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Filamentation instability of electromagnetic waves in compensated magnetoactive semiconductors

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The filamentation instability of electromagnetic waves in semiconductors in a uniform magnetic field is investigated. The nonlinear responses of the medium, including the nonparabolic dispersion of the conduction carriers as well as the second-order density and magnetic field perturbations that are created by the radiation pressure, are taken into account. The spatial amplification rate and threshold field associated with the filamentation instability are presented. The relevance of our work to electromagnetic-wave propagation in a sample of narrow-band-gap semiconductors is pointed out.

It is well known that interaction of intense electromagnetic radiation or microwave electric fields with mobile carriers in narrow-band-gap semiconductors such as InSb and InAs, etc., can lead to a host of nonlinear effects.[1–7] The latter involve the phenomena of optical mixing, the Kerr effect, electrostriction, thermal perturbation of the sample, nonlinear electronic polarization, and stimulated Brillouin scattering. In the past, it has been suggested[2] that in crystals such as InSb and InAs the observed nonlinearity comes from the nonparabolicity of the conduction band. The effect of second-order density perturbations that are driven by the radiation pressure has been incorporated[6] in the stimulated scattering of laser radiation in a piezoelectric semiconductor. Recently, the modulational instability of Alfvén waves in a compensated semiconductor such as germanium (Ge) has been considered, taking into account the nonuniform heating of electrons and holes.[7]

In this Brief Report, we present a unified description of the filamentation instability of finite amplitude electromagnetic waves in narrow-band-gap semiconductors, taking into account the combined effect of mass modulation of electrons and holes near the bottom of the conduction band as well as the second-order quasistationary density and magnetic field perturbations that are created by the radiation pressure. It is shown that the spatially modulated electromagnetic wave envelope is governed by a multidimensional cubic Schrödinger equation. The filamentation instability of a constant amplitude pump is investigated and expressions for the spatial amplification rates and thresholds are given. Our results may play a major role in the study of nonlinear interaction of lasers or microwave electric fields in semiconductors.

Consider a narrow-band-gap semiconductor embedded in an external magnetic field $\mathbf{B}_0$ which is directed along the z axis. The semiconductor specimen consists of electrons and holes whose mass ratio m_e/m_h need not be small. The masses of the electrons and holes may be changed due to the nonparabolicity of the conduction band. Thus, $m_\sigma = m_{0\sigma}/(1-v_\sigma^2/c_{v\sigma}^2)^{1/2}$, where $\mathbf{v}_\sigma$ is the velocity of species σ (e for the electrons and h for the holes), m_0 is the effective mass near the bottom of the conduction band, $c_v = (E_g/2m_0)^{1/2}$, and E_g is the gap energy. Here, c_v plays the same role in the dynamics that the speed of light c plays in relativistic mechanics. For realistic situations $c_v \ll c$.

Let us assume that the semiconductor sample is irradiated with right-hand circularly polarized electromagnetic waves whose electric field is represented as $\mathbf{E} = E(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\exp(ikz - i\omega t) + \text{c.c.}$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors transverse to $B_0\hat{\mathbf{z}}$. The frequency ω and wave vector $\mathbf{k} = k\hat{\mathbf{z}}$ are related by[8]

$$\omega^2 = k^2c^2 + \omega \sum_\sigma \omega_{p\sigma}^2/(\omega + \omega_{c\sigma}) , \quad (1)$$

where $\omega_{p\sigma} = (n_\sigma q_\sigma^2/m_\sigma \epsilon_0)^{1/2}$ and $\omega_{c\sigma} = q_\sigma B_0/m_\sigma$ are, respectively, the plasma and gyrofrequencies of particle species σ. Note that (1) is valid under the approximation $|\omega + \omega_{c\sigma}| \gg kv_{t\sigma}$, where $v_{t\sigma} = (T_\sigma/m_\sigma)^{1/2}$ is the thermal velocity. Nonlinear interaction of finite amplitude radiation with quasistationary perturbations in an electron-hole semiconductor plasma gives rise to an envelope of radiation which evolves according to the nonlinear Schrödinger equation[8]

$$iv_g\partial_z E + \tfrac{1}{2}S\nabla_\perp^2 E - \Delta E = 0 , \quad (2)$$

where the group velocity v_g and the perpendicular group dispersion S are given by

$$v_g = kc^2 \Big/ \left[\omega - \sum_\sigma \omega_{p\sigma}^2\omega_{c\sigma}/2(\omega+\omega_{c\sigma})^2\right] \quad (3a)$$

and

$$S = \left[1 + \frac{\sum_\sigma \dfrac{\omega_{p\sigma}^2\omega_{c\sigma}}{(\omega+\omega_{c\sigma})}}{2\left[\omega^2 - \sum_\sigma \omega_{p\sigma}^2\right]}\right]\frac{v_g}{k} . \quad (3b)$$

Note that in (2) we have assumed $\partial_t = 0$ and $\partial_z^2 \ll \nabla_\perp^2$. Furthermore, the nonlinear shift Δ comes from the excitation of the second-order density $n_{1\sigma}$ and magnetic field B_{1z} perturbations, as well as the particle mass modulation near the bottom of the conduction band. For $v_\sigma^2 \ll c_{v\sigma}^2$, we have[8]

$$\Delta = \frac{v_g}{2kc^2}\sum_\sigma \frac{\omega\omega_{p\sigma}^2}{(\omega+\omega_{c\sigma})}\left[\frac{n_{1\sigma}}{n_{0\sigma}} - \frac{\omega_{c\sigma}}{\omega+\omega_{c\sigma}}\frac{B_{1z}}{B_0} - \frac{q_\sigma^2\omega}{m_{0\sigma}^2 c_{v\sigma}^2}\frac{|E|^2}{(\omega+\omega_{c\sigma})^3}\right] . \quad (4)$$

The second-order density and field-aligned (parallel to $B_0\hat{\mathbf{z}}$) magnetic field perturbations are governed by

$$\frac{q_\sigma}{m_\sigma}(\mathbf{E}_S + \mathbf{v}_\sigma \times \mathbf{B}_0) - \frac{T_\sigma}{m_\sigma}\frac{\nabla n_{1\sigma}}{n_{0\sigma}} = \mathbf{F}_\sigma , \quad (5a)$$

and

$$\nabla \times \mathbf{B}_1 = \mu_0 \sum_\sigma n_{0\sigma} q_\sigma \mathbf{v}_\sigma , \quad (5b)$$

where $-m_\sigma \mathbf{F}_\sigma$ is the ponderomotive force on species σ. Here[8]

$$\mathbf{F}_\sigma = \frac{q_\sigma^2}{m_\sigma^2(\omega+\omega_{c\sigma})^2}\left[\nabla_\perp + \frac{\omega+\omega_{c\sigma}}{\omega}\hat{\mathbf{z}}\partial_z\right]|E|^2 . \quad (6)$$

Assuming quasineutrality ($n_{1e}/n_{0e} = n_{1h}/n_{0h}$, where n_{0e} and n_{0h} are the unperturbed density of the electrons and holes) in (5) and eliminating the ambipolar electric field $\mathbf{E}_s$ that is associated with the quasistationary response, we find from (5)

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}_1) \times \mathbf{B}_0 - \left[\sum_\sigma n_{0\sigma} T_\sigma\right]\frac{\nabla n_1}{n_0} = \sum_\sigma n_{0\sigma} m_\sigma \mathbf{F}_\sigma . \quad (7)$$

The z component of (7) in conjunction with (6) yields the second-order density perturbation

$$\frac{n_{1\sigma}}{n_{0\sigma}} = -\frac{\epsilon_0|E|^2}{\omega\left[\sum_\sigma n_{0\sigma} T_\sigma\right]}\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega+\omega_{c\sigma}} . \quad (8)$$

On the other hand, applying $\nabla_\perp$ on (7) and making use of (6), and $\partial_z^2 \ll \nabla_\perp^2$, we readily get

$$B_{1z} + \frac{n_{1\sigma}\mu_0}{n_{0\sigma}B_0}\sum_\sigma n_{0\sigma}T_\sigma = -\frac{|E|^2}{c^2 B_0}\sum_\sigma \frac{\omega_{p\sigma}^2}{(\omega+\omega_{c\sigma})^2} . \quad (9)$$

Eliminating $n_{1\sigma}$ from (8) and (9) we find the second-order field-aligned magnetic field perturbation

$$B_{1z} = \frac{|E|^2}{\omega c^2 B_0}\sum_\sigma \frac{\omega_{p\sigma}^2\omega_{c\sigma}}{(\omega+\omega_{c\sigma})^2} . \quad (10)$$

Inserting (8) and (10) in (4) we obtain $\Delta = -Q|E|^2$, and (2) takes the form

$$iv_g\partial_z E + \tfrac{1}{2}S\nabla_\perp^2 E + Q|E|^2 E = 0 , \quad (11)$$

where the expression for Q is

$$Q = (v_g/2kc^4B_0^2)(C_n + C_B + C_r) , \quad (12)$$

with

$$C_n = \frac{4}{\beta}\left[\sum_\sigma \frac{\omega_{p\sigma}^2}{\omega+\omega_{c\sigma}}\right]^2 , \quad (13a)$$

$$C_B = \left[\sum_\sigma \frac{\omega_{p\sigma}^2\omega_{c\sigma}}{(\omega+\omega_{c\sigma})^2}\right]^2 , \quad (13b)$$

and

$$C_r = \omega^2 c^2 \sum \frac{\omega_{p\sigma}^2\omega_{c\sigma}^2}{c_{v\sigma}^2(\omega+\omega_{c\sigma})^4} . \quad (13c)$$

We have defined $\beta \equiv 4\mu_0(\sum n_{0\sigma}T_\sigma)/B_0^2$.

The filamentation instability of a constant amplitude (E_0) electromagnetic wave against the mass modulation of electrons and holes as well as the excitation of second-order density and magnetic field perturbations can be investigated on the basis of (11). The standard analysis yields a dispersion relation[9]

$$K_z^2 = \frac{SK_\perp^2}{4v_g^2}(SK_\perp^2 - 4Q|E_0|^2) , \quad (14)$$

where $\mathbf{K} = (K_x, K_y, K_z)$ is the wave vector of the quasistationary perturbations. Assuming $K_z = -iK_i$ ($K_i > 0$), we obtain the amplification rate. The minimum spatial scale length L ($=2\pi/K_i$) for $K_\perp^2 = (2Q/S)|E_0|^2$ is

$$L = 2\pi v_g/Q|E_0|^2 \equiv 4\pi kc^4B_0^2/(C_n + C_B + C_r)|E_0|^2 . \quad (15)$$

Equation (14) shows that the filamentation instability starts at

$$E_0^2 \geq SK_\perp^2/4Q . \quad (16)$$

The critical power for the beginning of the filamentation process is

$$P = IA \equiv (kc^2|E_0|^2\epsilon_0/\omega)(\pi\lambda_\perp^2/4) , \quad (17)$$

where I is the intensity of the radiation, A is the cross-sectional area, and $\lambda_\perp = 2\pi/K_\perp$ is the wavelength. Substituting for $|E_0|^2$ from (16) into (17) and making use of the expression for S and Q, we write (17) as

$$P = \frac{\pi^3 kc^6B_0^2\epsilon_0}{2\omega(C_n + C_B + C_r)}\left[1 + \frac{\sum_\sigma \frac{\omega_{p\sigma}^2\omega_{c\sigma}}{(\omega+\omega_{c\sigma})}}{2\left[\omega^2 - \sum_\sigma \omega_{p\sigma}^2\right]}\right] . \quad (18)$$

We have thus presented two formulas (15) and (18) which should be useful in the study of electromagnetic wave filamentation in an electron-hole plasma. Specific results for helicons (Alfvén waves) can be obtained by introducing the limit $\omega \ll |\omega_{ce}|$ ($\omega \ll \omega_{ch}$).

In summary, we have reported an investigation of the filamentation instability of finite amplitude electromagnetic waves in an electron-hole plasma. The important nonlinearities which are included here come from the particle mass modulation near the bottom of the conduction band as well as second-order quasistationary density and magnetic field perturbations that are created by the radiation pressure. Physically, the nonlinear effects change the index of refraction of the electron-hole plasma. A change in the local density due to the presence of the electromagnetic waves gives rise to a gradient in the electron density which then acts as a convex lens and produces a curved phase front so that the laser electric field gradient is enhanced.

Our results are valid for an arbitrary radiation frequency and ratio m_e/m_h. In order to illustrate the physics of filamentation processes in semiconductor plasmas, we have kept the analysis simple and retained the nonlinear effects associated with the modulation of the mass of the electrons and holes as well as the ponderomotive force of the radiation. We have ignored the differential Joule heating that might arise from the interparticle collisions in solids. In fact, the differential joule heating nonlinearity produces a spatial modulation of the equilibrium particle temperature. Consequently, the second-order density and magnetic field perturbations are modified. Hence, the regimes as well as increments of filamentation instability could be affected.

The nonlinear phenomena discussed here could have a profound influence on the propagation of electromagnetic waves in solid-state plasmas. The filamentation instability might break up the radiation into pipes which could cause local heating of the electrons and holes. Accordingly, various optical phenomena which could be useful for diagnostic purposes ought to be reconsidered in light of the present investigation.

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