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Two types of conductance minima in electrostatic Aharonov-Bohm conductance oscillations

M. Cahay
Scientific Research Associates, Inc., Glastonbury, Connecticut 06033

S. Bandyopadhyay
Department of Electrical and Computer Engineering, University of Notre Dame, Notre Dame, Indiana 46556

H. L. Grubin
Scientific Research Associates, Inc., Glastonbury, Connecticut 06033

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We predict the existence of *two different* types of conductance minima, arising from *different* interference conditions, in the conductance oscillation of a one-dimensional ring due to the electrostatic Aharonov-Bohm effect. The occurrence of *two* types of minima doubles the frequency of the conductance troughs in the oscillations, making it *twice* that predicted by the Aharonov-Bohm effect. This feature, which is not inhibited by elastic scattering in the weak-localization regime, can, however, be observed only at sufficiently low temperatures. At elevated temperatures, one of the two types of minima is bleached out and the normal Aharonov-Bohm frequency is restored.

In this Rapid Communication, we point out an intriguing feature in the conductance oscillation of a one-dimensional ring due to the electrostatic Aharonov-Bohm effect.[1] Unlike in the magnetostatic effect, the conductance oscillation of a ring due to the electrostatic effect exhibits *two distinct* sets of minima arising from two *different* interference conditions. One set of minima is caused by the usual destructive interference of transmitted electrons, and the other arises due to the constructive interference of an electron traveling completely around the ring and interfering with itself at its point of entry into the ring. In the next paragraphs we establish the existence of this feature and discuss various issues related to it.

For purposes of analysis, we represent a one-dimensional ring structure as shown in Fig. 1. We assume that phase randomization in the two contacts (termed "source" and "drain") occur sufficiently far away from the junctions between the contacts and the paths.[2] The (two-terminal) conductance of the structure, in the linear-response regime, is given by[3]

$$G=\frac{e^2}{2hkT}\int dE\,|T_{\mathrm{total}}(E)|^2 \mathrm{sech}^2\left(\frac{E-E_F}{2kT}\right), \quad (1)$$

where T_{total} is the transmission coefficient of an electron through the entire structure, E is the kinetic energy of the electron, and E_F is the Fermi level.

The conductance G depends on the transmission T_{total}. The transmission T_{total} can be found from the overall scattering matrix for the ring structure determined by cascading three scattering matrices.[4] They represent propagation from the source to the two paths, propagation along the paths and propagation from the paths to the drain, respectively. For simplicity, we represent the first and the last of these scattering matrices (for junctions A-B and C-D in Fig. 1) by the so-called Shapiro matrix[5] which relates the incident, reflected, and transmitted amplitudes according to (see Fig. 1)

$$\begin{pmatrix} A^- \\ B_1^+ \\ B_2^+ \end{pmatrix} = \begin{pmatrix} -(a+b) & (\sqrt{\epsilon})^* & (\sqrt{\epsilon})^* \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b^* & a \end{pmatrix} \begin{pmatrix} A^+ \\ B_1^- \\ B_2^- \end{pmatrix}, \quad (2)$$

where the asterisk denotes complex conjugate.

The scattering matrix representing propagation along the two paths (i.e., across the junction B-C in Fig. 1) is given by[4]

$$\begin{pmatrix} B_1^- \\ B_2^- \\ C_1^+ \\ C_2^+ \end{pmatrix} = \begin{pmatrix} r_1 & 0 & t_1' & 0 \\ 0 & r_2 & 0 & t_2' \\ t_1 & 0 & r_1' & 0 \\ 0 & t_2 & 0 & r_2' \end{pmatrix} \begin{pmatrix} B_1^+ \\ B_2^+ \\ C_1^- \\ C_2^- \end{pmatrix}, \quad (3)$$

where t and r stand for the transmission and reflection coefficients within the paths. The subscripts 1 and 2 identify the corresponding path and the unprimed and primed quantities are associated with forward and reverse propagation of an electron from the source to the drain.

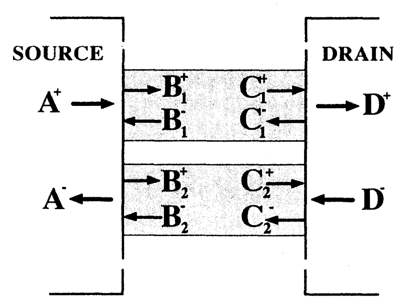

FIG. 1. Schematic representation of a one-dimensional ring-like structure showing the incident, reflected, and transmitted electron amplitudes.

If we assume ballistic transport in the two paths, in which case $r_1 = r_2 = r_1' = r_2' = 0$, then cascading the three scattering matrices for the three junctions A-B, B-C, and C-D yields the overall scattering matrix and thence the transmission T_{total} $(= D^+/A^+)$ as

$$T_{\text{total}} = \frac{\epsilon[(t_1+t_2)-(b-a)^2 t_1 t_2 (t_1'+t_2')]}{[1-t_1(a^2 t_1' + b^2 t_2')][1-t_2(a^2 t_2' + b^2 t_1')] - a^2 b^2 t_1 t_2 (t_1'+t_2')^2} . \tag{4}$$

The above equation is a perfectly general expression for the transmission through a *ballistic* one-dimensional ring. It may be pointed out that ballistic transport, although difficult to achieve, is not totally unexpected in strictly one-dimensional semiconductor microstructures at low enough temperatures since elastic scattering events are highly unlikely in one-dimensional structures.[3] The case of diffusive transport, when elastic scattering is present, is discussed later.

Let us now consider the conductance of the ballistic ring in a magnetic field. In the presence of a magnetic flux inducing the *magnetostatic* Aharonov-Bohm effect, t_1, t_2, t_1', and t_2' in Eq. (4) transform according to the following rule:[6]

$$\begin{bmatrix} t_1 \to \hat{t}_1 e^{-i\theta/2} & t_1' \to \hat{t}_1' e^{i\theta/2} \\ t_2 \to \hat{t}_2 e^{i\theta/2} & t_2' \to \hat{t}_2' e^{-i\theta/2} \end{bmatrix} , \tag{5}$$

where the quantities with carets represent transmission amplitudes in the absence of any magnetic flux and θ is the magnetostatic Aharonov-Bohm phase shift given by $\theta = (e/\hbar)\Phi$, Φ being the flux threaded by the ring. Using the transformations given by Eq. (5) in Eq. (4) and assuming that the two arms of the ring are identical in all respects so that (in the absence of any flux) $\hat{t}_1 = \hat{t}_2$ and $\hat{t}_1' = \hat{t}_2'$, we get

$$T_{\text{total}}(\theta) = \frac{\epsilon \hat{t}_1 e^{i\theta/2}(1+e^{-i\theta})[1-(b-a)^2 \hat{t}_1 \hat{t}_1']}{[1-\hat{t}_1\hat{t}_1'(a^2+b^2 e^{-i\theta})][1-\hat{t}_1\hat{t}_1'(a^2+b^2 e^{i\theta})] - a^2 b^2 \hat{t}_1^2 \hat{t}_1'^2 (e^{i\theta/2}+e^{-i\theta/2})^2} . \tag{6}$$

The numerator in $T_{\text{total}}(\theta)$ goes to zero and hence the conductance of the ring [see Eq. (1)] reaches a minimum[7] whenever

$$\theta = \frac{e}{\hbar}\Phi = (2n+1)\pi . \tag{7}$$

This gives the usual conductance minima in the magnetostatic Aharonov-Bohm oscillations associated with destructive interference of transmitted electrons. Note, however, that the numerator in $T_{\text{total}}(\theta)$ also becomes identically zero *(independent of the magnetic flux)* if the following condition is satisfied,

$$(b-a)^2 \hat{t}_1 \hat{t}_1' = 1 . \tag{8}$$

It can be shown from the required unitarity of the Shapiro matrix that the quantity $b-a$ differs from unity by a constant phase factor, i.e., $b-a=e^{iv}$. Also, in ballistic transport, $\hat{t}_1 = \hat{t}_1' = e^{ikL}$ (where L is the length of each path and k is the electron's wave vector in either path at zero magnetic flux). Therefore, Eq. (8) really corresponds to the condition,

$$2kL + 2v = 2n\pi . \tag{9}$$

It appears that if condition (9) is satisfied [in which case the numerator in $T_{\text{total}}(\theta)$ remains identically zero independent of θ], the conductance of the ring should always remain at its minimum, regardless of the magnetic flux. However, that is not quite true since the denominator in $T_{\text{total}}(\theta)$ could also become zero at some values of the magnetic flux. It is easy to see that the denominator does vanish whenever $\theta = 2n\pi$ or $\Phi = nh/e$ ($n=0$ or an integer) in which case, application of L'Hospital's rule shows that $|T_{\text{total}}(\Phi = nh/e)| = 1$. It is interesting to note that if the ring's parameters (wave vector and length) are such that Eq. (9) is satisfied (which actually implies that the ring is "Fabry-Perot resonant" at zero magnetic flux), then $|T_{\text{total}}(\Phi)| = \delta_{\Phi, nh/e}$, where the δ is a Krönicker δ. In that case, at a temperature of 0 K, the magnetoconductance $G(\Phi)$ of the ring will appear as a series of "spikes" occurring at $\Phi = nh/e$; the spikes, however, will broaden with increasing temperature.

In the case of the *electrostatic* effect, the transformations in Eq. (5) are replaced by

$$\begin{bmatrix} t_1 \to \hat{t}_1 & t_1' \to \hat{t}_1' \\ t_2 \to \hat{t}_1 e^{i\phi} & t_2' \to \hat{t}_1' e^{i\phi} \end{bmatrix} , \tag{10}$$

where ϕ is the electrostatic Aharonov-Bohm phase shift between the two paths given by

$$\phi = \frac{e}{\hbar} V\langle\tau_t\rangle = \frac{\sqrt{2m^*E}}{\hbar}\left[\left[1+\frac{eV}{E}\right]^{1/2} - 1\right] L . \tag{11}$$

Here $\langle\tau_t\rangle$ is the harmonic mean of the transit times through the two paths which depends on the incident energy E of the electrons and also the potential difference V between the paths.[8]

The difference between the transformations in Eqs. (5) and (10) accrue from the fact that the magnetostatic Aharonov-Bohm phase shifts suffered by an electron in traveling along opposite directions (time-reversed paths) have *opposite* signs, whereas the electrostatic phase shifts will have the *same* sign. This is an important distinction which ultimately causes two different sets of minima to appear in the electrostatic effect but not in the magnetostatic effect. Is is also this difference that precludes the existence of an electrostatic analog of the magnetostatic

Aronov-Al'tshuler-Spivak effect.

Using the transformations given by Eq. (10) in Eq. (4), we obtain (for the electrostatic case),

$$T_{\text{total}}(\phi)=\frac{\epsilon\hat{t}_1(1+e^{i\phi})[1-(b-a)^2\hat{t}_1\hat{t}'_1e^{i\phi}]}{[1-\hat{t}_1\hat{t}'_1(a^2+b^2e^{i\phi})][1-\hat{t}_1\hat{t}'_1(a^2e^{2i\phi}+b^2e^{i\phi})]-a^2b^2\hat{t}_1^2\hat{t}_1'^2e^{i\phi}(1+e^{i\phi})^2}\;. \tag{12}$$

The numerator of $T_{\text{total}}(\phi)$ vanishes and the conductance reaches a minimum[8] whenever

$$\phi=(2n+1)\pi$$

or (13)

$$\frac{\sqrt{2m^*E}}{\hbar}\left[\left(1+\frac{eV}{E}\right)^{1/2}-1\right]L=(2n+1)\pi\;.$$

This gives the usual conductance minima (which we call the *primary* minima) associated with destructive interference of transmitted electrons. Note, however, that the numerator of $T_{\text{total}}(\phi)$ also vanishes if

$$(b-a)^2\hat{t}_1\hat{t}'_1e^{i\phi}=1\;. \tag{14}$$

In ballistic transport, this corresponds to the condition

$$2kL+\phi+2\nu=\frac{\sqrt{2m^*E}}{\hbar}\left[\left(1+\frac{eV}{E}\right)^{1/2}+1\right]L+2\nu=2m\pi\;. \tag{15}$$

It is obvious that whenever condition (15) is satisfied, the numerator of $T_{\text{total}}(\phi)$ goes to zero and the conductance should fall to a minimum unless the denominator of $T_{\text{total}}(\phi)$ also happens to go to zero at the same time. The denominator vanishes whenever $\phi=2n\pi$. Hence, unless Eq. (15) is satisfied only by those values of ϕ that are even multiples of π (which requires $2kL+\nu$ to be an even multiple of π or the ring to be Fabry-Pérot resonant at $V=0$), the conductance of the ring should reach a minimum whenever ϕ satisfies Eq. (15). This gives rise to an additional set of minima which we call the *secondary minima.* The physical origin of the secondary minima is the following: Eq. (15) represents the condition that *an electron, entering one of the paths from the left contact, gets reflected into the other path at the right contact, travels full circle around the ring and interferes constructively with itself at its point of entry at the left contact. This maximizes the reflection and hence minimizes the transmission and conductance.* This phenomenon could also cause a secondary set of minima to appear in the magnetostatic oscillations, but there the conditions for the occurrence of the primary and secondary minima are exactly identical (they occur at exactly the same value of the magnetic flux), so that they are always indistinguishable. But in the electrostatic case, the two conditions are different so that the two minima are *distinguishable.*

Let us now establish the requirements for the distinguishability. For this, we first find the difference between the phase shifts that give rise to the primary and secondary minima. From Eqs. (13) and (15),

$$\phi_{\text{primary}}-\phi_{\text{secondary}}=(2n+1)\pi-(2m\pi-2kL-2\nu)\;. \tag{16}$$

This difference becomes an even multiple of π (or, equivalently zero) if $2(kL+\nu)$ is an odd multiple of π, i.e., if the ring happens to be Fabry-Pérot antiresonant at $V=0$. In that case, the primary and secondary minima will overlap and remain indistinguishable. Barring this case, and the case of the ring being Fabry-Pérot resonant at $V=0$, both types of minima will not only occur in the oscillations, but also remain distinguishable.

It is clear that the appearance of *two different* sets of minima doubles the frequency of the conductance troughs in the oscillations. It is interesting to examine whether this can ever give rise to *exactly* half-periodic $(h/2e)$ oscillations. For this to happen, the secondary minima must occur when $\phi=2m\pi$ since the primary minima always occur when $\phi=(2n+1)\pi$. But the secondary minima cannot occur when $\phi=2m\pi$ since [from Eq. (15)] that would require $2(kL+\nu)$ to be an even multiple of π in which case the secondary minima do not even appear. Hence, exactly half-periodic oscillations can never arise from this effect in ballistic transport.

We now examine the effect of nonzero temperature on the two types of minima. Nonzero temperature gives rise to a thermal spread in the electron's energy which results in a bleaching out of the conductance minima due to ensemble averaging over the electron's energy. The primary minima are bleached out when the spread in the quantity on the left-hand side of Eq. (11) (due to a spread in the electron's energy) exceeds π and the secondary minima are bleached out when the spread in the quantity on the left-hand side of Eq. (15) exceeds π. These two spreads are

$$\Delta_{\text{primary}}=\left(\frac{m^*}{2\hbar^2}\right)^{1/2}L\left(\frac{1}{\sqrt{E+eV}}-\frac{1}{\sqrt{E}}\right)kT\;,$$

$$\Delta_{\text{secondary}}=\left(\frac{m^*}{2\hbar^2}\right)^{1/2}L\left(\frac{1}{\sqrt{E+eV}}+\frac{1}{\sqrt{E}}\right)kT\;, \tag{17}$$

where kT is the thermal spread in the energy.

From Eq. (17), we can find two critical temperatures T^c_{primary} and $T^c_{\text{secondary}}$ above which the primary minima and the secondary minima, respectively, are bleached out. These two temperatures are estimated by equating Δ_{primary} and $\Delta_{\text{secondary}}$ to π which gives (assuming the electron energy E to be the Fermi energy E_F)

$$kT^c_{\text{primary}}\sim\frac{h}{L}\frac{1}{\sqrt{2m^*}}\left(\frac{1}{\sqrt{E_F}}-\frac{1}{\sqrt{E_F+eV_p}}\right)^{-1}, \tag{18}$$

and

$$kT^c_{\text{secondary}}\sim\frac{h}{L}\frac{1}{\sqrt{2m^*}}\left(\frac{1}{\sqrt{E_F}}+\frac{1}{\sqrt{E_F+eV_s}}\right)^{-1}, \tag{19}$$

where V_p is the potential at which the first primary minimum occurs and V_s is the potential at which the first secondary minimum occurs in the oscillations.

Note that both T^c_{primary} and $T^c_{\text{secondary}}$ increase with increasing E_F or carrier concentration and decrease with increasing length of the structure. It is therefore necessary to have short structures with high-carrier concentration in order to observe both minima at sufficiently high temperatures.

In Fig. 2, we show the effect of temperature on both types of minima in the oscillations. While the primary minima can persist up to rather high temperatures, the secondary minima bleach out at much lower temperatures since [as we can see from Eqs. (18) and (19)] $T^c_{\text{secondary}} < T^c_{\text{primary}}$. This means that in an experimental situation, raising the temperature will gradually wash out the secondary minima and the oscillations will gradually revert to the normal Aharonov-Bohm oscillations with only the primary minima visible at higher temperatures.

Finally, another interesting feature, which is clearly visible in the oscillations, is that the primary minima tend to bleach out more and more in the higher cycles of the oscillations whereas the secondary minima exhibit the *opposite* behavior. This allows one to distinguish between the two types of minima in experimental data. It is a very interesting behavior and is easily understood from Eq. (17) which shows that at a given temperature, Δ_{primary} *increases* with increasing V while $\Delta_{\text{secondary}}$ actually *decreases* with increasing V. The significance of this is that at elevated temperatures, even if the secondary minima are not visible in the first few cycles of the oscillations, *they could eventually show up in the later cycles.*

Before concluding this Rapid Communication, we briefly discuss the effect of elastic scattering. We have carried out an analysis in the presence of elastic scattering following Ref. 4 and found that elastic scattering does not inhibit the twin-minima feature in the weak localization regime as long as the temperature is well below $T^c_{\text{secondary}}$.[9] We have also found that the feature is not completely inhibited in multichanneled transport as long as the number of channels (propagating modes) is not too large.[9] It therefore appears that the feature predicted in this Rapid Communication is quite robust and should be observable in realistic semiconductor structures at sufficiently low temperatures.

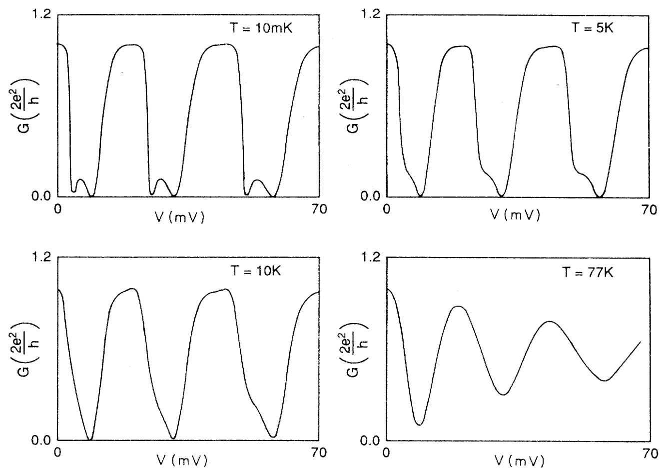

FIG. 2. The electrostatic Aharonov-Bohm conductance oscillations in a "ballistic ring" made of GaAs showing both types of minima. Each type of minima recur with the usual Aharonov-Bohm periodicity, but the separation between two adjacent minima (belonging to the two different types) is smaller than and unrelated to the Aharonov-Bohm periodicity. Note that the secondary minima are bleached out at much lower temperatures than the primary. These curves were obtained by performing the integral in Eq. (1) numerically. The parameters for the ring were carrier concentration equals 1.55×10^{6} cm^{-1}, path length equals 1000 Å, $\epsilon=0.5$, and $\nu=0$.

In conclusion, we have established the existence of a hitherto unsuspected feature in the conductance oscillation of a one-dimensional ring due to the electrostatic Aharonov-Bohm effect. We have identified the origin of this feature and discussed the conditions for its observability.

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[1]For a discussion of the electrostatic Aharonov-Bohm effect which is a type-II nonlocal effect, see, for example, G. Matteucci and G. Pozzi, Phys. Rev. Lett. **54**, 2469 (1985).

[2]We thank S. Washburn for pointing out the importance of this assumption.

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[7]The minimum value of the conductance in the magnetostatic effect is exactly zero in ballistic transport even at nonzero temperatures. However, in the electrostatic case, the minimum value is zero only at 0 K.

[8]Note that since the electrostatic Aharonov-Bohm phase shift ϕ is not a linear function of the potential V, the conductance oscillations are not exactly periodic in V. Actually, the period increases continuously with increasing V.

[9]M. Cahay, S. Bandyopadhyay, and H. L. Grubin (unpublished); *Nanostructure Physics and Fabrication,* edited by M. A. Reed and W. P. Kirk (Academic, Boston, in press).

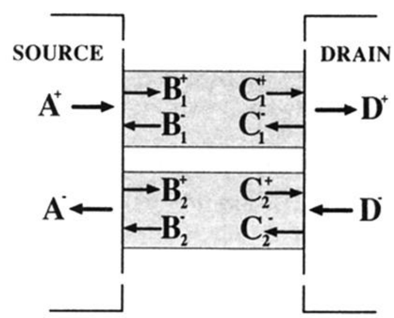

FIG. 1. Schematic representation of a one-dimensional ring-like structure showing the incident, reflected, and transmitted electron amplitudes.