Variational calculation of two-dimensional-polaron energy levels in a magnetic field

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Energy levels of two-dimensional Fröhlich polarons in a magnetic field are calculated variationally by use of a generalization of the Lee-Low-Pines ansatz for the variational wave function allowing the phonon distribution function to depend on the electronic coordinates. It is shown that this choice leads to the correct interpolation between the weak- and strong-field limits. Comparing our ground-state energies with those obtained within second-order perturbation theory and the path-integral method, we argue that neither of these methods gives a satisfactory description of the dynamics of the electron-phonon interaction under the influence of an external field.

I. INTRODUCTION

Polarons have recently gained revitalizing interest because of the ability to form two-dimensional (2D) electron gases in semiconducting heterostructures. In particular, the polaronic effects in the presence of a magnetic field plays a crucial role in the interpretation of experimental data such as cyclotron resonances. From the experimental point of view, 2D polarons have been observed in several structures. Some examples are GaAs-$Ga_{1-x}Al_xAs$ heterojunctions[1] and metal-oxide-semiconductor (MOS) structures.[2]

As far as theoretical calculations of polaron Landau levels are concerned, there has been great interest for the perturbative approach because of the possibility to give exact closed-form expressions for the second-order perturbation-theory shifts.[3–5] Corrections up to the fourth order have been calculated numerically.[6] Moreover, ground-state energies have also been calculated within the path-integral method,[7,8] although there exists no mathematical proof that in the presence of a magnetic field the calculated ground-state energy is an upper bound to the exact value.

When a polaron is placed in a magnetic field we can immediately consider two opposite limits comparing the cyclotron frequency $\omega_c = eB/mc$ and the longitudinal optical-phonon frequency ω_L. For simplicity we introduce the dimensionless magnetic field $\lambda = \omega_c/\omega_L$ and use as units of length and energy the polaron radius $R = (\hbar/2m\omega_L)^{1/2}$ and the phonon quantum $\hbar\omega_L$, respectively. In the weak field limit $\lambda \ll 1$ the ground-state energy is given by[3,4]

$$E = -\frac{\alpha\pi}{2} + \frac{\lambda}{2}\left[1 - \frac{\alpha\pi}{8}\right], \tag{1}$$

where α is the Fröhlich coupling constant. The above equation is a well-known result of the second-order perturbation theory. Since $\lambda \ll 1$, the orbit of the Landau level is greater than the polaron radius and hence the polarization field can follow the electron motion. In fact, in Eq. (1) we can see that the ground-state energy is given by the sum of the polaron self-energy $(-\alpha\pi/2)$ and of the Landau energy $(\lambda/2)$ corrected by the polaron effective mass $(1-\alpha\pi/8)$. Weak-field formulas for any Landau level have been given in the literature.[4–6]

On increasing the magnetic field the orbit of the Landau level becomes comparable and eventually smaller than the polaron radius. There are two high-field limits: $\lambda \to \infty$ with $\alpha\sqrt{\lambda} \to 0$ and $\lambda \to \infty$ with $\alpha\sqrt{\lambda} \to \infty$. In the first case $(\lambda \to \infty, \alpha\sqrt{\lambda} \to 0)$ the second-order perturbation theory ground-state energy is[3]

$$E \to \frac{\lambda}{2} - \frac{\alpha}{2}\sqrt{\pi\lambda}\ . \tag{2}$$

In the second case $(\lambda \to \infty, \alpha\sqrt{\lambda} \to \infty)$ the polarization field can no longer follow the electron motion; the effective electron-phonon interaction reduces to that between phonons and a static distribution of charge and the electronic and phonon wave functions are decoupled. The ground-state energy is given by[9]

$$E = \frac{\lambda}{2} - \frac{\alpha}{2}\sqrt{\lambda\pi/2}\ . \tag{3}$$

Larsen[9] has observed that neither the second-order perturbative ground-state energy nor that calculated by Xiaoguang, Peeters, and Devreese[8] within the path-integral method are able to account for the continuous change of the nature of the interaction going from $\alpha\sqrt{\lambda} \to 0$ to $\alpha\sqrt{\lambda} \to \infty$. In fact, for the case $\alpha \ll 1$ he has calculated the fourth-order perturbative correction to the ground-state energy showing that this correction goes positive for strong field and that the true ground-state energy exceeds both second-order perturbative results and those obtained with the path-integral method.

A theory which is capable of correctly describing the continuous change of the effective electron-phonon interaction from weak to strong fields with α constant can be based on a variational scheme. This is the problem addressed here. The variational calculation presented in this paper has two main features. First, the conservation of the total angular momentum[10] in the direction of the magnetic field is explicitly exploited introducing bosonic variables symmetry adapted to the axial symmetry of the

problem. This has the advantage of simplifying the mathematics and gives the possibility of studying the excited states with definitive values of the total angular momentum. Second, a Lee-Low-Pines ansatz for the trial wave function is introduced in which the phonon distribution function depends on the electronic coordinates. This choice, already made in the theory of polaronic excitons[11] and surface polarons,[12] has the desirable feature of giving a satisfactory description of the dynamics of the electron-phonon interaction under the influence of an external field.

The plan of the paper is as follows. In Sec. II we define the Hamiltonian and the transformation to the angular variables, in Sec. III we present our variational scheme, and in Sec. IV we give the numerical results and our conclusions.

II. THE HAMILTONIAN

The model Hamiltonian that we use for describing the motion of a 2D polaron under the influence of a static and uniform magnetic field oriented in the z direction reads

$$H=P_\rho^2+(P_\phi+\tfrac{1}{4}\lambda\rho)^2+\sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}+\sum_{\mathbf{k}}(V_k e^{i\mathbf{k}\cdot\boldsymbol{\rho}}a_{\mathbf{k}}+\text{H.c.}) , \tag{4}$$

where $P_\phi=-(i/\rho)\partial/\partial\phi$, $P_\rho=-i(\partial/\partial_\rho)$, $\lambda=\omega_c/\omega_s$, $V_k=-i(2\pi\alpha/kS)^{1/2}$, $\boldsymbol{\rho}$ and $\mathbf{k}$ are the two-dimensional coordinate and wave vector, and S is the normalization area. From Eq. (4) it can be seen that we adopt a strictly two-dimensional approximation in the sense that we neglect the finite extension of the electronic wave function in the z direction.

The electronic part of the Hamiltonian in Eq. (4) is written in polar coordinates in the plane perpendicular to the field. In order to take full advantage of the axial symmetry, it is convenient to rewrite the phonon terms in an alternative form by transforming canonically the boson operators $a_{\mathbf{k}}$ into the angular momentum representation

$$a_{k,n}=\sqrt{k}\left[\frac{S}{(2\pi)^2}\right]^{1/2}\int_0^{2\pi}\Theta_n^*(\phi_k)a_{\mathbf{k}}\,d\phi_k , \tag{5}$$

where

$$\Theta_n(\phi)=\frac{1}{\sqrt{2\pi}}e^{in\phi} . \tag{6}$$

Using Eq. (5) and the expansion

$$e^{i\mathbf{k}\cdot\boldsymbol{\rho}}=2\pi\sum_n i^n J_n(k\rho)\Theta_n(\phi)\Theta_n^*(\phi_k) , \tag{7}$$

where $J_n(x)$ are the Bessel functions of integer order and ϕ,ϕ_k are the $\boldsymbol{\rho}$ and $\mathbf{k}$ polar angles, respectively, the Hamiltonian in Eq. (4) becomes

$$H=P_\rho^2+(P_\phi+\tfrac{1}{4}\lambda\rho)^2+\int_0^\infty dk\sum_n a_{k,n}^\dagger a_{k,n}+\int_0^\infty dk\sum_n[V_k T_{k,n}(\rho)\Theta_n(\phi)a_{k,n}+\text{H.c.}] , \tag{8}$$

where

$$T_{k,n}(\rho)=i^n\sqrt{kS}J_n(k_\rho) . \tag{9}$$

The Hamiltonian in Eq. (8) now exhibits the axial symmetry for rotations around the direction of the magnetic field. In fact, it is easy to show that the angular momentum operator

$$L_z=-i\frac{\partial}{\partial\phi}+\int_0^\infty dk\sum_n n a_{k,n}^\dagger a_{k,n} \tag{10}$$

commutes with the Hamiltonian in Eq. (8) and we can therefore classify the energy levels according to the values of L_z.

It is well known that the problem of an electron in a static magnetic field can be solved exactly.[13] The eigenfunctions in polar coordinates are given by

$$\Phi_{n_\rho,m}(\boldsymbol{\rho})=A_m e^{-\rho^2/4\beta^2}\rho^{|m|}e^{im\phi}M(-n_\rho,|m|+1,\rho^2/4\beta^2) \tag{11}$$

with the eigenvalues

$$E=\lambda\left[n_\rho+\frac{|m|+m+1}{2}\right] . \tag{12}$$

In Eqs. (11) and (12) n_ρ and m are the radial and the angular-momentum quantum numbers, respectively. In Eq. (11) $\beta^2=2/\lambda$, A_m is a normalization constant, and $M(a,b,x)$ is the standard confluent hypergeometric function. We shall see in the next sections how a variational calculation is able to determine the polaronic corrections to the energy levels of Eq. (12) for the set of states with $n_\rho=0$ and for any value of m.

III. VARIATIONAL CALCULATION

The electron-phonon interaction in Eq. (8) depends on the electronic coordinates ρ and ϕ. Since our aim is to implement a variational calculation of the Lee-Low-Pines type which consistently take into account the (ρ,ϕ) dependence of the electron-phonon interaction, our trial wave function is chosen to be

$$|\Psi_m\rangle=e^{S(\rho)}|0\rangle\Phi_m(\boldsymbol{\rho}) , \tag{13}$$

where

$$S(\boldsymbol{\rho})=\int_0^\infty dk\sum_n[f_{k,n}(\boldsymbol{\rho})^*a_{k,n}-f_{k,n}(\boldsymbol{\rho})a_{k,n}^\dagger] \tag{14}$$

and $|0\rangle$ is the phonon vacuum. The electronic wave function $\Phi_m(\boldsymbol{\rho})$ is defined in Eq. (11) where we set $n_\rho=0$ and consider β as a variational parameter.

In order to have a trial wave function which is an eigenstate of L_z, the commutator between L_z and $S(\boldsymbol{\rho})$ must vanish:

$$[L_z,S(\boldsymbol{\rho})]=0 . \tag{15}$$

This condition is satisfied by choosing

$$f_{k,n}(\boldsymbol{\rho})=F_{k,n}(\rho)\Theta_n^*(\phi)\ , \tag{16}$$

where the functions Θ_n are defined in Eq. (6). With Eq. (15) it is easy to show that the trial state Ψ_m is an eigenstate of L_z with eigenvalue m. The phonon distribution function $f_{k,n}(\boldsymbol{\rho})$ and β are determined by minimizing the expectation value of the Hamiltonian in Eq. (8) which reads

$$\langle\Psi_m|H|\Psi_m\rangle=\langle\Phi_m|(P_\rho+j_\rho)^2+(P_\phi+j_\phi)^2+\frac{\lambda}{2}\rho(P_\phi+j_\phi)+\frac{\lambda^2}{16}\rho^2+\int_0^\infty dk\sum_n|\nabla f_{k,n}(\boldsymbol{\rho})|^2$$
$$+\int_0^\infty dk\sum_n|f_{k,n}(\boldsymbol{\rho})|^2-\int_0^\infty dk\sum_n[V_kT_{n,k}(\boldsymbol{\rho})\Theta_n(\phi)f_{k,n}(\boldsymbol{\rho})+\text{c.c.}]|\Phi_m\rangle\ , \tag{17}$$

where

$$j_\rho=\frac{i}{2}\int_0^\infty dk\sum_n\left[f_{k,n}(\boldsymbol{\rho})\frac{\partial f_{k,n}^*(\boldsymbol{\rho})}{\partial\rho}-f_{k,n}^*(\boldsymbol{\rho})\frac{\partial f_{k,n}(\boldsymbol{\rho})}{\partial\rho}\right]\ , \tag{18}$$

$$j_\phi=\frac{i}{2}\int_0^\infty dk\sum_n\frac{1}{\rho}\left[f_{k,n}(\boldsymbol{\rho})\frac{\partial f_{k,n}^*(\boldsymbol{\rho})}{\partial\phi}-f_{k,n}^*(\boldsymbol{\rho})\frac{\partial f_{k,n}(\boldsymbol{\rho})}{\partial\phi}\right]\ . \tag{19}$$

The current densities j_ρ and j_ϕ in Eqs. (18) and (19) vanish provided $f_{k,n}(\boldsymbol{\rho})$ is chosen in the form of Eq. (16) with the radial part $F_{k,n}(\boldsymbol{\rho})$ either real or purely imaginary and independent on the sign of n. Assuming this to be true, we shall see that the resulting $f_{k,n}(\boldsymbol{\rho})$ is consistent with this assumption. The functional variation of Eq. (17) with respect to $f_{k,n}^*(\boldsymbol{\rho})$ leads to the following partial differential equation for $f_{k,n}(\boldsymbol{\rho})$:

$$-\nabla^2 f_{k,n}(\boldsymbol{\rho})-\frac{\nabla|\Phi_m|^2}{|\Phi_m|^2}\cdot\nabla f_{k,n}(\boldsymbol{\rho})+f_{k,n}(\boldsymbol{\rho})=V_k^*T_{k,n}^*\Theta_n^*\ . \tag{20}$$

This equation shows that the electronic charge distribution and the polarization field are linked in such a way that they should be determined self-consistently. This is the key point in our variational theory.

The self-consistent solution of Eq. (20) for any Φ_m is a very difficult task. However, if we make a variational choice for Φ_m, we can solve Eq. (20) and then minimize the resulting total energy. A very good choice for Φ_m is given by Eq. (11), that is, the exact wave function in absence of the electron-phonon interaction. In Eq. (11) we now set $n_\rho=0$ in order to have the lowest energy levels for a fixed m. Inserting Eqs. (16) and (11) in Eq. (20) we have an ordinary differential equation for the radial part of $f_{k,n}(\boldsymbol{\rho})$ which reads

$$\frac{d^2F_{k,n}}{d\rho^2}+\left[\frac{2|m|+1}{\rho}-\frac{\rho}{\beta^2}\right]\frac{dF_{k,n}}{d\rho}-\left[\frac{n^2}{\rho^2}+1\right]F_{k,n}=-V_k^*T_{k,n}^*\ . \tag{21}$$

The exact solution of the above differential equation can be found in terms of the confluent hypergeometric functions M and U (Ref. 14)

$$F_{k,n}(\rho)=V_k^*\frac{\beta}{\sqrt{2}}\left[\frac{\rho^2}{2\beta^2}\right]^s\frac{\Gamma(a)}{\Gamma(b)}\left[M(a,b,\rho^2/2\beta^2)\int_\rho^\infty\left[\frac{x^2}{2\beta^2}\right]^\nu e^{-x^2/2\beta^2}T_{k,n}^*(x)U(a,b,x^2/2\beta^2)dx\right.$$
$$\left.+U(a,b,\rho^2/2\beta^2)\int_0^\rho\left[\frac{x^2}{2\beta^2}\right]^\nu e^{-x^2/2\beta^2}T_{k,n}^*(x)M(a,b,x^2/2\beta^2)dx\right]\ , \tag{22}$$

where Γ is the γ function and

$$\begin{aligned}s&=\tfrac{1}{2}[-|m|+(m^2+n^2)^{1/2}]\ ,\\ a&=\tfrac{1}{2}[-|m|+(m^2+n^2)^{1/2}+\beta^2]\ ,\\ b&=1+(m^2+n^2)^{1/2}\ ,\\ \nu&=\tfrac{1}{2}[1+|m|+(m^2+n^2)^{1/2}]\ ,\end{aligned} \tag{23}$$

The final expression of the expectation value of Eq. (17) is

$$E_\nu(\beta)=\left[\frac{1}{2\beta^2}+\frac{\beta^2\lambda^2}{8}\right](m+1)+\frac{m\lambda}{2}+\frac{1}{m!2^m\beta^{2(m+1)}}\int_0^\infty v_{\text{eff}}(\rho)e^{-\rho^2/2\beta^2}\rho^{2m+1}d\rho\ , \tag{24}$$

where we have defined the effective potential

$$v_{\rm eff}(\rho)=\alpha\int_0^\infty dk\sum_n\left[|F_{k,n}|^2\left[1+\frac{n^2}{\rho^2}\right]+\left|\frac{dF_{k,n}}{d\rho}\right|^2-2V_k^*J_nF_{k,n}\right] \quad (25)$$

which describes the effect of the electron-phonon interaction on the Landau levels. The minimization of Eq. (24) with respect to β allows us to determine self-consistently the magnetopolaron energy levels and wave functions.

Equation (22) holds for any magnetic field strength and for any value of the total angular momentum m. It is nevertheless interesting to look for simple close-form asymptotic expressions for $F_{k,n}$. We shall discuss the weak and strong magnetic field limits for the ground state only obtained setting $m=0$. Let us first consider the case $\lambda\ll 1$ with α constant. Since in this case $\beta\to\infty$, we can use the following asymptotic limits for the confluent hypergeometric functions:[14]

$$\lim_{a\to\infty} M(a,b,x/a)\frac{1}{\Gamma(b)}=x^{(1-b)/2}I_{b-1}(2\sqrt{x}), \quad (26)$$

$$\lim_{a\to\infty} U(a,b,x/a)\Gamma(1+a-b)=x^{(1-b)/2}K_{b-1}(2\sqrt{x}), \quad (27)$$

TABLE I. A table showing the ground-state energies for $\alpha=0.1$, 1.0, 4.0, and for different values of the dimensionless magnetic field λ. The column headed E_f are the energies calculated with the path-integral method and are taken from Ref. 8; column E_p lists the results of second-order perturbation theory [see Eq. (31)], and column E_v gives the results of our variational theory. For $\alpha=4.0$ we list the variational and path-integral energies only. The column headed E_a are the energies calculated with Eq. (30) and $\langle n\rangle$ is the average number of virtually excited phonons. The numerical accuracy on the values of E_v is estimated to be ±0.005, ±0.05, and ±0.1 for $\alpha=0.1$, 1.0, and 4.0, respectively. The energies are in units of $\hbar\omega_{\rm L}$.

λ	E_f	E_p	E_v	E_a	$\langle n\rangle$
0.1	−0.109 51	−0.109 05	−0.108		0.0787
0.2	−0.061 49	−0.061 05	−0.057		0.0804
0.4	0.034 51	0.034 92	0.044		0.0816
0.6	0.130 49	0.130 86	0.141		0.0874
0.8	0.226 47	0.226 80	0.238		0.0901
1.0	0.322 47	0.322 75	0.336	0.4373	0.0943
1.5	0.562 61	0.562 80	0.579	0.6732	0.1019
2.0	0.803 00	0.803 13	0.823	0.9114	0.1097
4.0	1.767 59	1.767 63	1.801	1.8747	0.1362
10.0	4.682 67	4.682 67	4.755	4.8018	0.2000
20.0		9.576 85	9.707	9.7197	0.2782
			$\alpha=1.0$		
0.1	−1.592 02	−1.540 55	−1.52		0.791
0.2	−1.560 80	−1.510 49	−1.45		0.812
0.4	−1.498 15	−1.450 81	−1.38		0.840
0.6	−1.435 09	−1.391 40	−1.32		0.876
0.8	−1.371 52	−1.332 01	−1.23		0.909
1.0	−1.307 39	−1.272 45	−1.14	−0.1266	0.943
1.5	−1.145 16	−1.122 04	−0.99	−0.0175	1.020
2.0	−0.983 30	−0.968 70	−0.82	0.1138	1.097
4.0	−0.327 48	−0.323 74	−0.02	0.7467	1.362
10.0	1.826 27	1.826 71	2.56	3.0183	2.003
20.0		5.768 46	7.08	7.1975	2.701
			$\alpha=4.0$		
0.1	−8.2067		−6.1		3.163
0.2	−8.2057		−6.1		3.280
0.4	−8.2031		−6.1		3.371
0.6	−8.1996		−6.1		3.517
0.8	−8.1952		−6.0		3.635
1.0	−8.1899		−6.0	−2.0066	3.773
1.5	−8.1728		−6.0	−2.3199	4.080
2.0	−8.1502		−6.0	−2.5449	4.390
4.0	−8.0090		−5.9	−3.0132	5.448
10.0	−7.7004		−4.8	−2.9266	8.104
20.0			−2.4	−1.2099	11.129

where $I_n(x)$ and $K_n(x)$ are the modified Bessel functions. Inserting Eqs. (26) and (27) in Eq. (22), a lengthy but straightforward calculation leads to

$$F_{k,n}(\rho)=\frac{V_k^*}{1+k^2}J_n(k\rho)\ . \tag{28}$$

This result shows that the phonon distribution function and therefore the polarization depends on the electronic coordinates, that is, the polarization follows adiabatically the electron motion. The ground-state energy obtained with Eq. (28) and with $\lambda=0$ is $-\alpha\pi/2$, that is, the polaron self-energy without field. For a weak but nonvanishing field we did not find a simple expression for the energy. The mathematical difficulty is that such calculation requires the sum of several series containing Bessel functions. This can be done analytically only for $\lambda=0$. However, numerical calculations show that for $\alpha\rightarrow 0$ the variational ground-state energy is the same as given by Eq. (1), the weak-field limit of second-order perturbation theory.

The strong field limit $\lambda\rightarrow\infty$ with α constant is obtained by observing that when $\beta\rightarrow 0$ in Eqs. (22) and (25) we can retain only the $n=0$ term. Numerical calculations show that all the contributions from the $n\neq 0$ terms are negligible. Since $a=\beta^2/2$ and $b=1$ when $\beta\rightarrow 0$, it is easy to show that Eq. (22) reduces to

$$F_{k,0}(\rho)=V_k^*\sqrt{kS}\,e^{-k^2/\lambda^2}\ . \tag{29}$$

From this equation we can see immediately that the polarization cloud forming the polaron is, in this case, independent on the electronic coordinates. This result corresponds to the adiabatic approximation for the total wave function, that is, the electronic and phonon wave functions are separated. In fact, the ground-state energy which follows from Eq. (29) and Eq. (24) reads

$$E_a=\frac{\lambda}{2}-\frac{\alpha}{2}\sqrt{\lambda\pi/2} \tag{30}$$

which is identical with Eq. (3) obtained with a product wave function.

IV. NUMERICAL RESULTS AND CONCLUSIONS

The general expression for the phonon distribution function given in Eq. (22) allows the calculation of polaronic effects on Landau levels with $n_\rho=0$ and any m. This set of states includes the ground state ($m=0$) and the excited states with $m\neq 0$. However, in this paper we confine our discussion to the ground state only.

The second-order perturbation theory ground-state energy is given by[3]

$$E_p=\frac{\lambda}{2}-\frac{\pi\alpha}{2\sqrt{\lambda}}\frac{\Gamma\left[\frac{1}{\lambda}\right]}{\Gamma\left[\frac{1}{\lambda}+\frac{1}{2}\right]}\ . \tag{31}$$

It has already been shown that the above result is also obtained within the path-integral method.[8,9]

In Table I we present the variational ground-state energies (E_v) calculated with the numerical minimization of Eq. (24) together with those resulting from the second-order perturbation theory [E_p in Eq. (31)] and path-integral calculations (E_f) described in Ref. 8. We also give the average number $\langle n\rangle$ of virtually excited phonons and the energy E_a calculated from Eq. (30). Table I shows that E_v does not differ from E_p and E_f in the weak-field limit. However, for strong field E_v is larger than both E_p and E_f. The field strength at which E_v becomes larger than E_p and E_f increases on decreasing the Fröhlich coupling constant α. We also note that $\langle n\rangle$ increases with the magnetic field suggesting that the effective electron-phonon interaction is an increasing function of the field strength.

The above considerations show that our variational theory is capable of giving a consistent description of the dynamics of the electron-phonon interaction under the influence of a static magnetic field. The correct asymptotic limits for weak- and strong-field strength are recovered. A comparison of our numerical results with those obtained within second-order perturbation calculations indicates that this approach may fail in giving the correct ground-state energy. This failure should not be surprising if one recognizes that the electronic charge distribution associated to the electronic motion and the polarization field are linked in such a way [see Eq. (20)] that the determination of energies and wave functions requires the solution of a self-consistent problem. Moreover, since $\langle n\rangle$ increases with the field strength, it is clear that the validity of the perturbative approach is doubtful particularly $\lambda\geq 1$.

Recently Larsen[9] has questioned the variational character of the path-integral method of Xiaoguang, Peeters, and Devreese[8] for polarons in a magnetic field. He has shown, evaluating the fourth-order perturbative corrections to the ground-state energy,[9] that the path-integral method ceases to give an upper bound to the exact ground-state for $\lambda>1.5$ and in the limit $\alpha\rightarrow 0$. Our variational calculations confirm this behavior and show that the path-integral method is unreliable even for large value of α. As a matter of fact, it is well known the path-integral method is not well defined from a mathematical point of view. The problem is that the action is a complex rather than a real function and this implies that a variational principle cannot be proved. However, Xiaoguang, Peeters, and Devreese[8] have found that in the weak-field region the polaron behaves as a rigid entity while in the strong-field region the polarization can no longer follow the electron motion. Even if their conclusion is correct on physical grounds, the actual ground-state energies disagree with our variational calculations.

[1]U. Merkt, Phys. Scr. **T23**, 63 (1988).
[2]M. A. Brummel, D. R. Leadley, M. A. Hopkins, and R. J. Nicholas, Phys. Scr. **T23**, 73 (1988).
[3]D. M. Larsen, Phys. Rev. B **30**, 4807 (1984).
[4]S. Das Sarma, Phys. Rev. Lett. **52**, 859 (1984).
[5]F. M. Peeters and J. T. Devreese, Phys. Rev. B **31**, 3689 (1985).
[6]D. M. Larsen, Phys. Rev. B **33**, 799 (1986).
[7]F. M. Peeters and J. T. Devreese, Phys. Rev. B **25**, 7201 (1982);

25, 7302 (1982).
[8]Wu Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **32**, 7964 (1985).
[9]D. M. Larsen, Phys. Rev. B **32**, 2657 (1985).
[10]G. Strinati, J. Math. Phys. **28**, 981 (1987).
[11]G. Iadonisi and F. Bassani, Nuovo Cimento **D2**, 1541 (1983); G. Iadonisi, F. Bassani, and G. Strinati, Phys. Status Solidi B (to be published).
[12]D. Ninno and G. Iadonisi, Phys. Rev. B **38**, 3803 (1988).
[13]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics Non-Relativistic Theory* (Pergamon, New York, 1965), p. 424.
[14]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1970).