Low-field transport coefficients in $GaAs/Ga_{1-x}Al_xAs$ heterostructures

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We have analyzed the current theories of the conductivity of a two-dimensional electron gas in low magnetic fields to determine the correct usage of the transport and quantum lifetimes. The analysis is shown to correctly predict the low-field amplitudes and phases of the de Haas–Shubnikov oscillation in both the longitudinal and Hall resistivities of two samples. Deviations at high fields are attributed to localization.

I. INTRODUCTION

In a two-dimensional (2D) electron gas both the diagonal and off-diagonal elements of the resistivity (or conductivity) tensor oscillate as a function of magnetic field with a periodicity determined by the electron density. It is observed experimentally, at least near the quantum Hall regime, that the two sets of oscillations are in quadrature. However, the theoretical treatments of Ando,[1] Ando, Matsumoto, and Uemura,[2] and Isihara and Smrčka,[3] valid for low fields, predict that the two terms will oscillate in antiphase. It has not been clear whether (i) there is a transition from quadrature behavior at high fields to a low-field regime where the oscillations are in antiphase and agree with theory or (ii) the discrepancy persists to low fields and is a consequence of invalid assumptions in the theories. If *deviations* from these theories are to be considered, e.g., when investigating different kinds of scattering or considering quasi-one-dimensional systems, it is important to establish whether they are valid and when.

The theories consider only one lifetime, but in practice it is important to distinguish between the quantum lifetime τ_q which is given by the total scattering rate and the transport lifetime τ_ρ which is weighted by the scattering angle θ, i.e.,

$$\frac{1}{\tau_q}=\int P(\theta)d\Omega \ , \tag{1}$$

$$\frac{1}{\tau_\rho}=\int P(\theta)(1-\cos\theta)d\Omega \ . \tag{2}$$

For short-range potentials these two lifetimes are identical, but for $GaAs/Ga_{1-x}Al_xAs$ heterostructures, where the dominant scattering mechanism is usually the long-range potential associated with donors which are set back from the 2D electron gas and which produce predominantly small-angle scattering, they typically differ by a factor of 10 or more (e.g., Refs. 4 and 5). A number of *ad hoc* assumptions have been introduced into the literature when experimental results involving two lifetimes are compared with theories that consider only one. As we will demonstrate, an examination of the theories shows that there is a correct and consistent way of treating the two lifetimes and we present experimental results which confirm that the theories are valid for low fields.

II. ANALYSIS

In a magnetic field the density of states g acquires an oscillatory component Δg which, at low fields, can be written[3]

$$\frac{\Delta g(\varepsilon)}{g_0}=2\sum_{s=1}^{\infty}\exp(-\pi s/\omega_c\tau_q)\cos\left[\frac{2\pi s\varepsilon}{\hbar\omega_c}-s\pi\right] , \tag{3}$$

where g_0 is the zero-field density of states, ω_c is the cyclotron frequency $|e|B/m^*$, and ε is the electron energy. The above expression assumes that the Landau levels are broadened and each can be represented by a Lorentzian with a width Γ independent of energy or magnetic field, such that $\tau_q=\hbar/2\Gamma$. Because of the oscillations in g, corresponding oscillatory components develop in all the physical properties and in particular in the conductivity σ. Isihara and Smrčka[3] show that these can be written

$$\sigma_{xx}=\frac{e^2}{m^*}\frac{\tau}{(1+\omega_c^2\tau^2)}N_{\rm eff} \ , \tag{4}$$

$$\sigma_{xy}=e\frac{\partial N}{\partial B}-\omega_c\tau\sigma_{xx} \ , \tag{5}$$

where $N_{\rm eff}$, the effective number of electrons participating in the transport, and N, the number of states below the Fermi energy, are obtained in terms of the Green function of the system of electrons and impurities.

In the particular theoretical formulations[1-3] leading to Eqs. (4) and (5), a short-range scattering potential was used: with this assumption the τ appearing in these equations becomes identical with the τ_q in Eq. (3) and leads to the difficulties outlined in the Introduction. However, Eqs. (4) and (5) must reduce to the standard semiclassical results in zero field. In this case, $N_{\rm eff}$ is just N_0 the density of electrons, $\partial N/\partial B=0$, and τ is the *transport* lifetime.

Because of this equivalence we identify τ in Eqs. (4) and (5) with τ_ρ, the field-dependent transport lifetime. In the low-field regime, Eqs. (4) and (5) can be evaluated using the Boltzmann equation but with an oscillatory part to the density of states given by Eq. (3). Both $N_{\rm eff}$ and $1/\tau_\rho$ are proportional to the density of states at the Fermi energy ε_F, so

$$N_{\rm eff}=N_0\left[1+\frac{\Delta g(\varepsilon_F)}{g_0}\right], \tag{6}$$

$$N_{\rm eff}\tau_\rho=N_0\tau_0 , \tag{7}$$

where τ_0 is the zero-field value of τ_ρ. It should be noted, in connection with the discussion below, that these two equations do *not* require Δg to be small. We also have

$$\begin{aligned}\frac{\partial N}{\partial B}&=\frac{\partial}{\partial B}\int_{-\infty}^{\varepsilon_F}\Delta g(\varepsilon)d\varepsilon\\&=\frac{-2g_0\varepsilon_F}{B}\sum_s \exp(-\pi s/\omega_c\tau_q)\cos\left[\frac{2\pi s\varepsilon_F}{\hbar\omega_c}-s\pi\right]\\&=\frac{eN_0\,\Delta g(\varepsilon_F)}{m^*\omega_c g_0} ,\end{aligned} \tag{8}$$

where we have discarded two small terms by using $\varepsilon_F \gg \hbar/\tau_q$ and $\varepsilon_F \gg \hbar\omega_c$. Equations (6)–(8) are the same as those obtained by Isihara and Smrčka.[3] Inserting these equations into Eqs. (4) and (5) and assuming $\Delta g(\varepsilon_F)/g_0$ is small leads immediately to the results[3]

$$\sigma_{xx}=\frac{\sigma_0}{1+\omega_c^2\tau_0^2}\left[1+\frac{2\omega_c^2\tau_0^2}{1+\omega_c^2\tau_0^2}\frac{\Delta g(\varepsilon_F)}{g_0}\right], \tag{9}$$

$$\sigma_{xy}=-\frac{\sigma_0\omega_c\tau_0}{1+\omega_c^2\tau_0^2}\left[1-\frac{3\omega_c^2\tau_0^2+1}{\omega_c^2\tau_0^2(1+\omega_c^2\tau_0^2)}\frac{\Delta g(\varepsilon_F)}{g_0}\right], \tag{10}$$

where σ_0 is the zero-field value of σ_{xx}. We stress that everywhere in these equations τ_0 is the transport lifetime at zero field and that τ_q appears only through Δg. Equivalent results were first obtained by Ando,[1] and Ando and Uemura,[2] using different methods compared to those of Isihara and Smrčka[3] outlined above. Finally, Eqs. (9) and (10) can be readily inverted to give simple expressions for the resistivity ρ:

$$\rho_{xx}=\frac{1}{\sigma_0}\left[1+2\frac{\Delta g(\varepsilon_F)}{g_0}\right], \tag{11}$$

$$\rho_{xy}=\frac{\omega_c\tau_0}{\sigma_0}\left[1-\frac{1}{\omega_c^2\tau_0^2}\frac{\Delta g(\varepsilon_F)}{g_0}\right]. \tag{12}$$

In their present form all these equations are valid only for $T=0$. At finite T the oscillations in the conductivities and resistivities will be damped and this can be taken into account in the usual manner by multiplying the oscillatory terms by $D(sX)=sX/\sinh(sX)$, where $X=2\pi^2kT/\hbar\omega_c$. It is important to note that Eqs. (9)–(12) rely on $\Delta g/g_0 \ll 1$ but there is no restriction on $\omega_c\tau_0$. As noted above, the two resistivities are predicted to oscillate in antiphase whereas they are usually observed to be in quadrature. Our interpretation of the various τ as they appear in the above theory is based on a plausibility argument and is clearly not rigorous, but we will show that it is consistent with the observed behavior.

III. EXPERIMENTAL RESULTS

Two specimens were used for the experiments. Sample 1 had $N_0 \sim 8.8\times10^{11}$ cm^{-2} and was a conventional GaAs/Al$_x$Ga$_{1-x}$As ($x \approx 0.3$) heterostructure with relatively low mobility (81 000 cm^2 V^{-1} s^{-1}) because it had an undoped spacer layer only 17 Å thick. Sample 2 had $N_0 \sim 9.1\times10^{11}$ cm^{-2} and was an inverted heterostructure, i.e., GaAs on top of Ga$_{1-x}$Al$_x$As, that was purposely grown without any attempt to reduce interface roughness. The mobility of 9700 cm^2 V^{-1} s^{-1} was presumably dominated by interface roughness scattering. We chose low-mobility samples so that at 1.2 K, the temperature of the measurements, the amplitude of the quantum oscillations was dominated by impurity broadening of the Landau levels [i.e., $\exp(-s\pi/\omega_c\tau_q)$ in Eq. (3)] and we were able to correct for the thermal broadening term $D(sX)$ with little error being introduced. At the same time we were able to investigate two samples where the ratios of τ_0/τ_q were quite different.

In both cases a Hall bar geometry was fabricated and measurements made using phase-sensitive detection at 85 Hz. Magnetic fields were determined from the current through the superconducting magnet with a small correction for current flowing through the (normal) superconducting switch. Phase measurements are particularly sensitive to small field errors so in all cases the data were averaged for field sweeps up and down. After subtraction of a linear background term, filtering and smoothing were done with a software digital filter.[6] Not only could the frequency of the filter be precisely chosen to pass only the desired fundamental signal, but also, unlike analog filters, there were no filter-induced phase lags. Because of the filter, the measured quantity is the fundamental component of the resistivity oscillations (i.e., $s=1$), not the peak values.

An examination of Eqs. (11) and (12) shows that the oscillatory parts of the resistivities, say, $\Delta\rho$, should be related by

$$\frac{1}{2}\frac{\Delta\rho_{xx}}{\rho_0}=-\omega_c\tau_0\frac{\Delta\rho_{xy}}{\rho_0}=D(X)\frac{\Delta g(\varepsilon_F)}{g_0} , \tag{13}$$

where $\rho_0=1/\sigma_0$. With these relations in mind we have used our data at each oscillation extremum to calculate the magnitudes of the quantities $[1/2D(X)]\Delta\rho_{xx}/\rho_0$ and $[\omega_c\tau_0/D(X)]\Delta\rho_{xy}/\rho_0$, with m^* taken to be $0.068m_e$, and these are shown in Figs. 1(a) and 1(b) as a function of $1/B$. Equations (3) and (13) indicate that these reduced resistivities should be linear in $1/B$ with an intercept of 2 as $1/B\rightarrow0$ and slopes given by $-\pi m^*/|e|\tau_q$. It can be seen that $\Delta\rho_{xx}$ follows this behavior up to fields for which $\Delta\rho_{xx}\sim\rho_0$, i.e., until ρ_{xx} oscillates close to zero and the quantum Hall steps are well defined. When these $\Delta\rho_{xx}$ plots are used to obtain τ_q it is found that $\tau_0/\tau_q=9.1$ for sample 1 and 3.9 for sample 2. In the case of $\Delta\rho_{xy}$, we obtain agreement with theory only at low fields. It is important to note that the data for $\Delta\rho_{xx}$ and $\Delta\rho_{xy}$ will only coincide at low fields if, in Eq. (13), τ_0 is the transport lifetime, i.e., not τ_q. The agreement that we obtain in this regard, and the intercept of the $\Delta\rho_{xx}$ data at 2, both indicate that we have correctly identified the relevant τ to be used in the equations. We also note that the deviations of $\Delta\rho_{xy}$ from the theoretical predictions occur in both samples at $\omega_c\tau_q \sim 0.5$ although the values of $\omega_c\tau_0$ are quite different in the two cases (approximately 5 for sample 1 and 2 for sample 2). For comparison, $\Delta\rho_{xx}$ follows Eq. (11) until $\omega_c\tau_q \sim 1$.

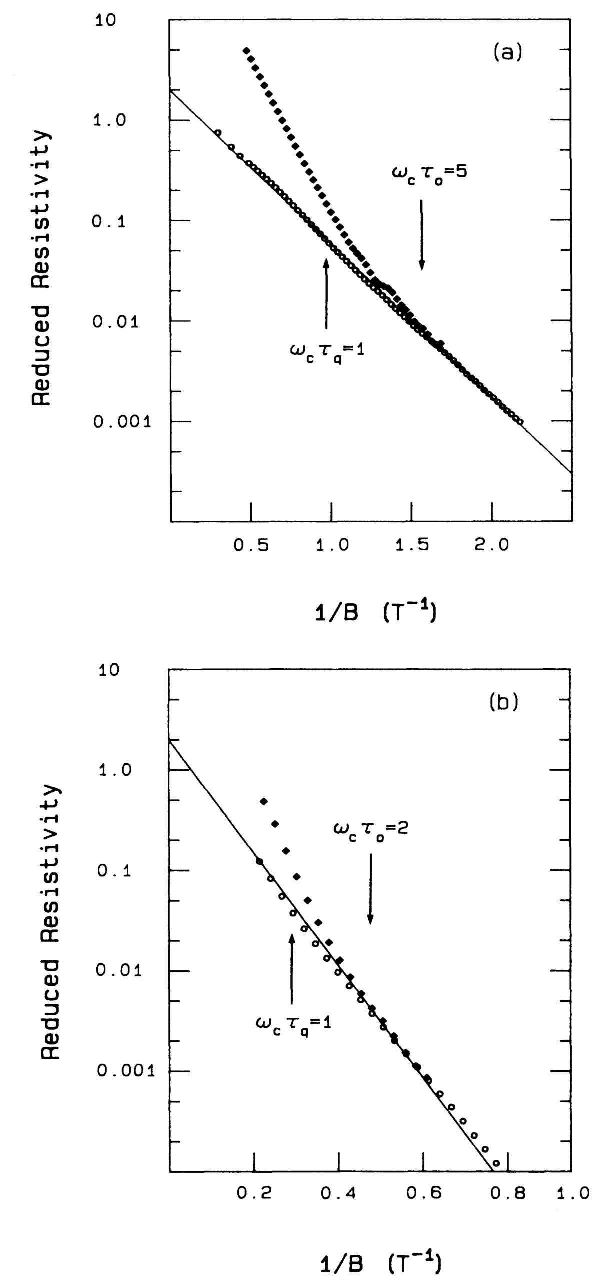

FIG. 1. (a) Reduced resistivities for specimen 1 plotted against reciprocal field. The quantities plotted are $\Delta\rho_{xx}/2D(X)\rho_0$ (open circles) and $\omega_c\tau_0\Delta\rho_{xy}/D(X)\rho_0$ (solid diamonds) where $\Delta\rho$ is the amplitude of the oscillations, ρ_0 is the zero field value of ρ_{xx}, and $D(X)$ is the temperature correction term $[X/\sinh(X)]$ as defined in the text. The straight line through 2 at $1/B=0$ is the one-parameter fit used to obtain a value of τ_q. (b) Similar plot for specimen 2.

Similar behavior has been observed in many samples but the reduced $\Delta\rho_{xx}$ data are not always so linear and sometimes do not appear to extrapolate to the value of 2. We suspect these problems are caused by non-Lorentzian broadening of the Landau levels. However, in all cases the Hall term is larger than the diagonal term but approaches it as the field is reduced. When $\omega_c\tau_q < 0.5$ then the Hall term is always either close to the diagonal term or the signals are no longer visible.

We now turn to the phases of the oscillation. From Eq. (3) the phase in cycles is $s(\varepsilon/\hbar\omega_c-\frac{1}{2})$. For the fundamental we write the experimental value (in cycles) as $\phi=(F/B+\phi_0)$ where the frequency F is $N_0h/2e$. The total phase ϕ was determined at each oscillation extremum but it is convenient to plot $\phi-F_t/B$ as a function of $1/B$, where F_t is a trial frequency chosen to be close to the correct value of F. If ϕ_0 is independent of field, the plot should be a straight line of slope $F-F_t$ with an intercept at $1/B=0$ of ϕ_0. Figures 2(a) and 2(b) show such plots for the two specimens. At low fields the accuracy is limited by noise, which is worse for the Hall signals because not only are they experimentally weaker but they must also be separated from the large linear background; at high fields it is limited mainly by the uncertainties involved in separating long-period oscillating terms with a rapidly varying amplitude from the monotonic background. Small errors in the field affect both channels equally, so the difference in phase between the two terms is expected to be more accurate than the absolute phase. For both samples the phase of $\Delta\rho_{xx}$ is 0.5, within experimental error, as expected from Eq. (3), but for $\Delta\rho_{xy}$ it moves from a high-field value of 0.75 towards the predicted value of 1 at low fields. This shift occurs for both samples at $\omega_c\tau_q \sim 0.5$ and so mirrors the behavior of the amplitude already discussed.

IV. DISCUSSION

The main point that emerges from the analysis is that Eqs. (11) and (12) are both valid at sufficiently low fields. The latter equation appears to have a much more limited range of validity than the former, and the main purpose of this discussion will be to examine why this might be so. We notice that the only term producing oscillations in σ_{xx} is $\omega_c^2\tau_p^2$ in the denominator of Eq. (4). As we have already noted, both $N_{\rm eff}$ and $1/\tau_p$ are proportional to the density of states at the Fermi level and Eq. (7) should be valid even when $\Delta g/g_0$ is not small. Hence, if we write $1/\tau_p=1/\tau_0+\Delta(1/\tau)\sim g_0+\Delta g$ and assume $\omega_c^2\tau_p^2 >> 1$, then Eq. (4) yields, for the difference between an adjacent maximum and minimum of σ_{xx},

$$2\,\Delta\sigma_{xx}=\frac{\sigma_0}{\omega_c^2}\left\{\left[\frac{1}{\tau_0}+\Delta\left[\frac{1}{\tau}\right]\right]^2-\left[\frac{1}{\tau_0}-\Delta\left[\frac{1}{\tau}\right]\right]^2\right\}$$

$$=4\frac{\sigma_0}{\omega_c^2\tau_0^2}\frac{\Delta g}{g_0}\ . \tag{14}$$

This is the same result as from Eq. (9) in the same limit. Furthermore, when $\Delta\rho_{xx}$ is obtained by inversion, the oscillatory contribution from σ_{xy} is smaller by a factor $1/\omega_c^2\tau_0^2$ than that from σ_{xx}. This argument indicates that within the semiclassical approximation and neglecting localization, Eqs. (9) and (11) should remain accurate even for large $\Delta g/g_0$. In other words, these equations might be valid not only for low fields ($\Delta g/g_0 \ll 1$) but also at high fields ($\omega_c\tau_q \sim 1 \sim \Delta g/g_0$), and hence they might be good approximations at intermediate fields, as is observed in Figs. 1(a) and 1(b).

By contrast the Hall terms are not so simple. If the same approach is used, then for $\omega_c\tau_0 \gg 1$, the oscillating term in Eq. (10) is the difference between two terms, each of which is larger by a factor of order $\omega_c^2\tau_0^2$. There is a further cancellation of large terms when ρ_{xy} is determined by inversion. This suggests that any discrepancy between theory and experiment of a factor of two may not be as serious as it initially appears in Figs. 1(a) and 1(b).

However, because the deviations in both specimens occur at the same value of $\omega_c\tau_q$, but different values of $\omega_c\tau_0$, and because they are correlated with the change of phase we feel a more fundamental explanation is indicated, and it is more likely that they are associated with localized states between the Landau levels. As the field increases, the levels separate and the effect of the localized states becomes progressively more important. On the quantum Hall steps, where the Fermi level lies in the localized states, the phase of ρ_{xy} is 0.75 so the shift of phase from 1.0 at low fields to 0.75 at high fields should be considered as a signature of localization. This is expressed quite clearly in a field theoretical treatment[7] where expressions are obtained showing how the bare conductances σ^0 are renormalized by localization, i.e.,

$$\sigma_{xx}=\sigma_{xx}^0+C_0+\sum_{n=1}^{\infty}C_n\cos(2\pi n\sigma_{xy}^0 h/e^2)\ , \tag{15}$$

$$\sigma_{xy}=\sigma_{xy}^0+\sum_{n=1}^{\infty}\theta_n\sin(2\pi n\sigma_{xy}^0 h/e^2)\ . \tag{16}$$

The coefficients C_n and θ_n depend on the bare conductance σ_{xx}^0 through a factor $\exp(-4\pi n\sigma_{xx}^0 h/e^2)$. When the field is low, these coefficients are small and the oscillatory terms are determined by σ_{xx}^0 and σ_{xy}^0. At higher fields there is a shift to the right-hand terms in Eqs. (15) and (16), which in the case of σ_{xy} results in a shift of phase. (For $\omega_c\tau_0>1$, $\sigma_{xy}^0\simeq N_0e/B$, so $2\pi\sigma_{xy}^0h/e^2 \simeq 2\pi F/B$. Notice that the theory ignores spin, so $F=N_0h/e$.) This shows quite explicitly that as a result of localization the shift in phase occurs only in σ_{xy}. It is also of interest that these theoretical results are consistent with σ_{xx} and ρ_{xx} showing relatively minor deviations from Eqs. (9) and (11). A feature of the theory is the occurrence of *two* sets of fixed points, at the quantum Hall plateaux and between the plateaux, i.e., at both the minima and maxima of σ_{xx} and ρ_{xx} which are exactly the points where the amplitude of the oscillation is evaluated experimentally. For the minima, σ_{xx} must remain positive so the renormalized values cannot move too far from the bare values. At the maxima, the coefficients C_n have their smallest values so the renormalization will be small here also. Hence it is not unreasonable that the semiclassical expressions might provide a good description of the experimental data on ρ_{xx} over a wide field range.

Finally it may be useful to briefly examine the errors

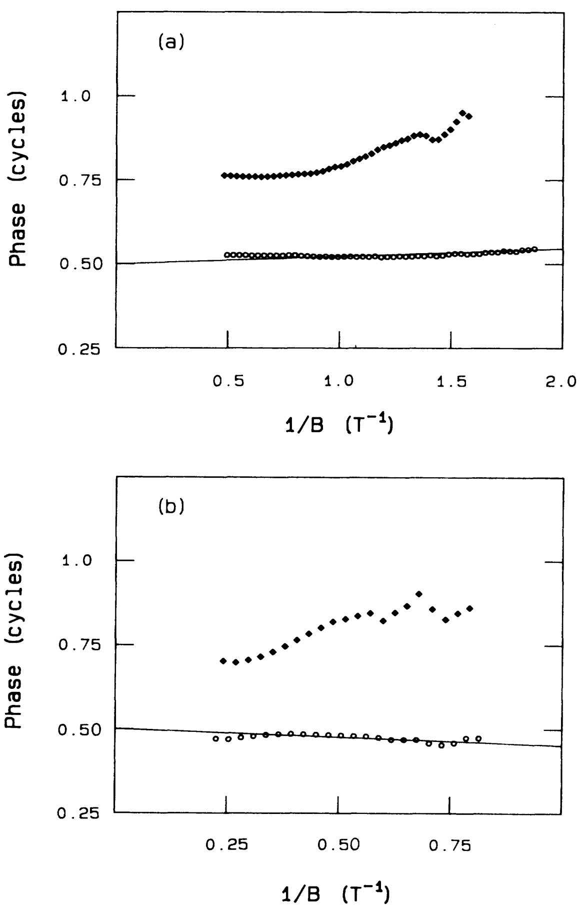

FIG. 2. (a) Phase ($\phi-F_t/B$) in cycles for the oscillating part of the resistivities in specimen 1 plotted against reciprocal field for $\Delta\rho_{xx}$ (open circles) and $\Delta\rho_{xy}$ (solid diamonds). The trial frequency F_t is 18.13 T and the straight line is a single-parameter fit giving a correction frequency of 0.02 T. The resultant frequency (18.15 T) corresponds to a density of 8.78×10^{11} cm^{-2}. The difference in phase of $\frac{1}{4}$ cycle at high fields corresponds to the two sets of oscillations being in quadrature and a difference of $\frac{1}{2}$ a cycle at low fields to antiphase. (b) Similar plot for specimen 2. In this case the trial frequency is 18.8 T and the corrected frequency of 18.75 T corresponds to a density of 9.07×10^{11} cm^{-2}.

introduced by the assumption, often used in the literature, that the τ_0 which appears in Eqs. (9) and (10) should be replaced by τ_q when fitting experimental data. If the experiments are done in the regime $\omega_c \tau_0 \gg 1$, as is generally the case, the values of $1/\tau_q$ derived in this way will be in error by a term $\sim d[\ln(1+x^2)]/dx$ with $x = 1/\omega_c \tau_q$. For typical values of $x \sim 1$, the measured τ_q will be in error by a factor of about $(\pi - 1)/\pi$. In practice we do not observe any significant deviation from linearity in the relevant plots and we do indeed find the values of $1/\tau_q$ to be about 30% too low with intercepts at $1/B \rightarrow 0$ that are significantly changed.

V. CONCLUSIONS

We have argued that the quantum lifetime appears only in the broadening of the Landau levels and that the lifetime which appears in the expressions for the conductivity is the transport lifetime at zero field. With this interpretation, the calculation of ρ_{xx} correctly predicts both the absolute magnitude and phase of the oscillations. It also appears, semiempirically but perhaps not unexpectedly, that ρ_{xx} accurately reflects the density of states at the Fermi level over a wide range of field. The oscillations in ρ_{xy} are more complex and tend to the low-field theoretical behavior only in the limit of very small oscillations in the density of states, a regime which is difficult to probe experimentally; it is likely that the deviations from theory might be a useful probe of the development of the localized states between Landau levels.

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[1]T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).

[2]T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. **39**, 279 (1975).

[3]A. Isihara and L. Smrčka, J. Phys. C **19**, 6777 (1986).

[4]J. P. Harrang, R. J. Higgins, R. K. Goodall, P. R. Jay, M. Laviron, and P. Delescluse, Phys. Rev. B **32**, 8126 (1985).

[5]S. Das Sarma and F. Stern, Phys. Rev. B **32**, 8442 (1985).

[6]J. F. Kaiser and W. A. Reed, Rev. Sci. Instrum. **49**, 1103 (1978).

[7]A. M. M. Pruisken, in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987), p. 117.