Coupling of plasmons to polar phonons in GaAs quantum wells

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Mode coupling between the degenerate electron-hole plasma and longitudinal-optical phonons in GaAs quantum wells is treated theoretically. Both the plasmons and coupled modes are found to be confined in long-wavelength ranges. In contrast to bulk crystals, the condition that the dielectric function of the coupled system is negative is necessary. The wave numbers of the coupled modes in the GaAs well of 50 Å in width are smaller than 2×10^6 and 1.5×10^6 cm^{-1} at the electron density of 1×10^{17} and 1×10^{18} cm^{-3}, respectively, where the modes exhibit no Landau damping. These results indicate that the plasma does not affect carrier heating through the Fröhlich interaction.

I. INTRODUCTION

The energy relaxation of hot electrons in semiconductors provides information about fundamental physical processes such as electron-phonon and electron-electron interactions. At high carrier density a strong reduction of the energy loss rate of hot electrons has been observed in GaAs quantum wells (QW's) as well as in bulk GaAs, and this reduction is ascribed mainly to the formation of hot phonons.[1-4]

Though the effect of hot phonons may be dominant, theoretical works still have predicted that the Fröhlich interaction is screened appreciably by the density fluctuation of carriers.[1-3] Recently, however, Rühle and Polland[5] and Polland *et al.*[6] reported that heating rates of cold carriers in GaAs were independent of their densities at least up to 4×10^{17} cm^{-3} in the bulk and to 1×10^{18} cm^{-3} in the QW's, thereby concluding that the screening of the Fröhlich interaction was negligible.

In this paper we show a theoretical treatment of the mode coupling between the plasmon and longitudinal-optical (LO) phonon in the GaAs QW's and discuss the screening problem. Concerning the electron-phonon interactions in the QW most of the work reported so far has assumed that the phonons are three dimensional in nature,[7,8] and then the screening of the Fröhlich interaction between quasi-two-dimensional electrons and bulk-like LO phonons has been discussed first with the Debye model.[9] In subsequent studies[10-14] the screened scattering potential has been assumed to be represented by the bare potential divided by the dielectric function of the carrier plasma. In this method the interactions between the plasmons and LO phonons do not seem to be correctly taken into account.

As discussed in a previous paper,[15] the best way to resolve the screening problem is presumably to begin with the study of the mode coupling, since at least the long-range part of the plasmon-LO phonon interactions is absorbed into the coupled modes. In the bulk GaAs it has been shown[15] that at the electron density of 1×10^{18} cm^{-3} the frequency of the LO phonons in a long-wavelength range becomes nearly equal to the transverse optical-phonon frequency, the fact of which means that the longitudinal electric field associated with the LO phonon in this wavelength range is almost screened. On the other hand, LO phonons having shorter wavelengths are not affected by the plasma and hence the scattering of electrons by these phonons is unchanged.

The mode coupling in QW's has been studied on the assumption that the total dielectric function vanishes.[16,17] This is, however, inapplicable to the present problem since the boundary conditions at heterointerfaces of the electromagnetic fields should be considered both for plasmons[18-21] and phonons.[22]

In Sec. II we treat the coupled mode as a guided mode in a three-layer waveguide which makes a model of the GaAs QW's. Dispersion relations of and phonon contribution to the coupled modes are calculated numerically. The results of the calculations are discussed in Sec. III in connection with the screening problem.

II. COUPLED MODE IN QUANTUM WELL

We consider a double heterostructure consisting of a GaAs layer sandwiched by $Al_{0.3}Ga_{0.7}As$ layers as illustrated in Fig. 1. This is a symmetric three-layer slab waveguide, for which we seek guided modes propagating in the direction parallel to the interfaces, the z direction in the coordinate axis shown in Fig. 1. We take the

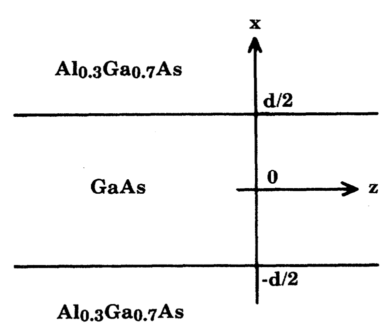

FIG. 1. Quantum well structure and the coordinate axis.

dielectric continuum model for the response of lattice ions to external fields, where the dielectric functions $\epsilon_0(\omega)$ and $\epsilon_1(\omega)$ of GaAs and $Al_{0.3}Ga_{0.7}As$ may be written, respectively, as

$$\epsilon_0(\omega)=\epsilon_\infty(\omega_l^2-\omega^2)/(\omega_t^2-\omega^2) , \tag{1}$$

$$\epsilon_1(\omega)=\epsilon_{1\infty}\frac{(\omega_{l1}^2-\omega^2)(\omega_{l2}^2-\omega^2)}{(\omega_{t1}^2-\omega^2)(\omega_{t2}^2-\omega^2)} . \tag{2}$$

Here, ω is the frequency, ω_l and ω_{li} are longitudinal optical frequencies, and ω_t and ω_{ti} are transverse-optical frequencies.[23]

In the QW are there free carriers resulting from doped impurities or generated by optical means, and hence the dielectric function of the GaAs layer becomes anisotropic. We assume that the carriers are degenerate in the ground subband and polarizable in the *yz* plane only, thus neglecting intersubband excitations. This assumption is valid in the limit of vanishingly small well width. Then, the dielectric function $\epsilon_\perp$ perpendicular to the GaAs layer may be approximated as

$$\epsilon_\perp=\epsilon_0(\omega) , \tag{3}$$

but the dielectric function $\epsilon_\parallel$ in the parallel direction becomes

$$\epsilon_\parallel=\epsilon_0(\omega)+4\pi\chi , \tag{4}$$

where χ is the susceptibility of the quasitwo-dimensional electron gas. In calculating the dispersion relations of the coupled modes we take the real part χ_1 of χ in Eq. (4).

Since the longitudinal electric field due to the plasmons is in the well layer, it is sufficient to consider the transverse magnetic (TM) mode[19,20] of the form $\exp[i(\omega t-qz)]$ with q as a wave number. From the Maxwell equations we have the following relations among the electric fields E_x, E_z, and the magnetic field H_y in the well

$$(i\omega\epsilon_\perp/c)E_x=iqH_y , \tag{5}$$

$$(i\omega\epsilon_\parallel/c)E_z=\partial H_y/\partial x , \tag{6}$$

$$(i\omega/c)H_y=iqE_x+\partial E_z/\partial x , \tag{7}$$

where c is the light velocity. Substituting Eqs. (5) and (6) into (7) we obtain

$$\epsilon_\perp\frac{\partial^2 H_y}{\partial x^2}+\epsilon_\parallel\left[\frac{\epsilon_\perp\omega^2}{c^2}-q^2\right]H_y=0 \tag{8}$$

and a similar equation in the barrier layers in which $\epsilon_\perp=\epsilon_\parallel=\epsilon_1(\omega)$.

We are looking for a guided wave whose fields are sinusoidal inside the well and decay exponentially outside. The field H_y can be expressed as either an even or an odd function, of which the even one is excluded here because it gives photonlike modes. Thus we have

$$H_y=A\sin(kx)\exp[i(\omega t-qz)] \tag{9}$$

for $|x|<d/2$, where d is the width of the well, and

$$H_y=(x/|x|)B\exp(-\gamma|x|)\exp[i(\omega t-qz)] \tag{10}$$

for $|x|>d/2$. From Eqs. (8)–(10) we find

$$\epsilon_\perp k^2=\epsilon_\parallel[(\epsilon_\perp\omega^2/c^2)-q^2] , \tag{11}$$

$$\gamma^2=q^2-\epsilon_1(\omega)\omega^2/c^2 , \tag{12}$$

and from the boundary conditions at $x=\pm d/2$,

$$\gamma=-[\epsilon_1(\omega)/\epsilon_\parallel]k\cot(kd/2) . \tag{13}$$

Combining Eqs. (11) and (12) leads to

$$\epsilon_0(\omega)K^2+\epsilon_\parallel\Gamma^2=\epsilon_\parallel[\epsilon_0(\omega)-\epsilon_1(\omega)](\omega d/2c)^2 , \tag{14}$$

where $K=kd/2$ and $\Gamma=\gamma d/2$. In our case ω is at most 10^{14} s^{-1} and hence the right-hand side of Eq. (14) is negligibly small. From Eqs. (13) and (14) we find that the TM modes can be obtained when $\epsilon_\parallel<0$, $\epsilon_0(\omega)>0$, and $\epsilon_1(\omega)>0$. With $\epsilon_\parallel=-|\epsilon_\parallel|$ and the approximation of neglecting terms proportional to $(\omega/c)^2$, Eqs. (12) and (14) lead, respectively, to

$$\Gamma=Q\equiv qd/2 , \tag{15}$$

$$\epsilon_0(\omega)K^2=|\epsilon_\parallel|\Gamma^2 , \tag{16}$$

and Eq. (13) is rewritten as

$$\Gamma\tan K=[\epsilon_1(\omega)/|\epsilon_\parallel|]K . \tag{17}$$

Substituting Eq. (16) into Eq. (17) and using Eq. (15), we find the dispersion relation

$$\tan\{[|\epsilon_\parallel|/\epsilon_0(\omega)]^{1/2}Q\}=\epsilon_1(\omega)[|\epsilon_\parallel|\epsilon_0(\omega)]^{-1/2} . \tag{18}$$

For small q and small ω, $|\epsilon_\parallel|$ approaches to ω_p^2/ω^2, where ω_p is the plasma frequency, and then Eq. (18) leads to

$$\omega=\omega_p[qd/2\epsilon_1(0)]^{1/2} . \tag{19}$$

The square-root dependence on q of ω has been already reported for the quasitwo-dimensional plasma.[18–21] For $d\to 0$, Q will be small and Eq. (18) leads to

$$\epsilon_1(\omega)-|\epsilon_\parallel|Q=0 , \tag{20}$$

which corresponds to some earlier results.[18,19] In the case of the bulk or the two-dimensional crystal we have $\epsilon_\parallel=0$ from Eq. (6) as usual, since $\partial H_y/\partial x=0$ in the bulk or $E_x=H_y=0$ in the two-dimensional case.

Figure 2 shows the dispersion relations calculated from Eq. (18) for the plasma of electrons and holes of equal densities, where the dashed and solid lines correspond to the electron or hole density $N=1\times10^{17}$ cm^{-3} and 1×10^{18} cm^{-3}, respectively. In the calculations we take the well width of 50 Å, the electron and hole effective masses of 0.067 and 0.45 m, the Stern's result[18] to the susceptibility χ, and the parameter values in $\epsilon_0(\omega)$ and $\epsilon_1(\omega)$ from the paper of Kim and Spitzer.[23] The upper branch in Fig. 2 represents the plasmon-LO phonon coupled mode, while the lower branch is the well-known two-dimensional plasmon mode though a little mixing of phonons may be found. The range of the wave number in each branch is determined from the fact that the modes exist only when $\epsilon_\parallel<0$. In these ranges the imaginary part

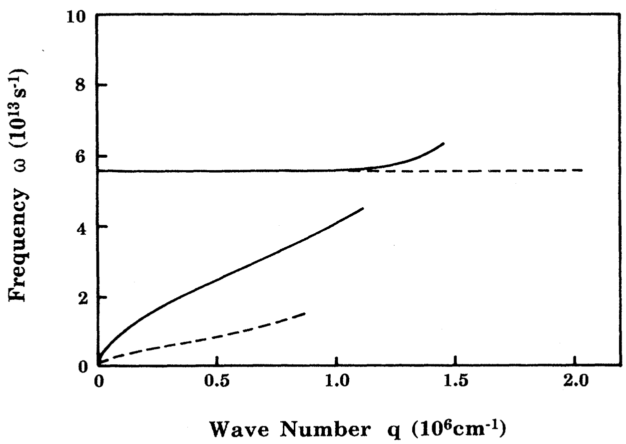

FIG. 2. Dispersion relations of the coupled plasmon and phonon modes for degenerate electron-hole plasma in the GaAs quantum well of 50 Å in width. The dashed and solid lines correspond to the plasma density $N=1\times10^{17}$ and 1×10^{18} cm^{-3}, respectively.

of χ is found to be zero, and therefore these modes exhibit no Landau damping.

As in the previous paper,[15] we define the ratio of phonon strength R

$$R=[\epsilon_0(\omega)-\epsilon_\infty]^2/\{[\epsilon_0(\omega)-\epsilon_\infty]^2+(4\pi\chi_1)^2\} , \quad (21)$$

since the charge fluctuations are proportional to the susceptibilities. The dashed ($N=1\times10^{17}$ cm^{-3}) and solid ($N=1\times10^{18}$ cm^{-3}) curves in Fig. 3 represent the phonon strength ratio of the coupled modes (the upper branch) in Fig. 2, indicating that at $N=1\times10^{17}$ cm^{-3} the coupled mode is almost phononlike, while it becomes rather plasmonlike at $N=1\times10^{18}$ cm^{-3}.

III. DISCUSSION

The magnetic field H_y of the coupled mode has been given as the odd function of x, so that the transverse electric field E_x is odd and the longitudinal electric field E_z is even. This indicates that the coupled mode can be taken as longitudinal on the average about x. However, it can also be regarded as propagating with zigzag ways in the well through the following expressions of the electric fields:

$$E_x=E_x^+\exp[-i(kx+qz)]-E_x^-\exp[i(kx-qz)] , \quad (22)$$

$$E_z=E_z^+\exp[-i(kx+qz)]+E_z^-\exp[i(kx-qz)] . \quad (23)$$

Figure 4 shows the ratio of the transverse to the longitudinal electric fields for the coupled modes in Fig. 2 calculated from

$$|E_x^\pm/E_z^\pm|=k/q , \quad (24)$$

which is derived from Eqs. (5), (6), and (9). In long wavelengths ($q<1\times10^6$ cm^{-1}) the coupled modes are found to be nearly transverse though there the frequency is equal to the longitudinal-optical frequency ω_l.

Our model for the GaAs QW can be easily extended to GaAs-AlGaAs heterojunctions by replacing $\epsilon_1(\omega)$ of one of the barrier layers with $\epsilon_0(\omega)$. Dispersion relations of the coupled modes for this heterojunction model are found to be very similar to those for the GaAs QW. Besides, we have examined a fictitious model in which the dielectric functions of both barrier layers are equal to $\epsilon_0(\omega)$, that is, the model in which the carriers are confined in a potential well but the dielectric function is homogeneous. In this case also we have obtained similar results to the GaAs QW except some difference in the ratio $|E_x^\pm/E_z^\pm|$.

As shown in Fig. 2 the mode coupling occurs only in the long-wavelength ranges. Because the long-range interactions between the plasmons and LO phonons are included in forming the coupled mode, the Fröhlich in-

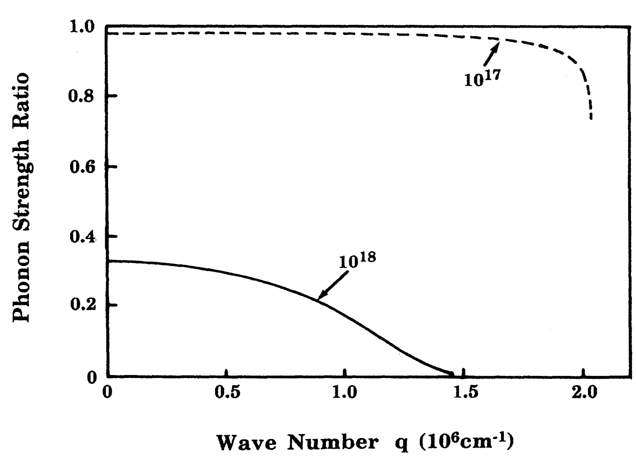

FIG. 3. Phonon strength ratio defined by Eq. (21) for the coupled modes (the upper branch in Fig. 2) as a function of the wave number.

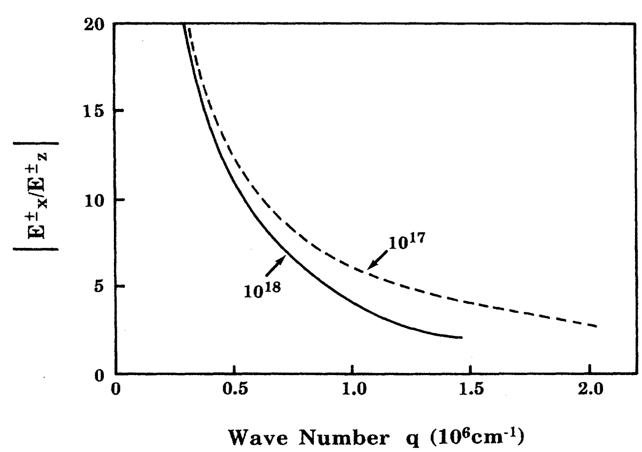

FIG. 4. Wave number dependence of the ratio of transverse to longitudinal electric fields for the coupled modes (the upper branch in Fig. 2).

teractions due to uncoupled LO phonons are unchanged. The wave number of phonons absorbed by the electrons and holes at the Fermi surface is in a limited range related to the Fermi wave number and the energy of the phonons. With $d = 50$ Å and $\omega_l = 5.52\times10^{13}$ s^{-1} we obtain the range of the wave number component q in the yz plane of the phonons absorbed by the electrons as follows: $2.03\times10^6 < q < 3.15\times10^6$ cm^{-1} at $N = 1\times10^{17}$ cm^{-3} and $1.31\times10^6 < q < 4.86\times10^6$ cm^{-1} at $N = 1\times10^{18}$ cm^{-3}. These ranges shift to higher values for the holes because of the large effective mass. The wave number of the coupled mode is found in Fig. 2 to be smaller than 2×10^6 and 1.5×10^6 cm^{-1} for $N = 1\times10^{17}$ and 1×10^{18} cm^{-3}, respectively. Consequently, we conclude that the plasmons do not have any significant effect on the carrier heating process due to the Fröhlich interaction.

We have neglected intersubband excitations in the calculation of χ. This may not be a good approximation to holes, where intersubband energy differences are relatively small. In the long-wavelength approximation, contribution of the intersubband excitation to the dielectric function is given as

$$-\omega_P^2\omega_F\omega_{\nu0}/\omega^2(\omega^2-\omega_{\nu0}^2) \ ,$$

where $\hbar\omega_F$ is the Fermi energy and $\hbar\omega_{\nu0}$ is the energy difference between the ground and the νth subbands.[20] Since the hole effective mass is large, this contribution may be negligible as compared to the dielectric function of the electron plasma, except cases of $\omega_l \simeq \omega_{\nu0}$.

In conclusion, we have calculated the dispersion relations of the coupled plasmon-LO phonon mode in the GaAs QW with assuming the dielectric continuum model for the lattice ions and the degenerate two-dimensional gas model for the carriers. To obtain the coupled mode, it is found that $\epsilon_\parallel < 0$ is necessary instead of $\epsilon_\parallel = 0$ in the exactly two-dimensional model. Since main phonons absorbed by the carriers at the Fermi surface remain uncoupled, the Fröhlich interaction participating with the carrier heating process are not screened.

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