Anomalies in the cyclotron resonance in high-mobility GaAs-$Ga_{1-x}Al_xAs$ heterojunctions

R. J. Nicholas, M. A. Hopkins,* D. J. Barnes, and M. A. Brummell†
Clarendon Laboratory, Parks Road, Oxford OX1 3PU, United Kingdom

H. Sigg, D. Heitmann, and K. Ensslin
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany

J. J. Harris and C. T. Foxon
Philips Research Laboratories, Cross Oak Lane, Redhill, Surrey RH1 5HA, United Kingdom

G. Weimann‡
Forschungsinstitut der Deutschen Bundespost beim Fernmeldetechnischen Zentralamt, Postfach 5000, Darmstadt, D-6100, Federal Republic of Germany
(Received 9 December 1988)

Frequency-dependent cyclotron-resonance measurements are reported on a series of GaAs-$Ga_{1-x}Al_xAs$ heterojunctions with some of the highest mobilities achieved. Both the effective mass and resonance linewidth show anomalies which appear to be related to a level-filling factor of $\nu=n_sh/eB$ close to 2. A modified Kramers-Kronig analysis demonstrates that the two are directly linked and have a common origin. At high fields where only one Landau level is occupied ($\nu<2$), the resonance position is shifted to higher frequencies (lower mass values) than would be expected for simple free-carrier behavior. This is thought to be due to potential fluctuations giving rise to weakly bound states for electrons in the lowest Landau level, implying that "extrinsic" effects are significant even in these very-high-mobility samples.

I. INTRODUCTION

There have been a number of studies of cyclotron resonance in two-dimensional (2D) systems in which anomalies in the effective mass and resonance linewidth have been observed at low temperatures. The first results were obtained using silicon metal-oxide-semiconductor (MOS) structures. As the electron density n_s was decreased, an initial rise in the cyclotron mass was seen and attributed to electron-electron interactions.[1–3] At lower densities, below the metal-insulator transition, a sample-dependent fall in the mass was observed, accompanied by a rapid narrowing of the linewidth.[1–4]

This was interpreted as an "impurity shift" of the cyclotron resonance due to localization by the random disorder potential,[4,5] although a single-particle picture did not seem able to explain all the results.[2,3] Wilson *et al.*[6] were the first to demonstrate a direct correlation of these effects with the Landau-level filling factor $\nu=n_sh/eB$, and suggested the formation of a pinned charge-density wave or "Wigner glass" state as the most plausible interpretation.

More recent work has concentrated on GaAs-$Ga_{1-x}Al_xAs$ heterojunctions. Englert *et al.*[7] observed maxima in the resonance linewidth when an integral number of Landau levels were filled ($\nu=2,4,6,\ldots$) which are believed to be caused by a filling-factor dependence of the screening of the impurity scattering.[7–10] A careful study of the linewidth in InAs quantum wells showed oscillations up to $\nu=14$.[11] An anticrossing behavior of the cyclotron mass, with a splitting of the resonance, was seen at frequencies proportional to $n_s^{1/2}$.[12] This was attributed to coupling to magnetoplasmons; however, similar structure has been found to depend upon the component of magnetic field parallel to the 2D layer and to vanish with increasing mobility, which seems inconsistent with this explanation.[13] Cyclotron-mass anomalies which occur at constant filling factors have also been observed.[13–16] Schlesinger *et al.*[15] saw a decrease in the mass and narrowing of the linewidth for $\nu<1$, which they interpreted in terms of reduced screening of the disorder-induced coupling to magnetoplasmons at $\nu=1$. Ensslin *et al.*[16] found a steplike decrease in the cyclotron mass together with a linewidth maximum as the filling factor was decreased below 2 with a similar, but considerably weaker, structure near $\nu=1$. It was suggested that this could be caused by the effect of filling-factor-dependent screening upon the degree of localization of the carriers, combined with coupling of the cyclotron transitions originating from different levels by electron-electron interactions.[17–19] Much larger shifts in the cyclotron mass for $\nu<2$ have been seen in samples where disorder has been deliberately introduced into the region of the 2D layer by electron irradiation[20,21] or the incorporation of a δ-doped layer of donors or acceptors.[21–22]

In this paper we present an investigation of the frequency dependence of the cyclotron resonance in some of the highest-quality GaAs-$Ga_{1-x}Al_xAs$ heterojunctions

available. Particular attention was paid to the frequency range where the transition from the low-field regime ($\nu > 2$) to the quantum limit ($\nu < 2$) occurs. The anomalous effects discussed above must somehow be connected with the presence of impurities or disorder, since Kohn's theorem[23] shows that electron-electron interactions cannot change the cyclotron mass in a translationally invariant system. One might, therefore, hope to see an "intrinsic" cyclotron resonance free of anomalies in these very pure samples. Instead, a large discontinuity in the cyclotron mass and a sharp maximum in the linewidth are seen near $\nu = 2$, particularly at low electron densities, and the resonance position at higher fields is found to be shifted to higher energies relative to the behavior expected for free carriers.

II. EXPERIMENTS AND RESULTS

The samples studied were grown by molecular-beam epitaxy at Philips Research Laboratories, Redhill, and at Fernmeldetechnisches Zentralamt (FTZ), Darmstadt. They consisted of conventional heterojunctions with wide spacer layers (750–3200 Å) and very high mobilities, typically of order 10^6 cm^2 V^{-1} s^{-1}. The individual sample characteristics are given in Table I. Shubnikov–de Haas and Hall measurements were used to determine the electron densities, which were varied using the persistent photoconductivity induced by illumination with a red light-emitting diode (LED). For the sample with the widest spacer layer (G161) cyclotron resonance could only be measured after the electron density had been increased by illumination, probably due to a vanishingly small carrier density in the dark. The cyclotron resonance was measured in transmission, using both far-infrared (FIR) lasers and a Fourier transform spectrometer. The sample substrates were wedged to avoid interference effects.

Figure 1 shows the frequency-field dependence of the cyclotron effective mass, defined as $m^* = eB/\omega$, and resonance half-width for sample 1532 at electron densities of $n_s = 0.8 \times 10^{11}$ and 1.5×10^{11} cm^{-2}. Landau-level filling factors ($\nu = n_s h/eB$) of 1 and 2 are indicated in the figure. These results were obtained at 2.1 K using a Fourier transform spectrometer, but no significant differences were found when the experiments were repeated at 1.3 K using a FIR laser. The half-widths were taken as half the width of the resonance when the change in transmission is half that at resonance. This does not give

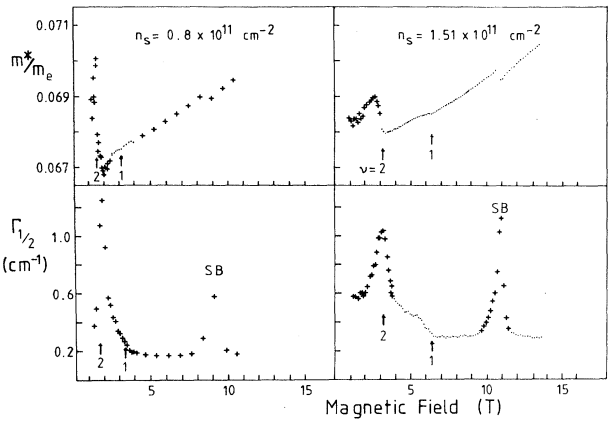

FIG. 1. The magnetic field dependence of the cyclotron effective mass m^*/m_e and measured half-width at half maximum absorption $\Gamma_{1/2}$ for sample 1532 (750-Å spacer) at carrier concentrations n_s of 0.8 and 1.51×10^{11} cm^{-2}. The fields corresponding to filling factors ν of 1 and 2 are indicated. Both dots and crosses are used, depending on the density of experimental point.

a direct measure of the true resonance broadening when the linewidth is very narrow due to the very large reflection coefficient. Theoretically this could easily be accounted for by plotting the quantity[24]

$$\hbar/\tau = \Gamma - \hbar\omega_p \ , \tag{1}$$

where

$$\omega_p = n_s e^2/\epsilon_0 c m^* (1 + n_G) \ , \tag{2}$$

Γ is the measured resonance half-width and n_G is the refractive index of GaAs. In practice, the linewidths measured in the quantum limit are very close to the saturation value $\hbar\omega_p$, and so the uncertainties in the scattering times deduced become very large. We have therefore plotted the experimental transmission half-widths $\Gamma_{1/2}$ (cm^{-1}) or $W_{1/2}$ (mT), which are always an overestimate of the true resonance broadening $\hbar/\tau$.

Above a frequency of about 50 cm^{-1} ($B = 3.5$ T) the effective mass increases approximately linearly with frequency, with a small change in gradient at the fields corresponding to a filling factor of 1, which becomes more pronounced at higher electron densities. Such an in-

TABLE I. Sample characteristics.

Sample no.	Spacer thickness (nm)	Dark μ (10^6 cm^2/V s)	Dark N_s (10^{11} cm^{-2})	Light[a] μ (10^6 cm^2/V s)	Light[a] N_s (10^{11} cm^{-2})
G161	320			0.1[b]	0.5[b]
G139	160	0.25	0.25	0.6	0.6
G63	80	0.75	0.85	2.1	2.0
G156	80	0.8	0.8	2.0	1.7
1532	75	0.45	0.8	1.1	1.5

[a]As illuminated for this experiment. These are not necessarily the saturation values.
[b]Estimates due to difficulties in establishing good electrical connections to the layer.

crease can be understood from the band nonparabolicity and polaron coupling,[25] as discussed below. The small discontinuity in the mass and peak in the linewidth around 10 T was only weakly dependent upon the electron density and is due to the well-known subband–Landau-level coupling.[26,27] This results from the presence of a small magnetic-field component in the plane of the layers due to misorientation of the sample, which is difficult to avoid experimentally. Comparison with deliberately misoriented samples allows us to estimate the orientation errors to be of order 0.5°.

However, the most striking features of the data shown in Fig. 1 are the large and rapid variations in the effective mass at low fields, accompanied by large peaks in the linewidth. These features shift to higher fields with increasing carrier concentration and appear to be tied to a particular value of the filling factor ν close to 2. When the carrier density is increased, the size of the mass discontinuity decreases, but the peak value of the linewidth is almost unchanged.

In Figs. 2–4 we show FIR laser results from three other samples with electron densities in the range 0.3×10^{11}–0.8×10^{11} cm^{-2}, all of which exhibit these effects. These samples were grown under similar conditions and differ only in the thickness of their $Ga_{1-x}Al_xAs$ spacer layers, which varied between 800 and 3200 Å. Figure 2 shows the frequency dependence of the effective mass and the halfwidth of sample G156 with an electron density of $n_s=0.8\times10^{11}$ cm^{-2}, so that the

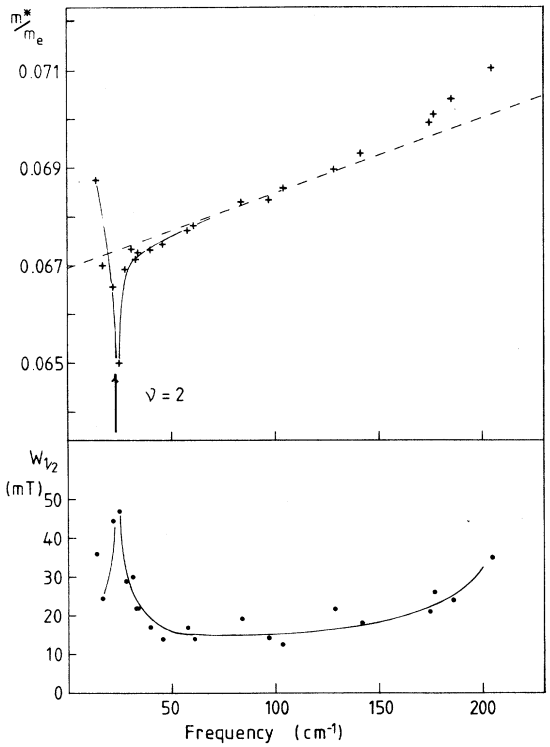

FIG. 2. The frequency dependence of m^*/m_e and $W_{1/2}$, the half-width in magnetic field, for sample G156 (800-Å spacer) at $n_s=8\times10^{10}$ cm^{-2}. The dashed line shows the expected nonparabolicity and the solid lines are guides to the eye.

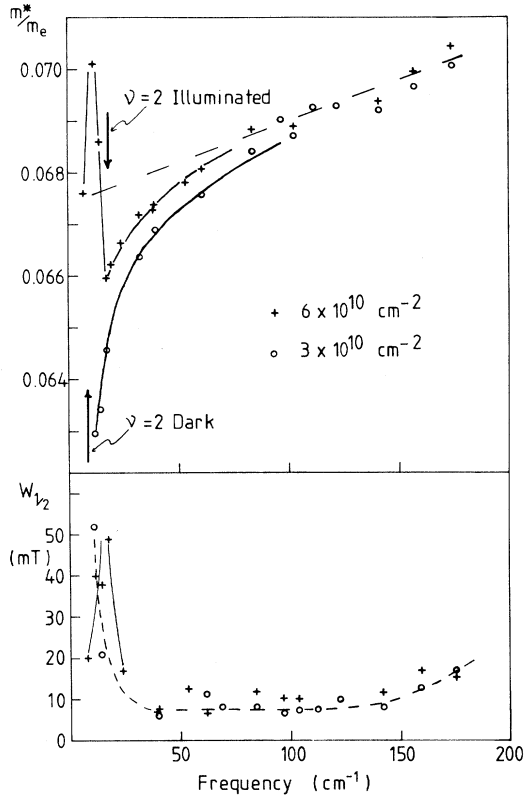

FIG. 3. The frequency dependence of m^*/m_e and $W_{1/2}$ for sample G139 (1600-Å spacer) at $n_s=2.5$ and 6×10^{10} cm^{-2}. The dashed line shows the expected nonparabolicity. The solid lines are guides to the eye.

fundamental frequency corresponding to $\nu=2$ is 22 ± 2 cm^{-2} as indicated in the figure. In the frequency range 50–150 cm^{-1}, the effective mass again increases nearly linearly with frequency due to the nonparabolicity, with a steeper increase at higher frequencies due to the onset of resonant polaron coupling.[25] However, at frequencies below 50 cm^{-1} the mass falls rapidly below a linear extrapolation of the higher-frequency results (indicated by a dashed line in Fig. 2) until, at a frequency of 24 cm^{-1}, the mass increases again, very sharply. The mass discontinuity is again accompanied by a broadening of the resonance, which has a maximum width where the discontinuity occurs.

The data taken from the two samples with the lowest electron densities and widest spacer layers, shown in Fig. 3 and 4, are particularly striking. Results are shown for sample G139 (Fig. 3) both before and after illumination and for sample G161 (Fig. 4) after illumination. These samples had spacer layers of 1600 and 3200 Å, respectively, and electron densities as low as 0.3×10^{11} cm^{-2}, corresponding to the quantum limit being reached at a frequency of 8 cm^{-1} ($B=0.6$ T). The cyclotron mass can be seen to decrease rapidly with decreasing frequency as $\nu=2$ is approached. The low electron densities lead to very narrow linewidths, and hence accurate mass values even at these low fields, except near $\nu=2$. In this region the greater broadening and consequently reduced maximum transmission change cause larger uncertainties.

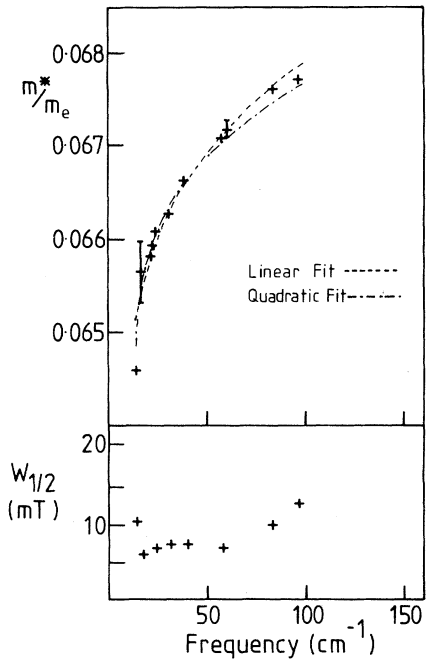

FIG. 4. The frequency dependence of m^*/m_e and $W_{1/2}$ for sample G161 (3200-Å spacer) at a carrier concentration of approximately 5×10^{10} cm^{-2}. The dashed and dotted-dashed lines show fits to the data assuming an energy offset in the resonance of linear and quadratic form, respectively. The "error bars" indicate the measured half-width of the resonance; the actual uncertainty in the cyclotron mass is an order of magnitude smaller.

The bars in Fig. 4 show the full widths of the resonances, and it can be seen that for $\nu<2$ the divergence of the mass from the expected linear frequency dependence is substantially larger than the linewidth. In sample G161, the half-width remains at 6–7 mT down to a frequency of 17 cm^{-1}, where the resonance is shifted from its expected position by $\sim$25 mT.

Some typical experimental recordings from sample G139 after illumination are shown in Fig. 5, covering the frequencies at which the mass oscillation and resonance

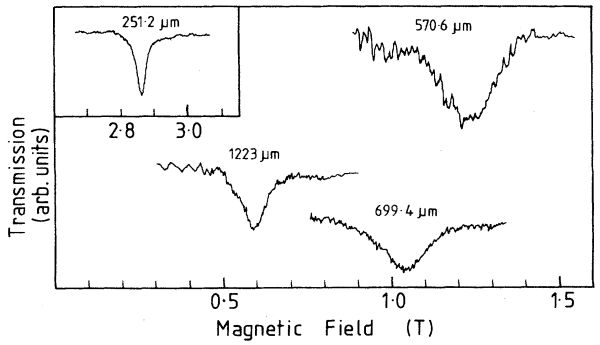

FIG. 5. Some typical experimental recordings of the cyclotron resonance in sample G139 after illumination, showing the linewidth broadening near $\nu=2$ (corresponding to a wavelength of 600 μm).

broadening occurred. The broadening of the resonance near $\nu=2$ can be clearly observed, but the resonances do not become noticeably asymmetric in this region.

Several trends can be inferred from the data given in Figs. 1–4. Increasing the electron density, either by illuminating the sample or by decreasing the spacer layer thickness, increases the frequency at which these features occur. In all cases this frequency appears to be tied to a filling factor of 2, although the resonance broadening is weighted toward the lower-frequency (higher ν) side of the fundamental field. The resonance broadened, in most cases, to a maximum halfwidth of $\sim$50 mT, but the magnitude of the mass discontinuity increased with decreasing electron density. In addition, the sharpness of the linewidth maximum and the mass discontinuity increased with decreasing n_s. These trends have also been observed in several other samples grown at Philips Research Laboratories and FTZ.

III. ANALYSIS AND DISCUSSION

Before discussing these results, let us first consider the behavior we might expect to see for an "intrinsic" cyclotron resonance. It is well known that the Landau-level separation is not exactly proportional to the resonance field and varies with Landau quantum number because of the band nonparabolicity and polaron coupling.[25,28–31] In the quantum limit, only transitions from the lowest Landau level are possible and the mean energy of these transitions, and hence the cyclotron mass, should increase approximately linearly with frequency. Below the fundamental field, higher Landau levels are populated and transitions from the two highest occupied levels can occur. The observed cyclotron resonance should thus consist of two transitions (which may not be resolved) with relative intensities determined by the level populations and matrix elements. However, the average energy of these transitions will remain approximately constant at $E_F(B=0)=\pi\hbar^2 n_x/m^*$, and so the cyclotron mass should be almost field independent, with only small oscillations as a function of filling factor. These oscillations should be stronger for higher fundamental fields, and small frequency-dependent oscillations in the cyclotron mass have indeed been observed[31] in samples with higher electron densities than those considered here, and were in reasonable agreement with theory.[29] In addition, the energy dependence of the electron g factor should lead to a slight difference in energy between the spin-up and spin-down transitions between a given pair of Landau levels.[28] In the limit of vanishing temperature, this should cause the appearance of a second resonance for $1<\nu<2$ and a third resonance for higher filling factors.

In our measurements we observe a single resonance over the whole frequency range studied. This is not altogether surprising, since the predicted splittings between the various transitions are in all cases smaller than the resonance halfwidths. The approximately linear increase in the cyclotron mass with frequency for $\nu<1$ apparent in Figs. 1 and 2 is consistent with five-band $\mathbf{k}\cdot\mathbf{p}$ theories of the band nonparabolicity[28,29] and a nonresonant polaron coupling which is smaller than the bulk value due to

the finite thickness of the 2D layer and many-body effects.[25,30] The change in gradient of the frequency dependence of the cyclotron mass at $\nu=1$ (Fig. 1) could be explained by the increasing influence of the cyclotron transition from the upper spin component of the lowest Landau level as the filling factor increases from 1 to 2. However, there is some suggestion of a minimum in the mass at $\nu=1$, and a stronger minimum was seen by Ensslin *et al.*[16] at higher electron densities. In other cases a steplike decrease in the mass has been seen.[15,16] It is difficult to see how nonparabolicity could account for these structures, and it certainly cannot explain either the rapid fall in the cyclotron mass as ν increases towards 2 or the magnitude of the mass discontinuity near $\nu=2$. This is particularly apparent for the lower-electron-density samples, where the mass in fact falls below the bulk GaAs band-edge mass of $m^*=0.0660m$,[28] as has also been in other studies.[14,16]

An approximate analysis suggests that the drop in the effective mass $m^*(B)$ as ν increases towards 2 can be accounted for by an offset energy for the resonance of the form

$$\hbar\omega=\hbar eB/m^*(B)=(\hbar\omega_c+\Delta_L) \tag{3}$$

appropriate for binding to an isolated shallow impurity when $\hbar\omega_c \gg \Delta_L$, or

$$(\hbar\omega)^2=(\hbar\omega_c)^2+\Delta_Q^2 \tag{4}$$

corresponding to binding by a harmonic-oscillator potential, where ω_c is the unperturbed cyclotron frequency. The frequency dependence of the resonance is complicated by the presence of the band and polaron nonparabolicities. This has been studied in detail at higher energies[25] where it was found that for $\omega \leq 150$ cm^{-1} the energy dependence of m^* for the lowest cyclotron transition is well described by the relation

$$\frac{1}{m_*}=\frac{1}{m_0^*}\left[1+\frac{2K_2}{E_g}\hbar\omega\right], \tag{5}$$

where m_0^* is the subband-edge mass and K_2 is approximately -1.4. Since the energy offset is much smaller than the measurement frequency ω, we can reduce Eq. (3) and (4) to the form

$$\hbar\omega'=\hbar\omega_c^0+\Delta_L \tag{6}$$

and

$$(\hbar\omega')^2=(\hbar\omega_c^0)^2+\Delta_Q^2 , \tag{7}$$

where $\omega_c^0=eB/m_0^*$ and ω has been corrected to

$$\omega'=\frac{\omega}{1+\frac{2K_2}{E_g}\hbar\omega} \tag{8}$$

to take into account the nonparabolicity. This allows us to use linear plots to determine the offset parameters Δ. A typical curve is given in Fig. 6 for sample G139. The values of Δ deduced from the two types of analysis (the linear and quadratic sums) are shown in Table II. These

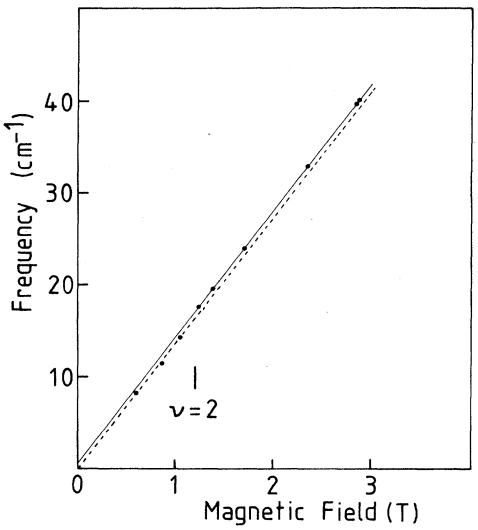

FIG. 6. A plot of the resonance positions against frequency for sample G139 after illumination, corrected for nonparabolicity using Eq. (7). The solid line is a linear fit to the points for a filling factor ν of less than 2, and the intercept at $B=0$ gives the offset Δ_L. The dashed line has the same slope but no offset.

give values of Δ between 0.03 and 0.10 meV for a linear offset and between 0.38 and 0.55 meV for a quadratic sum. It is not possible to make an unequivocal decision from these fits as to which offset sum is more appropriate, although the quadratic sums give values which vary less between the different samples. The results of fits using the parameters Δ_L and Δ_Q of Table II are shown in Fig. 4 for sample G161.

The offset resonance is only observed at fields where the filling factor has fallen below 2 and the only cyclotron transitions possible originate from the lowest Landau level. There is a transition back to a direct proportionality between frequency and field at lower fields (Fig. 6), and the peak in the linewidth coincides with the change in behavior. By introducing a modified form of Kramers-Kronig analysis we can in fact show that the two are directly related. For a given resonance the shift and broadening may be described by a self-energy shift of the form

$$E(B)=\Delta(B)+i\Gamma(B) . \tag{9}$$

Provided that $E(B)$ is an analytic function of B, as would appear to be the case from Fig. 1, we can use complex analysis to show that

$$\Delta(B)=\frac{1}{\pi}\int\frac{\Gamma(B')dB'}{B'-B} \tag{10}$$

and

$$\Gamma(B)=-\frac{1}{\pi}\int\frac{\Delta(B')dB'}{B'-B} . \tag{11}$$

Using this analysis and the assumption that $\Gamma(B)=\Gamma(-B)$, the linewidth data plotted in Fig. 1 for the higher electron density have been transformed to give the resulting shift in the energy $\Delta(B)$. This is shown in Fig. 7. It is clear that the effect of the linewidth peak

TABLE II. Offset cyclotron-resonance fits.

Sample no.	Condition	Linear fit $\hbar\omega=\hbar\omega_c+\Delta_L$ Δ_L (meV)	m_0/m_3	Quadratic fit $(\hbar\omega)^2=(\hbar\omega_c)^2+\Delta_Q^2$ Δ_Q (meV)	m_0/m_e
G161	light	0.05	0.06662	0.39	0.06618
G139	dark	0.10	0.06742	0.55	0.06663
G139	light	0.06	0.06761	0.53	0.06722
G63	dark	0.03	0.06722	0.38	0.06706
G156	dark	0.07	0.06757	0.54	0.06704
1532	dark	0.05	0.06741	0.46	0.06710

around $\nu=2$ is to generate a corresponding shift Δ in the resonance position of magnitude 0.07 meV, with a second shift appearing around the subband coupling. The shift is similar to the values of Δ deduced from the linear shift analysis of the resonance position (Table II), with the proviso that the origin of $\Delta(B)$ is arbitrary since $\Gamma(B)$ was assumed to be constant below the lowest field at which measurements could be made (~ 1 T).

The conclusion from this analysis is that the physical origin of the high-field, low-occupancy energy shift of the resonance and the broadening at $\nu=2$ are related, and are a consequence of the change in the behavior at $\nu=2$. A change in the position of the resonance is by itself sufficient to explain the appearance of the linewidth maximum. This idea has already been suggested by the theoretical analysis of Gold,[32] in connection with impurity-induced coupling to plasmons and single-particle excitations, where the memory function approach was used to show that the broadening and self-energy shift were directly related. However, filling-factor-dependent effects were not considered.

Broadening of the cyclotron resonance in a 2D electron gas when an integral number of Landau levels are filled has been observed by several workers.[7,10,11] When the Fermi energy lies between two Landau levels, screening can only occur through interlevel excitations and is thus strongly suppressed at low temperatures, causing the resonance to broaden. Such a reduction in the screening should also be possible at odd integer filling factors, where the Fermi energy lies between the two spin-split halves of a Landau level, but no observations of this have been reported. As discussed above, the broadening of the resonance will be associated with a fall in the resonance field and the cyclotron mass due to the interdependence of the real and imaginary parts of the dielectric function through the Kramers-Kronig relations. Filling-factor-dependent screening cannot, however, provide a complete explanation for our results since one would then expect the cyclotron mass to be unperturbed in the high-field limit ($\nu \ll 2$), whereas an offset resonance is in fact observed.

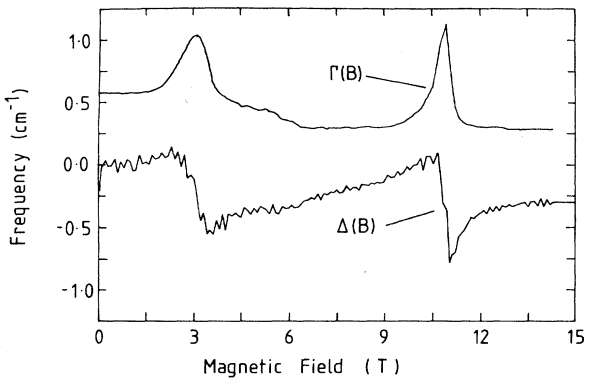

FIG. 7. The experimentally measured resonance half-width in sample 1532 after illumination (upper trace, from Fig. 1) and the corresponding offset of the resonance position (lower trace) deduced from the modified Kramers-Kronig relations as described in the text. The half-width has been assumed constant below the lowest field measurements (~ 1 T) and the offset has arbitrarily been set to zero at zero magnetic field.

Changes in the screening as a function of filling factor may indirectly affect the cyclotron mass by modifying the interactions between the cyclotron motion and other modes in the system. Schlesinger *et al.*[15] attributed the fall in the mass and narrowing of the linewidth they observed for $\nu<1$ to enhanced coupling to higher-energy magnetoplasmons at $\nu<1$. Such coupling can only occur through the disorder potential, since all the magnetoplasmons are degenerate with the cyclotron resonance at $\mathbf{k}=\mathbf{0}$ (Ref. 33) as required by Kohn's theorem.[23] The Coulomb interaction then acts to increase the splittings between the modes, thus depressing the cyclotron frequency. This mechanism should cause a maximum in the cyclotron mass at $\nu=1$, rather than the steplike behavior seen,[15,16] while at higher electron concentrations a minimum in the cyclotron mass was in fact observed.[16] Ensslin *et al.*[16] also observed structure near $\nu=2$ similar to that reported here, and suggested that reduced screening of the random impurity potential, leading to an increase in electron localization, could play a role in this.

The most striking similarities with the results presented here come from the work of Sigg *et al.*,[20,21] where cyclotron resonance was studied in GaAs-$Ga_{1-x}Al_xAs$ heterojunctions with disorder deliberately introduced by electron bombardment, and the early work on silicon inversion layers.[4,6] In both cases a shifted resonance, corresponding to a decrease in the mass, was observed for occupancies of $\nu<2$, and the magnitude of the shift was dependent upon the sample or disorder level. These results were interpreted using the coupled harmonic-oscillator equation [Eq. (4)]. The effect of disorder on the cyclotron mass has not been extensively studied theoretically, but a simple model[4,34] assuming potential fluctua-

tions varying slowly on the scale of the cyclotron radius $l_c=(\hbar/eB)^{1/2}$ shows that the change in frequency of the cyclotron motion is given by the local curvature of the potential:

$$\hbar\omega-\hbar\omega_c=\tfrac{1}{2}(l_c\nabla)^2V(r)\ . \tag{12}$$

Once the ground Landau level has been completely filled, the shift vanishes because the curvature of the potential for the occupied states becomes zero on average. For a weak harmonic potential, Eq. (12) reduces to a quadratic shift of the form given by Eq. (4). The cyclotron resonance in the presence of an assembly of such potentials was studied theoretically by Mikeska and Schmidt,[5] who found that for low densities of localization centers $N_c \ll n_s$ there could be an abrupt transition in the cyclotron mass at $\nu=2$. The critical parameter is the filling factor, not the number of impurities generating the disorder potential, because nearly all the electron states are expected to be localized.[35] Once the second Landau level begins to fill, the localized states must each accomodate a second electron, which will be much less strongly bound than the first. This causes the discontinuous jump in the resonance position near $\nu=2$. The analogy for an isolated impurity is the binding of a second electron to a neutral donor to form the D^- state. We believe that this model provides a good basis to interpret our results in terms of a cyclotron resonance originating from weakly localized states in the lowest Landau level, even in these very high mobility samples.

Ando[36] has shown that for low densities of scattering centers discrete impurity states or bands can be formed, splitting off states from the main Landau levels. This does not seem to be occurring in our measurements, since we do not observe separate impurity-shifted transitions. Such transitions should only be clearly seen when the number of impurities is comparable to the electron density, which is unlikely to be the case in our high mobility samples. However, they have been observed in quantum-well samples with deliberately doped wells[37] and in cyclotron emission experiments on GaAs-$Ga_{1-x}Al_xAs$ heterojunctions,[38] where the low density of excited electrons and the relaxation times involved favor their observation.

The origin of the localization potential could be related to the remote impurities in the n-$Ga_{1-x}Al_xAs$ beyond the spacer layer, interface fluctuations, or the residual impurities in the nominally undoped GaAs. The remote impurities are not thought to be significant, as the offsets shown in Table II do not appear to have any systematic dependence upon the spacer layer thickness. An interesting feature of the results of Sigg *et al.*,[20,21] where disorder was induced by electron irradiation and subsequent annealing to create controlled numbers of repulsive scattering centers, was the apparent relation between the offset frequencies Δ determined from a quadratic fit to the resonance positions [Eq. (4)] and the density of scattering centers N_c. They found that Δ^2 (meV^2) $=(1.3\times10^{-10})N_c$ (cm^{-2}). Using this formula the offsets given in Table II suggest that $N_c=1.1\times10^9-2.3\times10^9$ cm^{-2}, almost 2 orders of magnitude smaller than n_s. The condition $N_c \ll n_s$ required for the validity of the theoretical description of Mikeska and Schmidt[5] is thus easily fulfilled. In fact, this value of N_c is of the same order as the number of residual impurities in the GaAs within the 2DEG wave function. These, like the electron bombardment damage, are expected to consist mainly of repulsive scattering centers. The doping level of the GaAs is believed to be of order $N_A+N_D=4\times10^{14}$ cm^{-3} (Ref. 25) which, when combined with a wave-function extent of $\sim$250 Å, gives a sheet impurity concentration of 1×10^9 cm^{-2} within the 2D electron layer.

In conclusion, we have observed strong anomalies in the cyclotron resonance in very high mobility heterojunctions. These appear to be related to a filling factor of $\nu=2$, with a transition to "classical" behavior in the high-density, low-field limit. The use of a modified Kramers-Kronig type of analysis demonstrates that the change in the character of the resonance at $\nu=2$ can lead directly to the maximum in the linewidth. This broadening does not require any separate explanation, as suggested by some authors, but is a direct consequence of the rapid change in resonance position at the critical occupancy of $\nu=2$. The experimental results can be qualitatively interpreted using a model based on shifts in the resonance position induced by localization due to random potential fluctuations. A complete explanation of the data would also require filling-factor-dependent screening, coupling to magnetoplasmons and hybridization of the different cyclotron transitions to be taken into account. However, such effects can have no influence upon the cyclotron mass in the absence of disorder, and so the presence of impurities must be invoked to explain the infrared properties of even the highest-quality heterojunctions currently available.

ACKNOWLEDGMENTS

This work has been partially supported by the Science and Engineering Research Council. One of us (R.J.N.) would like to thank the Max-Planck-Institut für Festkorperforschung for hospitality during a very productive visit. We would like to thank P. van der Wel and J. Singleton for useful discussions.

*Present address: Service National des Champs Intenses, Centre National de la Recherches Scientifique, 25 avenue des Martyrs, 166-X, 38042 Grenoble CEDEX, France.

†Present address: Department of Physics, The University, Highfield, Southampton S09 5NH, United Kingdom.

‡Present address: Walter-Schottky Institut, Technische Universität München, D-8046 Garching, Federal Republic of Germany.

[1] G. Abstreiter, J. P. Kotthaus, J. F. Koch, and G. Dorda, Phys. Rev. B **14**, 2480 (1976).

[2] T. A. Kennedy, R. J. Wagner, B. D. McCombe, and D. C. Tsui, Solid State Commun. **22**, 459 (1977).

[3]R. J. Wagner, T. A. Kennedy, B. D. McCombe, and D. C. Tsui, Phys. Rev. B **22**, 945 (1980).
[4]J. P. Kotthaus, G. Abstreiter, J. F. Koch, and R. Ranvaud, Phys. Rev. Lett. **34**, 151 (1975).
[5]H. J. Mikeska and H. Schmidt, Z. Phys. B **20**, 43 (1975).
[6]B. A. Wilson, S. J. Allen, and D. C. Tsui, Phys. Rev. Lett. **44**, 479 (1980); Phys. Rev. B **24**, 5887 (1981).
[7]Th. Englert, J. C. Maan, Ch. Uhilein, D. C. Tsui, and A. C. Gossard, Solid State Commun. **46**, 545 (1983).
[8]R. Lassnig and E. Gornik, Solid State Commun. **47**, 959 (1983).
[9]T. Ando and Y. Murayama, J. Phys. Soc. Jpn. **54**, 1519 (1985).
[10]R. Lassnig, W. Seidenbusch, E. Gornik, and G. Weimann, in *Proceedings of the 18th International Coference on the Physics of Semiconductors, Stockholm, 1986,* edited by O. Engström (World Scientific, Singapore, 1987), p. 593.
[11]D. Heitmann, M. Ziesmann, and L. L. Chang, Phys. Rev. B **34**, 7463 (1986).
[12]Z. Schlesinger, S. J. Allen, J. C. M. Hwang, P. M. Platzmann, and N. Tzoar, Phys. Rev. B **30**, 435 (1984).
[13]E. Batke, H. L. Störmer, A. C. Gossard, and J. H. English, Phys. Rev. B **37**, 3093 (1988).
[14]S. Narita, K. Muro, S. Mori, S. Hiyamizu, and K. Nanbu, in *Application of High Magnetic Fields in Semiconductor Physics,* Vol. 177 of *Lecture Notes in Physics,* edited by G. Landwehr (Springer-Verlag, Berlin, 1983), p. 194.
[15]Z. Schlesinger, W. I. Wang, and A. H. MacDonald, Phys. Rev. Lett. **58**, 73 (1987).
[16]K. Ensslin, D. Heitmann, H. Sigg, and K. Ploog, Phys. Rev. B **36**, 8177 (1987).
[17]J. Appel and A. W. Overhauser, Phys. Rev. B **18**, 758 (1987).
[18]Y. Takada and T. Ando, J. Phys. Soc. Jpn. **44**, 905 (1978).
[19]Y. Takada, Surf. Sci. **98**, 442 (1980).
[20]H. Sigg, D. Weiss, and K. von Klitzing, Surf. Sci. **196**, 293 (1988).
[21]H. Sigg, J. Richter, K. von Klitzing, and K. Ploog, in *Proceedings of the International Conference on High Magnetic Fields in Semiconductor Physics, Würzburg, 1988, Springer Series in Solid State Sciences* (Springer-Verlag, Berlin, in press).
[22]J. Richter, H. Sigg, K. von Klitzing, and K. Ploog, unpublished.
[23]W. Kohn, Phys. Rev. **123**, 1242 (1961).
[24]T. A. Kennedy, R. J. Wagner, B. D. McCombe, and J. J. Quinn, Solid State Commun. **8**, 275 (1976).
[25]M. A. Hopkins, R. J. Nicholas, M. A. Brummell, J. J. Harris, and C. T. Foxon, Phys. Rev. B **36**, 4789 (1987).
[26]Z. Schlesinger, J. C. M. Hwang, and S. J. Allen, Phys. Rev. Lett. **50**, 2098 (1983).
[27]G. L. J. A. Rikken, H. Sigg, C. J. G. M. Langerak, H. W. Myron, J. A. A. J. Perenboom, and G. Weimann, Phys. Rev. B **34**, 5590 (1986).
[28]M. A. Hopkins, R. J. Nicholas, P. Pfeffer, W. Zawadzki, D. Gauthier, J. C. Portal, and M. A. diForte-Poisson, Semicond. Sci. Technol. **2**, 568 (1987).
[29]G. Lommer, F. Malcher, and U. Rössler, Superlatt. Microstruct. **2**, 273 (1986).
[30]Wu Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **36**, 9760 (1987).
[31]F. Thiele, U. Merkt, J. P. Kotthaus, G. Lommer, F. Malcher, U. Rössler, and G. Weimann, Solid State Commun. **62**, 841 (1987).
[32]A. Gold, Z. Phys. B **63**, 1 (1986).
[33]C. Kallin, and B. I. Halperin, Phys. Rev. B **31**, 3635 (1985).
[34]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[35]H. Aoki and T. Ando, Phys. Rev. Lett. **54**, 831 (1985).
[36]T. Ando, J. Phys. Soc. Jpn. **36**, 1521 (1974).
[37]E. Glaser, B. V. Shanabrook, R. L. Hawkins, W. Beard, J. M. Mercy, B. D. McCombe, and D. Musser Phys. Rev. B **36**, 8185 (1987).
[38]E. Gornik, W. Seidenbusch, R. Christianell, R. Lassnig, and C. R. Pidgeon, Surf. Sci. **196**, 339 (1988).