Fluctuations of critical current and phase slippage in Josephson junctions

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(Received 6 January 1989)

The effect of critical current noise on the phase-slip- rate in an overdamped Josephson junction is calculated. A significant increase of the Kramers escape rate is predicted for submicron metal-insulator-metal junctions. The temperature dependence of the rate shows deviations from the Arrhenius law at low temperatures if the junction conductance fluctuations are produced by quantum ionic tunneling processes in the junction barrier. Implications for magnetization decay in high-T_c superconductors are discussed.

Metal-insulator-metal junctions of small area are known to exhibit copious amounts of low-frequency conductance noise.[1] Such tunnel junctions, when made superconducting, are expected to show considerable fluctuations of the critical current. Previous theoretical works have considered the effect of thermal fluctuations of the quasiparticle current on the dc Josephson effect.[2-4] The case of a shunted (overdamped) junction has been considered by Ambegaokar and Halperin,[3] who used the analogy with the Brownian particle in a one-dimensional potential. The quantity of current experimental importance is the transition probability of the metastable state corresponding to the escape of the Brownian particle out of the potential well. Such thermally induced transitions cause disruption of the coupling of the phase of the order parameters of two superconductors comprising the junction. This phase slippage limits the lifetime of persistent currents in superconducting rings interrupted by the junctions[5] and produces rounding of the dc current-voltage characteristics.[6] The relevant problem of the Brownian motion of a particle in one dimension in a field of force has been studied by Kramers,[7] who showed that the probability distribution in the phase space satisfies a single-variable Smoluchowski equation, which can be solved to calculate the rate of particle flow over the barrier—a quantity often called the Kramers rate of escape.

In the present work we investigate the effect of the critical current noise on the Kramers escape rate in superconducting junctions. We consider a Josephson junction of critical current I_1, shunted by a capacitance C and resistance R and driven by a constant current I. Introducing the hysteresis parameter $\beta=2\pi I_1 R^2 C/\phi_0$, where ϕ_0 is the flux quantum, we assume $\beta \ll 1$, which corresponds to an overdamped junction. We also neglect quantum fluctuations of the phase difference θ. This can be justified,[8] in spite of the small capacitance, if the shunting resistance R is well below the universal value $R_0=\hbar/e^2=4.1\ \mathrm{k}\Omega$.

Under these conditions the Langevin equation,[3,4] describing the junction in the presence of both the thermal and the critical current noise, is

$$\frac{d\theta}{dt}=L(\theta)+F(t)+G(\theta)F_1(t) , \tag{1}$$

where

$$L(\theta)=\gamma(I-I_1\sin\theta) ,\qquad G(\theta)=-\sin\theta , \tag{2}$$

and where $\gamma=2eR/\hbar$. The term $F(t)=\gamma i(t)$ represents the thermal noise source arising from normal current fluctuations $i(t)$. The last term of Eq. (1) is associated with the critical current fluctuations $i_1(t)$. The correlation functions of the noise terms are assumed in the form

$$\langle F(t_1)F(t_2)\rangle=\epsilon\delta(t_1-t_2) ,\qquad \epsilon=2k_B T\gamma^2/R , \tag{3}$$

and

$$\langle F_1(t_1)F_1(t_2)\rangle=\gamma^2\langle i_1(t_1)i_1(t_2)\rangle = \left[\frac{Q}{2\tau_1}\right]\exp(-|t_1-t_2|/\tau_1) , \tag{4}$$

where τ_1 is the correlation time of the critical current fluctuation. If $\tau_1 \ll \tau_\theta$, where τ_θ is the correlation time of the fluctuation of θ associated with the flow over the barrier, a Smoluchowski equation for the probability distribution $P(\theta,t)$ can be derived from the Langevin equation (1).[9] Because of the multiplicative noise term $G(\theta)F_1(t)$, this equation is the type of stochastic differential equations studied in connection with the problem of noise-induced phase transitions.[10] There is some controversy as to which algorithm should be applied to derive the Smoluchowski equation associated with Eq. (1).[11] Some light has been shed on this controversy by applying the adiabatic elimination procedure to the multivariable Fokker-Planck equation.[12,13] In particular, if the correlation time of the noise $F_1(t)$ is much longer than the correlation time of the velocity $v=d\theta/dt$, the Stratonovich algorithm should be applied.[13] In what follows we assume this inequality to hold for the junction problem, which implies $\tau_1 \gg RC$. Then the probability distribution $P(\theta,t)$ satisfies the equation[9,13]

$$\frac{\partial P(\theta,t)}{\partial t}=-\frac{\partial}{\partial\theta}\left[\left[L(\theta)+(Q/4)\frac{\partial G^2(\theta)}{\partial\theta}\right]P(\theta,t)\right]+\frac{\partial^2}{\partial\theta^2}\{\tfrac{1}{2}[\epsilon+QG^2(\theta)]P(\theta,t)\}\;. \qquad (5)$$

Following Kramers,[7] we calculate the escape rate r from the equation $r=j/n_1$, where j is the stationary diffusion current and n_1 is the "number of particles" (i.e., representative points of the ensemble) near the point θ_1 at which the particle is initially caught. With the use of Eq. (5) we find

$$r^{-1}=2\left[\int_{\theta_1}^{\theta_2}\exp[-H(\theta)]d\theta\right]\times\int_{-\infty}^{\infty}\frac{\exp[H''(\theta_1)(\theta-\theta_1)^2/2]}{\epsilon+QG^2(\theta)}d\theta\;, \qquad (6)$$

where

$$H(\theta)=2\int_{\theta_1}^{\theta}\frac{L(\theta)+(Q/4)\partial G^2(\theta)/\partial\theta}{\epsilon+QG^2(\theta)}d\theta\;, \qquad (7)$$

where θ_1 and θ_2 are the two neighboring minima of $H(\theta)$. The first integral in Eq. (6) is dominated by a small region near the maximum θ_m of the function $-H(\theta)$. Applying the method of steepest descent to this integral, we obtain from Eq. (7) the rate

$$r(Q)=\frac{1}{4\pi}[\epsilon+QG^2(\theta_1)][|H''(\theta_1)H''(\theta_m)|]^{1/2}\times\exp[H(\theta_m)]\;. \qquad (8)$$

Equation (8) is the main result of this work. First, consider the case of $Q=0$, which corresponds to the standard case of a junction driven by the thermal quasiparticle noise only.[3,4] In this case the values of θ_1 and θ_m are $\theta_{10}=\sin^{-1}x$ and $\theta_{m0}=\pi-\theta_{10}$, where $x=I/I_1$. Evaluating $H(\theta_m)$ and the second derivatives of $H(\theta)$ using Eq. (7), we obtain from Eq. (8)

$$r(Q=0)=\frac{1}{2\pi}I_1\gamma(1-x^2)^{1/2}\times\exp\left[\frac{I_1\hbar}{2ek_BT}[x(\pi-2\sin^{-1}x)-2(1-x^2)^{1/2}]\right] \qquad (9)$$

which exhibits the Arrhenius law, with the attempt frequency

$$\omega_a=\omega_0^2(1-x^2)^{1/2}\tau_v/(2\pi)\;,$$

where $\tau_v=RC$ is the "phase velocity" correlation time and ω_0 is the Josephson plasma frequency of the junction. The exponent in Eq. (9) is equal to $(k_BT)^{-1}$ times the potential energy difference $U(\theta_{10})-U(\theta_{m0})=-\Delta U$. This is in agreement with the Kramers rate obtained for the overdamped junctions[3,5] for $\Delta U/k_BT\gg 1$.

To estimate the modification of this rate produced by the critical current noise, we consider the case of small Q and apply the perturbation expansion in Q to Eq. (8). To first order in Q we find an enhancement of the rate given by the expression

$$r(Q)=r(Q=0)\left[1+\frac{2QI_1}{\epsilon^2}\gamma|F(x)|\right]\;, \qquad (10)$$

where

$$F(x)=[x(\pi/2-\sin^{-1}x)+\tfrac{2}{3}(1-x^2)^{3/2}-(2-x^2)(1-x^2)^{1/2}]\;. \qquad (11)$$

As x increases from zero to one, the function $|F(x)|$ decreases monotonically from $|F(0)|=1.33$ to $|F(1)|=0$. Equation (10) implies that the order of magnitude of the rate enhancement is given by the factor

$$E=\frac{2QI_1\gamma}{\epsilon^2}\;. \qquad (12)$$

It should be pointed out that this enhancement stems, to first order in Q, entirely from the modification of $H(\theta_m)$ in the exponential term in Eq. (8). The prefactor remains unaffected, due to the cancellation of the Q-proportional terms.

To exemplify the predictions of the present calculation we consider a superconducting metal-insulator-metal junction and make an estimate of the enhancement rate of Eq. (12). Clarke and Hawkins[14] have studied the critical current fluctuations in Nb-NbO_x-Sn junctions. The low-frequency power spectral density of the excess critical current has been observed to increase typically as the inverse of the frequency. According to Eq. (4) the spectral density of the critical current assumed in our calculation is a single Lorentzian and an application to the $1/f$ spectrum found in Ref. 14 is not straightforward. Also the area of these junctions is not small enough to yield sufficiently large resistance fluctuations. We thus turn to the more recent experiments on the Nb-Nb_2O_5-PbBi submicron junctions.[1] The spectral density of the resistance fluctuations $S_R(\omega)$ has been found to exhibit a interesting behavior as a function of temperature. For the junction of very small area ($A\simeq 10^{-9}$ cm^2), $S_R(\omega)$ goes roughly as $1/\omega$ in the temperature interval $80<T<300$ K, but for $T\lesssim 80$ K the spectrum can be resolved into a few Lorentzians. Moreover, for $T\lesssim 15$ K the intensity and correlation time of the resistance fluctuation are nearly temperature independent, suggesting that the resistance changes are caused by trapping processes in the barrier involving configurational tunneling of the ions. To extract the information on the resistance fluctuations needed for the estimate of the parameter Q we replace the actual measured spectrum by a single Lorentzian

$$S_R(\omega)=S_1\tau_{\text{eff}}/(1+\omega^2\tau_{\text{eff}}^2)\;, \qquad (13)$$

where S_1 and τ_{eff} are (for $T\lesssim 15$ K) temperature-independent constants. Since the critical current is inversely proportional to the normal-state resistance R_n of the junction, we obtain from Eq. (4)

$$Q=2\gamma^2\int_0^\infty \langle i_1(t)i_1(0)\rangle dt=(\gamma I_1/R_n)^2 S_R(\omega=0) . \quad (14)$$

With the use of Eqs. (13) and (14), the expression (12) becomes

$$E=\frac{1}{4}(eRR_0I_1^3)\tau_{\text{eff}}(S_1/R_n^2)(k_BT)^{-2} . \quad (15)$$

In this expression, R is the effective shunt resistance, including the normal-state junction resistance and the external shunt. For the Nb-Nb_2O_5-PbBi junction of area $A\simeq10^{-9}$ cm^2, the normal-state resistance $R_n=110$ Ω. We assume $T=1$ K and use the low-temperature resistance noise data of Ref. 1 from which $(S_1/R_n^2)\simeq10^{-7}$, $\tau_{\text{eff}}\simeq10^{-5}$ sec. Allowing the critical current density to be 10^3 A/cm^2 the critical current is $I_1\simeq10^{-6}$ A. With the shunt resistance $R=1$ Ω, the hysteresis parameter $\beta\simeq3\times10^{-4}$, and the junction is well overdamped. Taking $C=10^{-13}$ F, the correlation time $\tau_v=RC=10^{-13}$ sec. The phase correlation time $\tau_\theta\simeq r^{-1}$ is about 20 sec for $Q=0$ and $x=0$. Hence the criteria for the validity of the Eq. (5) are well satisfied. With the above values Eq. (15) yields an enhancement $E=0.8$. Even larger values of E are obtained for lower temperatures or by increasing the shunt resistance. The perturbation expansion, used to derive Eq. (10), however, puts a limit on the ratio Q/ϵ, which must remain well below one. For the parameters used above, we have $Q/\epsilon\simeq3.6\times10^{-2}$, which justifies the first-order perturbation result. There is, however, no apparent physical reason, which would limit the rate $r(Q)$ given in Eq. (8), for larger values of Q/ϵ, when the perturbation expansion breaks down. In fact, dramatic increase of the Kramers escape rate has been predicted by Faetti *et al.*[15] in their study of an analogous problem of a Brownian particle in a double-well potential in the presence of multiplicative noise of large intensity.

The temperature dependence of $r(Q)$ is also of some interest, as it may serve as a diagnostics of the predicted effects of the critical current noise. If the fluctuations of the barrier are due to thermally activated processes, the noise intensity Q is proportional to the temperature, which according to Eq. (10), produces an effective decrease of the activation energy but preserves the temperature dependence given by expression (9). On the other hand, if the barrier fluctuations are due to quantum tunneling of the ions (as shown for low temperatures in Ref. 1), Q is temperature independent and the escape rate is larger for low temperatures, than predicted by the Arrhenius law. If the transition temperature of the superconductors in the junction is sufficiently high, the crossover from the Arrhenius to a weaker temperature dependence, due to quantum dominated barrier fluctuations, may be observable. These effects may play some role in the decay of the magnetization of superconducting glasses, in which superconducting grains are weakly coupled into closed loops.[16] Motta *et al.*[17] have measured the relaxation of the magnetization in high-T_c superconductors. In the Sr-La-Cu-O powder, the logarithmic rate of the magnetization is approximately linear with temperatures for $2<T<10$ K, but it tends to a much weaker T dependence for $T<1$ K. If quantum tunneling processes are involved in the tunneling barriers and weak links form the superconducting loops, the observed crossover may be qualitatively understood by taking into account the decay of the metastable current states induced by the fluctuations of the critical current.

In summary, we calculated the rate of the phase slip in an overdamped Josephson junction in the presence of the critical current noise. The rate is calculated by applying the high barrier approximation to the Smoluchowski equation derived with the use of the Stratonovich algorithm. For submicron metal-insulator-metal junctions a significant increase over the standard Kramers rate is predicted. Critical current noise with temperature-independent intensity produces deviations of the T dependence of the rate from the Arrhenius law at low temperatures. It is argued that the proposed phase-slip mechanism plays a role in the magnetization decay in superconducting glasses. The low-temperature anomaly of the logarithmic decay rate observed recently in Sr-La-Cu-O powder is attributed to the critical current fluctuations of the weak links.

ACKNOWLEDGMENTS

The author would like to thank Professor K. A. Müller for stimulating correspondence on the magnetization decay in high-T_c oxides.

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