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Resonant conduction in ballistic quantum channels

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The conductance of a short, narrow ballistic constriction in a two-dimensional electron gas in a semiconductor heterostructure is calculated *exactly* for a simple but plausible model. A novel phenomenon is predicted: Such quantum channels should exhibit resonant conduction in high-mobility samples at low temperatures. Their conductance should oscillate strongly as a function of Fermi level and gate voltage. This behavior can be used to measure directly the aspect ratio of the channels.

Recently, van Wees *et al.*[1] and Wharam *et al.*[2] discovered experimentally that the conductance G of a narrow constriction in the two-dimensional electron gas (2D EG) in a GaAs-Al_xGa_{1-x}As heterostructure is quantized in integer multiples of $2e^2/h$, $G=\nu 2e^2/h$, if the length of the constriction is much shorter than the electron mean-free path, so that conduction is ballistic. It was argued[1,2] on the basis of Sharvin's ideas[3] on point contacts and the Landauer theory[4] of one-dimensional (1D) conduction, that the quantization index ν is just the number of populated 1D electron subbands or transverse quantum states in the channel. These arguments are convincing. However, existing theories (including the recent work of Johnston and Schweitzer)[5] do not address the processes by which electrons are injected into the constriction and emitted from it. For a short ballistic channel these processes are very important. The purpose of this Rapid Communication is to present *exact* calculations of the conductance for a simple but plausible model of the constriction. A new and unexpected quantum transport effect is predicted: At low T the conductance of high-mobility samples should exhibit regular oscillations as a function of gate voltage and Fermi energy. These oscillations are due to resonant longitudinal electron states in the constriction, and could be used to measure directly the aspect ratio of the channel.

The present model is of a heterostructure in the x-y plane with a 2D EG occupying the left (L) and right (R) half-spaces, $x<-d$ and $x>d$, respectively, and a narrow ballistic channel (C) of length $2d$ centered on the x axis connecting the 2D regions (see lower right inset, Fig. 1). In the 2D regions, the electron Hamiltonian is

$$H_{2D}=-\hbar^2(\partial^2/\partial x^2+\partial^2/\partial y^2)/2m^*\ ,$$

with m^* the effective mass. In between, the Hamiltonian is

$$H_C=-\hbar^2(\partial^2/\partial x^2+\partial^2/\partial y^2)/2m^*+U(y)\ ,$$

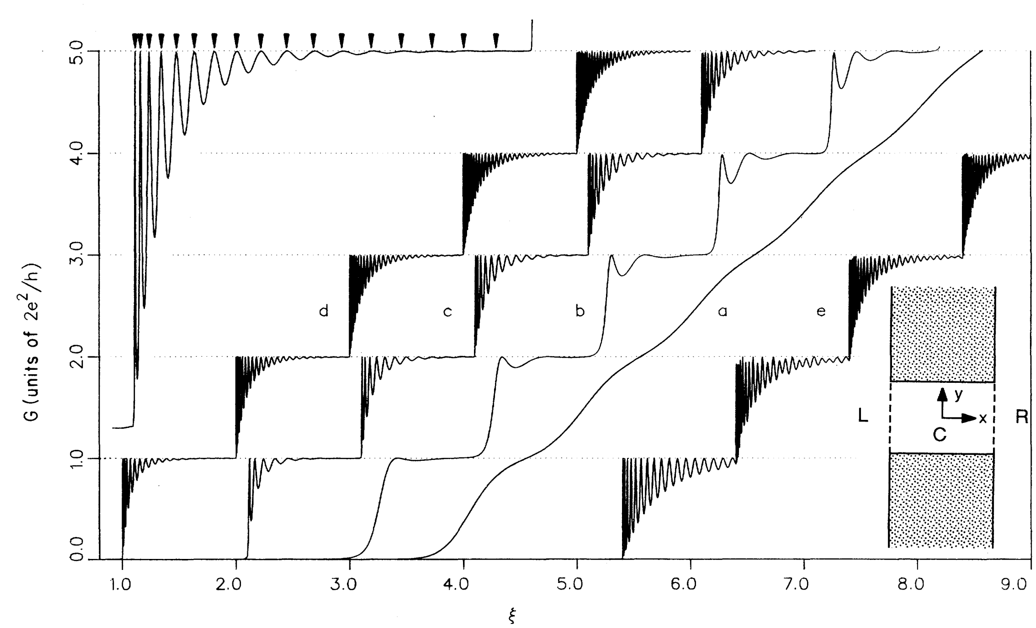

FIG. 1. G at $T=0$ vs ξ. E_F varies; W,U_0,d are fixed. $U_0=0$, $\hat{d}=0,1,5,10$ for curves a,b,c,d. Curve e: $\hat{U}=2.5$, $\hat{d}=10$.

where $U(y)$ is the potential confining the electrons to the channel. Consider an electron with wave vector $\mathbf{k}=(k,K)$ and energy $\varepsilon_\mathbf{k}$ incident on the channel opening from the left. For $x<-d$ its wave function can be written

$$\psi_\mathbf{k}^L(\mathbf{r})=e^{ikx}\phi_K^{2D}(y)+\sum_{K'}a_{K'}^L e^{-ik'x}\phi_{K'}^{2D}(y)\,,$$

where $\phi_K^{2D}(y)=e^{iKy}$, $k'=(2m^*\varepsilon_\mathbf{k}/\hbar^2-K'^2)^{1/2}$. The sum is over *all* transverse momenta K' so that imaginary values of k' (evanescent partial waves) are included. The convention $(-1)^{1/2}=+i$ is used throughout. In the channel the wave function is

$$\psi_\mathbf{k}^C(\mathbf{r})=\sum_n(a_n^{C+}e^{iq_nx}+a_n^{C-}e^{-iq_nx})\phi_n^C(y)\,,$$

where $\phi_n^C(y)$ is the eigenfunction of the nth transverse eigenstate of the confining potential $U(y)$ satisfying $H_C\phi_n^C(y)=\varepsilon_n\phi_n^C(y)$, $q_n=[2m^*(\varepsilon_\mathbf{k}-\varepsilon_n)/\hbar^2]^{1/2}$, and the sum is over all levels n including those for which q_n is imaginary. In the right-hand 2D region the transmitted wave is of the form

$$\psi_\mathbf{k}^R(\mathbf{r})=\sum_{K'}a_{K'}^R e^{ik'x}\phi_{K'}^{2D}(y)\,,$$

where $k'=(2m^*\varepsilon_\mathbf{k}/\hbar^2-K'^2)^{1/2}$. The meaning of the symbols $\mathbf{k}$, k, K, k', K', n, and q_n will be as defined above throughout this paper.

The continuity of $\psi_\mathbf{k}$ at $x=-d$ yields

$$a_{K'}^L=-\delta_{K'K}e^{-i(k'+k)d}+\sum_n M_{-K'n}(a_n^{C+}e^{-i(q_n+k')d}+a_n^{C-}e^{i(q_n-k')d})\,, \tag{1}$$

where $M_{Qn}=\int_{-\infty}^{\infty}\phi_Q^{2D}(y)\phi_n^C(y)dy$. Choosing $\phi_n^C(y)$ to be real, the continuity of $\partial\psi_\mathbf{k}/\partial x$ at $x=-d$ yields

$$\sum_{K'}a_{K'}^L k'M_{K'n}e^{ik'd}=kM_{Kn}e^{-ikd}-q_n(a_n^{C+}e^{-iq_nd}-a_n^{C-}e^{iq_nd})\,. \tag{2}$$

Eliminating $a_{K'}^L$ from (1) and (2) and repeating the same procedure at $x=d$ yields, respectively,

$$\sum_n[(T_{mn}+q_n\delta_{mn})e^{-iq_nd}a_n^{C+}+(T_{mn}-q_n\delta_{mn})e^{iq_nd}a_n^{C-}]=2ke^{-ikd}M_{Km}\,, \tag{3}$$

$$\sum_n[(T_{mn}-q_n\delta_{mn})e^{iq_nd}a_n^{C+}+(T_{mn}+q_n\delta_{mn})e^{-iq_nd}a_n^{C-}]=0\,, \tag{4}$$

where $T_{mn}=\sum_{K'}k'M_{K'm}M_{-K'n}$. The reflected waves in the left 2D region and the transmitted waves on the right have been eliminated, leaving a system of linear Eqs. (3) and (4) involving only the coefficients a_n^{C+} and a_n^{C-} which describe the state $\psi_\mathbf{k}$ within the channel. In terms of a_n^{C+} and a_n^{C-}, the electric current carried through the channel by $\psi_\mathbf{k}$ is

$$\begin{aligned}\langle\psi_\mathbf{k}|j_x|\psi_\mathbf{k}\rangle&=i\hbar e/2m^*\int_{-\infty}^{\infty}(\psi_\mathbf{k}^*\partial\psi_\mathbf{k}/\partial x-\psi_\mathbf{k}\partial\psi_\mathbf{k}^*/\partial x)dy\\&=-\hbar e/m^*\left[\sum_n{}^R q_n(a_n^{C+*}a_n^{C+}-a_n^{C-*}a_n^{C-})+\sum_n{}^I q_n(a_n^{C-*}a_n^{C+}-a_n^{C+*}a_n^{C-})\right]\,.\end{aligned} \tag{5}$$

The integral is taken across the channel. $\sum_n^R$ ($\sum_n^I$) runs over those values of n for which q_n is real (imaginary). The second sum is the contribution of the evanescent partial waves to the current. The total current J through the channel at $T=0$ is given by the sum of the contributions of all states $\psi_\mathbf{k}$ incident on the channel from the left in the energy interval eV near the Fermi energy E_F, where V is the potential difference between the two 2D EG regions.[6] The conductance is then

$$G=|J/V|=-2\int_{-a}^{a}m^*e/(h^2k)\langle\psi_\mathbf{k}|j_x|\psi_\mathbf{k}\rangle dK\,,$$

where $a=(2m^*E_F/\hbar^2)^{1/2}$, $k=(2m^*E_F/\hbar^2-K^2)^{1/2}$, and V is small.

This solution of the model is exact. However, the confining potential $U(y)$ of the constriction is not known accurately. In modeling 1D channels in heterostructures, $U(y)$ is usually assumed to be either a parabolic or square-well potential. The self-consistent calculations of Laux, Frank, and Stern[7] suggest that the parabolic well is appropriate when only the lowest one or two subbands contain electrons while the square well should be closer to reality when several subbands are populated. Here I will present results for the square-well model potential: $U(y)=U_0$ for $-W/2<y<W/2$, $U(y)=\infty$ for $|y|>W/2$. The results for the parabolic case will be published elsewhere.[8] Using the well-known eigenfunctions $\phi_n^C(y)$ of the square-well potential, the overlap integrals M_{Km} which enter (3) and (4) were evaluated analytically. Equations (3) and (4) were solved numerically for a_n^{C+} and a_n^{C-} by truncating n and m at a high transverse level N and the solutions were used to calculate the conductance. The effect on the conductance of the transverse levels lying well above the Fermi energy was found to decrease rapidly with increasing energy and the calculated conductance converged very well with increasing cut-off N. A numerical accuracy of 0.1%–0.01% including all truncation errors was readily obtainable in most cases.

The $T=0$ conductance G for square-well confinement depends on three variables: the normalized 2D EG Fermi energy $\hat{E}_F=E_F/\Delta$, the normalized height of the potential step encountered by the electron on entering the constriction $\hat{U}=U_0/\Delta$, and the aspect ratio $\hat{d}=2d/W$ of the channel. Here $\Delta=h^2/8m^*W^2$; $\varepsilon_n=n^2\Delta+U_0$. In Fig. 1, G calculated at $T=0$ is plotted, varying the 2D EG Fermi energy, and holding the channel parameters d, W, and U_0 fixed. The Fermi energy is parametrized by $\xi=(\hat{E}_F-\hat{U})^{1/2}$. The horizontal scale is for curve d; the other curves are offset to the right by multiples of 1.1. Curves a–d are for $\hat{U}=0$. Curve a is the limiting case of zero aspect ratio, $\hat{d}\rightarrow 0$. Interestingly, for this "ideal" point contact there is no quantization of the conductance; a channel of nonzero length is necessary for quantization. Curves b,c,d are for $\hat{d}=1$, 5, and 10, respectively. In these cases the conductance is close to $G=\nu 2e^2/h$ near the right-hand side of each plateau although the accuracy of quantization decreases with increasing ν. But the rise to each plateau is oscillatory, the strength of the oscillations increasing with $\hat{d}$ and with ν, and the peak-to-peak amplitude can approach $2e^2/h$. The $\nu=1$ step of curve d is shown enlarged at the upper left. The pointers mark the values of ξ which satisfy the resonance condition $i\lambda_F/2=2d\gamma$, for $i=1,2,\ldots,17$, where λ_F is the de Broglie wavelength of the Fermi electrons in the channel, and γ ($=1.044$ *in this case*) is an adjustable parameter chosen to fit the positions of the peaks of the conductance curve. Clearly the oscillations are due to resonances which occur when the length of the channel, adjusted for end effects, is an integral number of half wavelengths of the Fermi electrons in the channel. That is, as well as the level structure associated with transverse confinement, the open-ended channel supports *longitudinal* resonant electron states, the electronic quantum analog of the acoustic resonant modes of an open organ pipe. Conductance measurements are a spectroscopy of these resonant states. For the square-well potential, the location ξ of the ith resonance associated with electrons in the nth subband is given by $\xi_{\rm res}^2=i^2/(\hat{d}\gamma)^2+n^2$ so that observations of these resonances could be used to find the aspect ratio of the channel. Curve e in Fig. 1 is for $\hat{d}=10$, $\hat{U}=2.5$. The main effect of a nonzero $\hat{U}$ is to increase the amplitude of the oscillations. Notice the beats in the $\nu=2$ region which occur because the oscillations due to the electrons in the $n=1$ subband have not yet died out when the $n=2$ subband beings to fill.

While it is possible to vary the Fermi energy of the 2D EG by illuminating the sample,[2] the quantization of G is observed by varying the gate voltage which controls the width of the channel. This is modeled in Fig. 2, where E_F and d are held fixed but W is assumed to vary, controlling Δ and hence $\hat{E}_F$ and $\hat{d}$. $\hat{U}$ is arbitrarily set to be zero. The aspect ratio $\hat{d}$ is taken to be five when the Fermi level is at the bottom of $n=11$ subband. $\hat{d}$ increases as the channel is "squeezed" and the successive subbands are emptied. The dependence of the channel width on the gate voltage is not known, so that the conductance is again plotted against ξ. (Since the conductance plateaus are equally spaced in ξ in the present model, and are approximately equally spaced in gate voltage in the data of van Wees *et al.*,[1] it is tempting to suppose that ξ is a roughly linear function of gate voltage.) The $\nu=1$, 5, and 10 conductance plateaus are shown at $T=0$ in Fig. 2(a). Figure 2(b) shows the temperature dependence of the $\nu=3$ plateau calculated using the well-known result $G_T(\mu)=-\int G_0(\varepsilon)\partial f/\partial\varepsilon d\varepsilon$, where G_T is the conductance at temperature T, $f=(e^{(\varepsilon-\mu)/kT}+1)^{-1}$ is the Fermi function and μ is the chemical potential. Since in the experimental systems $\mu\sim 10$ meV, $kT=0.001\mu$ corresponds to $T\sim 0.1$ K. Note however, that the T dependence of the oscillations is sensitive to the channel aspect ratio which is not accurately known. The oscillations are smoothed out when kT becomes comparable to the spacing of the peaks of G_0 as a function of Fermi energy *at fixed W*, which can be estimated from the above result $\xi_{\rm res}^2=i^2/(\hat{d}\gamma)^2+n^2$. Since the amplitude and period of the oscillations in the plateau region increase with plateau index ν, the oscillations may be expected to survive to somewhat higher T for higher ν plateaus. If T is high enough to smooth out the oscillations, the measured conductance in the plateaus *for higher* ν will be *less* than $\nu 2e^2/h$ and the plateau will have a finite slope. Such a qualitative trend is clearly visible in the data of van Wees *et al.*[1] A sizable U_0, which could occur at very low W,[7] would tend to depress G for similar reasons (cf. Fig. 1, curve e).

Electron energy-level broadening due to impurity

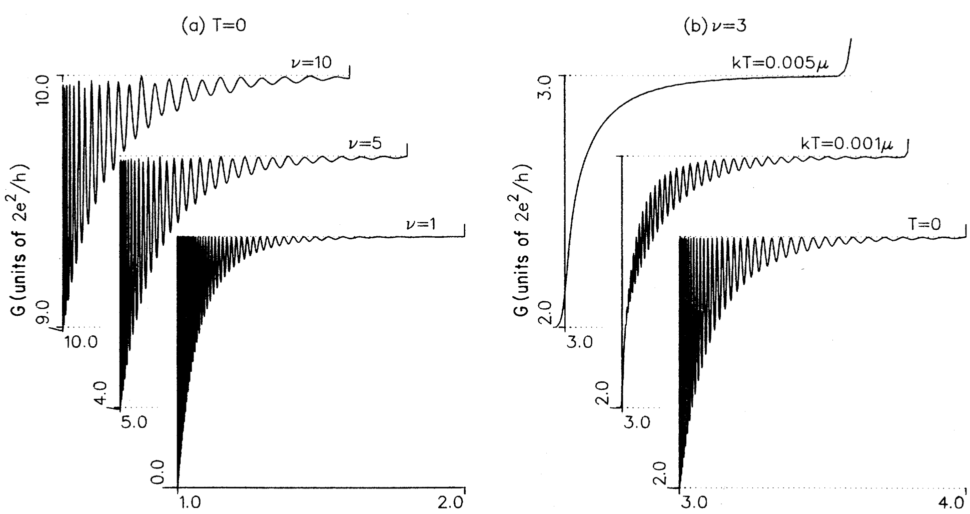

FIG. 2. G vs ξ. W varies; E_F,d are fixed. $U_0=0$. (a) Plateaus 1,5,10 at $T=0$. (b) T dependence of plateau 3.

scattering in the 2D EG regions will also tend to smooth the conductance oscillations. This effect is highly sample dependent and difficult to treat quantitatively, but given the current pace of improvements in sample mobilities, its importance should decrease with time. It will be necessary to experimentally extract the resonances predicted here from the background of universal conductance fluctuations (UCF) at low T.[4(b)] Since the resonant oscillations are regular and the UCF are not, it should be possible to do this in much the same way that the Aharonov-Bohm oscillations in small metallic rings are extracted from the UCF background.

In conclusion, a novel quantum transport effect, resonant conduction in narrow ballistic channels, has been predicted theoretically. The observation of this phenomenon will challenge experimentalists.

Note added. After this work was submitted for publication, B. J. van Wees informed me that he and his collaborators may have confirmed the existence of resonant ballistic conduction experimentally in *some* of their samples. The sample dependence of the strength of the resonant oscillations may be due in part to uncontrolled variations in the channel geometry. The more sharply defined the channel orifices are, the stronger the resonances in the conductance should be. In the present model these orifices are sharply defined.

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