omega)=[\epsilon(\omega)(\omega^2/c^2)-p^2]^{1/2}$, with $\mathrm{Re}[\alpha(p,\omega)]>0$, $\mathrm{Im}[\alpha(p,\omega)]>0$, and

$$G_0(p)=\frac{i\epsilon(\omega)}{\epsilon(\omega)\alpha_0(p,\omega)+\alpha(p,\omega)} \tag{6}$$

is the surface-polariton Green's function for a planar surface. The scattering matrix, $T(p|k)$, in Eq. (4) is the solution of the equation

$$T(p|k)=V(p|k)+\int\frac{dq}{2\pi}V(p|q)G_0(q)T(q|k)\ , \tag{7}$$

where to leading order in $\zeta(x_1)$

$$V(p|k)=\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\hat{\zeta}(p-k)[\epsilon(\omega)pk-\alpha(k,\omega)\alpha(p,\omega)]\ , \tag{8}$$

with

$$\hat{\zeta}(p)=\int dx_1\,\zeta(x_1)\exp(-ipx_1)\ . \tag{9}$$

In terms of the T matrix in Eq. (7) we can also write the surface-polariton Green's function, $G(p|k)$, for a rough surface as

$$G(p|k)=2\pi\delta(p-k)G_0(k)+G_0(p)T(p|k)G_0(k)\ . \tag{10}$$

This Green's function will be very useful in the work presented below, as we shall see that the intensity of diffusely scattered light and the intensity correlation function of diffusely scattered light can be written in terms of configurationally averaged products of these functions. These configurationally averaged products of surface-polariton Green's functions can then be treated using standard diagrammatic techniques for the study of excitations in disordered media.[28]

The time-averaged intensity or energy density in an electromagnetic plane wave whose magnetic field is $H_2(x_1,x_3|\omega)$ is given by

$$I(x_1,x_3|\omega)=\frac{1}{8\pi}|H_2(x_1,x_3|\omega)|^2\ . \tag{11}$$

Using Eq. (11), the intensity correlation function of the diffusely scattered light, measured on the plane defined by $x_3>[\zeta(x_1)]_{\max}$, can then be written as

$$C(x_1,x_1',x_3|\omega)=C_r\{\langle|[H_2^>(x_1,x_3|\omega)]_{sc}|^2|[H_2^>(x_1',x_3|\omega)]_{sc}|^2\rangle-\langle|[H_2^>(x_1,x_3|\omega)]_{sc}|^2\rangle\langle|[H_2^>(x_1',x_3|\omega)]_{sc}|^2\rangle\}\ , \tag{12}$$

where $C_r=1/64\pi^2$, and $[H_2^>(x_1,x_3|\omega)]_{sc}$ is the scattered portion of the total magnetic field which is given in Eq. (2). In Eq. (12) we are only interested in the correlation of the diffusely scattered light, so we shall ignore the specular component of $[H_2^>(x_1,x_3|\omega)]_{sc}$ in the evaluation of Eq. (12). If the specular components of $H_2^>(x_1,x_3|\omega)$ were retained in Eq. (12), the resulting correlation function would include contributions from the specular beam and from the interference between the specular and diffusely reflected light. From Eqs. (2), (4), and (10) we have

$$[H_2^>(x_1,x_3|\omega)]_{sc}=\int\frac{dq}{2\pi}\{2\pi\delta(q-k)[R_0(k)+2iG_0(k)\alpha_0(k)]-2iG(q|k)\alpha_0(k)\}e^{iqx_1+i\alpha_0(q\omega)x_3}\ , \tag{13}$$

so that upon substitution of Eq. (13) into Eq. (12), we obtain

$$C(x_1,x_1',x_3|\omega)=16C_r\alpha_0^4(k,\omega)\int\frac{dq}{2\pi}\int\frac{dq'}{2\pi}\int\frac{dp}{2\pi}\int\frac{dp'}{2\pi}[\langle G(q|k)G^*(q'|k)G(p|k)G^*(p'|k)\rangle-\langle G(q|k)G^*(q'|k)\rangle\times\langle G(p|k)G^*(p'|k)\rangle]_D e^{i(q-q')x_1}e^{i(p-p')x_1'}e^{i[\alpha_0(q)-\alpha_0^*(q')+\alpha_0(p)-\alpha_0^*(p')]x_3}\ . \tag{14}$$

The subscript D on the square brackets on the right-hand side of Eq. (14) indicates that all terms containing factors of $2\pi\delta(q-k)$, $2\pi\delta(p-k)$, etc., are dropped, as these represent contributions from the specular beam and from interference of the specular and diffuse beams.

In a similar manner, we can obtain an expression for the time-averaged diffuse field intensity at x_1 on the plane defined by $x_3 > [\zeta(x_1)]_{\max}$. We find

$$I(x_1,x_3|\omega)=C_r^0\langle |[H_2^>(x_1,x_3|\omega)]_{sc}|^2\rangle$$
$$=4C_r^0\alpha_0^2(k,\omega)\int \frac{dq}{2\pi}\int \frac{dq'}{2\pi}\langle G(q|k)G^*(q'|k)\rangle_D e^{i(q-q')x_1}e^{i[\alpha_0(q)-\alpha_0^*(q')]x_3} , \tag{15}$$

where $C_r^0=1/8\pi$ and, in the evaluation of Eq. (15), we again omit the specular terms contained in $[H_2(x_1,x_3|\omega)]_{sc}$. The evaluation, then, of the speckle intensity correlation function and the average diffuse intensity is now reduced to the treatment of the four-particle Green's function contained in the square brackets on the right-hand side of Eq. (14) and the two-particle Green's function on the right-hand side of Eq. (15). In the following we shall turn to the determination of the average single, two- and four-particle Green's functions given above.

The average single-particle Green's function, $\langle G(q|k)\rangle = 2\pi\delta(q-k)G(k)$, is calculated in Eqs. (17)–(23) of Ref. 11. In the approximation used there only self-energy corrections to leading order in $\langle V(q|p)V(p|k)\rangle$, where V is given in Eq. (8), were retained. The resulting expression for $G(k)$ is then

$$G(k)\simeq \frac{C_1}{k-K_{sp}-i\Delta_{tot}}-\frac{C_1}{k+K_{sp}+i\Delta_{tot}} \tag{16}$$

for $|k\pm K_{sp}|\lesssim \Delta_{tot}$, where

$$C_1=\frac{\epsilon_1(\omega)[-\epsilon_1(\omega)]^{1/2}}{1-\epsilon_1^2(\omega)} , \tag{17}$$

$$K_{sp}=\frac{\omega}{c}\left[\frac{\epsilon_1(\omega)}{\epsilon_1(\omega)+1}\right]^{1/2} , \tag{18}$$

$$\Delta_{tot}=\Delta_\epsilon+\Delta_{sp} , \tag{19}$$

$$\Delta_\epsilon=\frac{\epsilon_2(\omega)K_{sp}}{2\epsilon_1(\omega)[\epsilon_1(\omega)+1]} , \tag{20}$$

$$\Delta_{sp}\simeq 2\sqrt{\pi}a\sigma^2C_1^2\left[\frac{\epsilon_1(\omega)-1}{\epsilon_1(\omega)}\right]^2K_{sp}^4\exp(-a^2K_{sp}^2) , \tag{21}$$

and

$$G(k)\simeq G_0(k) \tag{22}$$

for $|k\pm K_{sp}|\gg\Delta_{tot}$. We notice that the small self-energy correction Δ_{tot} is important only near the poles of the Green's function. These poles describe the surface polaritons propagating along the random interface and, as in Ref. 11, only self-energy correlations to the imaginary parts of the poles are retained as these determine the surface-polariton lifetimes. Corrections to the real parts cause only small frequency shifts in the polariton dispersion curves which, as in Ref. 11, are not of interest to us in the following.

The two-particle Green's function, $\langle G(q|k)G^*(p|k)\rangle$, was calculated in Eqs. (24)–(33) of Ref. 11 in the ladder-diagram approximation. From Eqs. (27), (29), and (30) of Ref. 11, we find that

$$\langle G(q|k)G^*(p|k)\rangle=(2\pi)^2\delta(q-p)F(p|k) , \tag{23}$$

where

$$F(p|k)=\delta(p-k)|G(k)|^2+|G(p)|^2\frac{\tau(p|k)}{2\pi}|G(k)|^2 \tag{24}$$

[notice that in the second term on the right-hand side we include a factor of 2π which was left out of Eq. (29) of Ref. 11], and

$$\tau(p|k)=\Gamma_0(p,k|p,k)+\int\frac{ds}{2\pi}\Gamma_0(p,s|p,s)|G(s)|^2\tau(s|k) . \tag{25}$$

In Eq. (25) $\Gamma_0(p,k|p,k)$ comes from the irreducible four-vertex function in Eqs. (25) and (28) of Ref. 11, and to lowest order in the interaction is given by [see Eq. (36) of Ref. 11]

$$\Gamma_0(p,k|p,k)=K(p,k)=\pi^{1/2}a\sigma^2\left|\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\right|^2|\epsilon(\omega)pk-\alpha(p,\omega)\alpha(k,\omega)|^2\exp\left[-\frac{a^2}{4}(k-p)^2\right] . \tag{26}$$

Also, in the evaluation of integrals involving $|G(s)|^2$, such as occur in Eq. (25), we make the approximation

$$|G(s)|^2\to\frac{\pi C_1^2}{\Delta_{tot}}[\delta(s-K_{sp})+\delta(s+K_{sp})] . \tag{27}$$

This treats the major contributions to such integrals as arising from the poles of the surface polaritons in $G(s)$.

If we substitute Eq. (27) into Eq. (25), we obtain

$$\tau(p|k)=K(p,k)+\frac{C_1^2}{2\Delta_{\rm tot}}\frac{1}{1-(\Delta_{\rm sp}/\Delta_{\rm tot})^2}\times\{K(p,K_{\rm sp})K(K_{\rm sp},k)+K(p,-K_{\rm sp})K(-K_{\rm sp},k)+\frac{\Delta_{\rm sp}}{\Delta_{\rm tot}}[K(p,K_{\rm sp})K(-K_{\rm sp},k)+K(p,-K_{\rm sp})K(K_{\rm sp},k)]\}\ , \tag{28}$$

where Eqs. (22) and (18) of Ref. 11 have been used to write

$$\Delta_{\rm sp}=\frac{C_1^2}{2}K(-K_{\rm sp},K_{\rm sp})\ . \tag{29}$$

The two-particle Green's function is then obtained by substituting Eq. (28) into Eqs. (23) and (24). This is the so-called ladder diagram approximation for the two-particle Green's function. In nondissipative systems these ladder terms are responsible for the diffusion poles which appear in the random system reducible four-vertex and two-particle Green's functions.

The average intensity of diffusely scattered light at the plane above the scattering surface given by $x_3>\zeta(x_1)_{\rm max}$ can now be calculated in the ladder approximation. From Eqs. (23), (24), and (15) we find

$$I(x_1,x_3|\omega)\simeq 4C_r^0|G(k)|^2\alpha_0^2(k\omega)\times\int\frac{dq}{2\pi}|G(q)|^2\tau(q|k)e^{-2\,{\rm Im}\alpha_0(q)x_3}\ , \tag{30}$$

where $\tau(p|k)$ for the ladder diagrams is given in Eq. (28). We note that the right-hand side of Eq. (30) is independent of x_1 due to the restoration of translational invariance along the x_1 axis by the surface averaging.

We now turn to the evaluation of the four-particle Green's function given by

$$\langle G(q|k)G^*(q'|k)G(p|k)G^*(p'|k)\rangle_D\ . \tag{31}$$

The evaluation of this term amounts to summing all possible Feynman diagrams (see Ref. 11) with factors of $|G(k)|^4$ entering from the right and factors of $G(q)G^*(q')G(p)G^*(p')$ leaving the left. Using the same notation as in Fig. 1 of Ref. 11, Green's functions are represented by solid lines, and dotted lines represent the scattering interactions. Here we must be careful to distinguish between solid lines representing G and G^* terms. This later point was not a problem in the work in Ref. 11 as only one set of G,G^* entered into those calculations.

If we are only interested in the leading-order correlations in Eq. (31), however, then great simplification occurs, as we can write it as a linear combination of products of the averages of lower-order Green's functions.[1-3] In particular, we have

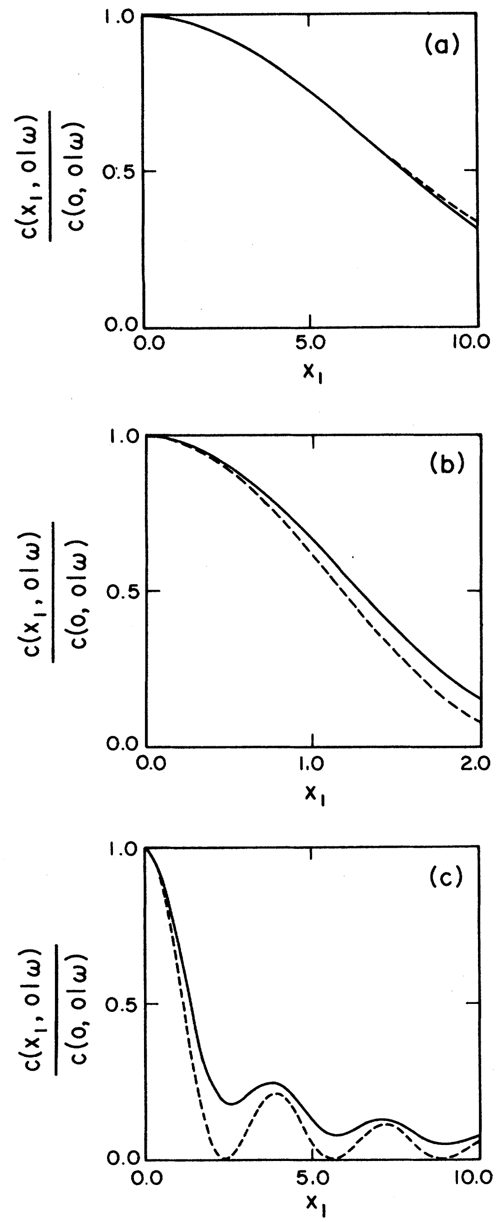

FIG. 1. Plot of $C(x_1,0|\omega)/C(0,0|\omega)$ from Eq. (40) vs x_1 given in units of $\lambda/2\pi$, where λ is the wavelength of the incident light. Results are presented for light whose wavelength is 4579 Å incident on a silver surface with $\epsilon_1(\omega)=-7.5$, $\epsilon_2(\omega)=0.24$ for (a) $a=10\,000$, $\sigma=15.18$ Å; (b) $a=1000$, $\sigma=50$ Å; (c) $a=500$, $\sigma=25$ Å. Results for $\Theta_i=0°$ (dashed line) and $\Theta_i=45°$ (solid line) are shown on each plot.

$$\langle G(q|k)G^*(q'|k)G(p|k)G^*(p'|k)\rangle_D \simeq \langle G(q|k)G^*(q'|k)\rangle_D \langle G(p|k)G^*(p'|k)\rangle_D + \langle G(q|k)G^*(p'|k)\rangle_D \langle G(p|k)G^*(q'|k)\rangle_D + \langle G(q|k)G(p|k)\rangle_D \langle G^*(q'|k)G^*(p'|k)\rangle_D \,, \tag{32}$$

where terms involving $\langle G\rangle_D=0$ or $\langle G^*\rangle_D=0$ do not enter into our consideration of the diffuse scattering. [Note that if we drop the subscript D on the $\langle\ \rangle$ in Eq. (31), then the approximation in Eq. (32) would be incorrect, as we would need to consider terms involving $\langle G\rangle$ and $\langle G^*\rangle$ which would represent the interference of the diffuse and specularly scattered fields.] The two-particle Green's functions occurring in Eq. (32) of the form $\langle GG^*\rangle$ have already been determined above in the ladder approximation, but the forms $\langle GG\rangle$ and $\langle G^*G^*\rangle$ must now be considered.

From the single-particle Green's-function equation given in Eqs. (7)–(10) above, we can easily write the Bethe-Salpeter equation in the lowest-order ladder-diagram approximation for the two-particle Green's function $\langle G(q|k)G(p|k)\rangle$. We find

$$\langle G(q|k)G(p|k)\rangle = 2\pi\delta(q-k)2\pi\delta(p-k)G^2(k) + G(q)G(p)\int\frac{dr}{2\pi}\int\frac{ds}{2\pi}\langle V(q|r)V(p|s)\rangle\langle G(r|k)G(s|k)\rangle \,, \tag{33}$$

with

$$\langle V(q|r)V(p|s)\rangle = 2\pi\delta(q-r+p-s)V_0(q,r|p,s) \,, \tag{34a}$$

where

$$V_0(q,r|p,s) = \sqrt{\pi}a\sigma^2\left[\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\right]^2 [\epsilon(\omega)qr-\alpha(q,\omega)\alpha(r,\omega)][\epsilon(\omega)ps-\alpha(p,\omega)\alpha(s,\omega)]\exp\left[-\frac{a^2}{4}(q-r)^2\right] . \tag{34b}$$

If we assume that $\langle G(q|k)G(p|k)\rangle = 2\pi\delta(q+p-2k)G_1(q,p,k)$, then substituting this ansatz into Eq. (34) gives

$$G_1(q,p,k) = 2\pi\delta(p-q)2G^2(k) + G(q)G(p)\int\frac{dr}{2\pi}V_0(q,r|p,q-r+p)G_1(r,q-r+p,k) \,, \tag{35}$$

which can be written in the form

$$G_1(r,p+q-r,k) = 2\pi\delta(p+q-2r)2G^2(k) + G(r)G(p+q-r)\int\frac{dr'}{2\pi}V_0(r,r'|p+q-r,p+q-r')G_1(r',p+q-r',k) \,. \tag{36}$$

Looking at Eq. (36) we see that for $\Delta_\epsilon,\Delta_{sp} \ll K_{sp}$ there is no overlapping of the poles in the product $G(r)G(p+q-r)$ as occurs in the product $|G(s)|^2$ in Eqs. (25) and (27). Consequently, Eqs. (35) and (36) represent series expansions in increasing powers of $\Delta_{sp}/(\omega/c) \ll \Delta_{sp}/\Delta_{tot}$, whereas the expansion in Eq. (25) for $\langle GG^*\rangle$ is in powers of Δ_{sp}/Δ_{tot}. As a result of this we need only keep the first interaction term in the ladder sum given in Eqs. (35) and (36). Doing this we find that

$$\langle G(q|k)G(p|k)\rangle_D \simeq 2\pi\delta(q+p-2k)G(q)G(p)V_0(q,k|p,k)G^2(k) \,, \tag{37}$$

where $p\neq q$ in the above.

Substituting Eqs. (23), (24), and (37) into Eq. (32), and the latter into Eq. (14), we find for the intensity correlation function

$$C(x_1,x_1',x_3|\omega) = 16C_r\alpha_0^4(k,\omega)|G(k)|^4 \times\left[\left|\int\frac{dq}{2\pi}|G(q)|^2\tau(q|k)e^{iq(x_1-x_1')}e^{-2\,\mathrm{Im}\alpha_0(q)x_3}\right|^2 + \left|\int\frac{dq}{2\pi}G(2k-q)G(q)V_0(2k-q,k|q,k)e^{-iq(x_1-x_1')}e^{i[\alpha_0(2k-q)+\alpha_0(q)]x_3}\right|^2\right] . \tag{38}$$

The first integral in the large parens in Eq. (38) can be written as

$$\int\frac{dq}{2\pi}|G(q)|^2\tau(q|k)e^{-2\,\mathrm{Im}\alpha_0(q)x_3}e^{iq(x_1-x_1')} = \int_{-\omega/c}^{\omega/c}\frac{dq}{2\pi}|G(q)|^2\tau(q|k)e^{iq(x_1-x_1')} + \frac{C_1^2}{2\Delta_{tot}}[\tau(K_{sp}|k)e^{iK_{sp}(x_1-x_1')} + \tau(-K_{sp}|k)e^{-iK_{sp}(x_1-x_1')}]e^{-2\mathrm{Im}\alpha_0(K_{sp})x_3} \,, \tag{39}$$

where we have used the pole approximation of Eq. (27) in integrating over the regions $|q| > \omega/c$. In Eq. (39) the integral over the interval $|q| < \omega/c$ gives the speckle correlations in the diffusely scattered radiation fields, whereas the contribution from the intervals $|q| > \omega/c$ refers to the fields generated by surface polaritons propagating along the random interface. The surface-polariton contributions are found to decay exponentially, with a decay length of $[2\,\mathrm{Im}\alpha_0(K_{sp})]^{-1}$, as we move perpendicularly away from the mean surface. The second integral in the large parens of Eq. (38) also has both radiative and surface-wave-related terms, both of which exhibit an x_3 dependence. For complex values of α_0 the integral decays exponentially with increasing distance from the surface. The range of q over which the α_0's are real is finite, and of length $2\omega/c$. Hence, if the second integral is well defined, then its x_3 dependence in the region of real α_0 must be that of a decreasing function in x_3 which, using an uncertainty-principle argument, must be confined to a region of order $(\omega/c)^{-1}$ above the mean surface. Consequently, for $x_3 \to \infty$ we find that the intensity correlation function reduces to

$$C(x_1,x_1'|\omega)=16C_r\alpha_0^4(k,\omega)|G(k)|^4\left|\int_{-\omega/c}^{\omega/c}\frac{dq}{2\pi}|G(q)|^2\tau(q|k)e^{iq(x_1-x_1')}\right|^2 , \tag{40}$$

which can be evaluated numerically. It is noted from Eqs. (30) and (40) that for $x_1=x_1'$

$$C(x_1,x_1,x_3|\omega)=[I(x_1,x_3|\omega)]^2 , \tag{41}$$

a result that is obtained in the statistics of many speckle systems.[1-3,14,29,30]

Equation (40) has been evaluated by us for the case of light of wavelength 4579 Å incident on a rough Ag surface at angles of incidence of 0° and 45°. The complex dielectric constant of the Ag surface[31] was taken to be $\epsilon(\omega)=-7.5+i0.24$, which is the value for bulk Ag. Calculations for three different values of the surface-roughness parameters a and σ are presented in Fig. 1.

In the limit that $a \gg \lambda$, Eq. (40) can be evaluated analytically, and a comparison with these numerical results can be made. We find

$$C(x_1,x_1',x_3|\omega)\xrightarrow[a\to\infty]{}16|G(k)|^8\,C_r\alpha_0^4(k)\sigma^4\left|\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\right|^4|\epsilon(\omega)k^2-\alpha^2(k)|^4\exp\left[-\frac{2(x_1-x_1')^2}{a^2}\right] , \tag{42}$$

which is in excellent agreement with the results in Fig. 1(a). From Eq. (42) the correlation length of the speckle pattern is seen to depend only on the correlation length of the surface roughness. The speckle correlation length, though, in the other two plots for $a \le \lambda$, presented in Fig. 1, agrees less well with that given in Eq. (42), and in this limit depends in some more complicated way on a and λ.

In general, we see in Fig. 1 that for $a > \lambda$ the intensity correlation function is reasonably well described by a Gaussian form such as that in Eq. (42). In this limit, then, the shape of the intensity correlation function is independent of the dielectric properties of the surface and depends only on its roughness. For $a < \lambda$, on the other hand, the intensity correlation function exhibits an underdamped type of behavior with oscillations of wavelength of the order of half the optical wavelength. This type of underdamped behavior has been found for bulk speckle by Shapiro[1] and by Stephen and Cwilich.[3] The wavelength of the underdamped intensity correlation function in the $a \ll \lambda$ limit of Eq. (40) is found to be mildly sensitive to the value of ϵ_2, which describes dissipation in the surface medium. This can be seen in Fig. 2, where we present results for the intensity correlation function for a silver surface with $a=500$ Å, $\sigma=25$ Å, $\lambda=4579$ Å, and for $\epsilon_1=-7.5$, $\epsilon_2=0.24$, $0.5\times(0.24)$, and $5\times(0.24)$. It is observed that upon increasing ϵ_2 from its correct value of 0.24, the apparent wavelength of the underdamped correlation function increases slightly.

We next turn to the evaluation of the intensity correlation function of diffusely scattered light for two incident beams of frequencies ω and $\omega+\Delta\omega$, where $\Delta\omega \ll \omega$. The angle of incidence, Θ_i, of these two beams will be taken to be the same, and we will be interested in the intensity correlation function at the point (x_1,x_3) as a function of the frequency difference $\Delta\omega$. It is given by

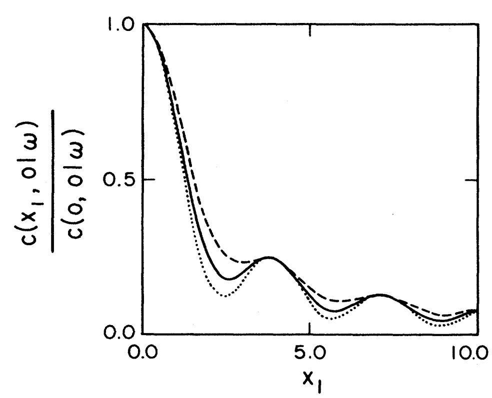

FIG. 2. Plot of $C(x_1,0|\omega)/C(0,0|\omega)$ as in Fig. 1(c) but now for (a) $\epsilon_2(\omega)=0.24$ (solid line); (b) $\epsilon_2(\omega)=5\times(0.24)$ (dashed line); (c) $\epsilon_2(\omega)=0.5\times(0.24)$ (dotted line).

$$C(x_1,x_3|\Theta_i,\omega,\Delta\omega)=C_r\{\langle |[H_2(x_1,x_3|\omega)]_{sc}|^2\,|[H_2^{>}(x_1,x_3|\omega+\Delta\omega]_{sc}|^2\rangle - \langle |[H_2^{>}(x_1x_3|\omega)]_{sc}|^2\rangle\langle |[H_2^{>}(x_1x_3|\omega+\Delta\omega)]_{sc}|^2\rangle\} , \tag{43}$$

where specular contributions from $|(H_2)_{sc}|^2$ are to be ignored. We must now explicitly indicate the ω dependence of the incident light and the Green's function for surface polaritons on a rough surface defined in Eqs. (7)–(10). This is easily done by just appending a subscript to the functions $G(p|k)$, $G(k)$, $G_0(k)$, $V(p|k)$, $T(p|k)$, and $R(p|k)$ in Eqs. (2), (7)–(10), and (12)–(16), so these now read $G_\omega(p|k)$, $G_\omega(k)$, $G_{0\omega}(k)$, $V_\omega(p|k)$, $T_\omega(p|k)$, and $R_\omega(p|k)$, for the frequency ω. The Dyson equation for $G_\omega(p|k)$ from Eqs. (7)–(10) is then

$$G_\omega(p|k)=2\pi\delta(p-k)G_{0\omega}(k)+G_{0\omega}(p)\int\frac{dq}{2\pi}V_\omega(p|q)G_\omega(q|k) . \tag{44}$$

For an angle of incidence Θ_i measured from the normal to the mean surface, the components of the wave vector of the incident beams parallel to the mean surface will be given by $k=(\omega/c)\sin\Theta_i$ and $k'=[(\omega+\Delta\omega)/c]\sin\Theta_i$. Using Eq. (2) for these two incident beams in Eq. (43) above, we find

$$\begin{aligned}C(x_1,x_3|\Theta_i,\omega,\Delta\omega)=&16C_r\alpha_0^2(k,\omega)\alpha_0^2[k',(\omega+\Delta\omega)]\\&\times\int\frac{dq}{2\pi}\int\frac{dq'}{2\pi}\int\frac{dp}{2\pi}\int\frac{dp'}{2\pi}[\langle G_\omega(q|k)G_\omega^*(q'|k)G_{\omega+\Delta\omega}(p|k')G_{\omega+\Delta\omega}^*(p'|k')\rangle\\&\qquad-\langle G_\omega(q|k)G_\omega^*(q'|k)\rangle\langle G_{\omega+\Delta\omega}(p|k')G_{\omega+\Delta\omega}^*(p'|k')\rangle]_D\\&\qquad\times e^{i(q-q')x_1}e^{i(p-p')x_1}e^{i[\alpha_0(q,\omega)-\alpha_0^*(q',\omega)+\alpha_0(p,\omega+\Delta\omega)-\alpha_0^*(p',\omega+\Delta\omega)]x_3} .\end{aligned} \tag{45}$$

We can, again, to leading order terms in the interaction, write the four-particle Green's function as a product of two-particle Green's functions [see Eq. (32)], so that as $x_3\to\infty$, for $\Delta\omega/\omega\ll 1$, we have from Eq. (45)

$$C(x_1,x_3|\Theta_i,\omega,\Delta\omega)\simeq\left|4C_r^{1/2}\alpha_0(k,\omega)\alpha_0(k',\omega+\Delta\omega)\times\int\frac{dq}{2\pi}\int\frac{dp}{2\pi}\langle G_\omega(q|k)G_{\omega+\Delta\omega}^*(p|k')\rangle_D\,e^{i(q-p)x_1}e^{i[\alpha_0(q,\omega)-\alpha_0^*(p,\omega+\Delta\omega)]x_3}\right|^2 . \tag{46}$$

In writing Eq. (46) we have ignored terms of the type of $\langle GG\rangle$ and $\langle G^*G^*\rangle$ since, by the use of arguments similar to those following Eq. (39), these can be shown to decay rapidly with increasing distance from the surface.

To evaluate $\langle G_\omega(q|k)G_{\omega+\Delta\omega}^*(p|k')\rangle$ in the ladder-diagram approximation, we use Eq. (44) to write the Bethe-Salpeter equation. We find

$$\begin{aligned}\langle G_\omega(q|k)G_{\omega+\Delta\omega}^*(p|k')\rangle\simeq&2\pi\delta(q-k)2\pi\delta(p-k')G_\omega(k)G_{\omega+\Delta\omega}^*(k')\\&+G_\omega(q)G_{\omega+\Delta\omega}^*(p)\int\frac{dr}{2\pi}\int\frac{ds}{2\pi}\langle V_\omega(q|r)V_{\omega+\Delta\omega}^*(p|s)\rangle\langle G_\omega(r|k)G_{\omega+\Delta\omega}(s|k')\rangle ,\end{aligned} \tag{47}$$

where, from Eqs. (1) and (8),

$$\langle V_\omega(q|r)V_{\omega+\Delta\omega}^*(p|s)\rangle=2\pi\delta(q-r-p+s)V_{0,\omega,\omega+\Delta\omega}(q,r|p,s) , \tag{48a}$$

$$\begin{aligned}V_{0,\omega,\omega+\Delta\omega}(q,r|p,s)=&\sqrt{\pi}a\sigma^2\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\left[\frac{\epsilon(\omega+\Delta\omega)-1}{[\epsilon(\omega+\Delta\omega)]^2}\right]^*[\epsilon(\omega)qr-\alpha(q,\omega)\alpha(r,\omega)]\\&\times[\epsilon(\omega+\Delta\omega)ps-\alpha(p,\omega+\Delta\omega)\alpha(s,\omega+\Delta\omega)]^*\exp\left[-\frac{a^2}{4}(q-r)^2\right] .\end{aligned} \tag{48b}$$

If we write

$$\langle G_\omega(q|k)G_{\omega+\Delta\omega}^*(p|k')\rangle=(2\pi)^2\delta(q-k-p+k')F_{\omega,\omega+\Delta\omega}(q,k|p,k') , \tag{49}$$

then substituting Eqs. (49) and (48) into Eq. (47) gives

$$\begin{aligned}F_{\omega,\omega+\Delta\omega}(q,k|p,k')=&\delta(p-k')G_\omega(k)G_{\omega+\Delta\omega}^*(k')\\&+G_\omega(q)G_{\omega+\Delta\omega}^*(p)\int\frac{dr}{2\pi}V_{0,\omega,\omega+\Delta\omega}(q,r|p,k'-k+r)F_{\omega,\omega+\Delta\omega}(r,k|k'-k+r,k') ,\end{aligned} \tag{50}$$

where

$$F_{\omega,\omega+\Delta\omega}(r,k|k'-k+r,k')=\delta(r-k)G_\omega(k)G^*_{\omega+\Delta\omega}(k') + G_\omega(r)G^*_{\omega+\Delta\omega}(k'-k+r)\int\frac{dr'}{2\pi}V_{0,\omega,\omega+\Delta\omega}(r,r'|k'-k+r,k'-k+r') \times F_{\omega,\omega+\Delta\omega}(r',k|k'-k+r',k') . \quad (51)$$

For small frequency shifts $\Delta\omega$, such that $\Delta\omega \ll \Delta_{\text{tot}}$, we can take $G_\omega(r)G^*_{\omega+\Delta\omega}(k'-k+r)\simeq|G_\omega(r)|^2$ in Eq. (51). Equation (51) can then be written as

$$F_{\omega,\omega+\Delta\omega}(r,k|k'-k+r,k')\simeq\delta(r-k)G_\omega(k)G^*_{\omega+\Delta\omega}(k')+|G_\omega(r)|^2\frac{\tau(r|k,k')}{2\pi}G_\omega(k)G^*_{\omega+\Delta\omega}(k') , \quad (52)$$

where

$$\tau(r|k,k')=V_{0,\omega,\omega+\Delta\omega}(r,k|k'-k+r,k')+\int\frac{dr}{2\pi}V_{0,\omega,\omega}(r,r'|r,r')|G_\omega(r)|^2\tau(r|k,k') , \quad (53)$$

and we have taken $V_{0,\omega,\omega+\Delta\omega}(r,r'|k'-k+r,k'-k+r')\simeq V_{0,\omega,\omega}(r,r'|r,r')$ in the integral on the left-hand side of Eq. (53). A solution for $\tau(r|k,k')$ can easily be obtained from Eq. (53) by using the approximation for $|G_\omega(r)|^2$ given in Eq. (27). Then using Eqs. (52) and (53) in Eq. (50) gives

$$F_{\omega,\omega+\Delta\omega}(q,k|p,k')\simeq\delta(p-k')G_\omega(k)G^*_{\omega+\Delta\omega}(k')+G_\omega(q)G^*_{\omega+\Delta\omega}(p)\frac{\tau(q|k,k')}{2\pi}G_\omega(k)G^*_{\omega+\Delta\omega}(k') . \quad (54)$$

From Eq. (49), $\langle G_\omega(q|k)G^*_{\omega+\Delta\omega}(p|k')\rangle$ follows.

Substituting Eqs. (49) and (54) into Eq. (46) for the intensity correlation function, we find

$$C(x_1,x_3|\Theta_i,\omega,\Delta\omega)\simeq\left|4C_r^{1/2}\alpha_0^2(k,\omega)\int\frac{dq}{2\pi}|G_\omega(q)|^2\tau(q|k,k')|G_\omega(k)|^2e^{i[\alpha_0(q,\omega)-\alpha_0^*(q-k+k',\omega+\Delta\omega)]x_3}\right|^2 . \quad (55)$$

In general, this expression exhibits a complicated dependence on x_3 which must be determined numerically, but, in the limit $a\to\infty$, Eq. (55) can be evaluated analytically.

In this limit only the first term in Eq. (53) contributes to $\tau(q|k,k')$ as all higher-order terms contain factors of $e^{-a^2K_{\text{sp}}^2/4}\to 0$, so that we have

$$\tau(q|k,k')\xrightarrow[a\to\infty]{}\sqrt{\pi}a\sigma^2\left[\frac{\epsilon(\omega)-1}{[\epsilon(\omega)]^2}\right]^2|\epsilon(\omega)qk-\alpha(q,\omega)\alpha(k,\omega)|^2\exp\left[-\frac{a^2}{4}(q-k)^2\right]$$
$$=T(q,k,\omega)\exp\left[-\frac{a^2}{4}(q-k)^2\right] \quad \text{as } a\to\infty . \quad (56)$$

Here we have ignored terms of order $\Delta\omega$ in the coefficient multiplying the exponential. Using Eq. (56) in Eq. (55) we have

$$C(x_1,x_3|\Theta_i,\omega,\Delta\omega)\simeq\left|4C_r^{1/2}\alpha_0^2(k,\omega)T(k,k,\omega) \times|G_\omega(k)|^2\int\frac{dq}{2\pi}\exp\left[-\frac{a^2}{4}(q-k)^2\right]\exp\{i[\alpha_0(q,\omega)-\alpha_0(q-k+k',\omega+\Delta\omega)]x_3\}\right|^2 . \quad (57)$$

If we now expand $\alpha_0(q,\omega)-\alpha_0^*(q-k-k',\omega+\Delta\omega)$ in Eq. (57) in powers of $\Delta\omega/\omega\ll 1$, we find from Eq. (3a)

$$\alpha_0(q,\omega)-\alpha_0^*(q-k+k',\omega+\Delta\omega)=-\frac{\omega/c-q\sin\Theta_i}{[(\omega/c)^2-q^2]^{1/2}}\frac{\Delta\omega}{c}+O((\Delta\omega/\omega)^2) . \quad (58)$$

Retaining only the leading-order term in $\Delta\omega/c$, we can expand the coefficient of $\Delta\omega/c$ in powers of $q-k$ to give

$$\alpha_0(q,\omega)-a_0^*(q-k+k',\omega+\Delta\omega)\simeq-\cos\Theta_i\{1+\tfrac{1}{2}[(\omega/c)^2\cos^4\Theta_i]^{-1}(q-k)^2\}\frac{\Delta\omega}{c} , \quad (59)$$

to terms in $(q-k)^2$. Substituting Eq. (59) into Eq. (57), we find that, for $\Delta\omega/\omega\ll 1$, $a\gg\lambda$, and $|(2/\cos^3\Theta_i)(\Delta\omega/\omega)(c/\omega)(x_3/a^2)|\lesssim 1$,

$$C(x_1,x_3|\Theta_i,\omega,\Delta\omega)\to\frac{16C_r\alpha_0^4(k,\omega)}{\pi a^2}|G_\omega(k)|^8|T(k,k,\omega)|^2\left[1+\frac{4}{\cos^6\Theta_i}\left[\frac{c}{\omega}\frac{\Delta\omega}{\omega}\right]^2x_3^2/a^4\right]^{-1/2} . \quad (60)$$

The expression in Eq. (60) was obtained by assuming that $\exp[-(a^2/4)(q-k)^2]$ was small for q outside the radiative region (i.e., for $|q|>\omega/c$); for this to hold we need in addition that $1-\sin\Theta_i \gg \lambda/\pi a$. The dependence on $\Delta\omega/\omega$ observed in Eq. (60) is somewhat different than that found by Shapiro,[1] and Stephen and Cwilich[3] for speckle in bulk media. Shapiro finds the intensity-intensity correlation function to depend exponentially on $\sqrt{\Delta\omega/\omega}$, whereas Stephen and Cwilich obtain additional corrections to the work of Shaprio which involve integrals of Bessel functions whose arguments depend on $\sqrt{\Delta\omega/\omega}$.

III. CONCLUSIONS

We have calculated the speckle statistics of light scattered from a randomly rough metallic surface in the limit of partially developed speckle. The random surface in our theory is a grating which is described by Gaussian random statistics, and we have employed a multiple-scattering theory which takes into account the dielectric nature of the randomly rough surface.

Previous work on speckle (Refs. 14–22) has treated the statistics of light reflected by perfectly conducting surfaces or has taken the phase of the reflected light to be proportional to the surface-profile function, and then has only determined the statistics of the speckle contrast (i.e., the variance of the intensity fluctuations in the speckle pattern). Our work has gone further in that it has determined the spatial correlations which exist in the speckle pattern within the context of the more realistic model of the interaction of light with a rough dielectric surface. Furthermore, our work gives the same dependence of the variance of the intensity fluctuations in the speckle pattern on the average intensity of reflected light as in Refs. 14–22, in spite of the inclusion, in our theory, of losses due to the imaginary part of the dielectric constant. These Ohmic losses are found to affect the spatial correlations in the speckle pattern, particularly in the $a \ll \lambda$ limit in which the wavelength of the underdamped behavior of the correlation function is seen to increase with increasing ϵ_2. The new aspect of the present approach is that it allows for the loss of total intensity of the reflected light due to the imaginary part of the complex dielectric constant, and for the changes in the correlation length of the intensity correlation function due to the surface profile and the dielectric nature of the surface. Neither of these two features are treated in the work presented in Refs. 14–22.

The calculation of the four-particle Green's function, upon which our results are based, is similar to that done by Shapiro[1] for the speckle of light propagating in bulk random media. Corrections to Shapiro's theory due to Stephen and Cwilich[3] need not concern us for the silver surface considered in this paper, as in all cases studied here $\Delta_{sp}/\Delta_{tot} \leq 0.14$ so that Ohmic losses dominate the losses due to the roughness-induced conversion of the surface polaritons into volume waves in the vacuum. The high-order diffusive corrections studied by Stephen and Cwilich[3] become important only in the limit $\Delta_{sp}/\Delta_{tot} \simeq 1$.

ACKNOWLEDGMENTS

This research was supported in part by U.S. Army Research Office Grant No. DAAL03-88-K-0067. A.R.M. would like to thank Professor Roger Elliott and the Department of Theoretical Physics at Oxford University for its hospitality during his recent visit at which time part of this work was done.

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