Nonlinear optics and transport in laser-excited semiconductors

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The theory of the spectral and kinetic properties of the excited electron-hole plasma and of the electromagnetic field is developed in the framework of a nonequilibrium many-body theory. Particular emphasis is given to the effect of the laser-induced energy branches on the transport (intraband kinetics) and nonlinear-optical properties (interband kinetics) of the system.

I. INTRODUCTION

The aim of this paper is the description of a direct-gap semiconductor which is excited by an intensive, monochromatic laser in the spectral range of the fundamental gap. We will treat the excitation of an electron-hole (e-h) plasma and its kinetics, the nonlinear quantum optics of the strong, exciting laser light field in the medium, as well as the linear optical response of the highly excited system. Many aspects of this general problem have already been investigated, particularly the many-body aspects in a quasiequilibrium description (for reviews see Refs. 1 and 2), but only recently a consistent, unified nonequilibrium many-body theory begins to emerge.[3–6] The reason for the difficulty of a general theory is that the spectral and kinetic properties of the electrons and photons influence each other and have to be determined self-consistently. Many-body aspects as well as nonequilibrium aspects, particularly under short-pulse excitation, are equally important. The mixing of the valence and conduction band states by the coherent laser light leads to phenomena which resemble the optical Stark effect in atomic systems and in other respects the polariton effect in exciton systems.

We start with a general nonrelativistic Hamiltonian for the externally perturbed system of electrons in a semiconductor and of photons,

$$H = H_e + H_p + H_I + H_{\rm ext}(t) \ . \tag{1.1}$$

H_e and H_p describe the noninteracting electrons and photons, respectively; H_I describes their interactions and $H_{\rm ext}(t)$ the coupling to an external, time-dependent perturbation.

For simplicity, we treat the semiconductor in a two-band model

$$H_e = \sum_{a=e,h} \int d^3r \, \psi_a^\dagger(\mathbf{r}) \left[\frac{E_g}{2} - \frac{\hbar^2 \Delta}{2m_a} \right] \psi_a(\mathbf{r}) \ , \tag{1.2}$$

where $\psi_a(\mathbf{r})$, $\psi_a^\dagger(\mathbf{r})$ are the field operators of the electrons and holes, respectively, which are obtained from an expansion of the general field operators into Wannier functions.[7] E_g is the energy gap, m_a are the effective masses of the electrons and holes. A parabolic band edge has been assumed. The spin is not explicitly treated.

The Hamiltonian of the transverse photons is

$$H_p = \sum_{\nu,\mathbf{k}} \hbar\omega_k c_{\nu,\mathbf{k}}^\dagger c_{\nu,\mathbf{k}} \ . \tag{1.3}$$

Here, ν and $\mathbf{k}$ describe the polarization and momentum of the photons, respectively. The interaction Hamiltonian is

$$\begin{aligned} H_{\rm int} = \tfrac{1}{2} \sum_{a,b} e_a e_b \int d^3r \int d^3r' \, \psi_a^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}') \\ \times V(|\mathbf{r}-\mathbf{r}'|) \psi_b(\mathbf{r}') \psi_a(\mathbf{r}) \\ - \frac{1}{c} \int d^3r \, \mathbf{j}(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r},t) \\ + \frac{1}{2mc^2} \int d^3r \, \rho(\mathbf{r}) \mathbf{A}^2(\mathbf{r},t) \ . \end{aligned} \tag{1.4}$$

The Coulomb potential is

$$V(r) = \frac{1}{\epsilon_0 r} \ .$$

ϵ_0 is the dielectric constant of the unexcited crystal. The last two terms describe the interaction with the light field. The vector potential is

$$\mathbf{A}(\mathbf{r}) = \sum_{\nu,\mathbf{k}} \left[\frac{2\pi\hbar c^2}{V\omega_k} \right]^{1/2} \mathbf{e}_\nu(\mathbf{k}) (c_{\nu,\mathbf{k}}^\dagger + c_{\nu,-\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{r}} \ . \tag{1.5}$$

The charge density is given by

$$\rho(\mathbf{r}) = \sum_a e_a \psi_a^\dagger(\mathbf{r}) \psi_a(\mathbf{r}) \tag{1.6}$$

and the current density by

$$\mathbf{j}(\mathbf{r}) = \int d^3r [\mathbf{j}_{\rm cv}(\mathbf{r}-\mathbf{r}') \psi_e^\dagger(\mathbf{r}) \psi_h^\dagger(\mathbf{r}') + \text{H.c.}] \ , \tag{1.7}$$

where $\mathbf{j}_{\rm cv}(\mathbf{r})$ is the nonlocal interband-current-density matrix element.[8]

The external perturbations are written in form of externally controlled charges $\rho_{\rm ext}(\mathbf{r},t)$ and currents $\mathbf{j}_{\rm ext}(\mathbf{r},t)$,

$$H_{\rm ext}(t)=\int d^3r' \left\{\rho_{\rm ext}(\mathbf{r}',t)\Phi(\mathbf{r}')-\frac{1}{c}\mathbf{j}_{\rm ext}(\mathbf{r}',t)\cdot\mathbf{A}(\mathbf{r}') + \frac{1}{2mc^2}\rho_{\rm ext}(\mathbf{r}',t)\,\mathbf{A}^2(\mathbf{r}')\right\} . \tag{1.8}$$

The scalar potential $\Phi(\mathbf{r})$ is the solution of the Poisson equation with the total charge density $\rho_t=\rho+\rho_{\rm ext}$,

$$\Phi(\mathbf{r})=\int d^3r'\, V(|\mathbf{r}-\mathbf{r}'|)\rho_t(\mathbf{r}') . \tag{1.9}$$

Physically, the external charges and currents belong to the exciting laser. However, they will not be specified further, but will be used formally to develop the equations for the nonequilibrium Green's function by the functional derivative technique.[9-13]

In Sec. II the basic equations of motion for the expectation values of the field operators as well as for all necessary Green's functions will be given. The approximations, which will be used for the material functions in these equations, will be discussed. We will assume, that the vector potential $\mathbf{A}$ has due to the laser action a finite expectation value,

$$\langle\,\mathbf{A}(\mathbf{r})\,\rangle=\mathbf{A}_+(\mathbf{r},t)e^{i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)}+\text{c.c.} , \tag{1.10}$$

where $\mathbf{k}_0$ and ω_0 are the wave number and frequency of the coherent, monochromatic laser beam, respectively. The amplitude $\mathbf{A}_\pm$ will be treated as a slowly varying function.

In Sec. III we develop the nonequilibrium description of an electron-hole plasma. In addition to the many-body effects, which are already present in a quasiequilibrium description,[1,2] such as gap shrinkage and excitonic enhancement, there is a renormalization brought about by the $\mathbf{j}\cdot\mathbf{A}$ interband coupling. The renormalization of the electron and hole dispersion due to this coherent optical coupling is proportional to the local intensity A^2 of the laser light beam.[6,14-16] The kinetic equations for the Wigner distribution of the quasiparticles with this renormalized dispersion are derived. Here, all four branches, which reduce for $\langle\,\mathbf{A}\,\rangle\to 0$ to ε_e, ε_h, and $\hbar\omega_0-\varepsilon_h$, and $\hbar\omega_0-\varepsilon_e$, are taken into account. By symmetry arguments, one can show that only two branches are independent, but the existence of these additional laser induced bands influences the physical properties of the system. For example, there arise transitions between all four branches. The electron-hole pair amplitude resembles the anomalous propagators of a superconductor. In the static screening approximation for the interband Coulomb self-energy, a closed equation for the one-time *e-h* propagator can be obtained,[7] which contains, however, in addition to earlier formulations[8,17] the high-field renormalization effects. In analogy with superconductivity one obtains an integral equation for the gap function, which describes the strength of the interband mixing.[3,17] In contrast to superconductivity, the gap equation has an inhomogeneity given by $\lambda_0=\mathbf{j}_{\rm cv}\cdot\langle\,\mathbf{A}_\pm\,\rangle$; the solution of the gap equation can thus be interpreted in terms of a vertex renormalization of the interband dipole matrix element. In the limit $\langle\,\mathbf{A}\,\rangle\to 0$, the gap equation degenerates and has to be understood as an eigenvalue problem for the now-undetermined frequency ω_0, which serves in the quasiequilibrium as chemical potential for the incoherently excited plasma.[18] Another interesting problem in this context is the question whether or not in the framework of a many-body theory does the inset of laser action in the semiconductor lead to a spontaneous symmetry breaking, i.e., to $\langle\,\mathbf{A}\,\rangle\neq 0$ for the lasing mode (compare Refs. 19 and 20).

In order to investigate the propagation of the laser beam in the medium, it turns out to be simpler to introduce the terms $\langle\,\mathbf{j}\,\rangle\cdot\langle\,\mathbf{A}\,\rangle$ and $\langle\,\mathbf{j}\,\rangle\cdot\langle\,\mathbf{E}\,\rangle$,[5] instead of the *e-h* propagator, which determines the interband polarization. In Sec. IV, these quantities are introduced and related to the electron-hole kinetics. Particularly, the energy dissipation rate $\langle\,\mathbf{j}\,\rangle\cdot\langle\,\mathbf{E}\,\rangle$ of the laser light is shown to be given by that portion of the Boltzmann collision integrals for the carriers which contain laser-induced transitions.[6] Furthermore, the Green's functions for the longitudinal (plasmons) and transverse ("incoherent" photons) Bose excitations are investigated. The scattering of these excitations determines the *e-h* kinetics and the propagation and degree of coherence of the light beam. In this paper we dispense with the dynamical treatment of the plasmons and treat them as a bath. However, the dynamical treatment of the photons is crucial. According to their definition, the Green's functions describe only the fluctuations of the light field. Thus they contain just the linear optical properties of the laser-excited semiconductors. For the *e-h* kinetics and the light beam propagation it is often appropriate to neglect the finite photon population and to consider only the spontaneous emission into a photon vacuum. In this approximation one neglects the feedback of the scattered light on the excitation processes. Nevertheless, a kinetic equation for the photons is needed for the determination of the light scattered out of the coherent beam. The resulting photon distribution shows interesting features at the resonance frequency ω_0.

II. FUNCTIONAL DERIVATIVE TECHNIQUE FOR NONEQUILIBRIUM SYSTEMS

A. Expectation values and the double time contour

For a systematic investigation of our system described by the Hamiltonian (1.1)–(1.9), we follow closely the techniques of Refs. 5 and 9–13, and introduce the following formal definition of an expectation value of an arbitrary field operator:

$$\langle O(\underline{1})\rangle_C=\frac{\mathrm{tr}\{\rho(-\infty)T_C[S_C\tilde{O}(\underline{1})]\}}{\mathrm{tr}[\rho(-\infty)S_C]} , \tag{2.1}$$

where the underlined argument $\underline{1}=\mathbf{r},\underline{t}_1$ is defined on the double-time contour C: $-\infty\to+\infty\to-\infty$. $\rho(-\infty)=\rho_0$ is the statistical equilibrium operator. T_C is the contour-time ordering operator and S_C is the time evolution operator

$$S_C = T_C \exp\left[-\frac{i}{\hbar}\int_C d\underline{\tau}\,\tilde{H}_{\rm ext}(\underline{\tau})\right] \tag{2.2}$$

for the external perturbation. The tilde indicates the interaction representation. The functions $\rho_{\rm ext}(\underline{1})$ and $\mathbf{j}_{\rm ext}(\underline{1})$ contained in $H_{\rm ext}$ will also be defined on C. In the physical limit, the functions on the positive $(-\infty\rightarrow\infty)$ and negative branch are equal to each other, e.g., $\rho_{\rm ext}(\mathbf{r},t_+)=\rho_{\rm ext}(\mathbf{r},t_-)$. In this limit $S_C=1$, so that (2.1) reduces to the usual expectation value. In the following we will derive all equation of motion by functional derivatives. At the end, the physical limit is taken, i.e., $\langle O(\underline{1})\rangle_C\rightarrow\langle O(1)\rangle$. The averaged potentials obey in the Coulomb gauge the following equations:

$$\Delta_1\langle\Phi(\underline{1})\rangle_C=-4\pi\langle\rho(\underline{1})\rangle_C\ , \tag{2.3}$$

$$\left[\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]_1\langle\mathbf{A}(\underline{1})\rangle_C = -\frac{4\pi}{c}\langle\mathbf{j}(\underline{1})\rangle_C+4\pi\langle\rho(\underline{1})\mathbf{A}(\underline{1})\rangle_C\ , \tag{2.4}$$

where the last two terms combine to give the proper current expectation value in the presence of a vector potential. The last term will be neglected in the following.

B. Green's functions

The one-particle Green's function (GF) for an electron-hole plasma is defined by

$$G_{ab}(\underline{1},\underline{2})=-\frac{i}{\hbar}\langle\psi_a(\underline{1})\psi_b^\dagger(\underline{2})\rangle_C\ , \tag{2.5}$$

where $\psi_a=(\psi_e,\psi_h^\dagger)$ for $a=(e,h)$. Thus the matrix structure of (2.5) with respect to the particle index is

$$G(\underline{1},\underline{2})=-\frac{i}{\hbar}\begin{bmatrix}\langle\psi_e(\underline{1})\psi_e^\dagger(\underline{2})\rangle_C & \langle\psi_e(\underline{1})\psi_h(\underline{2})\rangle_C\\ \langle\psi_h^\dagger(\underline{1})\psi_e^\dagger(\underline{2})\rangle_C & \langle\psi_h^\dagger(\underline{1})\psi_h(\underline{2})\rangle_C\end{bmatrix}.$$

In the physical limit, each GF is a 2×2 matrix,[9] depending whether $\underline{t}_1$ and $\underline{t}_2$ lie on the positive or negative branch of the contour,[11]

$$G(\underline{1},\underline{2})=\begin{bmatrix}G(1_+,2_+) & G(1_+,2_-)\\ G(1_-,2_+) & G(1_-,2_-)\end{bmatrix} =\begin{bmatrix}G(1,2) & G^<(1,2)\\ G^>(1,2) & \bar{G}(1,2)\end{bmatrix}, \tag{2.6}$$

where G and $\bar{G}$ are the time- and antitime-ordered GF's, respectively. Similarly, $G^<$ and $G^>$ are the particle and hole propagators.

In notation of Refs. 11 and 12, one has a Dyson equation in the form of[5]

$$[G_{ac}^{0\text{-}1}(\underline{1},\underline{1}')-\Sigma_{ac}(\underline{1},\underline{1}')]G_{cb}(\underline{1}'\underline{2})=\delta_{ab}(\underline{1},\underline{2})\ . \tag{2.7}$$

The summation convention is used for all repeated indices and variables, which appear only on one side of the equation, i.e., on the left-hand side (lhs) an integration over

$$\int d^3r_1'\int_C d\underline{t}_1'$$

is included.

The inverse of the free-particle GF is

$$G_{ab}^{0\text{-}1}(\underline{1},\underline{2})=\left[\left[i\hbar\frac{\partial}{\partial t_1}-h_a(\underline{1})\right]\delta_{ab}(\underline{1},\underline{2}) +\frac{1}{c}\mathbf{j}_{ab}(\underline{1},\underline{2})\cdot\mathbf{A}_{\rm eff}(\underline{2})\right], \tag{2.8}$$

with the one-particle Schrödinger operator

$$h_a(\underline{1})=\pm\left[\frac{E_g}{2}-\frac{\hbar^2}{2m_a}\Delta_1+e_a\left[\Phi_{\rm eff}(\underline{1})+\frac{A_{\rm eff}(\underline{1})^2}{2mc^2}\right]\right], \tag{2.9}$$

$$\Phi_{\rm eff}(\underline{1})=\Phi_{\rm ext}(\underline{1})+\langle\Phi(\underline{1})\rangle_C\ , \tag{2.10}$$

$$\mathbf{A}_{\rm eff}(\underline{1})=\langle\mathbf{A}(\underline{1})\rangle_C\ , \tag{2.11}$$

and $\mathbf{J}_{ee}=\mathbf{j}_{hh}=0$, as well as

$$\mathbf{j}_{eh}(\underline{1}\text{-}\underline{2})=\mathbf{j}_{\rm cv}(\mathbf{r}_1-\mathbf{r}_2)\delta(\underline{t}_1-\underline{t}_2)\ .$$

The self-energies are given by the following functional equation:

$$\Sigma_{ab}(\underline{1},\underline{2})=-4\pi i\hbar e_a G_{ac}(\underline{1},\underline{1}')\frac{\delta G_{cb}^{-1}(\underline{1}',\underline{2})}{\delta\Phi_{\rm eff}(\underline{3})}d(\underline{3},\underline{1}) -\frac{1}{c}\mathbf{j}_{ac}G_{cd}(\underline{1},\underline{1}')\frac{\delta G_{db}^{-1}(\underline{1}',\underline{2})}{\delta\langle\mathbf{A}(\underline{3})\rangle}\overleftrightarrow{\mathrm{D}}(\underline{3},\underline{1}) -\frac{e^2}{2mc^2}[\langle A^2(\underline{1})\rangle-\langle\mathbf{A}(\underline{1})\rangle^2]\delta_{ab}\delta(\underline{1}\text{-}\underline{2})\ . \tag{2.12}$$

In the following we will use Eq. (2.12) in the random-phase approximation (RPA) with $G_{ab}^{-1}\simeq G_{ab}^{0\text{-}1}$.

The longitudinal and transverse photon GF's in (2.11) are given by

$$d(\underline{1},\underline{2})=-\frac{1}{4\pi}\frac{\delta\Phi_{\rm eff}(\underline{1})}{\delta\rho_{\rm ext}(\underline{2})} =\frac{1}{4\pi}V(\underline{1}\text{-}\underline{2}) +\frac{1}{4\pi}[\langle\Phi(\underline{1})\Phi(\underline{2})\rangle_C-\langle\Phi(\underline{1})\rangle_C\langle\Phi(\underline{2})\rangle_C]\ , \tag{2.13}$$

$$D_{ik}(\underline{1},\underline{2})=-\frac{c}{4\pi}\frac{\delta A_{i\rm eff}(\underline{1})}{\delta j_{k\rm ext}(\underline{2})} =\frac{1}{4\pi}[\langle A_i(\underline{1})A_k(\underline{2})\rangle_C-\langle A_i(\underline{1})\rangle_C\langle A_k(\underline{2})\rangle_C]\ , \tag{2.14}$$

where

$$V(\underline{1}\text{-}\underline{2})=\frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}\delta(t_1-t_2)\ .$$

Equations (2.13) and (2.14) describe the fluctuations in the electromagnetic field, i.e., its incoherent part.

$$[d_0^{-1}(\underline{1},\underline{1}')-p(\underline{1},\underline{1}')]d(\underline{1}'\underline{2})=\delta(\underline{1}-\underline{2}) , \tag{2.15}$$

$$[\vec{\vec{D}}_0^{-1}(\underline{1},\underline{1}')-\vec{\vec{P}}(l,l')]\vec{\vec{D}}(\underline{1}',\underline{2})=\vec{\vec{\delta}}_T(\underline{1}-\underline{2}) , \tag{2.16}$$

where $\vec{\vec{\delta}}_T(\underline{1}-\underline{2})$ is the transverse δ function:

$$\vec{\vec{\delta}}_T(\underline{1},\underline{2})=\left[\nabla_1\nabla_2\frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}+\vec{\vec{1}}\delta(\mathbf{r}_1-\mathbf{r}_2)\right]\delta(\underline{t}_1-\underline{t}_2) . \tag{2.17}$$

The photon self-energies, called the polarization functions, are defined by

$$p(\underline{1},\underline{2})=-4\pi\frac{\delta\langle\rho(\underline{1})\rangle_C}{\delta\phi_{\text{eff}}(\underline{2})} , \tag{2.18}$$

$$\vec{\vec{P}}(\underline{1},\underline{2})=-\frac{4\pi}{c}\frac{\delta\langle\mathbf{j}(\underline{1})\rangle_C}{\delta\mathbf{A}_{\text{eff}}(\underline{2})} . \tag{2.19}$$

The expectation values of the charge and current densities can be expressed by particle Green's functions:

$$\langle\rho(\underline{1})\rangle=i\hbar e_a G_{aa}(\underline{1},\underline{2}^+)|_{2^+=1} , \tag{2.20}$$

$$\langle\mathbf{j}(\underline{1})\rangle=2\hbar\,\mathrm{Im}\mathbf{j}_{cv}(\underline{1}-\underline{2})G_{he}(\underline{1},\underline{2}^+)|_{2^+=1} . \tag{2.21}$$

With these expressions we find for the polarization functions:

$$p(\underline{1},\underline{2})=4\pi i\hbar e_a G_{ab}(\underline{1},\underline{3})\frac{\delta G_{bc}^{-1}(\underline{3},\underline{4})}{\delta\phi_{\text{eff}}(\underline{2})}G_{ca}(\underline{4},\underline{1}) , \tag{2.22}$$

$$\vec{\vec{P}}(\underline{1},\underline{2})=-\frac{4\pi i\hbar}{c}\mathbf{j}_{cv}(0)G_{ha}(\underline{1},\underline{4})\frac{\delta G_{ab}^{-1}(\underline{4},\underline{5})}{\delta\mathbf{A}_{\text{eff}}(\underline{2})}G_{be}(\underline{5},\underline{1}) - \text{c.c.} . \tag{2.23}$$

These expressions will again be used in the RPA. With this approximation Eqs. (2.3), (2.4), (2.7), (2.15), and (2.16) form a closed system of equations for the determination of $\langle\Phi\rangle$, $\langle A\rangle$, G_{ab}, d, and $\vec{\vec{D}}$, in which the material functions $\langle\rho\rangle$, $\langle\mathbf{j}\rangle$, Σ_{ab}, p, and $\vec{\vec{P}}$ have to be determined self-consistently due to Eqs. (2.20), (2.21), (2.12), (2.22) and (2.23).

III. NONEQUILIBRIUM ELECTRON-HOLE PLASMA

A. Reformulation of the Dyson equation

In order to investigate the intraband Green's functions G_{ee} and G_{hh}, we eliminate according to Ref. 5 in their Dyson equations the interband Green's functions G_{eh}. Introducing instead of the time- and antitime-ordered Green's functions, the retarded and advanced Green's functions by

$$G^r(1,2)=G(1,2)-G^<(1,2)=G^>(1,2)-\bar{G}(1,2) , \tag{3.1}$$

$$G^a(1,2)=G(1,2)-G^>(1,2)=G^<(1,2)-\bar{G}(1,2) . \tag{3.2}$$

we find the equations

$$[G_{ee}^{0\text{-}1}(1,1')-\tilde{\Sigma}_{ee}^{r,a}(1,1')]G_{ee}^{r,a}(1',2)=\delta(1\text{-}2) , \tag{3.3}$$

$$[G_{ee}^{0\text{-}1}(1,1')-\tilde{\Sigma}_{ee}^{r}(1,1')]G_{ee}^{\gtrless}(1',2)=\tilde{\Sigma}_{ee}^{\gtrless}(1,1')G_{ee}^{a}(1',2) , \tag{3.4a}$$

and

$$G_{ee}^{\gtrless}(1,1')[G_{ee}^{0\text{-}1}(1',2)-\tilde{\Sigma}_{ee}^{a}(1',2)]=G_{ee}^{r}(1,1')\tilde{\Sigma}_{ee}^{\gtrless}(1',2) , \tag{3.4b}$$

and corresponding equations for the holes ($e\rightarrow h$). The self-energy

$$\tilde{\Sigma}_{ee}(\underline{1},\underline{2})=\Sigma_{ee}(\underline{1},\underline{2})+G_{eh}^{-1}(\underline{1},\underline{1}')G_h(\underline{1}',\underline{2}')G_{eh}^{-1}(\underline{2}',\underline{2}) \tag{3.5}$$

contains in addition to the original diagonal self-energy Σ_{ee} a laser-induced part, which stems from the elimination of G_{eh}. The function $G_h(\underline{1},\underline{2})$ is defined by the relation

$$G_h(\underline{1},\underline{1}')G_{hh}^{-1}(\underline{1}',\underline{2})=\delta(\underline{1}-\underline{2}) . \tag{3.6}$$

In order to treat Eqs. (3.4) and (3.45) further, we introduce the "local" variables $\mathbf{r}=(\mathbf{r}_1+\mathbf{r}_2)/2$ and $t=(t_1+t_2)/2$, while we introduce Fourier transforms with respect to the relative variables $\mathbf{r}_1-\mathbf{r}_2$, t_1-t_2, respectively. We call $\mathbf{k},\omega$ the spectral variables. All functions depend now on the variables $\xi=(\mathbf{K};\mathbf{R})$, where we use the four-dimensional vectors $\mathbf{K}=(\mathbf{k},i\omega/c)$ and $\mathbf{R}=(\mathbf{r},ict)$. In a homogeneous, stationary situation the dependence on the local variables vanishes. Generally, we assume that the dependence on the local variables is so weak, that linear gradient expansions can be used. For a detailed derivation of spectral and kinetic equations we refer to Refs. 5, 6, and 21. In the following we will use some of the results of these derivations.

B. The quasiparticle spectrum

The poles of the retarded GF determine the quasiparticle spectra. From Eq. (3.3) we get in the ξ representation (compare also Refs. 5, 6, and 13)

$$\varepsilon_{e,1,2}(\mathbf{k},\mathbf{R})=\tfrac{1}{2}\{\varepsilon_e(\mathbf{k},\mathbf{R})+[\hbar\omega_0-\varepsilon_h(\mathbf{k}_0-\mathbf{k},\mathbf{R})]\} \pm\tfrac{1}{2}(\{\varepsilon_e(\mathbf{k},\mathbf{R})-[\hbar\omega_0-\varepsilon_h(\mathbf{k}_0-\mathbf{k},\mathbf{R})]\}^2 +\lambda^2(\mathbf{k},\mathbf{R}))^{1/2} , \tag{3.7}$$

$$\varepsilon_{h1,2}(\mathbf{k},\mathbf{R})=\hbar\omega_0-\varepsilon_{e1,2}(\mathbf{k}_0-\mathbf{k},\mathbf{R}) .$$

The energies $\varepsilon_a(\mathbf{k},\mathbf{R})$ contain already the renormalization due to the intraband self-energies, i.e., they are the complex solutions of the equations

$$\hbar\omega-\varepsilon_a(\mathbf{k})-\Sigma_{aa}^r(\xi)=G_{aa}^{r\,-1}(\xi)=0 , \tag{3.8}$$

where $\varepsilon_a(\mathbf{k})$ are according to (1.2) the free-particle energies of electrons and holes:

$$\varepsilon_a(\mathbf{k})=\left[\frac{\varepsilon_g}{2}+\frac{\hbar^2k^2}{2m_a}\right] . \tag{3.9}$$

Σ_{aa}^r does not contain interband coupling. The parameter λ in Eq. (3.7) represents the laser-induced interband coupling

$$\lambda(\mathbf{k},\mathbf{R})=\mathbf{j}_{cv}(\mathbf{k})\langle\,\mathbf{A}(\mathbf{R})\,\rangle-\Sigma^r_{eh}(\mathbf{k},\mathbf{R})\ . \tag{3.10}$$

Σ^r_{eh} is the RPA Coulomb interband self-energy which we take[7] in the nonretarded approximation

$$\begin{aligned}&\Sigma^r_{eh}(1,2)\simeq\Sigma^r_{eh}(\mathbf{r}_1,\mathbf{r}_2,t_1)\delta(t_1-t_2)\ ,\\&\Sigma^r_{eh}(\xi)\simeq\Sigma^r_{eh}(\mathbf{k},\mathbf{R})\ .\end{aligned} \tag{3.11}$$

The effective interband coupling causes a resonance interaction between the states with the energies $\varepsilon_a(\mathbf{k},\mathbf{R})$ and the laser-induced states with $\hbar\omega_0-\varepsilon_b(\mathbf{k}_0-\mathbf{k},\mathbf{R})$ $(a\neq b)$. Disregarding the small photon wave number k_0 in (3.7), and the damping of the valence- and conduction-band electrons, i.e., $\mathrm{Im}[\varepsilon_e(\mathbf{k},\mathbf{R})+\varepsilon_h(\mathbf{k},\mathbf{R})]\simeq 0$, one gets at the resonance wave number $\mathbf{k}_r$,

$$\varepsilon_e(\mathbf{k}_r,\mathbf{R})=\hbar\omega_0-\varepsilon_h(\mathbf{k}_r,\mathbf{R})\ , \tag{3.12}$$

a laser-induced gap of the size $|\lambda(\mathbf{k}_r,\mathbf{R})|$. Away from the resonance at $\mathbf{k}_r$ the dispersions have the following asymmetric behavior (see Fig. 1):

For $k>k_r$,

$$\begin{aligned}&\varepsilon_{e1}\to\varepsilon_e,\quad \varepsilon_{e2}\to\hbar\omega_0-\varepsilon_h\ ,\\&\varepsilon_{h1}\to\varepsilon_h,\quad \varepsilon_{h2}\to\hbar\omega_0-\varepsilon_e\ .\end{aligned} \tag{3.13a}$$

For $k<k_r$,

$$\begin{aligned}&\varepsilon_{e1}\to\hbar\omega_0-\varepsilon_h,\quad \varepsilon_{e2}\to\varepsilon_e\ ,\\&\varepsilon_{h1}\to\hbar\omega_0-\varepsilon_e,\quad \varepsilon_{h2}\to\varepsilon_h\ .\end{aligned} \tag{3.13b}$$

Each of the four dispersions (3.7) thus changes its character at the resonance $\mathbf{k}_r$. If it corresponds to states with the particle energies $\varepsilon_a(\mathbf{k},\mathbf{R})$ on one side, it corresponds to laser-induced states with $\hbar\omega_0-\varepsilon_b(\mathbf{k},\mathbf{R})$ on the other side $(a\neq b)$. This feature is well known from, e.g., the polariton effect in semiconductors. In contrast to the situations with polaritons, we have an exact pairwise symmetry between the four dispersions. Thus we will limit ourselves in the following to those branches of (3.7) which approach for $\lambda\to 0$ the particle dispersions (3.8). These "physical" dispersions are given for $a=e,h$ by

$$\tilde{\varepsilon}_a(\mathbf{k},\mathbf{R})=\begin{cases}\varepsilon_{a1}(\mathbf{k},\mathbf{R}) & \text{for } k>k_r\ ,\\ \varepsilon_{a2}(\mathbf{k},\mathbf{R}) & \text{for } k<k_r\ .\end{cases} \tag{3.14}$$

With (3.14) the retarded GF's have the following structure:

$$\begin{aligned}G^r_{ee}(\xi)&=\frac{a(\mathbf{k},\mathbf{R})}{\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})}+\frac{b(\mathbf{k},\mathbf{R})}{\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})}\ ,\\ G^r_{hh}(\xi)&=\frac{a(\mathbf{k},\mathbf{R})}{\hbar\omega+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})}\ \frac{b(\mathbf{k},\mathbf{R})}{\hbar(\omega+\omega_0)-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})}\ ,\end{aligned} \tag{3.15}$$

where the weight coefficients are given by

$$\begin{aligned}&a(\mathbf{k},\mathbf{R})+b(\mathbf{k},\mathbf{R})=1\ ,\\&b(\mathbf{k},\mathbf{R})=\frac{\tilde{\varepsilon}_e-\varepsilon_e}{\tilde{\varepsilon}_e+\tilde{\varepsilon}_h-\hbar\omega_0}=\frac{\tilde{\varepsilon}_h-\varepsilon_h}{\tilde{\varepsilon}+\tilde{\varepsilon}_h-\hbar\omega_0}\ .\end{aligned} \tag{3.16}$$

The original particle states and the laser-induced states have the weights a and b, respectively. In the approximation (3.10) a and b are real, and with the exception of the resonance region one has $a\simeq 1$, $b\simeq 0$.

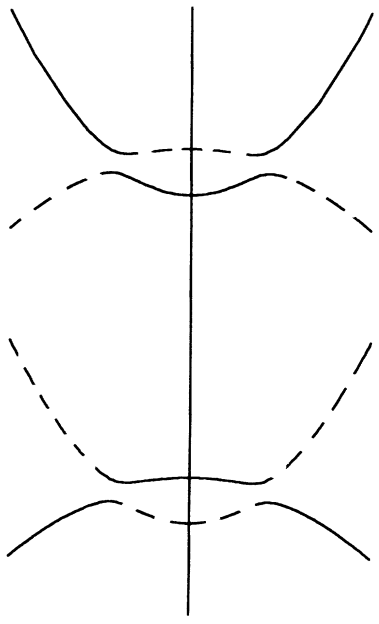

FIG. 1. Schematic band structure. Laser-induced bands, dashed lines; physical bands, solid lines.

C. Kinetic equations for electrons and holes

The intraband kinetics, e.g., of the electrons in the renormalized bands, can be obtained from Eqs. (3.4), which have the usual form for a single-band problem, but contain a redefined self-energy. From the difference of Eqs. (3.4a) and (3.4b) one obtains by standard methods the quantum Boltzmann equation[3,21]

$$\begin{aligned}i\left[\frac{\partial}{\partial t}+\frac{\mathbf{k}\nabla_r}{m}\right]G^<=&\tilde{\Sigma}^>G^<-\tilde{\Sigma}^<G^>+i\,\mathrm{Re}\{\tilde{\Sigma}^r,G^<\}\\&+i\{\tilde{\Sigma}^<,\mathrm{Re}G^r\}\ ,\end{aligned} \tag{3.17}$$

and

$$\{a,b\}=\frac{\partial a}{\partial R_i}\frac{\partial b}{\partial K_i}-\frac{\partial a}{\partial K_i}\frac{\partial b}{\partial R_i}$$

is the Poisson bracket written in terms of the Minkowski vectors $\mathbf{R}=\mathbf{r}$, ict, and $\mathbf{K}=\mathbf{k}$, $i\omega/c$.

The lhs can also be written as a Poisson bracket

$$\begin{aligned}i\left[\frac{\partial\omega}{\partial\omega}\frac{\partial G}{\partial t}+\frac{\partial\epsilon_k}{\partial k_i}\frac{\partial}{\partial r_i}\right]G^<&\\=i\left[\frac{\partial G^{0\text{-}1}}{\partial\omega}\frac{\partial G^<}{\partial t}-\frac{\partial G^{0\text{-}1}}{\partial k_i}\frac{\partial G^<}{\partial r_i}\right]&\\=i\{G^{0\text{-}1},G^<\}\ ,&\end{aligned}$$

where $G^{0\text{-}1}=\omega-\epsilon_k$. Neglecting the Poisson bracket between $\Sigma^<$ and $\mathrm{Re}G^r$ in (3.17), this equation can now be rewritten as

$$i\{G^{r\text{-}1},G^<\}=\tilde{\Sigma}^>G^<-\tilde{\Sigma}^<G^>\ . \tag{3.18}$$

A frequency integration over particle and hole propoagators $G^{\lessgtr}_{aa}$ yields the Wigner distributions of the electrons and holes,

$$\frac{\hbar}{i}\int\frac{d\omega}{2\pi}G^{<}_{ee}(\xi)=f_e(\mathbf{k},\mathbf{R}) ,$$
$$-\frac{\hbar}{i}\int\frac{d\omega}{2\pi}G^{>}_{hh}(-\xi)=f_h(\mathbf{k},\mathbf{R}) , \tag{3.19}$$

where $-\xi=(-\mathbf{k},-\omega,\mathbf{R})$.

The quantum Boltzmann equation (3.18) allows one, however, not to derive a closed system of equations for the Wigner distribution. Therefore, we will make the ansatz

$$iG^{<}_{ee}(\xi)=A_{ee}(\xi)f_e(\xi) , \tag{3.20}$$

where $A_{ee}(\xi)$ is the spectral function

$$A_{ee}(\xi)=-2\,\mathrm{Im}G^{r}_{ee}(\xi) . \tag{3.21}$$

It should be noted that the distribution function $f_e(\xi)$ does not have to be an equilibrium distribution because (3.21) is merely a definition of the still unknown function f_e. For the spectral function we use the quasiparticle approximation, so that

$$G^{<}_{ee}(\xi)=2\pi i[a\delta(\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R}))$$
$$+b\delta(\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R}))]f_e(\xi) . \tag{3.22}$$

Equation (3.22) shows that $f_e(\xi)$ is given by two frequency-independent distributions

$$\tilde{f}_e(\mathbf{k},\mathbf{R})=f_e(\xi)\,|_{\hbar\omega=\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})} ,$$
$$\tilde{g}_e(\mathbf{k},\mathbf{R})=f_e(\xi)\,|_{\hbar\omega=\hbar\omega_0-\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})} . \tag{3.23}$$

$\tilde{f}_e$ is the distribution of the physical branch, and $\tilde{g}_e$ is that of the laser-induced branch. Correspondingly, one finds for the holes

$$\tilde{f}_h(\mathbf{k},\mathbf{R})=f_h(\xi)\,|_{\hbar\omega=\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})} ,$$
$$\tilde{g}_h(\mathbf{k},\mathbf{R})=f_h(\xi)\,|_{\hbar\omega=\hbar\omega_0-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})} . \tag{3.24}$$

Inserting the ansatz (3.22) into the quantum Boltzmann equation (3.18), one gets by integrating the frequency over the vicinity of the various poles four equations for the distribution functions $\tilde{f}_e,\tilde{g}_e,\tilde{f}_h,\tilde{g}_h$. Within our approximations one finds

$$\tilde{g}_e=1-\tilde{f}_h ,$$
$$\tilde{g}_h=1-\tilde{f}_e . \tag{3.25}$$

Thus one has only two independent kinetic equations for $\tilde{f}_e$ and $\tilde{f}_h$. Following Refs. 5 and 6, the lhs of the equation, e.g., the electron distribution, can be obtained inserting the ansatz (3.22) together with

$$G^{r}_{ee}(\xi)=\left[\frac{a}{\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})}+\frac{b}{\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})}\right]$$
$$=\frac{a[\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})]+b[\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})]}{[\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})][\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})]} \tag{3.26}$$

into (3.18). Making use of the properties $\{F(A),Af\}=\{F(A),f\}A$ one gets after, a frequency integration around $\tilde{\varepsilon}_e$,

$$\left\{\frac{[\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})][\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})]}{a[\hbar(\omega-\omega_0)+\tilde{\varepsilon}_h(\mathbf{k},\mathbf{R})]+b[\hbar\omega-\tilde{\varepsilon}_e(\mathbf{k},\mathbf{R})]},f(\omega)\right\}a\Big|_{\hbar\omega=\tilde{\varepsilon}_e}=\{(\hbar\omega-\tilde{\varepsilon}_e),f(\omega)\}\,|_{\hbar\omega=\tilde{\varepsilon}_e}$$
$$=\left[\frac{\partial}{\partial t}-\frac{\partial\tilde{\varepsilon}_e}{\partial r_i}\frac{\partial}{\partial k_i}+\frac{\partial\tilde{\varepsilon}_e}{\partial k_i}\frac{\partial}{\partial r_i}\right]\tilde{f}_e , \tag{3.27}$$

where

$$\frac{\partial}{\partial t}\tilde{f}_e=\frac{\partial\tilde{\varepsilon}_e}{\partial t}\frac{\partial f_e(\omega)}{\partial\omega}\Big|_{\hbar\omega=\tilde{\varepsilon}_e}+\frac{\partial f(\omega)}{\partial t}\Big|_{\hbar\omega=\tilde{\varepsilon}_e}$$

has been used. The weight function a cancels exactly, and the renormalized energies appear in the drift term.

Next we turn to the evaluation of the rhs of the kinetic equation. The frequency integration yields

$$a[\tilde{\Sigma}^{>}_{ee}(\hbar\omega=\tilde{\varepsilon}_e)\tilde{f}_e-\tilde{\Sigma}^{<}_{ee}(\hbar\omega=\tilde{\varepsilon}_e)(1-\tilde{f}_e)] . \tag{3.28}$$

From (3.5) one sees that $\tilde{\Sigma}_{ee}$ has the structure

$$\tilde{\Sigma}_{ee}=\Sigma_{ee}+F_{ee} ,$$

where

$$F_{ee}(\underline{1},\underline{2})=G^{-1}_{eh}(\underline{1},\underline{1}')\,G_h(\underline{1}',\underline{2}')G^{-1}_{he}(\underline{2}',\underline{2}) .$$

$F^{<}_{ee}$ is obtained by taking the $+,-$ matrix element with respect to the time-ordering index

$$F^{<}_{ee}(1,2)=G^{-1,++}_{eh}(1,1')G^{<}_{h}(1',2')G^{-1,--}_{he}(2',2) , \tag{3.29}$$

where we have assumed again[7] that the screened Coulomb potential is nonretarded,

$$\Sigma_{eh}(\underline{1},\underline{2})\simeq[\mathbf{j}_{cv}(1\text{-}2)\,\mathbf{A}(\underline{1})$$
$$+d_s(1\text{-}2)G_{eh}(\underline{1},\underline{2})]\delta(\underline{t}_1-\underline{t}_2) .$$

In order to find $G_h^{<}$ in (3.29) we take again the $+,-$ ma-

trix element of the definition (3.6) of G_h and get

$$G_h^<(1,2)=-G_h^r(1,1')G_{hh}^{-1,<}(1',2')G_h^a(2',2)\ ,$$

and because $G_{hh}^{0-1,<}=0$

$$G_h^<(1,2)=-G_h^r(1,1')\Sigma_{hh}^<(1',2')G_h^a(2',2)\ . \tag{3.30}$$

Taking into account that $\mathbf{A}$ as well as the driven G_{eh} vary with $e^{\pm i\mathbf{K}_0\cdot\mathbf{R}}$, we find in the rotating wave approximation

$$F_{ee}^<(\xi)=|\,G_{eh}^{-1,r}(\mathbf{K},\mathbf{R})G_h^r(\mathbf{K}-\mathbf{K}_0,\mathbf{R})\,|^2\,\Sigma_{hh}^<(\mathbf{K}-\mathbf{K}_0,\mathbf{R})\ . \tag{3.31}$$

From (3.28) one sees that $F_{ee}^<$ has to be evaluated at the frequency $\hbar\omega=\tilde{\varepsilon}_e$. With $G_{eh}^{-1,r}(\mathbf{k},\tilde{\varepsilon}_e,\mathbf{R})=\lambda$ [see (3.10)] we get

$$F_{ee}^<=|\,\lambda\,|^2\,|\,G_h^r(\mathbf{k},\tilde{\varepsilon}_e-\hbar\omega_0,\mathbf{R})\,|^2\,\Sigma_{hh}^<(\mathbf{k},\tilde{\varepsilon}_e-\hbar\omega_0,\mathbf{R})\ , \tag{3.32}$$

i.e., the scattering rate $\Sigma_{hh}^<$ is weighted with $|\,\lambda\,|^2\,|\,G_{hh}^r(\tilde{\varepsilon}_e-\hbar\omega_0)\,|^2$. According to the last section, the retarded GF is given by

$$G_{hh}^r(\tilde{\varepsilon}_e-\hbar\omega_0)=\frac{1}{\frac{1}{2}[\Delta\pm(\Delta^2+4\,|\,\lambda\,|^2)^{1/2}]}\ ,$$

where

$$\Delta=\varepsilon_e+\varepsilon_h-\hbar\omega_0$$

so that

$$|\,G_{hh}^r(\tilde{\varepsilon}_e-\hbar\omega_0)\,|^2=\frac{4}{[\Delta\pm(\Delta^2+4\,|\,\lambda\,|^2)^{1/2}]^2}\ .$$

The weight coefficients a and b [(3.17)] are given by

$$a=\frac{1}{2}\left[1\pm\frac{\Delta}{(\Delta^2+4\,|\,\lambda\,|^2)^{1/2}}\right]\ ,$$

$$b=\frac{1}{2}\left[1\mp\frac{\Delta}{(\Delta^2+4\,|\,\lambda\,|^2)^{1/2}}\right]$$

so that

$$\frac{b}{a}=\frac{4\,|\,\lambda\,|^2}{[\Delta\pm(\Delta^2+4\,|\,\lambda\,|^2)^{1/2}]^2}\ .$$

Therefore we get finally

$$F_{ee}^<=\frac{b}{a}\Sigma_{hh}^<(\mathbf{k},\tilde{\varepsilon}_e-\hbar\omega_0,\mathbf{R})$$

and

$$\tilde{\Sigma}_e^<(\mathbf{k},\mathbf{R})=a\,\Sigma_{ee}^<(\hbar\omega=\tilde{\varepsilon}_e)+b\,\Sigma_{hh}^<(\hbar\omega=\tilde{\varepsilon}_e-\hbar\omega_0)\ . \tag{3.33}$$

Thus the resulting rate equations for the renormalized bands have the simple form[6]

$$\left[\frac{\partial}{\partial t}+(\nabla_k\varepsilon_a)\nabla_r-(\nabla_r\varepsilon_a)\nabla_k\right]\tilde{f}_a=\frac{1}{i\hbar}[\tilde{\Sigma}_a^>\tilde{f}_a-\tilde{\Sigma}_a^<(1-\tilde{f}_a)]\ , \tag{3.34}$$

where the scattering rates are defined by ($a\neq b$)

$$\tilde{\Sigma}_a^{\gtrless}(\mathbf{k},\mathbf{R})=a\,\Sigma_{aa}^{\gtrless}(\hbar\omega=\tilde{\varepsilon}_a)+b\,\Sigma_{bb}^{\gtrless}(\hbar\omega=\tilde{\varepsilon}_a-\hbar\omega_0)\ . \tag{3.35}$$

The self-energies $\Sigma_{aa}^{\gtrless}$ follow from Eq. (2.12) in the RPA as

$$\Sigma_{aa}^{\gtrless}(\xi)=\frac{2\hbar i}{V}\sum_{k'}\int d\omega'\left[e^2G_{aa}^{\gtrless}(\xi')d^{\gtrless}(\xi-\xi')+\frac{j^2}{c^2}G_{bb}^{\gtrless}(\xi')D^{>}(\xi-\xi')\right]\ . \tag{3.36}$$

The interpretation of the rates proportional to the weight coefficient b is that due to the coherent band mixing the scattering in band b contributes to a certain extent to the scattering in band a.

D. The *e-h* pair amplitude

In Sec. III C the interband GF G_{eh} has been eliminated and led to a laser-induced correction term in the intraband self-energies. Here we will investigated G_{eh} directly because of its connection with the optical polarization of the system and with excitonic properties. A full inclusion of the exciton kinetics would require the treatment of an electron-hole pair GF, i.e., of a four-point GF, which is beyond the scope of this paper. We limit ourselves here to the treatment of the laser driven polarization wave G_{eh}, and disregard the incoherent, relaxed part of the *e-h* pairs. This procedure is justified in the plasma limit; the pure excitonic limit has been treated in Refs. 22–25.

From the Dyson equation (2.7) for $a=e$ and $b=h$ one gets with Eqs. (2.6), (3.1), and (3.2) > AM

$$G_{ee}^{-1,r}(1,1')G_{eh}^r(1',2)+G_{eh}^{-1,r}(1,1')G_{hh}^r(1',2)=0\ , \tag{3.37a}$$

$$G_{ee}^{-1,r}(1,1')G_{eh}^<(1',2)+G_{ee}^{-1,<}(1,1')G_{eh}^a(1',2)+G_{eh}^{-1,r}(1,1')G_{hh}^<(1',2)+G_{eh}^{-1,<}(1,1')G_{hh}^a(1',2)=0\ . \tag{3.37b}$$

For all interband functions we will make in the ξ representation an ansatz analogous to (1.10),

$$\begin{aligned}G_{eh}^r(\xi)&=g_+^r(\xi)e^{i\mathbf{K}_0\cdot\mathbf{R}}+g_-^r(\xi)e^{-i\mathbf{K}_0\cdot\mathbf{R}}\ ,\\ G_{eh}^<(\xi)&=g_+^<(\xi)e^{i\mathbf{K}_0\cdot\mathbf{R}}+g_-^<(\xi)e^{-i\mathbf{K}_0\cdot\mathbf{R}}\ ,\\ G_{eh}^{-1,r}(\xi)&=\lambda_+(\xi)e^{i\mathbf{K}_0\cdot\mathbf{R}}+\lambda_-(\xi)e^{-i\mathbf{K}_0\cdot\mathbf{R}}\ ,\\ \Sigma_{eh}^r(\xi)&=\sigma_+(\xi)e^{i\mathbf{K}_0\cdot\mathbf{R}}+\sigma_-(\xi)e^{-i\mathbf{K}_0\cdot\mathbf{R}}\ ,\end{aligned} \tag{3.38}$$

where we used again the four-dimensional notation $\mathbf{K}_0=(\mathbf{k}_0,i\omega_0/c)$ and $\mathbf{R}=(\mathbf{r},ict)$ and $\xi=(\mathbf{K},\mathbf{R})$. Taking the Fourier transform with respect to the relative variables of, e.g., Eq. (3.36), one has to evaluate Fourier transforms of products according to the general rule

$$\exp\left[\frac{i}{2}(\nabla_{R_1}\cdot\nabla_{K_2}-\nabla_{R_2}\cdot\nabla_{K_1})\right]$$
$$\times F(\mathbf{K}_1,\mathbf{R}_1)G(\mathbf{K}_2,\mathbf{R}_2)\,|_{\mathbf{K}_1=\mathbf{K}_2=\mathbf{K};\ \mathbf{R}_1=\mathbf{R}_2=\mathbf{R}}\ . \tag{3.39}$$

For the second term, e.g., in (3.36), one gets with this formula

$$\exp\left[\frac{i}{2}(\nabla_{R_1}\cdot\nabla_{K_2}-\nabla_{R_2}\cdot\nabla_{K_1})\right]\lambda_{\pm}(\mathbf{K}_1,\mathbf{R}_1)$$
$$\times e^{\pm i\mathbf{K}_0\cdot\mathbf{R}_1}G^r_{hh}(\mathbf{K}_2,\mathbf{R}_2)\,|_{\mathbf{K}_1=\mathbf{K}_2=\mathbf{K};\ \mathbf{R}_1=\mathbf{R}_2=\mathbf{R}}\ . \tag{3.40}$$

For the gradient expansion we disregard the slow variation of $\lambda_{\pm}$ and G^r_{hh}, but take the rapidly oscillating terms exactly into account,

$$\lambda_{\pm}(\mathbf{K},\mathbf{R})e^{\mp(1/2)\mathbf{K}_0\cdot\nabla_K}G^r_{hh}(\mathbf{K},\mathbf{R})$$
$$=\lambda_{\pm}(\mathbf{K},\mathbf{R})G^r_{hh}\left[\mathbf{K}\mp\frac{\mathbf{K}_0}{2},\mathbf{R}\right]\ .$$

The total equation (3.36) yields

$$g^r_{\pm}(\xi)=-\lambda_{\pm}(\xi)G^r_{ee}\left[\mathbf{K}\pm\frac{\mathbf{K}_0}{2},\mathbf{R}\right]G^r_{hh}\left[\mathbf{K}\mp\frac{\mathbf{K}_0}{2},\mathbf{R}\right]\ . \tag{3.41}$$

For the derivation of the spectral equation for the electron-hole particle propagator we use in addition to (3.37) the related equation, which stems from $GG^{-1}=1$,

$$G^r_{ee}(1,1')G^{-1,<}_{eh}(1',2)+G^<_{ee}(1,1')G^{-1,a}_{eh}(1',2)$$
$$+G^r_{eh}(1,1')G^{-1,<}_{hh}(1',2)+G^<_{eh}(1,1')G^{-1,a}_{hh}(1',2)=0\ . \tag{3.42}$$

The difference of (3.37) and (3.42) yields

$$G^{-1,r}_{ee}G^<_{eh}-G^<_{eh}G^{-1,a}_{hh}=G^<_{ee}G^{-1,a}_{eh}-G^{-1,r}_{eh}G^<_{hh}+G^r_{ee}G^{-1,<}_{eh}$$
$$-G^{-1,<}_{eh}G^a_{hh}+G^r_{eh}G^{-1,<}_{hh}$$
$$-G^{-1,<}_{ee}G^a_{eh}\ .$$

Using the definitions (3.38) and (3.41) one gets

$$[\hbar\omega_0-\varepsilon_e(\mathbf{k},\mathbf{R})-\varepsilon_h(\mathbf{k},\mathbf{R})]g^<_{\pm}(\xi)$$
$$=\lambda_{\pm}(\xi)\left[G^<_{ee}\left[\mathbf{K}\pm\frac{\mathbf{K}_0}{2},\mathbf{R}\right]-G^<_{hh}\left[\mathbf{K}\mp\frac{\mathbf{K}_0}{2},\mathbf{R}\right]+\frac{\Sigma^<_{ee}\left[\mathbf{K}\pm\frac{\mathbf{K}_0}{2},\mathbf{R}\right]-\Sigma^<_{hh}\left[\mathbf{K}\mp\frac{\mathbf{K}_0}{2},\mathbf{R}\right]}{G^{-1,r}_{ee}\left[\mathbf{K}\pm\frac{\mathbf{K}_0}{2},\mathbf{R}\right]G^{-1,r}_{hh}\left[\mathbf{K}\mp\frac{\mathbf{K}_0}{2},\mathbf{R}\right]-|\lambda(\xi)|^2}\right]\ . \tag{3.43}$$

The coupling parameter $\lambda_{\pm}(\mathbf{K},\mathbf{R})$ depends because of (3.36) via $\Sigma^r_{ee}(\xi)$ on $g^<_{\pm}(\xi)$,

$$\lambda_{\pm}(\xi)=\mathbf{j}_{cv}(\mathbf{k})\cdot\mathbf{A}_{\pm}(\mathbf{R})-\sigma(\xi)\ , \tag{3.44}$$

$$\sigma_{\pm}(\xi)\simeq 4\pi i e^2\hbar\sum_{\mathbf{k}'}\int d\omega'\,g^<_{\pm}(\xi')d^r(\xi-\xi')\ . \tag{3.45}$$

Equation (3.43) is thus an integral equation for the electron-hole pair propagator $G^<_{eh}$ or $g^<_{\pm}$, respectively. The physical reason for this structure is the excitonic nature of the pair function. It obeys in k space an integral equation with the attractive screened Coulomb potential as integral kernel. A solution of this equation with a dynamically screened Coulomb potential has been developed in the framework of a Shindo approximation in the low-field limit by Refs. 26 and 4. For a statically screened Coulomb potential the plasmon propagator is approximated by

$$d^r(\xi)\simeq d^r_s(\mathbf{k},\mathbf{R})=\int d\omega\,d^r(\xi)\ . \tag{3.46}$$

For this approximation, we can introduce the frequency-independent polarization function[3]

$$P_{\pm}(\mathbf{k},\mathbf{R})=\int d\omega\,g^<_{\pm}(\xi)\ .$$

Neglecting the self-energy contributions $\Sigma^<_{aa}$, which vanish for a statically screened Coulomb potential, we get

$$[\hbar\omega_0-\varepsilon_e(\mathbf{k},\mathbf{R})-\varepsilon_h(\mathbf{k},\mathbf{R})]P_{\pm}(\mathbf{k},\mathbf{R})$$
$$=2\pi i\left[\mathbf{j}_{cv}(\mathbf{k})\cdot\mathbf{A}_{\pm}(\mathbf{R})-4\pi i\hbar e^2\sum_{\mathbf{k}'}P_{\pm}(\mathbf{k}',\mathbf{R})d_s(\mathbf{k}'-\mathbf{k},\mathbf{R})\right]$$
$$\times[a(\mathbf{k}',\mathbf{R})-b(\mathbf{k}',\mathbf{R})][1-\tilde{f}_e(\mathbf{k}',\mathbf{R})-\tilde{f}_h(\mathbf{k}',\mathbf{R})]\ . \tag{3.47}$$

In the low-field limit $a\simeq 1$, $b\simeq 0$, Eq.(3.47) reduces to the polarization equation derived in Ref. 3. There it has been shown that the corresponding susceptibility function obeys the Bethe-Salpeter equation which describes the exciton ionization with increasing laser excitation as has been shown by numerical solutions in Ref. 1. The polarization equation (3.47) is in fact identical with the result Eq. (2.4) of Ref. 7 if the relations (2.16) and (2.17) of Ref. 7 are inserted in their general form, i.e., without the specialization for zero temperature. Without screening this polarization equation has been used[27,28] to describe the recently observed optical Stark shift of excitons[29] and

spectral hole burning.[30] Furthermore, it has been shown in Ref. 7 that the real-space formulation of (3.47) in which the polarization is a two-point function, (3.47) reduces—at least for weak excitations—to the coherent band-edge equation of Ref. 8.

$d_s(\mathbf{k}-\mathbf{k},\mathbf{R})$ represents the (statically) screened Coulomb potential, $1-f_e-f_h$ is the blocking factor due to the Pauli principle. Only states which are not yet occupied can be used to form the excitonic state.

For the coupling parameter $\lambda_\pm$ one also obtains an integral equation using (3.44) with the approximation (3.46) in (3.45) and (3.43):

$$\lambda_\pm(\mathbf{k},\mathbf{R})=\mathbf{j}_{cv}(\mathbf{k})\cdot\mathbf{A}_\pm(\mathbf{R})+8\pi\hbar e^2\sum_{\mathbf{k}'}[1-\tilde{f}_e(\mathbf{k}',\mathbf{R})-\tilde{f}_h(\mathbf{k}',\mathbf{R})]\times\frac{\lambda_\pm(\mathbf{k}',\mathbf{R})[a(\mathbf{k}',\mathbf{R})-b(\mathbf{k}',\mathbf{R})]d_s(\mathbf{k}-\mathbf{k}',\mathbf{R})}{\varepsilon_e(\mathbf{k}',\mathbf{R})\,\varepsilon_h(\mathbf{k}',\mathbf{R})-\hbar\omega_0}\ . \tag{3.48}$$

Equation (3.48) determines the size of the laser-induced gap for the resonant excitation and thus corresponds to the gap equation in the theory of superconductivity. Such inhomogeneous laser-influenced gap equations have been discussed previously in Refs. 15 and 3. An inspection [e.g., by iteration of (3.48)] shows that the solution can be written in the form

$$\lambda_\pm(\mathbf{k},\mathbf{R})=j_{cv}(\mathbf{k},\mathbf{R})\cdot\mathbf{A}_\pm(\mathbf{R})\ , \tag{3.49}$$

where $\mathbf{j}'_{cv}$ is a renormalized interband matrix element. In the limit of vanishing laser amplitude $\langle\mathbf{A}\rangle\to 0$, Eq. (3.48) degenerates to a homogeneous integral equation. The energy $\hbar\omega_0$ loses its meaning as an externally determined laser frequency. $\hbar\omega_0$ then becomes the eigenvalue of (3.48), which determines the frequency at which spontaneously a macroscopic electromagnetic field amplitude can build up by laser action.

IV. ELECTROMAGNETIC FIELD

A. The coherent electromagnetic field

The coherent part of the electromagnetic field is given by the average of the vector potential $\langle\mathbf{A}(\mathbf{R})\rangle$. We assume that the scalar potential has a zero expectation value.

In the slowly varying amplitude approximation (SVAA) we get for

$$\langle\mathbf{A}(\mathbf{R})\rangle=\mathbf{e}_\lambda\sqrt{I(\mathbf{R})}e^{i\phi(\mathbf{R})} \tag{4.1}$$

from (2.4) for the intensity $I(\mathbf{R})$ and the phase $\phi(\mathbf{R})$ the equations

$$\left[\frac{\partial}{\partial t}+c\frac{\mathbf{k}}{k_0}\cdot\nabla_r\right]I(\mathbf{R})=-\frac{c^3}{2\omega_0^2}W(\mathbf{R})\ , \tag{4.2}$$

$$\left[\frac{\partial}{\partial t}+c\frac{\mathbf{k}}{k_0}\cdot\nabla_r\right]\phi(\mathbf{R})=-\frac{c^2}{\omega_0}\frac{\langle\mathbf{A}(\mathbf{R})\rangle\cdot\langle\mathbf{j}(\mathbf{R})\rangle}{4I(\mathbf{R})}\ . \tag{4.3}$$

The derivative

$$\frac{\partial}{\partial t}+c\frac{\mathbf{k}}{k_0}\cdot\nabla_r=\frac{d}{dt}$$

is just the total time derivative, c is the light velocity in the medium. On the rhs there appears the energy dissipation rate

$$W(\mathbf{R})=\langle\mathbf{j}(\mathbf{R})\rangle\cdot\langle\mathbf{E}(\mathbf{R})\rangle\ . \tag{4.4}$$

The rhs in Eq. (4.3) is the effect of this absorption on the dispersion. The interband current density is given by

$$\langle\mathbf{j}(1)\rangle=-i\hbar[\mathbf{j}_{cv}(1\text{-}2)G^<_{he}(2^+,1)+\mathbf{j}^*_{cv}(1\text{-}2)G^<_{eh}(1,2^+)]\,|_{(2^+=1)} \tag{4.5}$$

$$=\mathbf{j}_+(\mathbf{R})e^{i\mathbf{K}\cdot\mathbf{R}}+\mathbf{j}_-(\mathbf{R})e^{-i\mathbf{K}\cdot\mathbf{R}}\ , \tag{4.6}$$

with

$$\mathbf{j}_-(\mathbf{R})=-i\hbar\sum_{\mathbf{k}}\mathbf{j}_{cv}(\mathbf{k})\int d\omega\,G^<_{eh}(\xi ec)=\mathbf{j}^*_+(\mathbf{R})\ , \tag{4.7}$$

where (4.6) corresponds to a rotating wave approximation.

In order to treat the optical quantities $W(\mathbf{R})$ and $\langle\mathbf{j}\rangle\cdot\langle\mathbf{A}\rangle$ on the same footing as the plasma kinetics, we use again the Dyson equation (2.7). For the matrix elements $a=b=e$ and $\underline{1}=1+$, $\underline{2}=2-$ we use the ξ representation and neglect all derivatives of the slowly varying amplitudes with respect to the local variables $\mathbf{R}$. A momentum and frequency integration yields

$$\mathbf{j}_+(\mathbf{R})\cdot\mathbf{A}_-(\mathbf{R})=-i\hbar\sum_{\mathbf{k}}\sigma_-(\mathbf{k},\mathbf{R})q_+(\mathbf{k},\mathbf{R})-i\hbar\sum_{\mathbf{k}}\int d\omega[G^r_{ee}(\xi)G^{-1,<}_{ee}(\xi)+G^<_{ee}(\xi)G^{-1,a}_{ee}(\xi)]\ . \tag{4.8}$$

σ_- is the amplitude of the retarded interband self-energy (3.38). With the approximations of a static screening, see (3.45)–(3.47), we get

$$\sigma_-(\mathbf{k},\mathbf{R})=4\pi i\hbar e^2\sum_{\mathbf{k}'}p_-(\mathbf{k}',\mathbf{R})d_s(\mathbf{k}-\mathbf{k}',\mathbf{R})\ . \tag{4.9}$$

The amplitude $q_+(\mathbf{k},\mathbf{R})$ is defined in analogy with p_+ for $e\leftrightarrow h$:

$$G^<_{he}(\xi)(\mathbf{k},\mathbf{R})=f^<_+(\mathbf{k},\mathbf{R})e^{i\mathbf{K}_0\cdot\mathbf{R}}+f^<_-(\mathbf{k},\mathbf{R})e^{-i\mathbf{K}_0\cdot\mathbf{R}} \tag{4.10}$$

and

$$q_\pm(\mathbf{k},\mathbf{R})=\int d\omega\,f^<_\pm(\xi)\ . \tag{4.11}$$

Thus the first term on the rhs of Eq. (4.8) can be written as

$$-i\hbar\sum_{\mathbf{k}}\sigma_{-}(\mathbf{k},\mathbf{R})q_{+}(\mathbf{k},\mathbf{R}) = -4\pi\hbar e^2\sum_{\mathbf{k},\mathbf{k}'}q_{+}(\mathbf{k}',\mathbf{R})d_s(\mathbf{k}-\mathbf{k}',\mathbf{R})q_{+}(\mathbf{k},\mathbf{R}) . \tag{4.12}$$

The dissipation rate is now obtained as

$$W(\mathbf{R}) = \langle \mathbf{j}(\mathbf{R})\rangle\cdot\langle \mathbf{E}(\mathbf{R})\rangle = \frac{2\omega_0}{c}\mathrm{Im}[\mathbf{j}_{+}(\mathbf{R})\cdot\mathbf{A}_{-}(\mathbf{R})] = 2\frac{\hbar\omega_0}{c}\sum_{\mathbf{k}}\int d\omega[\Sigma^{>}_{ee}(\xi)G^{<}_{ee}(\xi)-\Sigma^{<}_{ee}(\xi)G^{>}_{ee}(\xi)] , \tag{4.13}$$

the difference between the momentum- and frequency-integrated scattering rates out and into the state ξ. Similarly, we get the corresponding dispersive effects as

$$\langle \mathbf{j}(\mathbf{R})\rangle\cdot\langle \mathbf{A}(\mathbf{R})\rangle = 2\,\mathrm{Re}[\mathbf{j}_{+}(\mathbf{R})\cdot\mathbf{A}_{-}(\mathbf{R})] = -4\pi\hbar^2e^2\sum_{\mathbf{k},\mathbf{k}'}q_{+}(\mathbf{k},\mathbf{R})d_s(\mathbf{k}-\mathbf{k}',\mathbf{R})q_{+}(\mathbf{k}',\mathbf{R}) - 2i\hbar c\sum_{\mathbf{k}}\int d\omega\{\mathrm{Re}[G^{-1,r}_{ee}(\xi)]G^{<}_{ee}(\xi)-\Sigma^{<}_{ee}(\xi)\mathrm{Re}[G^{a}_{ee}(\xi)] . \tag{4.14}$$

In contrast to the kinetic equation (3.34), only the part Σ_{ee} of $\tilde{\Sigma}_{ee}$ enters in the optical functions (4.13) and (4.14). Furthermore, the interband Coulomb self-energy influences (4.14), while it drops out of the dissipation rate (4.13).

B. The incoherent electromagnetic field

The GF of the transverse and longitudinal components of the electromagnetic field, D and d, respectively, which represent the incoherent functions of the electromagnetic field, can be treated in correspondence with the particle GF's G_{ab}. In the framework of the corresponding approximations, namely, RPA, SVAA, quasiparticle approximation, and nonretarded static screening, one finds the spectrum and the distribution of the quasiparticles, i.e., of the photons and plasmons. The determination of the boson GF's is formally completely analogous to that of the fermion GF's,[5,11] and is even simpler due to the absence of nondiagonal GF's. However, due to their different physical contents, the derivations for the bosons are quite distinct from that for the fermions.

1. Photons

The Dyson equation (2.15) for the tensor function $\overleftrightarrow{D}$ of the photons will be simplified by assuming "transverse isotropy," i.e., the quantities $\overleftrightarrow{D}$ and $\overleftrightarrow{P}$ can be written as products of scalar times the transverse unit tensor $\overleftrightarrow{\delta}_T$. For the coupling we write correspondingly $\mathbf{j}_{cv}\otimes\mathbf{j}^{*}_{cv} = j^2\overleftrightarrow{\delta}_T$. Following Refs. 5 and 6, the poles of the retarded GF $D^r(\xi)$ determine the locally defined spectrum of the photons

$$\left[\frac{\omega^2}{c^2}-k^2-P^r(\xi)\right]\Bigg|_{\hbar\omega=\varepsilon_D(\mathbf{k},\mathbf{R})} = 0 . \tag{4.15}$$

The retarded polarization function, or photon self-energy, is given by

$$P^r(\xi) = \int\frac{d\omega}{2\pi i}\frac{P^{>}(\xi')-P^{<}(\xi')}{\omega-\omega'+i\varepsilon} , \tag{4.16}$$

where $\xi'=\mathbf{k}',\omega',\mathbf{r},t$.

The photon distribution $f_D(\xi)$ is introduced by

$$D^{<}(\xi) = f_D(\xi)A_D(\xi) , \tag{4.17}$$

where A_D is the spectral function of the photons,

$$A_D(\xi) = D^{>}(\xi)-D^{<}(\xi) = D^r(\xi)-D^a(\xi) , \tag{4.18}$$

which is peaked strongly at the poles of Eq. (4.15). Due to this fact, we introduce a frequency independent photon Wigner distribution

$$f_D(\xi)\,|_{\hbar\omega=\varepsilon_D(\mathbf{k},\mathbf{R})} = f_D(\mathbf{k},\mathbf{R}) . \tag{4.19}$$

From Eq. (2.15) we get then the following kinetic equation for the photons:

$$\left[\frac{\partial}{\partial t}+(\nabla_k\varepsilon_D)\cdot\nabla_R-(\nabla_R\varepsilon_D)\cdot\nabla_k\right]f_D(\mathbf{k},\mathbf{R}) = \int\frac{d\omega}{2\pi}\frac{(2\pi c)^2}{\varepsilon_D}[P^{>}(\xi)D^{<}(\xi)-P^{<}(\xi)D^{>}(\xi)] . \tag{4.20}$$

In the RPA we get from (2.17)

$$P^{\lessgtr}(\xi) = -4\pi i\hbar\frac{j^2}{c^2}\sum_{k'}\int d\omega'\,G^{\lessgtr}_{hh}(\xi')G^{\gtrless}_{ee}(\xi'-\xi) + \{e\leftrightarrow h\} . \tag{4.21}$$

By inserting the particle propagators (3.22)–(3.24) into (4.21), one sees that the collision integral in (4.20) describes the creation and annihilation of photons due to electronic transitions between the renormalized bands $\tilde{\varepsilon}_e$ and $\tilde{\varepsilon}_h$, as it is expected on the basis of Fermi's golden rule. However, in addition one also finds contributions

due to transitions between the laser-induced branches $\hbar\omega_0-\tilde{\varepsilon}_e$ and $\hbar\omega_0-\tilde{\varepsilon}_h$, which can be interpreted due to the symmetry (3.7) and (3.25) as transitions between $\tilde{\varepsilon}_h$ and $\tilde{\varepsilon}_e$ with different weight coefficients. Furthermore, one finds transitions between the physical branches and the laser-induced ones, which give rise to a singular property of $P^<$. For $\omega>0$ one gets altogether

$$P^<(\xi)=P_0^<(\xi)+q(\mathbf{R})\delta(\omega-\omega_0)\frac{4\pi i}{\hbar c^2} . \tag{4.22}$$

The function $q(\mathbf{R})$ is given by

$$q(\mathbf{R})=\sum_{\mathbf{k}} j^2(\mathbf{k})a(\mathbf{k},\mathbf{R})b(\mathbf{k},\mathbf{R}) \times\{\tilde{f}_e(\mathbf{k},\mathbf{R})[1-\tilde{f}_e(\mathbf{k},\mathbf{R})] -\tilde{f}_h(\mathbf{k},\mathbf{R})[1-\tilde{f}_h(\mathbf{k},\mathbf{R})]\} , \tag{4.23}$$

which describe direct conversions from coherent into incoherent excitations.

The explicit form of the kinetic photon equation is obtained by carrying out the frequency integration using (4.22)–(4.24) together with (3.20)–(3.23),

$$\left[\frac{\partial}{\partial t}+(\nabla_k\varepsilon_D)\cdot\nabla_R-(\nabla_R\varepsilon_D)\cdot\nabla_K\right]f_D =\frac{16\pi^2}{\hbar^2c^2}\eta_P\left[q(\mathbf{R})\delta(\hbar\omega_0-\varepsilon_D)+\sum_{\mathbf{k}'}\{a'^2\delta(\tilde{\varepsilon}'_e+\tilde{\varepsilon}'_h-\varepsilon_D)[\tilde{f}'_e\tilde{f}'_h(1+f_D)-(1-\tilde{f}'_e)(1-\tilde{f}'_h)f_D] +b'^2\delta(2\hbar\omega_0-\tilde{\varepsilon}'_e-\tilde{\varepsilon}'_h-\varepsilon_D)[(1-\tilde{f}'_e)(1-\tilde{f}'_h)(1+f_D)-\tilde{f}'_e\tilde{f}'_hf_D]\}\right] . \tag{4.24}$$

Here, the dashed quantities depend on $\mathbf{k}',\mathbf{R}$ while the others depend on $\mathbf{k},\mathbf{R}$. The $\mathbf{k}'$ sum on the rhs of Eq. (4.24) vanishes for $\mathbf{k}=\mathbf{k}_D$. Under this resonance condition (4.24) describes just the free drift of photons with the source term $q(\mathbf{R})$. Conversely, for $\mathbf{k}\neq\mathbf{k}_D$, the source term does not contribute and (4.24) describes the development of the photon distribution due to the rates of emission [$\propto$ to $(1+f_D)$] and of absorption ($\propto f_D$) of photons between the physical branches ($\propto a^2$) and the laser-induced branches ($\propto b^2$).

2. *Plasmons*

The longitudinal excitations of the electromagnetic field are described by the GF $d(\underline{1},\underline{1}')$. These longitudinal excitations are directly connected with the screening of the long-range Coulomb potential. As mentioned before, we will limit ourselves in this paper to the static screening, the dynamical aspect of screening has been persued, e.g., in Refs. 1, 4, and 6. Instead of treating plasmon dynamics and kinetics explicitly, we will simply consider them as a reservoir. Following Ref. 6 we use for the particle propagator for the plasmons

$$d^<(\xi)=-i\pi\eta_d(\mathbf{k},\mathbf{R})\{\delta(\hbar\omega\mp\varepsilon_d(\mathbf{k},\mathbf{R}))[1+f_d(\mathbf{k},\mathbf{R})] +\delta(\hbar\omega\pm\varepsilon_d(\mathbf{k},\mathbf{R}))f_d(\mathbf{k},\mathbf{R})\} . \tag{4.25}$$

Here, η_d are the spectral weight functions, ε_d the energies given by the plasmon poles of d^r, and f_d is the Wigner distribution of the plasmons.

Finally, we mention that the phonons which have not been treated explicitly in this paper can be treated formally in complete analogy with the phonons. The inclusion of phonons becomes important if one wants to study, e.g., the cooling of a hot electron-hole plasma on a longer time scale after a short pulse excitation.

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