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Magnetization by rotation and gyromagnetic gyroscopes

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We discuss how the general phenomenon of magnetization by rotation may be used to probe the angular velocity of the laboratory with respect to a local frame of inertia. We show that a gyroscope with no moving parts based on this phenomenon has to include a sensor and a shield with different magnetomechanic factors. This is necessary in order to suppress the sensitivity to a magnetic field without canceling the effect of rotation. We present a realization of the gyroscope where the magnetization of a ferromagnetic rod is measured by a superconducting quantum interference device (SQUID). The rod is rigidly held in the interior of a closed superconducting shield. Because of the difference between the spin magnetomechanic factor of the magnetic rod $\gamma = m/e$, with m and e respectively the electron mass and charge, and the orbital magnetomechanic factor of Cooper pairs in the shield $\gamma = 2m/e$, the SQUID output is linear in the absolute angular velocity of the device. By contrast, in the limit of perfect shielding, it turns out to be insensitive to magnetic fields. We experimentally demonstrate the principle of operation of this device.

I. INTRODUCTION

Magnetization by rotation is a phenomenon demonstrated by any kind of magnetic material.[1] For ferromagnetic materials it was first demonstrated by Barnett[2] in a series of classical experiments. For superconductors it was predicted by London[3] and demonstrated in subsequent experiments.[4] Analogous magnetomechanic phenomena are well known for systems of nuclei.[5]

The phenomenon demonstrates the fact that a body that becomes magnetized in the presence of an applied field $\mathbf{B}^{\text{ext}}$, can also be magnetized if it is set in rotation with respect to the local frame of inertia at an angular velocity $\mathbf{\Omega}$. In this paper, the term "applied field" means the field that the external sources would create in space in the absence of the body. The resulting variation of the magnetization is the same as that which would be induced by a variation of the applied magnetic field $\delta\mathbf{B}^{\text{ext}}$ with components $\delta B_i^{\text{ext}} = \gamma_{ij}\Omega_j$.[6]

The magnetomechanic three-tensor γ_{ij} is a property of the body and, as is well known, reflects the nature of the microscopic degree of freedom associated with the magnetization. Thus for isotropic ferromagnets $\gamma_{ij} = \gamma\delta_{ij}$, with δ_{ij} the Kronecker δ, and with γ very close to the spin value, $m/e \sim -5.69\times10^{-12}$ T s/rad. Here m and e are the electron mass and charge, respectively. The actual magnitude of γ is usually found to be higher than that of m/e by a few percent because of small orbital contributions to the total angular momentum of atoms.[7] In superconductors, magnetic phenomena are associated with the motion of Cooper pairs and then $\gamma = 2m/e$ with fairly high accuracy.[8]

In the last decades the precision of magnetic measurements has improved by many orders of magnitude thanks to the diffusion of superconducting quantum, interference devices (SQUIDS).[9] As a consequence, the idea becomes appealing to build a solid-state gyroscope based on the measurement of the magnetization induced by rotation of a magnetic body.

However, in order that such a gyroscope might be operated in practice, it should obviously be shielded from the effect of true magnetic fields. For instance, for the electron spin, the ratio between the equivalent magnetic field associated with the earth's rotation $\gamma\Omega_\oplus \sim 4\times10^{-16}$ T, and the true earth magnetic field $B_\oplus$, amounts to $\gamma\Omega_\oplus/B_\oplus \sim 10^{-11}$, an extremely low figure.

In this paper we describe how such a gyroscope can be realized. To do this we briefly review in Sec. II the thermodynamics of the phenomenon and we take the opportunity to put on more precise ground the equivalence between rotation and magnetic fields. In Sec. III, we discuss the effect of rotation on a magnetic shield. We show that the effect of rotation on a body rigidly enclosed in an ideal shield with the same γ value as the body itself, is canceled, while it is not so if the body and the shield have different γ's. The effect of sources of a magnetic field located inside the shield is also discussed. We then specifically sketch how the gyroscope may be made and operated. In Sec. IV we describe a realization of the gyroscope, where the magnetization of a ferromagnetic rod rigidly enclosed in a superconducting shield is measured by a radio-frequency SQUID, and we present the experimental test of the operation of the device. Finally, in Sec. V we summarize the results on a simple model for the intrinsic thermal noise of the device.

II. THERMODYNAMICS OF THE MAGNETIZATION BY ROTATION

In this section we review the thermodynamics of the phenomenon for those systems where a magnetization vector can be defined. We will derive a result that allows the calculation in any point of the space of the magnetic field due to the magnetization induced by the rotation of the system. The result pertains also to systems where γ_{ij} does not reduce to $\gamma\delta_{ij}$ and where it also varies from point to point. It will be derived without the need for introducing ad hoc terms in the free energy of the system, as is done, for instance, in Ref. 1.

Superconductors, for which a magnetization in a strict sense cannot be defined, are briefly discussed at the end of the section where we just recall the results obtained long ago by London.[3]

The work dW'/dt supplied per unit time by a distribution of free currents to a system composed by a rigid magnetic body and the electromagnetic field, is given by[1]

$$dW'/dt=\int \mathbf{H}\cdot(\partial\mathbf{B}/\partial t)dV+\int \mathbf{E}\cdot(\partial\mathbf{D}/\partial t)dV . \qquad (1)$$

Here we assume that the fields vary so slowly that the ohmic dissipation in the body, if any, may be neglected. In Eq. (1) $\mathbf{H}=\mathbf{B}/\mu_0-\mathbf{M}$ and $\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ with $\mathbf{P}$ the electrical polarization of the body and $\mathbf{M}$ its magnetization.

Suppose now that the net force acting on the body is zero so that its center of mass is held at rest in an inertial frame. This can be always accomplished by exerting on the body a proper system of forces to counterbalance the force exerted on it by the macroscopic electromagnetic fields. Let us call $\mathbf{K}$ the total momentum of the forces that are not due to the macroscopic electromagnetic fields. Then these forces would supply our system with an additional work per unit time $dW''/dt=\mathbf{K}\cdot\mathbf{\Omega}$. Here $\mathbf{\Omega}$ is the angular velocity of the body relative to an inertial frame.

If $\mathbf{L}$ is the total angular momentum of the system, body plus electromagnetic field, and $\mathbf{K}_{\rm em}$ is the torque that the electromagnetic field exerts on the system of free currents, then,

$$dW''/dt=\mathbf{K}\cdot\mathbf{\Omega}=\mathbf{\Omega}\cdot(d\mathbf{L}/dt)+\mathbf{K}_{\rm em}\cdot\mathbf{\Omega} , \qquad (2a)$$

with

$$\mathbf{K}_{\rm em}\cdot\mathbf{\Omega}=\int \mathbf{\Omega}\cdot\mathbf{r}\times[\nabla\cdot\mathbf{D})\mathbf{E}+(\nabla\times\mathbf{H}-\partial\mathbf{D}/\partial t)\times\mathbf{B}]dV , \qquad (2b)$$

where we have used the Maxwell equations $\nabla\cdot\mathbf{D}=\rho$ and $\nabla\times\mathbf{H}-\partial\mathbf{D}/\partial t=\mathbf{j}$ to express the density of free charge ρ and that of free current $\mathbf{j}$ in terms of the macroscopic fields.

Assume that both the free currents and the body have finite extent and that the fields vary slowly enough to allow us to neglect any electromagnetic radiation. Then for the total work supplied to the system $dW/dt=dW'/dt+dW''/dt$, from Eqs. (1), (2a), and (2b), and with some algebra, one gets

$$dW/dt=\mathbf{\Omega}\cdot(d\mathbf{L}/dt)+\int(\mathbf{E}+\mathbf{h}\times\mathbf{B})\cdot(\partial\mathbf{D}/\partial t+\mathbf{h}\cdot\nabla\mathbf{D}+\mathbf{D}\times\mathbf{\Omega})dv+\int(\mathbf{H}-\mathbf{h}\times\mathbf{D})\cdot(\partial\mathbf{B}/\partial t+\mathbf{h}\cdot\nabla\mathbf{B}+\mathbf{B}\times\mathbf{\Omega})dV , \qquad (3)$$

where $\mathbf{h}=\mathbf{\Omega}\times\mathbf{r}$ and where only terms linear in Ω have been kept.

Now, in an inertial system of coordinates one can usually define the fully contravariant four-tensor[10] $M^{\mu\nu}$ such that the components of the magnetization and electric polarization three-vectors $\mathbf{M}$ and $\mathbf{P}$ are respectively given by $M_i=\frac{1}{2}\varepsilon_{ijk}M^{jk}$ and $P_i=(1/c)M^{i0}$, with ε_{ijk} the Ricci symbol. It is then possible to define the four-tensor $H^{\mu\nu}=(1/\mu_0)F^{\mu\nu}-M^{\mu\nu}$, with $F^{\mu\nu}$ the fully contravariant electromagnetic tensor, such that the three-vectors $\mathbf{H}$ and $\mathbf{D}$ are given, respectively, by $H_i=\frac{1}{2}\varepsilon_{ijk}H^{jk}$ and $D_i=(1/c)H^{0i}$. In this same formalism $B_i=\frac{1}{2}\varepsilon_{ijk}F_{jk}$ and $E_i=cF_{i0}$.

Expressing the four-tensors $F_{\mu\nu}$ and $H^{\mu\nu}$ in a system of coordinates rotating with the body,[11] one can then define the four three-vectors $\mathbf{B}_\Omega$, $\mathbf{H}_\Omega$, $\mathbf{E}_\Omega$, and $\mathbf{D}_\Omega$ from the corresponding four-tensors using the same equations used in the inertial frame. It is easy to check that Eq. (3) may be expressed in terms of these vectors as

$$dW/dt=\int \mathbf{H}_\Omega\cdot(\partial\mathbf{B}_\Omega/\partial t)dV+\int \mathbf{E}_\Omega\cdot(\partial\mathbf{D}_\Omega/\partial t)dV+\mathbf{\Omega}\cdot d\mathbf{L}/dt , \qquad (4)$$

where now all the derivatives on the right-hand side of Eq. (4) are evaluated in the rotating frame. This last statement is also valid for the term $\mathbf{\Omega}\cdot d\mathbf{L}/dt$. In fact, it is known from classical analytical mechanics that the angular momentum vector $\mathbf{L}$ does not change moving to a rotating frame.[12] In addition, the time derivative $d\mathbf{L}/dt$, appearing in a scalar product with $\mathbf{\Omega}$, can be evaluated in any one of the two frames.

Equation (4) gives the total work supplied to the system per unit time. Then the internal energy U of the system increases at a rate $dU/dt=dW/dt+dQ/dt$, with dQ/dt the heat adsorbed per unit time. However, it is known[12] that the function that plays the role of the energy in a rotating system is $U'=U-\mathbf{\Omega}\cdot\mathbf{L}$. In terms of this function the energy balance of the system can be written as

$$dU'/dt=\int \mathbf{H}_\Omega\cdot(\partial\mathbf{B}_\Omega/\partial t)dV+\int \mathbf{E}_\Omega\cdot(\partial\mathbf{D}_\Omega/\partial t)dV-\mathbf{L}\cdot d\mathbf{\Omega}/dt+dQ/dt . \qquad (5)$$

At equilibrium all the physical fields that describe the system have obviously appeared stationary in a frame rotating with the body. Equation (5) ensures that in this case U' is a conserved quantity that can thus be identified with the internal energy of the system.

For infinitesimal transformations connecting equilibrium states, the differential δF of the free energy $F=U'-TS$, with T the temperature and S the entropy, may be evaluated in the rotating frame from Eq. (5) as

$$\delta F=\int \mathbf{H}_\Omega\cdot\delta\mathbf{B}_\Omega dV+\int \mathbf{E}_\Omega\cdot\delta\mathbf{D}_\Omega dV-\mathbf{L}\cdot\delta\mathbf{\Omega}-S\delta T . \qquad (6)$$

If the angular momentum can be written as $\mathbf{L}=\int \mathbf{l}dV$, with $\mathbf{l}$ an angular momentum density, then, in order that

δF is an exact differential, the following equality should hold:

$$\int [(\partial M_{\Omega i}/\partial \Omega_j)_{\mathbf{B}_\Omega,\mathbf{D}_\Omega,T} - (\partial l_j/\partial B_{\Omega i})_{\Omega,\mathbf{D}_\Omega,T}] \times \delta B_{\Omega i}\delta\Omega_j dV = 0 \ , \quad (7)$$

where, as we did in the inertial frame, we defined the three-vector $\mathbf{M}_\Omega$ as $\mathbf{M}_\Omega = (1/\mu_0)\mathbf{B}_\Omega - \mathbf{H}_\Omega$.

In Eq. (7) the variation of the field $\mathbf{B}_\Omega$ has to preserve the constraint $\nabla\cdot\mathbf{B}_\Omega = 0$ so that one gets

$$(\partial M_i/\partial \Omega_j)_{\mathbf{B},\mathbf{D},T} = (\partial l_j/\partial B_i)_{\Omega,\mathbf{D},T} + \partial \xi_j/\partial x_i \ , \quad (8)$$

where from now on we drop for convenience the subscript Ω, but it is still understood that all the quantities and the derivatives have to be taken in the system of coordinates rotating with the body.

In Eq. (8) ξ_j is an undetermined scalar function. However, it is easy to calculate that in the rotating frame the magnetic field $\mathbf{B}$ obeys the same couple of equations of magnetostatics $\nabla\cdot\mathbf{B}=0$ and $\nabla\times\mathbf{B}=\mu_0(\mathbf{j}+\nabla\times\mathbf{M})$ that it obeys in the inertial frame. Thus the physical quantity that acts as the source of the magnetic field $\mathbf{B}$ is $\nabla\times\mathbf{M}$ and the addition of a gradient is irrelevant. Therefore this term can be dropped from Eq. (8).

In Eqs. (7) and (8) we have assumed that both $\mathbf{M}(\mathbf{r})$ and $l(\mathbf{r})$ depend only on the local values of $\mathbf{B}(\mathbf{r})$ and $\mathbf{D}(\mathbf{r})$, $\mathbf{\Omega}$, and the temperature T, so that the partial derivatives in Eq. (8) have a definite meaning. This appears to be a legitimate assumption for all physical systems of interest here at least near $\Omega=0$.

Close to states where the local "susceptibility" matrix $(\partial M_i/\partial B_k)_{\Omega,\mathbf{D},T}$ has an inverse, recalling that this matrix is symmetric because of $(\partial M_i/\partial B_k)_{\Omega,\mathbf{D},T} = -(\partial^2 F/\partial B_k \partial B_i)_{\Omega,\mathbf{D},T}$, one gets

$$(\partial M_i/\partial \Omega_j)_{\mathbf{B},\mathbf{D},T} = [(\partial l_j/\partial M_k)_{\Omega,\mathbf{D},T}][(\partial M_i/\partial B_k)_{\Omega,\mathbf{D},T}] \ . \quad (9)$$

The total magnetic field $\mathbf{B}$ can be divided into a contribution due to free currents $\mathbf{B}^{\mathrm{ext}}$, i.e., the applied field, and one due to the magnetization of the body $\mathbf{B}^{\mathrm{mag}}$, so that $\mathbf{B}=\mathbf{B}^{\mathrm{ext}}+\mathbf{B}^{\mathrm{mag}}$. Then for the most general variation $\delta\mathbf{M}$ of $\mathbf{M}$ up to terms linear in the variations of the fields and of the angular velocity, using Eq. (9), one has

$$\delta M_i = (\partial M_i/\partial B_k)_{\Omega,\mathbf{D},T}[(\partial l_j/\partial M_k)_{\Omega,\mathbf{D},T}\delta\Omega_j + \delta B_k^{\mathrm{ext}} + \delta B_k^{\mathrm{mag}}] \ . \quad (10)$$

Equation (10) is in fact an integrodifferential equation for δM_i, since $\delta\mathbf{B}^{\mathrm{mag}}$ is a functional of $\nabla\times\mathbf{M}$. It is clear that in this equation the term $(\partial l_j/\partial M_k)_{\Omega,\mathbf{D},T}\delta\Omega_j$ appears on the same ground as the other term $\delta B_k^{\mathrm{ext}}$. Thus, one can conclude that the effect that an angular velocity variation has on the magnetic properties of a body is the same as the one due to the application, in a frame rotating with the body, of an equivalent magnetic field $\delta\mathbf{B}^{\mathrm{eq}}$ given by

$$\delta B_k^{\mathrm{eq}} = \gamma_{kj}\delta\Omega_j \ , \quad (11)$$

where the magnetomechanic coefficients γ_{kj} are defined as $\gamma_{kj} = (\partial l_j/\partial M_k)_{\Omega,\mathbf{D},T}$.

Let us comment on the results of this analysis, which go a bit beyond the more standard statements on the subject. First it is not irrelevant to stress that Eqs. (10) or (11) are valid only in a system of coordinates rotating with the body. In fact, these equations involve variations of three-vectors which are not all the same in the rotating and in the inertial systems. Moreover, the coefficients γ_{ij}, reflecting a local property of the body, do not depend on time only if evaluated in the rotating coordinates. This is of crucial relevance when calculating the variation of the magnetization of the body, during finite transformations connecting states with different values of the angular velocity.

A second point is the following. Usually the absolute value of the angular velocity is so low that all partial derivatives may be evaluated at $\mathbf{\Omega}=0$. If in addition l is a linear function of $\mathbf{M}$, then the above definition of the γ's coincides with the usual definition as the ratio between the value of the angular momentum density and that of the magnetization evaluated with the body at rest in an inertial frame. However, notice that the angular momentum density l contains also the contribution due to the electromagnetic field besides the one due to atomic momenta. Thus identifying $(\partial l_j/\partial M_k)_{\Omega,\mathbf{D},T}$ with the microscopically defined atomic magnetomechanic factors would be somewhat inaccurate. This is of no relevance for the great majority of practical situations, and in particular for the scope of the present paper, but it could be of some importance in the determination of the γ factors in systems of very low magnetic susceptibility in the presence, for instance, of a substantial amount of electric polarization. Notice, in addition, that outside the magnetic body, where the macroscopic fields coincide with their microscopic definition and where the angular momentum is entirely due to the field, Eq. (8) is a trivial identity. In fact, both $(\partial M_i/\partial \Omega_j)_{\mathbf{B},\mathbf{D},T}$ and $(\partial l_j/\partial B_i)_{\Omega,\mathbf{D},T}$ coincide in this case with $\epsilon_0(E_j r_i - E_k r_k \delta_{ij})$.

As a third point it has to be noticed that the equivalent field in Eq. (11) cannot be mimicked in every situation by a true magnetic field. This follows immediately from the fact that the divergence of $\delta\mathbf{B}^{\mathrm{eq}}$, for composite or inhomogeneous systems where γ_{ij} is a function of $\mathbf{r}$, is not necessarily zero everywhere. Thus we stress that Eq. (11) has to be understood in the meaning that $\delta\mathbf{B}^{\mathrm{eq}}$ has to be used in the equation of magnetostatics Eq. (10), in order to calculate $\delta\mathbf{M}$ or, more precisely, $\nabla\times\delta\mathbf{M}$. By no means then must it be considered a true magnetic field.

Equation (10) allows the calculation of the detailed distribution of the magnetization in the whole body. A much simpler formula can be derived, following analogous reasoning, for the variation of the total magnetic moment of the body. In fact, if the field $\mathbf{B}^{\mathrm{ext}}$ generated by the external sources can be considered to be uniform over the volume of the body, and takes the value $\mathbf{B}^0$ there, then one has, again in the rotating frame,

$$(\partial \Pi_i/\partial \Omega_j)_{\mathbf{B}^0,T} = (\partial L_j/\partial B_j^0)_{\Omega,T} \ , \quad (12)$$

where $\mathbf{\Pi} = \int \mathbf{M}\, dV$ is the total magnetic moment of the body.

Notice that Eq. (8), except for the irrelevant gradient on the right-hand side, represents the application of Eq. (12) to each elementary volume of the body.

If the body is superconducting, the magnetization loses its meaning at least in the ordinary sense and the magnetic properties of the body have to be described in terms of a supercurrent density. However, as we already said, London[3] predicted for a rotating superconductor, that in a system of coordinates rotating with the body the magnetic field and the supercurrent density obey the same set of equations as those they obey in an inertial frame, provided that the "known" term $\mathbf{B}^{\text{ext}}$ is substituted by $\mathbf{B}^{\text{ext}}+(2m/e)\mathbf{\Omega}$. Thus again an equivalence between magnetic field and rotation is established with $\gamma_{ij}=(2m/e)\delta_{ij}$. Obviously Eq. (12), that involves only the total magnetic moment of the body, a perfectly defined quantity for superconductors too, remains valid in this case.

Let us now discuss the physical meaning of the angular velocity vector $\mathbf{\Omega}$ above. Clearly not every system showing magnetization by rotation can be considered strictly a rigid body. For instance, in a steadily rotating superconductor, the electron superfluid in a surface layer with thickness of the order of the penetration depth, precesses with respect to the crystal lattice, and it is just this surface current that gives rise to the London magnetic moment. However, at thermodynamic equilibrium this current distribution is a function of the temperature, of the applied magnetic field, and uniquely of the angular velocity of the lattice. Thus, both the angular momentum of the lattice and that of the superfluid can be expressed as a function of this angular velocity.

On the other hand, for the systems described by a magnetization, in deriving the formulas above we have supposed that a nonelectromagnetic torque $\mathbf{K}$ acting on the system supplies a power $dW''/dt=\mathbf{K}\cdot\mathbf{\Omega}$. In order to fulfill this condition it is not necessary that all of the body be rigid but just that the part of it on which the torque acts performs a rigid rotation at an angular velocity $\mathbf{\Omega}$.

Thus, in general, $\mathbf{\Omega}$ will represent the angular velocity of some rigidly rotating part of the system provided that, at equilibrium, all thermodynamic functions may be expressed in terms of $\mathbf{\Omega}$ alone and not of the angular momenta of the various parts taken separately.

Here it has been supposed that the states that are connected by the thermodynamical transformations are states of equilibrium. As it is known,[12] this is possible for solid bodies if the rotation is uniform and takes place around a principal axis of inertia of the body. If, however, this is not the case, the variation of all thermodynamic quantities can still be considered adiabatic if the angular velocity is such that $\Omega\tau \ll 1$, with τ the longest relaxation time of the system. Then all the above formulas still hold in a quasiequilibrium sense.

Finally, notice that for ferromagnetic materials the total variations of the equivalent magnetic field are so low, for any realistic value of the angular velocity, that the resulting changes of the magnetization can safely be considered reversible, and thus the magnetic susceptibilities involved in the phenomenon are the so-called reversible or initial ones.

III. ROTATION AND MAGNETIC SHIELDS

The equations in the previous section allow us to calculate only the variation of the magnetization due to a variation of the angular velocity, the initial value of it being undetermined.

However, this does not prevent the use of this phenomenon, at least in principle, to measure steady rotations. In fact, suppose that the laboratory is rotating around an axis that we call $\hat{\mathbf{z}}'$ at a constant rate Ω. Suppose now that a long cylindrical magnetic rod with $\gamma_{ij}=\gamma\delta_{ij}$ and whose symmetry-axis unit vector we call $\hat{\mathbf{z}}$, rotates at a rate ω around another axis $\hat{\mathbf{x}}'$ fixed to the laboratory and normal to both $\hat{\mathbf{z}}'$ and $\hat{\mathbf{z}}$. As a consequence $\hat{\mathbf{z}}$, as seen in the laboratory, rotates in a plane containing $\hat{\mathbf{z}}'$.

Taking the time t equal to zero when $\hat{\mathbf{z}}$ and $\hat{\mathbf{z}}'$ are aligned, the component Ω_z of the angular velocity along the $\hat{\mathbf{z}}$ axis turns out to be $\Omega_z=\Omega\cos(\omega t)$. The component Π_z of the total magnetic moment $\mathbf{\Pi}$ along $\hat{\mathbf{z}}$, measured in a frame rotating with the body, is then

$$\Pi_z=\Pi_z^0+(\partial\Pi_z/\partial B_z^0)\gamma\Omega\cos(\omega t)\ , \tag{13}$$

with Π_z^0 a constant, so that the angular velocity Ω appears in the amplitude of the magnetic moment modulation at a frequency ω. A similar modulation will appear obviously also in the value of the magnetization in each point of the body.

Unfortunately, changing the orientation of the body in the laboratory also changes its orientation relative to the sources of magnetic field. As a consequence, changes of magnetization have to be expected to be much larger than the one in Eq. (13), at least for any interesting value of Ω.

To avoid this problem one has to resort to some kind of magnetic shielding. However, in the above ideal scheme of measurement, the body has not only to be enclosed in a shield, but the shield has to move rigidly with the body. In fact, if this one moves relative to the shield, it will also move relative to sources of magnetic field located inside the shield. In best superconducting shields,[13] residual fields not less than 10^{-12} T are present, corresponding, say for a ferromagnetic material, to an angular velocity of ~ 0.2 rad/s.

This poses the question of what is the behavior of a magnetic shield upon steady rotation or, more precisely, upon variations of its angular velocity.

Consider for the moment only a passive shield, i.e., a closed cavity in some magnetic material like a ferromagnet or a superconductor. The property that makes such a system a magnetic shield is the fact that a variation of the field generated by external sources $\delta\mathbf{B}^{\text{ext}}$ induces a change $\delta\mathbf{M}$ in the magnetization of the shield walls. This change is such that, calling again $\delta\mathbf{B}^{\text{mag}}$ the field generated by it, one has

$$|\delta\mathbf{B}^{\text{mag}}+\delta\mathbf{B}^{\text{ext}}| \ll |\delta\mathbf{B}^{\text{ext}}| \tag{14}$$

at least in some part of the shield. A perfect shield is obviously one for which $\delta\mathbf{B}^{\text{mag}}=-\delta\mathbf{B}^{\text{ext}}$ in every part of the shielded space.

Let us now suppose that the external field variation is uniform and let us call it $\delta\mathbf{B}^0$. $\delta\mathbf{B}^{\text{mag}}$ would then be an almost uniform field such that everywhere

$$\delta\mathbf{B}^{\text{mag}} = -\delta\mathbf{B}^0 + \Delta\mathbf{B} \tag{15}$$

with $\Delta B \ll \delta B^0$. Moreover, in the linear approximation we are using here, $\Delta\mathbf{B}$ will be a linear function of $\delta\mathbf{B}^0$, i.e., $\Delta B_k = c_{kj}\delta B_j^0$. Suppose that γ_{ij} is the same everywhere within the shield wall. As a consequence, according to Eq. (11), a variation of the angular velocity of the shield, being equivalent to a uniform field $\delta B_k^{\text{eq}} = \gamma_{kj}\delta\Omega_j$, would induce inside it a magnetic field:

$$\delta B_k^{\text{mag}} = -\gamma_{kj}\delta\Omega_j + c_{ki}\gamma_{ij}\delta\Omega_j \ . \tag{16}$$

Let us now consider a second body enclosed in the shield and rigidly rotating with it. Suppose for simplicity that this body is so small that the magnetic field generated by its magnetization does not affect the magnetization of the shield. Let us call γ'_{ij} its magnetomechanic factors and assume that they do not change over the volume of the body. In these conditions the body will be magnetized as in the presence of a total applied field $\delta\mathbf{B}^{\text{eq}}$ given by

$$\delta\mathbf{B}_k^{\text{eq}} = \gamma'_{kj}\delta\Omega_j - \gamma_{kj}\delta\Omega_j + c_{ki}\gamma_{ij}\delta\Omega_j \ . \tag{17}$$

In the limit of perfect shielding $c_{ik} = 0$, if $\gamma'_{kj} = \gamma_{kj}$ then $\delta\mathbf{B}^{\text{eq}} = \mathbf{0}$: inside the shield the effect of rotation on materials with the same γ's as the shield itself disappears. If instead the γ's are not equal to those of the shield then the effect of the true magnetic field is still suppressed, while that of rotation is not and amounts to an equivalent field $\mathbf{B}_k^{\text{eq}} = (\gamma'_{kj} - \gamma_{kj})\delta\Omega_j$. Thus a practical device could be set up by using for the shield and for an internal sensitive element two materials with different γ's.

For a realistic device the detailed response has to be calculated applying Eq. (11) to the whole system of the shield and the body. This can be done analytically in a few cases. For instance, if a sphere of a soft magnetic material with radius r_0, initial susceptibility χ and $\gamma_{ij} = (m/e)\delta_{ij}$, the pure spin value, is rigidly enclosed in a concentric superconducting spherical layer of internal radius r_1, then an angular velocity variation $\delta\mathbf{\Omega}$ would induce a variation of magnetization $\delta\mathbf{M}$ given by

$$\delta\mathbf{M} = -(\chi/\mu_0)(m/e)\{1 + \chi[1 + \tfrac{2}{3}(r_0^3/r_1^3 - 1)]\}^{-1}\delta\mathbf{\Omega} \ , \tag{18}$$

while no variation is induced by an external field change.

Notice that in Eq. (18) the reaction of the shield to the magnetization of the inner sphere contributes the factor proportional to r_0^3/r_1^3.

It is of some interest to point out that all the above derivation holds for active shields too. By the term "active shield" we mean a system composed by some sensor of magnetic field and a set of coils arranged in a feedback scheme. In order that this system acts as a shield, the coils have to produce a field that cancels the field variations measured by the sensor and due to the external sources. Thus an equation like Eq. (14) or Eq. (15) is obeyed, with $\delta\mathbf{B}^{\text{mag}}$ representing now the field due to the coils.

A magnetic field sensor again responds to rotation as in the presence of an equivalent external field. For instance SQUIDS (Ref. 14) have $\gamma = 2m/e$, while NMR sensors have the magnetomechanic factors of nuclear spin.[5] Thus the effect of rotation on the active shield will be described by an equation similar to Eq. (17), and all the magnetic bodies with the same γ as the sensor will feel no magnetic effect of rotation in the limit of perfect shielding.

Thus, in conclusion, a gyromagnetic gyroscope, in the meaning of the present paper, has to be constituted by a body rigidly enclosed in a shield with a different magnetomechanic factor. The magnetization of the body has to be measured by some magnetometric device also rigidly connected to the rest. In the detailed calculation of the response of the instrument the direct effect of the rotation on this measuring device too has to be taken in account. This turns out to be simple if the sensor has the same γ as the shield, in which case it will feel no direct effect of the rotation.[15]

For instance, in the previous example of the ferromagnetic sphere enclosed in the superconducting shield, if the sensor is a SQUID magnetometer, it will only sense the flux change due to the variation of the magnetization of the inner sphere. This comprises also a flux change due to the currents that circulate on the inner surface of the shield to prevent the magnetic field generated by the magnetization of the sphere from penetrating into the shield wall. Specifically, if the SQUID is thought of as a plane circular coil tightly wound onto the equator of the sphere, it will feel a change in flux given by $\delta\phi = \pi r_0^2(2\mu_0/3)(1 - r_0^3/r_1^3)\delta\mathbf{M}$ with $\delta\mathbf{M}$ taken from Eq. (18).

This kind of gyromagnetic gyroscope could be enclosed in any other system of shields, the response being unaffected by their presence. In fact, the fields due to the magnetization induced by the rotation in the outer shields would be canceled as any other magnetic fields by the innermost shield.

Finally, we stress again that this kind of gyroscope can sense only variations of $\mathbf{\Omega}$, even if slow at will, and thus to measure the absolute value of the angular velocity of a reference frame, one has to modulate the orientation of the device in the frame as we sketched above.

IV. EXPERIMENT

To test the concepts outlined in Sec. III, we have performed an experiment on a composite system made of a superconducting shield rigidly enclosing a ferromagnetic rod, whose magnetization is read by a SQUID.[16] We will show that this system works as a gyroscope.

A. Experimental setup

In the present version of the gyroscope (Fig. 1) a 1.5-mm-o.d., 130-mm long rod of Cryoperm10 (Ref. 17), a cryogenic soft ferromagnetic steel, is enclosed in a 50-mm-i.d., 200-mm long closed cylindrical superconducting shield. The geometry of the apparatus is different from the one given as an example in Sec. III. In fact, the demagnetizing factor for a sphere is quite high and would

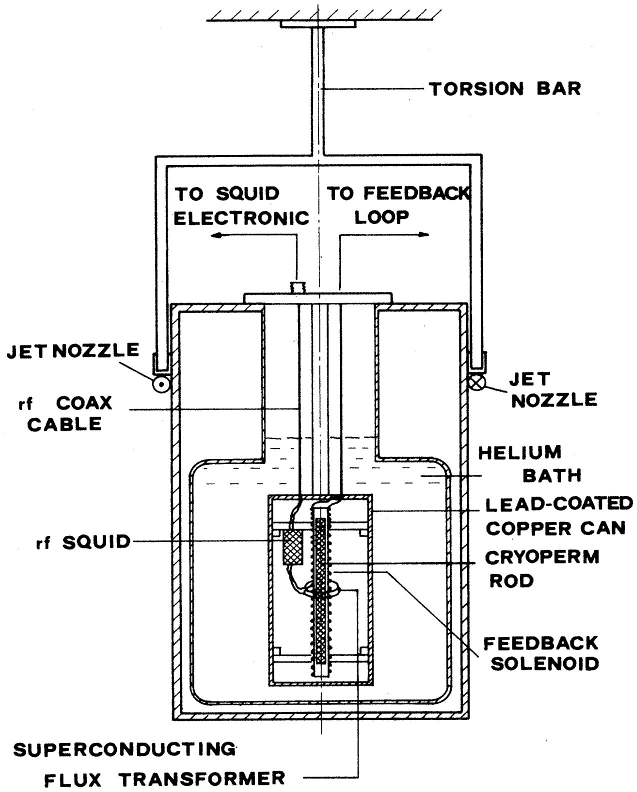


FIG. 1. Schematic view of the gyroscope and of the dewar torsional oscillation setup.

severely depress the sensitivity of the detector. The shield is made of a cylindrical copper can and two copper end flanges, which are coated by electrodeposition with a ~ 0.1-mm thick Pb layer on both the internal and the external surfaces. The superconducting electrical contact between the can and the end flanges is ensured by two Pb o-ring seals. Around the central section of the rod is wound a two-turn coil of niobium wire which constitutes one of the windings of a superconducting dc flux transformer.[9] The other winding of the transformer is inductively coupled to a commercial rf niobium SQUID. A 170-mm long solenoid with ~ 4400 turns per meter length is also wound onto the rod.

In the approximation of infinite length of both the rod and the shield, a variation of the component $\delta\Omega_z$ of the angular velocity of the apparatus along its symmetry axis would produce a change in the magnetization of the rod, whose component δM_z along the same axis would be given by

$$\delta M_z = -(\chi/\mu_0)(1-1/g)(1+\chi s/S)^{-1}(2m/e)\delta\Omega_z , \tag{19}$$

where s is the cross section of the rod and S is the inner section of the shield. χ is the magnetic susceptibility of the rod and $\mu=\mu_0(1+\chi)$ its permeability. Since the total equivalent magnetic field variations are always very small ($\delta B_z^{\text{eq}} < 10^{-12}$ T in all the experiments) the susceptibility value has to be taken as the one pertaining to small reversible changes of the magnetization around the initial state. In our case the χ of the rod was measured to be ~ 180 at room temperature and is expected to drop to $\sim 60\%$ of this value at liquid-helium temperature as shown by a Cryoperm toroidal sample in a separate experiment. Moreover, χ was found to be flat for slowly varying fields with a cutoff frequency of ~ 3.5 kHz due to skin effect. Therefore, χ can be taken to be equal to its zero-frequency limit, because the frequencies of all the signal involved in the experiment were always much less than this cutoff. In Eq. (19) it has been assumed that the ferromagnetic rod has $\gamma_{ij}=(2m/ge)\delta_{ij}$.

This variation of the magnetization and the related reaction of the shield would induce in the SQUID a change in flux $\delta\phi_\Omega$ given by

$$\delta\phi_\Omega = -T_0\chi s(1-1/g)(1-s/S)(1+\chi s/S)^{-1} \times(2m/e)\delta\Omega_z . \tag{20}$$

Here, T_0 is the so-called transformer ratio of the flux transformer, and it is defined as $T_0=NM(L_1+L_2)^{-1}$, where N is the number of turns of the coil wound on the rod, L_1 is the self-inductance of this same coil, L_2 is the self-inductance of the coil coupled to the SQUID, and M is the mutual inductance of this coupling. Although the transformer ratio is frequency dependent, as is χ and thus L_1, we can take again for T_0 the zero-frequency value everywhere in the analysis.

In our geometry, due to boundary effects caused by the finite length, the factor $\chi s(1-s/S)(1+\chi s/S)^{-1}$ in Eq. (20) has to be substituted by an effective area $\sigma(\chi)$ which is a function of the susceptibility of the rod.

The reason for the solenoid wound onto the rod is mainly to avoid the dependence of the response of the gyroscope from χ. In fact, in our apparatus (Fig. 2) the rf SQUID is used as a low-frequency magnetometer in the usual locked-loop operation to linearize the dependence of the output voltage to the applied flux. However, this is accomplished by feeding back the output voltage through a feedback resistor not as is usually done to the tank circuit, but instead to the solenoid wound on the magnetic rod. A current I circulating in this solenoid would induce a flux $\delta\phi_I$ at the SQUID given in the approximation of infinite length by

$$\delta\phi_I = T_0\mu_0 nI(1+\chi)s(1-s/S)(1+\chi s/S)^{-1} , \tag{21}$$

where n is the number of turns per unit length of the solenoid. In the limit of $\chi \gg 1$ the ratio $\delta\phi_\Omega/\delta\phi_I$ does not depend on χ, s, T_0 and S, providing for an instrument response independent of these parameters, as we will show below. Again Eq. (21) will be only approximately true in the actual geometry of the apparatus, where the factor $(1+\chi)s(1-s/S)(1+\chi s/S)^{-1}$ should be substituted by another effective area $\sigma'(\chi)$. This $\sigma'(\chi)$, in principle, does not coincide with $\sigma(\chi)$ mainly because the field generated by the solenoid does not have the same shape as the equivalent field $\mathbf{B}^{\text{eq}}$ due to rotation.

The small-signal open-loop gain $G(\omega)$ of the SQUID electronics shows a first cutoff at frequencies again much higher than those of the signals involved in the experiment. Therefore, calling G the zero-frequency limit of

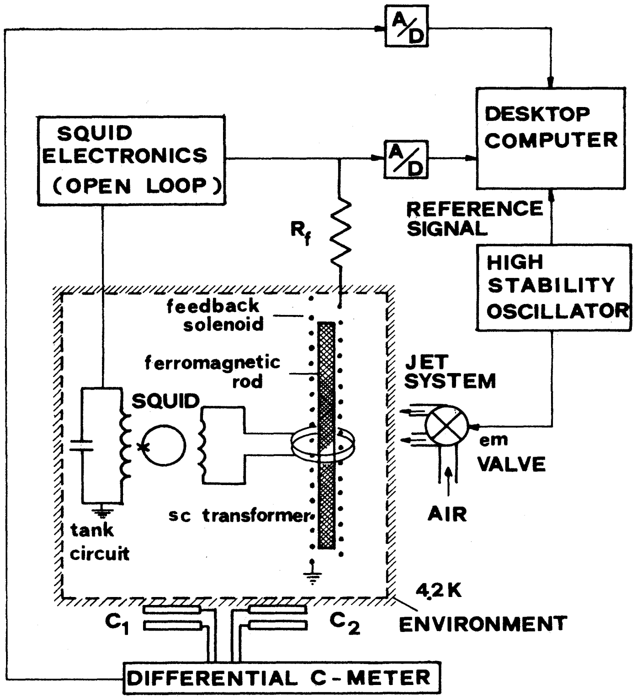


FIG. 2. Instrumentation scheme. (SC represents superconducting, em represents electromagnetic.) See text for details.

this gain, for our low-frequency signals, the Fourier transform of the output voltage $V(\omega)$ is given by $V(\omega)=G\phi_{\text{ext}}(\omega)$, with $\phi_e(\omega)$ the Fourier transform of the total external flux applied to the SQUID. Feeding this voltage into the solenoid through the resistor R_f, it will contribute to the total external flux with a feedback flux

$$\phi_f(\omega)=-T_0\mu_0 n\sigma'(\chi)V(\omega)/R_f \ .$$

The total flux will result from this feedback contribution and from the external signal $\phi_s(\omega)$ according to $\phi_{\text{ext}}(\omega)=\phi_f(\omega)+\phi_s(\omega)$. Suppose that this signal is due to a low-frequency variation of the angular velocity as in Eq. (20) and call $\Omega(\omega)$ its Fourier transform. Then the voltage output would be

$$V(\omega)=-GT_0(2m/e)(1-1/g)\sigma(\chi)\Omega_z(\omega)\times[1+GT_0\mu_0 n\sigma'(\chi)/R_f]^{-1} \ . \tag{22}$$

In the approximation of large gain, $\text{Re}[GT_0\mu_0 n\sigma'(\chi)/R_f]\gg 1$, one obtains

$$V(\omega)=-[\sigma(\chi)/\sigma'(\chi)](2m/e)\times(1-1/g)\Omega(\omega)[R_f/(n\mu_0)] \ . \tag{23}$$

In the limit of $\sigma(\chi)=\sigma'(\chi)$, as in the case of infinite length and $\chi\gg 1$, the dependence of the transfer function of the instrument on the magnetic susceptibility and on the geometry of the apparatus is canceled. This is very important because the magnetic susceptibility of ferromagnets is known to undergo slow but large changes, that would affect the calibration of the instrument. We will discuss in the next subsection the results of our real configuration.

The gyroscope was mounted into a superinsulated liquid-helium dewar so that its axis was aligned with the axis of the dewar (Fig. 1). In order to impress to the apparatus a controlled variation of its angular velocity, the dewar was suspended to the ceiling by means of a steel bar aligned with the same axis, thus making a torsion pendulum. The pendulum was set in torsional oscillations at the resonance frequency using a system of counter-directed tangential air jets controlled by an electromagnetic valve mastered by a low-frequency pulse generator (Fig. 2). The resonance frequency was changed in the range 0.2 to 0.8 Hz by replacing the torsion bar, and the resulting quality factor of the oscillator was typically ~ 500.

The angular displacement was monitored by a capacitive transducer designed to optimize the response linearity and its rejection to movements other than the torsional one. The capacitors consisted of two facing systems of plates, a lower one fixed to the floor and an upper one fixed to the bottom of the dewar. The lower system was composed of two electrically shorted plates occupying the two opposing 90° wide sectors of a 40-cm diameter circle centered on the torsion axis of the apparatus. The upper system was made of four adjacent plates each shaped in the form of an almost 90° wide circle sector. Each plate was insulated from the side neighbors and shorted to the opposite one. The difference between the capacitances of the two capacitors each resulting by the lower plates and by one couple of the upper plates, was measured by a capacitance meter.

The resulting transducer proved to be linear at better than 0.5% for angular displacements less than 60° around the equilibrium orientation. The transducer was calibrated against the angular oscillation of a laser beam reflected by a mirror mounted on the axis of rotation.

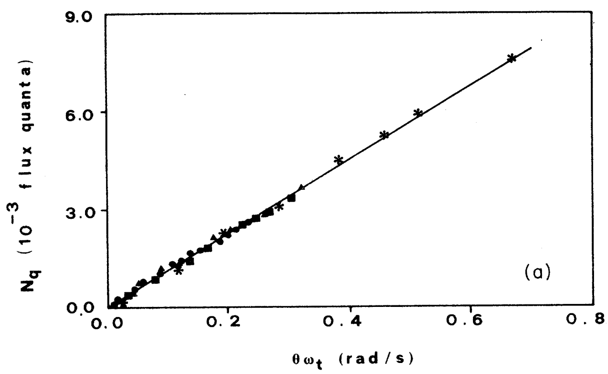
The output of the SQUID and that of the displacement transducer were sent to a data acquisition system and then transferred to a desktop computer. The in-phase and quadrature components of the SQUID signal, with respect to the angular displacement, were measured by standard numerical algorithms.

B. Results and discussion

Due to the high quality factor of the torsion pendulum, when the dewar is set in oscillation by the air-jet system, an almost monochromatic signal is obtained at the output of the angular-displacement transducer.

In Fig. 3 the Fourier components of the SQUID output at the frequency ω_t of the torsional oscillation of the dewar are reported. N_q in Fig. 3(a) is the component in quadrature with respect to the angular displacement signal. N_ϕ in Fig. 3(b) is instead the in-phase component. Both components are given in number of flux quanta and are reported as a function of the product $\omega_t\theta$ with θ the amplitude of the angular displacement signal. $\omega_t\theta$ obviously represents the amplitude of the angular velocity oscillation.

As can be seen, very good linear fits are obtained for



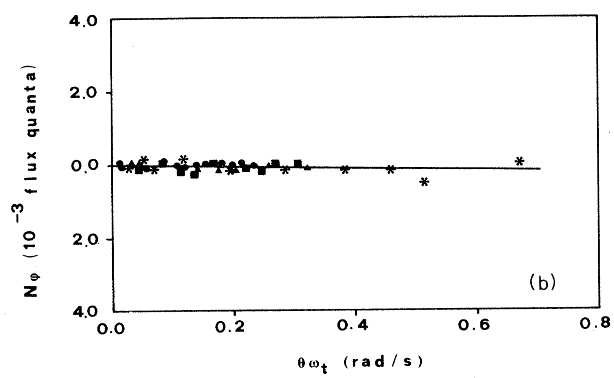


FIG. 3. Plots of (a) N_q and (b) N_ϕ vs $\omega_t\theta$ for different oscillation frequencies (see text). ●, $\omega_t=1.3$ rad/s; ■, $\omega_t=2.2$ rad/s; $*$, $\omega_t=3.3$ rad/s; ▲, $\omega_t=5.1$ rad/s.

both plots. These fits show a phase shift of $91.4°\pm0.7°$ between the SQUId output and the angular-displacement-transducer output. This is consistent with the SQUID signal being proportional to the angular velocity of the apparatus, and the barely significant shift with respect to the ideal 90° value is well accounted for by the mismatch between the electronics delays of the two signals. The sign of the effect is such that Eq. (19) is satisfied and thus the solenoid has to produce a magnetic field along the rotation axis with opposite direction with respect to the angular velocity. This and the magnitude of the effect prove that within the rotating superconducting shield a magnetic field $-(2m/e)\mathbf{\Omega}$ is present.

From the slope of the line in Fig. 3(a) and from Eq. (23), one gets

$$[\sigma(\chi)/\sigma'(\chi)](2m/e)(1/g-1) = (5.9\pm0.1)\times10^{-12}\ \mathrm{T\,s/rad}\ .$$

Since $-m/e=5.69\times10^{-12}$ T s/rad and assuming for Cryoperm10 the value $g\sim1.91$ as measured[18] for a similar alloy, Supermalloy, the effective-areas ratio turns out to be $\sigma(\chi)/\sigma'(\chi)\sim(1.05\pm0.02)$. Moreover, the residual dependence of the gyroscope response to the variations of χ was undetectable within the errors, while such a dependence was clearly seen in the former prototype of Ref. 19, in which the feedback solenoid was not used. The deviation from unity of the experimental figure of $\sigma(\chi)/\sigma'(\chi)$ includes as well the inaccuracies due to the shape of the solenoid, such as nonuniformity and misalignment. Therefore, the agreement between the above theory of the instruments response, even in the infinite-length approximation, and the experimental results has to be considered satisfactory.

The noise spectral density of the instrument can be estimated from the scatter of the points around the fit in Fig. 3(a) to be $S_\Omega=3\times10^{-2}$ rad/s$^{1/2}$. It appears to be contributed entirely by the intrinsic noise of the SQUID electronics of $\sim3.5\times10^{-4}\phi_0/\mathrm{Hz}^{1/2}$, with ϕ_0 the flux quantum. The resulting sensitivity is rather poor. However, this is due mainly to the fact that the permeability of the ferromagnetic rod turned out to be small $\chi\sim100$, and that the flux transformer ratio was not optimized[20] for this value. A more detailed discussion of the noise sources in this kind of gyroscope is given in the next section.

We rule out any significant contribution of spurious effects. In fact, a direct pickup of the ambient magnetic field by the flux transformer or by the SQUID itself would produce a signal proportional to the angular displacement and thus in-phase with it. One obvious effect that could instead simulate a signal proportional to the angular velocity of the apparatus is an inductive pickup of the ambient magnetic field in a normal circuit coupled to the SQUID such as, for instance, the rf bias line. In fact, the oscillation of the apparatus around a mean angular orientation θ_0 in the laboratory would induce an emf around the loop equal to

$$-d\phi(\theta)/dt \sim [(d\phi/d\theta)_{\theta=\theta_0}]d\theta/dt\ ,$$

where ϕ is the total magnetic flux in the loop due to the ambient field. ϕ and $d\phi/d\theta$ are obviously periodic functions of θ with period 2π. Thus if such a pickup would significantly contribute to the signal, then the slope of the line in Fig. 3(a) should depend periodically on θ_0. We performed the experiment changing the mean orientation of the apparatus in steps of 60° over one turn and we could not find any significant variation in that slope.

We also tested the attenuation of the shield by putting underneath the dewar a ring coil of 40-cm diameter that would produce at the location of the shield, but in the absence of it, a field up to 1 mT. A low-frequency alternating current was circulated into the coil in order to use the same detection procedure we used for the angular velocity experiment. The result was the detection of a signal in phase with the current and proportional to it, with a factor of proportionality independent of the frequency. Therefore, we concluded that this is, in fact, direct magnetic flux pickup by the superconducting circuitry. The resulting attenuation of the magnetic field by the shield was estimated to be $\sim10^{-10}$. Calculations show that the residual field leakage of the shield is well accounted for by a few holes of the order of 0.1 mm in diameter, such as those due to the feeds through the shield for wires and liquid helium. Improvements of the attenuation figure are possible by more careful design of the feeds, by multiple superconducting shields, and by a preshielding system with a ferromagnetic outer can. These precautions should easily improve the attenuation figure by orders of magnitude.

V. CONCLUDING REMARKS: NOISE AND PROSPECTIVE SENSITIVITY

We have recently[21] developed a model for the thermal magnetic noise in SQUID systems coupled to ferromagnetic cores. An experiment on SQUIDS coupled to high-permeability tori shows a very good agreement with the model.[21] A detailed discussion of this topic lies beyond the scope of the present paper. However, to discuss the prospective sensitivity of the gyroscope, we summarize here the principal conclusions.

A SQUID coupled to a ferromagnetic core may be thought of as possessing a frequency-dependent inductance. In fact, the magnetic permeability of ferromagnetic cores shows a variety of frequency behaviors that are reflected in similar frequency dependences of the inductance of any superconducting loop coupled to them. More specifically, the presence of the skin effect in the cores, that are also electrical conductors, sets a first cutoff, and usually the dominant one, in their effective permeability. In this case the cutoff frequency is, roughly speaking, the frequency at which the penetration depth of the magnetic field equates the thickness of the ferromagnetic material. The resulting inductance $L(\omega)$ of a coil wound on such a core will show a similar cutoff and will be then well described by a single-pole behavior like $L(\omega)=L_0(1+i\omega\tau)^{-1}+L_\infty$ with τ the cutoff time.

We have shown,[21,22] within the so-called resistively shunted Josephson junction (RSJ) model[9] and under the assumption that the frequency $1/\tau$ of this cutoff is much less than the rf pump frequency, that a SQUID coupled to a frequency dependence inductance of the form above can be modeled, by applying the usual Nyquist theory, as a noiseless device with an excess input flux noise whose spectral density in the limit of $\omega\to 0 S_\phi(0)$ is given by

$$S_\phi(0)=4k_B T L_0\tau\ , \tag{24}$$

with k_B the Boltzmann constant and T the temperature.

In the prototype of the gyroscope presented in the previous section the estimated value of $S_\phi(0)$ $\sim 10^{-8}\phi_0^2/\mathrm{Hz}^{1/2}$ turns out to be undetectable against the background noise of the SQUID electronics of $\sim 10^{-7}\phi_0^2/\mathrm{Hz}^{1/2}$.

The superconducting flux transformer that in the present version of the apparatus couples the SQUID to the ferromagnetic rod, is unnecessary and was adopted to allow an easy interchange of the rod. In fact, in a former experiment we already realized a version of the gyroscope where the rod was inserted directly into the hole of a single-hole SQUID.[19] In this case the transformer ratio T_0 in Eq. (20) may be taken equal to unity and the thermal noise would then set the signal-to-noise ratio at low frequencies to

$$(R_{\mathrm{SN}})=(2m/e)(1/g-1)\sigma(\chi)\delta\Omega(4k_B T L_0\tau)^{-1/2}\ . \tag{25}$$

The factor $(L_0\tau)^{1/2}$ in the denominator of the right-hand side is a function of the susceptibility of the rod, as it is the effective area $\sigma(\chi)$. The true values of these factors should be calculated numerically in the specific geometry adopted. Both these factors, however, have to contain a "feedback" denominator, as, for instance, the term $(1+\chi s/S)^{-1}$ in Eq. (20), representing the reaction of the superconducting shield to flux changes due to interior sources of field. A crude estimate of the ratio R_{SN} in Eq. (25) may be obtained assuming that these feedback denominators are the same for both terms and thus cancel out in the ratio R_{SN}. Insulated multilayers of amorphous ferromagnets have been reported[22] with cutoffs well above 10 MHz and still with reversible susceptibilities larger than $\sim 10^4$. If these specific magnetic properties are preserved at low temperatures, then it would be possible to anticipate, for a single rod detector, a value of R_{SN} in Eq. (25) of $\sim 10^{-7}$ rad/s$^{1/2}$ at liquid-helium temperature. This figure would compare with the sensitivity of the best lasers or mechanical gyros. Of course, configurations with N rods read in parallel would increase the signal-to-noise ratio by a factor of $N^{1/2}$ with respect to the single-rod configuration.

The realization of a device with this level of thermal noise would allow one to investigate other disturbances (mechanical and flicker noise, stray-field pickup, etc.) that could affect the instrument operation at this field of sensitivity.

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$$x' = x\cos(\Omega t) + y\sin(\Omega t) ,$$

$$y' = -x\sin(\Omega t) + y\cos(\Omega t) ,$$

and $z' = z$, taking $\boldsymbol{\Omega}$ aligned with the common z axis. The time coordinate is instead the same in both systems. Tensor quantities in this system are obviously obtained from the corresponding quantities in the inertial system, applying usual rules in the framework of general coordinate transformation.

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