Mesoscopic ring in a magnetic field: Reactive and dissipative response

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The response of a normal-metal ring threaded by a magnetic field and coupled to a dissipative external bath is studied via the Kubo formalism. Unlike in a wire, the diamagnetic contribution to the induced current in a multiply connected geometry is *not* completely canceled by the paramagnetic term, resulting in a persistent current in the ring. We find a nonzero reactive response $\mathrm{Im}\sigma$ which is a periodic function of the flux through the ring with period $\phi_0 = hc/e$ for a *single* ring, but a period of $\phi_0/2$ for an *array* of disconnected rings. The residual diamagnetic effects also contribute to intraband scattering in the ring. It is shown that the *minimum* in the dissipative response $\mathrm{Re}\sigma$ for zero flux seen in experiments is obtained upon a proper inclusion of intraband scattering (often omitted) in addition to the usual interband scattering. A new regime appears in the response functions where quantum size effects, i.e., a sensitivity to the discreteness of the energy levels, are probed when the broadening of levels γ and the temperature T are much less than the typical spacing between levels δE.

I. INTRODUCTION

A great deal of interest in the conductance of small devices has been sparked by the capability to fabricate systems having dimensions on the order of a few microns. At low temperatures the effective distance the electrons travel between inelastic collisions can exceed the sample dimensions, and the coherence of the single-particle wave function is thereby maintained across the entire sample. This leads to a number of interesting phenomena, e.g., sample-specific behavior,[1] fluctuations in the conductance as the magnetic field or chemical potential is varied,[1-3] and violations of Onsager relationships.[4,5]

An elementary example is a normal-metal ring through which a flux ϕ is threaded. General quantum-mechanical principles[6,7] require that the wave functions, eigenvalues, and hence all observables be periodic with flux with period $\phi_0 = hc/e$, the normal flux quantum. Büttiker, Imry, and Landauer[8] noted that the periodicity of the potential on a ring makes the problem formally the same as that of Bloch wave functions in a crystal, with wave vector k given by $kL = 2\pi\phi/\phi_0$, where L is the length of the loop. They also indicated the possibility of a persistent current in the presence of a static flux. The presence of such a persistent current implies the existence of Bloch oscillations[9] in response to a flux which increases linearly in time, with a "Josephson" frequency given by $eV/\hbar$, where V is the induced emf. These ideas have been analyzed by Landauer and Büttiker[10] with a simple phenomenological model for inelastic scattering, and show that the persistent current survives for weak scattering.

There have been two principal theoretical approaches to the problem of calculating the transport properties of a weakly dissipative system. The first, used extensively in many-body problems, is the Kubo formulation which relates the conductivity to a current-current correlation function.[11,12] This technique relates the dissipative response to the equilibrium fluctuations in an isolated system. Irreversibility is introduced by going to the infinite-volume limit and arguing that the random disorder scrambles the wave functions sufficiently to remove any coherence effects. The second is the Landauer formulation[13] that relates the conductance to the transmission probability for electrons across the sample. Dissipation is introduced by assuming that the sample is connected to reservoirs which emit and absorb electrons incoherently. The problem of coupling the ring to one or more reservoirs has been summarized by Büttiker[14] and by Imry.[15] We comment below on the equivalence of these conceptually rather different approaches.

Alt'shuler, Aronov, and Spivak[16] evaluated the dc conductivity of a hollow cylinder in a magnetic field via the Kubo formulation and showed that the coherent backscattering of time-reversed paths of electrons gives rise to a periodicity of $hc/2e$, half the normal value. An initial examination of the problem via the Landauer formulation[17] generally produced a periodicity of hc/e, later observed experimentally in very small systems, but did not find a period of $hc/2e$, except under very special conditions. It was later realized[18,19] that this extra periodicity appeared only after averaging the conductivity over the impurity distribution. A sufficient condition[20,21] for this averaging to occur in the experimental system is a process such as inelastic scattering which scrambles the phase of the electronic wave function. Elastic scattering, even when it produces an elastic mean free path much

smaller than the sample size, results in a complicated spatial dependence in the wave function which is not random. In fact, states nearby in energy are highly correlated.[3] Thus the electronic wave function that extends around the entire ring is coherent in the presence of elastic scattering. It is sensitive to a magnetic flux in such a fashion as to produce a periodicity of ϕ_0, not $\phi_0/2$ in a magnetic field. Phase-breaking processes, usually produced by inelastic scattering, truly randomize the phase of the wave function and cause the ring to break up into regions of size L_ϕ, the phase-breaking length, which are then independent "samples" that produce an averaging of the transport properties. Another source of averaging is a nonzero temperature since a thermal distribution of electrons uses all the electronic states within $k_B T$ of the Fermi surface and hence produces an average over energy. Stone and Imry[20] have investigated energy averaging and conclude that it also is equivalent to averaging over the impurity distribution.

Once the phase-breaking length (which is usually shorter than the inelastic scattering length since many inelastic scatterings can look nearly elastic and still disrupt the phase of the wave function) is of the order of the sample size, one can ask how dissipation occurs that is implied by a finite resistance. Landauer and Büttiker[10] and Büttiker[22] have taken the point of view that the resistive behavior is caused by coupling to an incoherent reservoir of electrons that represents the measuring apparatus, and that the conventional Kubo-Greenwood[12] formula is simply inapplicable in this situation. One can however, measure the conductivity of an isolated ring by inserting it into a microwave cavity and measuring the resulting absorption. It is not clear *a priori* whether such a measurement of the conductivity should give the same result as a four-probe measurement (neglecting for the moment the question of the different frequencies involved). Imry and Shiren[23] have examined the isolated metal ring from the point of view of the Kubo formulation and found a number of unusual features, including a conductivity that is a *maximum* at zero flux, in distinction to the predictions of weak localization.

The primary purpose of the present work is to go beyond previous discussions of a persistent current in equilibrium and to investigate the dynamical inductive response of a normal ring. We would also like to analyze the conductivity near zero flux to see if, within the linear-response formulation, it shows the behavior predicted by weak localization. Toward this end we study the current induced in an isolated metal ring in the presence of both a static and a time-dependent magnetic flux. Dissipation is included in the system by modeling the dynamics by a master equation with a relaxation term that phenomenologically accounts for the coupling of the electronic system to an external environment. In order to conserve the number of electrons in this model, the chemical potential must be a function of the flux and must also be time dependent. In addition, the quasiequilibrium state that the electrons relax to in the presence of the coupling must include the external perturbation. We discovered a number of subtleties in the evaluation of the response that have been overlooked in previous calculations. We find the following new features.

(i) In a multiply connected geometry, a generalized f-sum rule can be shown to hold, which leads to a finite persistent current even in the presence of dissipation.

(ii) The persistent currents contribute an intraband term to the dissipative response, which must be added to the usual interband term. The total dissipative response then, indeed has a *minimum* at $\phi=0$ for a loop. Such an intraband term is not present for a wire with open ends.

(iii) We find an extreme sensitivity of the response functions to the number of electrons in the ring or the chemical potential (depending on the ensemble used) in the weak damping limit. If the damping is increased or the temperature is raised, this effect disappears.

(iv) As the damping increases and the above-mentioned sensitivity disappears, the periodicity in flux of the dissipative response in the ring changes from ϕ_0 to $\phi_0/2$, while the reactive response continues to show a periodicity of ϕ_0. In an array of disconnected rings, the reactive response will also show a crossover to a periodicity of $\phi_0/2$.

(v) We find that the results given phenomenologically in Refs. 8 and 10 can also be obtained from the Kubo linear response to an applied flux if diamagnetic effects, which are usually ignored, are retained in the formalism.

This paper is divided into five sections. Section II describes our approach to the Kubo formula for a one-dimensional (1D) ring. We include inelastic effects within a number conserving formulation and consider the effects of diamagnetic and paramagnetic contributions to the response functions. The transverse f-sum rule for a loop is derived. In Sec. III we discuss the reactive response and mention some similarities to the response of superconductors. The dissipative part of the conductivity is described in Sec. IV and we conclude with some remarks in Sec. V. There are two appendices where the relaxation time approximation is discussed and the effect of the finite self-inductance of the loop is considered.

II. FORMALISM

The gauge-invariant tight-binding Hamiltonian of an electron constrained to move on a loop of N sites with lattice spacing a is given by

$$\hat{H}=\sum_n [(2t+V_n)c_n^\dagger c_n - te^{+i\theta_{n,n+1}}c_{n+1}^\dagger c_n - te^{-i\theta_{n,n+1}}c_n^\dagger c_{n+1}] , \tag{2.1}$$

where $t=\hbar^2/2ma^2$ is the hopping matrix element and will be taken as the unit of energy and $c_n^\dagger$ and c_n create and destroy particles on the nth site. Disorder is introduced via on site potentials $\{V_n\}$ chosen from some distribution. The phase of the hopping amplitude $\theta_{n,m}=e/(\hbar c)\int_{r_n}^{r_m}\mathbf{A}\cdot d\boldsymbol{l}$ arises from the flux contained in the ring. The numerical analysis is simplified if a gauge transformation is performed to put all of the static vector potential across the link between the Nth and first sites.[19] We will assume the vector potential is given by the exter-

nal flux alone and defer to Appendix B a discussion of the finite self-inductance of the loop. Next we will introduce a small time-varying flux through the ring. The new Hamiltonian is given by replacing A by $A+\delta A(t)$ in (2.1). To lowest order in δA, the perturbation is given by $\hat{H}_1(t)=-(1/c)\hat{J}_p\cdot\delta\mathbf{A}(t)$, where the paramagnetic current operator is defined by

$$\hat{J}_p=i\frac{e\hbar}{2ma}\sum_n (e^{i\theta_{n,n+1}}c^\dagger_{n+1}c_n-\text{H.c.}) . \tag{2.2}$$

Since the system is described by a Hamiltonian, it cannot show resistive behavior. To provide a relaxation mechanism, the electrons must be coupled to a thermal bath. This bath is not just a mathematical convenience. Since any measuring device must be reasonably classical in order to get a definite result, the process of measuring the response of the ring will inevitably produce an essentially uncontrolled interaction with the large number of degrees of freedom associated with the measuring device. In a sense, then, our approach has some of the flavor of the Landauer picture except that we will exchange only energy, and not particles, between the measuring instrument and the device. Of course, our model could also arise from truly inelastic processes in the ring, but we are assuming those are negligible.

If the coupling between the ring and the environment is small, the reduced density matrix $\hat{\rho}(t)$ for the electron can be described by a kinetic equation of the form

$$\frac{\partial\hat{\rho}(t)}{\partial t}+i[\hat{H}+\hat{H}_1(t),\hat{\rho}(t)]=-\gamma[\hat{\rho}(t)-\hat{\rho}_{\text{QE}}(t)] . \tag{2.3}$$

This kinetic equation neglects changes in the equilibrium density matrix induced by the coupling to the thermal bath. The above relaxation time approximation can be justified from a microscopic theory[24] when the coupling to the environment is weak. In (2.3), $\hat{\rho}_{\text{QE}}$ is the quasiequilibrium density matrix given by $\{\exp\beta[\hat{H}+\hat{H}_1(t)-\mu(t)]+1\}^{-1}$ and $\mu(t)$ is the chemical potential. Note that the quasiequilibrium state is governed by the total Hamiltonian. The chemical potential $\mu(t)$ is fixed by requiring that the number of electrons remain constant.[25,26]

Within linear response, the density matrix can be approximated by $\hat{\rho}=\hat{\rho}_0+\delta\hat{\rho}$, where the deviation from equilibrium $\delta\hat{\rho}$ is linear in the perturbation. In addition, if the chemical potential is expanded about its unperturbed value as $\mu(t)=\mu_0+\delta\mu(t)$, where $\delta\mu$ is a small shift proportional to the perturbation, the quasiequilibrium density matrix is given by

$$\langle\alpha|\hat{\rho}_{\text{QE}}|\beta\rangle=f_\alpha\delta_{\alpha\beta}-\frac{\delta A}{c}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\langle\alpha|\hat{J}_p|\beta\rangle - \frac{\partial f_\alpha}{\partial\epsilon_\alpha}\delta\mu\,\delta_{\alpha\beta} , \tag{2.4}$$

where $\hat{H}|\alpha\rangle=\epsilon_\alpha|\alpha\rangle$, $\epsilon_{\alpha\beta}=\epsilon_\alpha-\epsilon_\beta$, $\hat{\rho}_0|\alpha\rangle=f_\alpha|\alpha\rangle$, and $f_\alpha=[\exp(\epsilon_\alpha-\mu_0)+1]^{-1}$ is the Fermi distribution function. The response of the system can be given in terms of two kinds of changes, the first being real transitions caused by δA which give off-diagonal components to the density matrix of the form

$$\langle\alpha|\delta\hat{\rho}|\beta\rangle=-\frac{\delta A}{c}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\frac{\epsilon_{\alpha\beta}-i\gamma}{\epsilon_{\alpha\beta}-\omega-i\gamma}\langle\alpha|\hat{J}_p|\beta\rangle \tag{2.5}$$

and the second arising from the fact that the equilibrium state is flux sensitive which produces a change in the diagonal components of the density matrix by an amount

$$\langle\alpha|\delta\hat{\rho}|\alpha\rangle=\frac{\partial f_\alpha}{\partial\epsilon_\alpha}\frac{i\gamma}{\omega+i\gamma}\left[\frac{-\delta A}{c}\langle\alpha|\hat{J}_p|\alpha\rangle-\delta\mu\right] . \tag{2.6}$$

The chemical potential shift in (2.6) is obtained by imposing the conservation of probability which requires that $\text{Tr}\delta\hat{\rho}=0$. This constraint is sufficient to determine $\delta\mu$ and yields

$$\delta\mu=-\frac{\delta A}{c}\frac{\sum_\alpha(\partial f_\alpha/\partial\epsilon_\alpha)\langle\alpha|\hat{J}_p|\alpha\rangle}{\sum_\alpha(\partial f_\alpha/\partial\epsilon_\alpha)}\equiv-\frac{\delta A}{c}\langle J_p\rangle . \tag{2.7}$$

We can now calculate the induced current J_{ind} given by $\text{Tr}(\hat{J}\hat{\rho})$ where in general, the current operator is defined by $\hat{J}=-c\,\delta H/\delta A$. The induced current is composed of two parts; a "paramagnetic" contribution arising from the change in the density matrix $\hat{\rho}$ of the system caused by the perturbation $\hat{H}_1$ and a "diamagnetic" contribution from the change in the current operator due to the addition of δA. Thus $\hat{J}=\hat{J}_p+\hat{J}_d$ where the paramagnetic operator is defined in (2.2) and the diamagnetic contribution is given by

$$\hat{J}_d=-\frac{\delta A}{c}\frac{e^2}{2m}\sum_n (e^{i\theta_{n,n+1}}c^\dagger_{n+1}c_n+\text{H.c.})\equiv-\frac{\delta A}{c}\hat{D} . \tag{2.8}$$

In the continuum limit, the operator $\hat{D}$ is given by $(e^2/m)\delta(x-x')$. Using (2.2), (2.5), and (2.8), we find the conductivity defined as $J_{\text{ind}}=\sigma E=\sigma(i\omega/c)\delta A$ to be given by a generalization of the Kubo-Greenwood[12] formula

$$\sigma(\omega)=\frac{i}{\omega\mathcal{V}}\left[\sum_{\alpha} f_\alpha\langle\alpha\,|\,\hat{D}\,|\,\alpha\rangle+\frac{i\gamma}{\omega+i\gamma}\sum_{\alpha}\frac{\partial f_\alpha}{\partial\epsilon_\alpha}\langle\alpha\,|\,\hat{J}_p\,|\,\alpha\rangle(\langle\alpha\,|\,\hat{J}_p\,|\,\alpha\rangle-\langle J_p\rangle)\right.$$
$$\left.+\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\frac{\epsilon_{\alpha\beta}-i\gamma}{\epsilon_{\alpha\beta}-\omega-i\gamma}\,|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2\right], \tag{2.9}$$

where $\mathcal{V}$ denotes the volume of the wire. We show in Appendix A that the term $\langle J_p\rangle$, incorporated to conserve the number of particles in the system, though important in principle, is negligibly small.

The diamagnetic contribution in Eq. (2.9) can be rewritten using a generalized form of the f-sum rule.[27] To derive this generalization, consider the change caused in the Hamiltonian given in (2.1) by adding a small amount of flux $\delta\phi$. The corresponding change in the vector potential in the azimuthal direction is $A\rightarrow A+(\delta\phi/L)$, where $L=Na$ is the circumference of the loop. To $O((\delta\phi)^2)$ the change in the Hamiltonian is

$$\delta\hat{H}=-t\sum_n e^{i\theta_{n,n+1}}[i\delta\theta-\tfrac{1}{2}(\delta\theta)^2]c^\dagger_{n+1}c_n+\text{H.c.}\ , \tag{2.10}$$

where $\delta\theta=(ea/\hbar c)(\delta\phi/L)$. The change in the energy of the state $|\alpha\rangle$ caused by (2.10) is evaluated by second-order perturbation theory. By comparing terms with the Taylor expansion of the energy $\epsilon_\alpha(\phi+\delta\phi)$, we obtain the relations

$$\langle\alpha\,|\,\hat{J}_p\,|\,\alpha\rangle=-cL\frac{\partial\epsilon_\alpha}{\partial\phi} \tag{2.11}$$

and

$$\langle\alpha\,|\,\hat{D}\,|\,\alpha\rangle+2\sum_\beta\frac{|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2}{\epsilon_\alpha-\epsilon_\beta}=(cL)^2\frac{\partial^2\epsilon_\alpha}{\partial\phi^2}\ . \tag{2.12}$$

The latter relation, known as the generalized f-sum rule, reveals a very important difference between a wire and a ring. In a wire the diamagnetic term is completely canceled by the paramagnetic term and the right-hand side of (2.12) vanishes leading to the conventional f-sum rule. This happens because any change in the phase of the hopping matrix element can be removed by a simple gauge transformation and the eigenvalues are therefore independent of flux. In a ring, however, the geometry is not simply connected, so a change of the phase of the hopping matrix elements cannot in general be removed,[7] resulting in energy eigenvalues which depend on the magnetic flux. The sensitivity of the eigenvalues to the flux determines the degree to which the diamagnetic and paramagnetic terms in (2.12) fail to cancel.

In addition, (2.11) implies that a persistent current

$$J_{\text{per}}=-cL\sum_\alpha f_\alpha(\epsilon_\alpha(\phi)-\delta\mu(\phi))\frac{\partial\epsilon_\alpha}{\partial\phi} \tag{2.13}$$

can exist in equilibrium in the absence of any inelastic scattering because of the sensitivity of the energy levels to variations in the flux. In the absence of a magnetic field, the persistent current must vanish as required by time-reversal symmetry. Using (2.11) and (2.12), the conductivity can be rewritten as

$$\sigma(\omega)=\frac{1}{\mathcal{V}}\left[\frac{-icL}{\omega}\frac{\partial J_{\text{per}}}{\partial\phi}-\frac{(cL)^2}{\gamma-i\omega}\sum_\alpha\frac{\partial f_\alpha}{\partial\phi}\frac{\partial\epsilon_\alpha}{\partial\phi}\right]$$
$$+\frac{i}{\mathcal{V}}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\frac{|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2}{\epsilon_{\alpha\beta}-\omega-i\gamma}\ . \tag{2.14}$$

The first two terms are only present in a loop, while the last term is the same as the usual Kubo-Greenwood formula.[12] The two extra terms are directly attributable to the sensitivity of the system to a magnetic flux, and arise from the change in the persistent current in the loop, i.e., from the change in the equilibrium state as the flux is varied.

III. REACTIVE RESPONSE

It can be seen from (2.14) that $\text{Im}\sigma(\omega)\propto 1/\omega$ as $\omega\rightarrow 0$. This behavior is expected for free acceleration of electrons as is found in a superconductor. The unusual feature is that a mesoscopic normal ring by virtue of its topology can support a persistent current and hence when the system is perturbed by adding a small flux $\delta\phi$, there is an induced current $J_{\text{ind}}\propto\delta\phi$. Since J_{ind} is out of phase with the electric field, no power is absorbed from the electric field.

We have performed numerical calculations to study the response functions of a ring in a magnetic field, using the tight-binding Hamiltonian with 30 to 100 sites. On site disorder is drawn from a rectangular distribution of width $W/t=1$ and results are obtained for one configuration of potentials. From the eigenvalue spectrum in Fig. 1(a), it can be seen that the less localized nature of states in the middle of the band is reflected in a greater sensitivity of the eigenvalues to the flux compared to states near the bottom of the band.

Figure 2 shows a persistent current in the loop for two temperatures and for two different positions of the chemical potential. The persistent current, as expected, shows

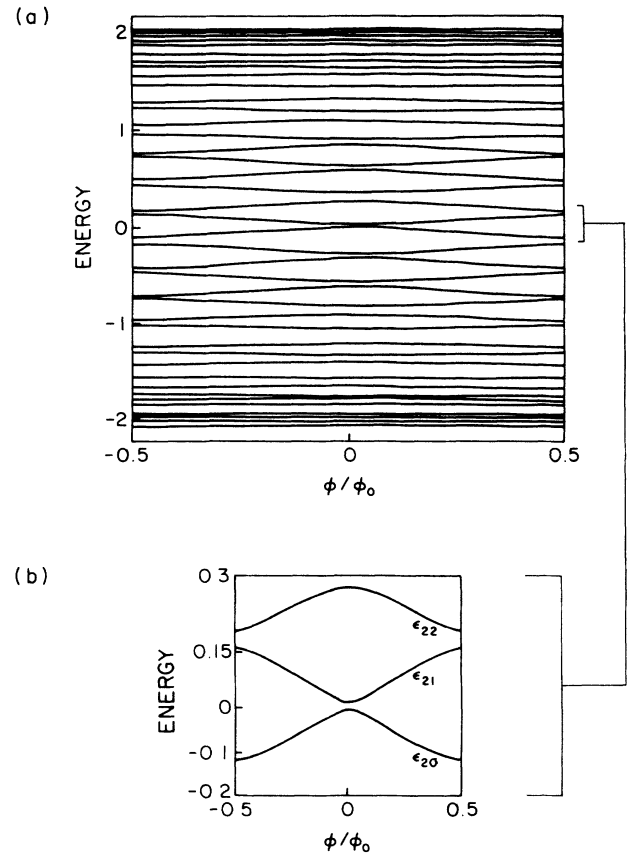

FIG. 1. Eigenvalue spectrum as a function of flux ϕ for a 40 site system with an onsite potential chosen randomly from a rectangular distribution of width $W=1.0t$, where $t=\hbar^2/(2ma^2)$ is the hopping amplitude and a is the lattice spacing. The inset shows the behavior of the 20th, 21st, and 22nd levels.

a periodicity of $\phi_0=hc/e$ and is odd in ϕ about zero flux. An interesting feature is that the persistent current can either lag or lead the electric field. In other words, the magnetic moment of the ring due to these currents can point either along the magnetic field or opposite to it depending on the number of electrons present.[15] By reference to the eigenvalue spectrum in Fig. 1, we see that adjacent pairs of states carry nearly equal and opposite currents so that the dominant contribution to the current comes from the topmost state. It has been argued that in a loop of finite transverse extent, the random sign coupled with a varying magnitude for different levels produces a persistent current of order $(ev_F/L)(k_F l)(N_{\rm ch})^{1/2}$, where $N_{\rm ch}$ is the number of available channels. While we have not investigated the effect of multiple channels in detail, however, in one dimension we find considerable correlation in the size of the current carried by adjacent states. It is therefore possible that the cancellation is much more complete than one would find from a random sum, so the estimate quoted above may in fact be an overestimate. In one dimension, however, we expect a persistent current of order ev_F/L. The shape of the persistent current as a function of the flux is highly asymmetric about $\phi=\phi_0/4$ in the low-temperature regime where $T\ll\delta E$. As T is increased and more states are included in the averaging, the asymmetry in the persistent current is reduced but at the same time the magnitude of the current drops significantly. Using the expression from Appendix B for the self-inductance of a loop, the flux ($=\mathcal{L}I$) induced by the persistent current in a loop of length 1 μm and thickness 0.01 μm is $\phi\approx(10^{-5}-10^{-4})\phi_0$.

The reactive response is given by the imaginary part of the conductivity in (2.14) and in the zero-frequency limit it is a measure of the change in the persistent current for

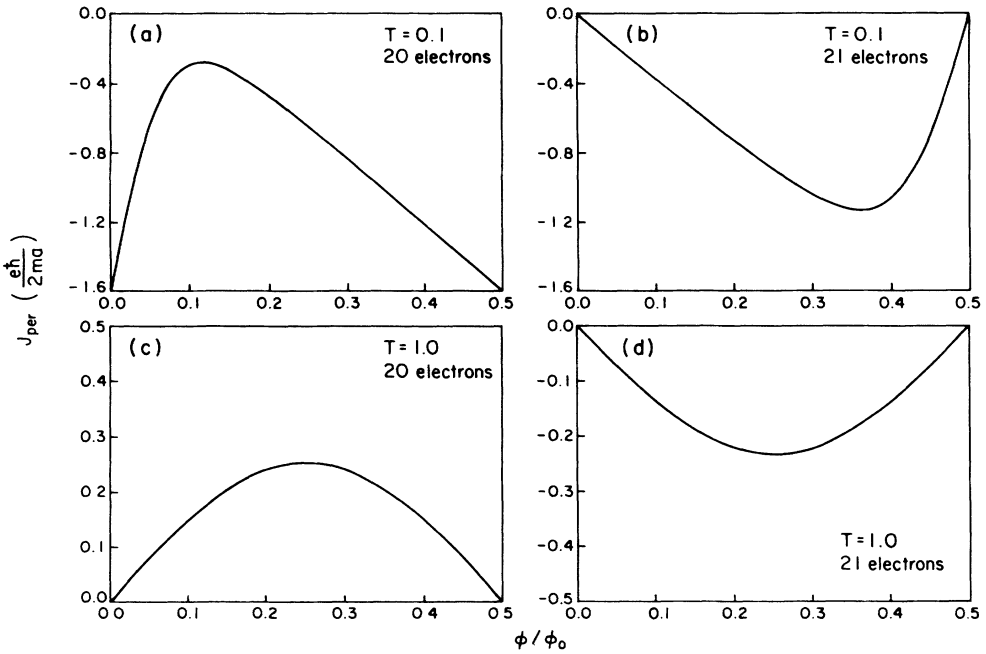

FIG. 2. Persistent currents in equilibrium for the system with the spectrum shown in Fig. 1. The current is given in units of $\hbar/(2ma)$ at two temperatures $T/\delta E=0.1$ and 1.0 where $\delta E=4t/N$ is a typical spacing between energy levels. (a) and (c) show the current for the chemical potential μ between the 20th and 21st levels (20 electrons) and (b) and (d) are for the case of 21 electrons where μ is between the 21st and 22nd levels. The current is odd in ϕ about $\phi=0$.

a small change in the static flux

$$\lim_{\omega\to 0}[\omega\,\mathrm{Im}\sigma(\omega)]=\frac{-cL}{\mathcal{V}}\frac{\partial J_{\mathrm{per}}}{\partial\phi}\ . \tag{3.1}$$

Note that in the zero frequency limit there is no change in the magnitude of the persistent current with damping, because the mixing of the uncoupled eigenstates of the electron due to the thermal bath has been neglected here. At finite frequencies, even within this approximation, damping reduces the amplitude of the response. The reactive response is shown in Fig. 3. For $T<\delta E$ quantum-size effects are evident, i.e., the response is sensitive to the position of μ. Inductive effects are most prominent near zero flux or near half flux quanta.

As T is increased and becomes of order δE, the response is averaged over a few electron states. However, unlike the dissipative response which is discussed in the next section, the reactive response continues to show a periodicity of ϕ_0 provided, of course, that the averaging does not destroy the phase coherence of the electron around the ring. We will first rewrite the reactive response as the sum of two terms:

$$\lim_{\omega\to 0}[\omega\,\mathrm{Im}\sigma(\omega]=\frac{-cL}{\mathcal{V}}\sum_{\alpha}f_{\alpha}(\epsilon_{\alpha}(\phi)-\mu(\phi))\frac{\partial^2\epsilon_{\alpha}}{\partial\phi^2}+\frac{-cL}{\mathcal{V}}\sum_{\alpha}\frac{\partial f_{\alpha}(\epsilon_{\alpha}(\phi)-\mu(\phi))}{\partial\phi}\frac{\partial\epsilon_{\alpha}}{\partial\phi}\ , \tag{3.2}$$

where the first term is related to the curvature of the energy levels and the other to the velocity of the levels. With the increase in temperature averaging, the periodicity of each of these terms individually changes to being *predominantly* $\phi_0/2$, however, a cancellation between these terms produces a net response in a *single* ring that is periodic with period ϕ_0. It must be noted that at some characteristic temperature, the two contributions to the reactive response attain a periodicity of half a flux quantum, however, the total response then is a constant and therefore, only in a trivial way has a periodicity of half flux quantum. In general for a single ring we find that the reactive response is periodic with the full flux quantum.

The situation is rather different in an array of disconnected rings. If the persistent current for the two positions of the chemical potential in Fig. 2 are averaged (equivalent to averaging the current in two rings differing in the number of electrons by one), the effect is to produce a strong even harmonic. This is shown in Fig. 4. Thus for an *array* of rings the dominant periodicity in the reactive response should be $\phi_0/2$. In addition, for an array the response must be averaged over disorder; which is found not to affect the weight in the even and odd harmonics substantially. This is shown in Fig. 5 for an average over ten configurations.

The noncancellation of the diamagnetic term described above in a normal ring bears a close resemblance to the behavior of a superconducting ring in a magnetic field or to a rotating torus containing a superfluid. In a superconductor the diamagnetic current arises from the phase

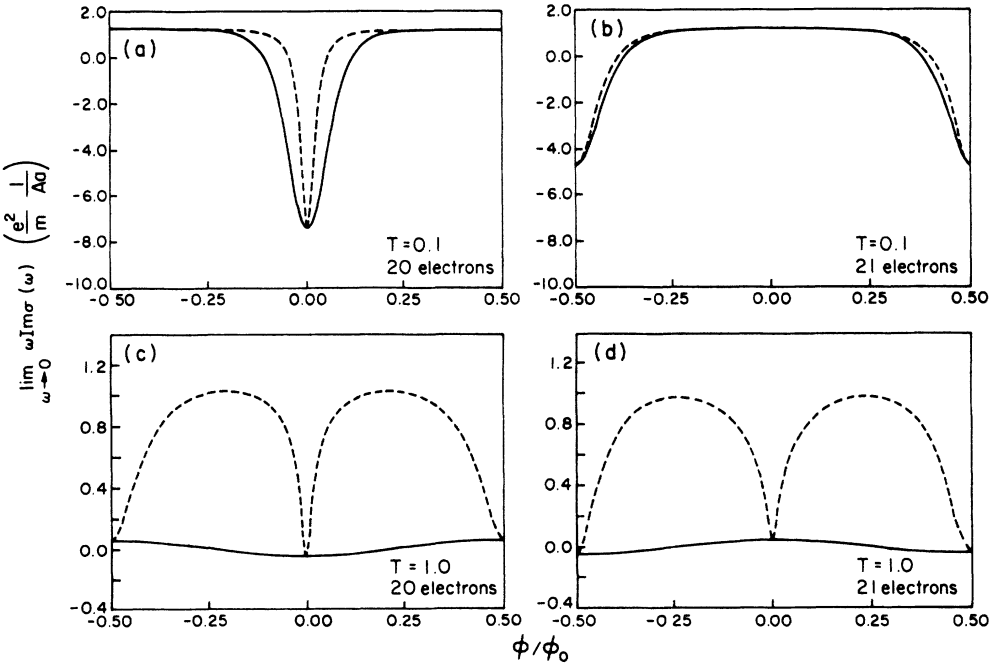

FIG. 3. Reactive response $\omega\,\mathrm{Im}\sigma$ in units of $e^2/(mAa)$ as a function of the flux ϕ for the same 40 site system as in the previous figures, where A is the cross-sectional area of the loop and "a" is the spacing between sites. In (a) and (c) the chemical potential μ is between the 20th and 21st levels (see Fig. 1) and in (b) and (d) μ lies between the 21st and 22nd levels. $\gamma/\delta E=0.05$, where $\delta E=4t/N$ is the typical spacing between levels. (a) and (b) $T/\delta E=0.1$; (c) and (d) $T/\delta E=1.0$. The dotted curve shows the behavior of the first term in (3.2), the "effective mass," and the solid curve is the total response.

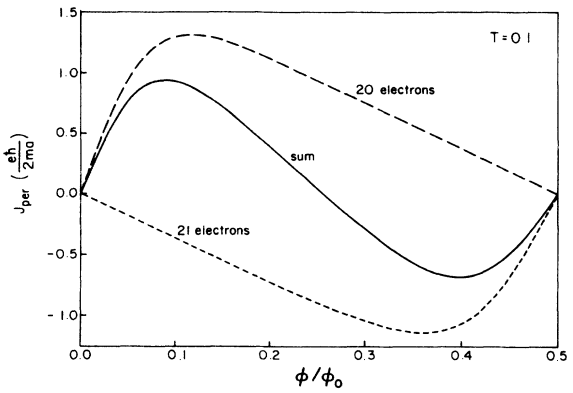

FIG. 4. Average of the persistent current over two different numbers of electrons (20 and 21) or, equivalently, over a range of chemical potential. The long dashed line is the result shown in Fig. 2(a) while the short dashed line is the result of Fig. 2(b). While each term is clearly periodic with period ϕ_0, the sum, which represents the situation in an array of rings, is periodic with period $\phi_0/2$.

rigidity of the order parameter due to the presence of off-diagonal long-range order.[6] This results in an asymmetry between the long-wavelength longitudinal and transverse responses. It is well known[28] that the long-wavelength limit of the longitudinal density-density response is the total density n, whereas that of the transverse response is the normal density n_n; the difference being the superfluid density $n_s = n - n_n$. A normal metal loop can also show this asymmetry when the coherence of the single-particle wave functions is preserved in the presence of sufficiently weak phase-breaking mechanisms. This can be understood if we consider a general spatially varying induced current, which if expanded as a Fourier series in the azimuthal angle θ yields $J = \sum_m J_m \exp(im\theta)$, where m is an integer. All terms

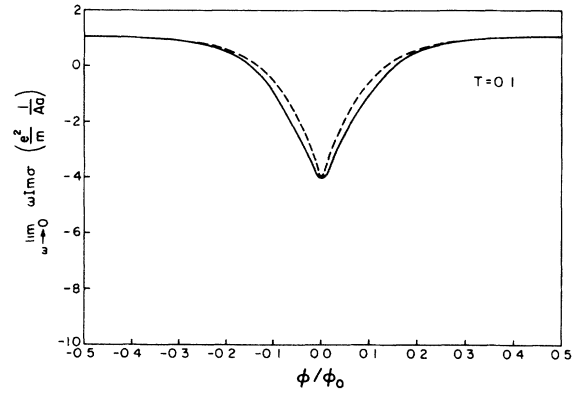

FIG. 5. The reactive response for 20 electrons averaged over ten configurations of on-site potentials. The temperature T and inelastic rate γ are the same as in Fig. 4(a); $T/\delta E = 0.1$ and $\gamma/\delta E = 0.05$. Note that although the amplitude of the response is reduced, the periodicity continues to remain ϕ_0. The dotted curve shows the behavior of the first term in (3.2), the effective mass, and the solid curve is the total response.

with $m \neq 0$ are longitudinal and satisfy the usual f-sum rule, which is Eq. (2.12) with the right-hand side equal to zero. The $m = 0$ term is however a transverse mode in a 1D loop, and according to (2.12), it behaves differently if the single-particle eigenfunctions are flux sensitive. The $m = 0$ mode is allowed only because of the multiply connected geometry, and does not exist for a wire with open ends. Hence the long-wavelength limit for a loop is not uniform. This behavior will persist for a loop of finite cross section, although the fact that the electrons will see different amounts of flux depending on their path will eventually wash out the effect.

From (2.9), the induced current can be written as

$$J_{\mathrm{ind}}(\omega) = -\frac{1}{c}\left[\frac{ne^2}{m} + \chi^T(\omega)\right]\delta A(\omega) , \tag{3.3}$$

where the continuum expression of D has been used and χ^T is the transverse current-current correlation function. In the zero frequency and zero wavelength limit, we obtain

$$\frac{ne^2}{m} + \chi^T(\omega \to 0) = \frac{-cL}{\mathcal{V}}\frac{\partial J_{\mathrm{per}}}{\partial \phi} , \qquad = \frac{e^2}{m} n_s , \tag{3.4}$$

where the analogy with superfluids has been used to identify $\chi^T(\omega = 0) = -e^2 n_n / m$. The presence of a net superfluid density will directly lead to a Meissner effect; however, in a one-dimensional ring it is not meaningful to talk of a penetration depth. Byers and Yang[6] have shown that the quantization of flux in a superconductor is a consequence of the perfect shielding of the Meissner effect. In a normal-metal ring, even one with a finite cross section, the induced current is much smaller than that needed to screen the magnetic field from the interior of the wire, since the "superfluid" density arises from the coherence in a few electron states. Thus there is no preference to an integral number of flux quanta in a normal-metal mesoscopic loop.

IV. DISSIPATIVE RESPONSE

The real part of (2.14) is the dissipative component of the response where the current is in phase with the applied electric field. We find

$$\mathrm{Re}\sigma(\omega) = \frac{1}{\mathcal{V}}\frac{\gamma}{\omega^2+\gamma^2}(cL)^2 \sum_\alpha \left[-\frac{\partial f_\alpha}{\partial \phi}\right]\frac{\partial \epsilon_\alpha}{\partial \phi} + \frac{1}{\mathcal{V}} \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} \frac{f_\alpha - f_\beta}{\epsilon_\beta - \epsilon_\alpha} |\langle \alpha | \hat{J} | \beta \rangle|^2 \times \frac{\gamma}{(\epsilon_{\alpha\beta}-\omega)^2+\gamma^2} . \tag{4.1}$$

Two primary sources of dissipation in this system can be discerned. (a) Intraband processes [first term in (4.1)], also known as Debye absorption in the context of atoms and molecules, where the scattering is within a single lev-

el. This arises because of the noncancellation of the diamagnetic term. (b) Interband processes [second term in (4.1)], where the scattering is between different levels. As shown here, for a loop both the intraband and interband terms must be included, however, that in a wire only the interband term (b) is present. The dissipative part of the conductivity must satisfy the following sum rule:[29]

$$\int_0^\infty d\omega \, \mathrm{Re}\sigma(\omega) = \frac{ne^2\pi}{2m} . \tag{4.2}$$

However, if we substitute (4.1) in (4.2) we find that some of the weight is missing. The missing weight $[=(-cL/\mathcal{V})\partial J_{\mathrm{per}}/\partial\phi]$ is precisely the free acceleration of electrons that must appear as a δ function at $\omega=0$.

There has been much controversy in the past on the inclusion of inelastic scattering in the Kubo-Greenwood formula for the real part of the conductivity, given in the absence of inelastic effects by

$$\mathrm{Re}\sigma(\omega) = \frac{1}{(\omega\mathcal{V})} \sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} (f_\beta - f_\alpha) \, |\langle \alpha | \hat{J} | \beta \rangle|^2 \times \delta(\epsilon_\alpha - \epsilon_\beta - \hbar\omega) \tag{4.3}$$

which is nonzero only when the energy difference of the initial and final states equals $\hbar\omega$. Imry and Shiren,[23] Thouless and Kirkpatrick,[30] and others have proposed a generalization of (4.3) to include inelastic scattering. Their form is obtained by assuming that in the presence of inelastic scattering, the δ function in (4.3) would get broadened into a Lorentzian of width γ. Their expressions, derived explicitly for a wire, amount to replacing ω by $\omega+i\gamma$ in the presence of inelastic scattering. This simple prescription, while justified in a wire where only the *interband* processes contribute to the conductivity, does *not* work when there are diamagnetic effects present, as in a ring, for which as seen from (2.14), the first term only has an ω and not $\omega+i\gamma$. In fact, a somewhat incautious application of the above-mentioned ansatz can lead to unphysical conclusions in the presence of diamagnetic effects. Within a linear-response formalism, the expression in (4.1) can be obtained from a relaxation time approximation only when the system is allowed to relax to a state of quasiequilibrium that includes the perturbation. In Appendix A we share some of the insight we gained by dealing with different approximations for the quasiequilibrium distribution function.

The behavior of Reσ as a function of the level broadening is shown in Fig. 6. If γ is much less than the typical level spacing δE at the Fermi level, the conductivity is proportional to γ; in this regime the discrete nature of the levels is probed. A larger γ simply leads to a larger rate of absorption of energy because the electrons spend most of their time in a single state before giving up their energy. For $\gamma \gg \delta E$, the discreteness of the levels is unimportant and the inelastic scattering into other levels dominates the absorption of energy resulting in a Drude-like response where $\mathrm{Re}\sigma \propto 1/\gamma$. We would like to emphasize that the scattering rate that appears in the Drude

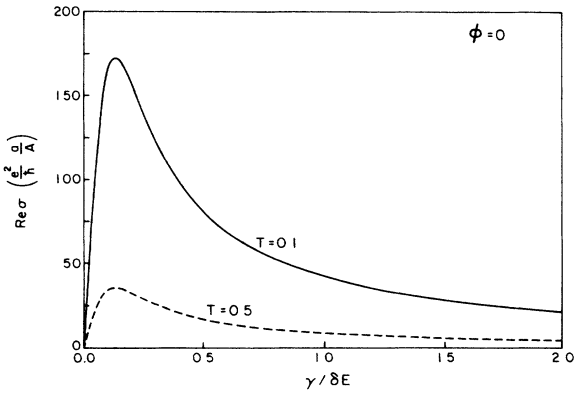

FIG. 6. dc conductivity in zero magnetic field as a function of the size of the inelastic scattering rate for two different temperatures $T/\delta E = 0.1$ and 0.5. Maximum dissipation occurs for $\gamma \approx \delta E$. For $\gamma \gg \delta E$ the behavior is Drude-like, with $\mathrm{Re}\sigma \propto 1/\gamma$.

expression is an *inelastic* and not an elastic rate. It can be seen that the maximum absorption occurs at $\gamma \approx \delta E$ (see Fig. 6).

As a function of the dc flux threading the ring, ϕ, we find that the conductivity is very sensitive to the position of the chemical potential μ when the temperature $k_B T$ and inelastic scattering rate $\hbar\gamma$ are much smaller than the typical level spacing δE. In this regime, where quantum-size effects dominate, the conductivity is clearly periodic in ϕ with period equal to the normal flux quantum $\phi_0 = hc/e$, as has been noted by other workers.[23,20] This is shown in Fig. 7 for two cases: (L) where μ is exactly at the center of the band, and (R) where μ is moved up by one level so that there is one more electron in the loop. Most of the absorption takes place either around zero flux or half flux quanta. The interband contribution to the conductivity switches from a maximum at $\phi=0$ to a minimum depending on the number of electrons. This behavior can be understood from the eigenvalue spectrum in Fig. 1 as a consequence of the different energy-level denominators involved in the absorption process. The total absorption must include intraband processes arising from the incomplete cancellation of diamagnetic terms. Shown by the solid curves in Fig. 7, the dissipative response including both interband and intraband contributions indeed has a *minimum* at $\phi=0$. The importance of these diamagnetic terms has also been discussed by Büttiker[22] within a Landauer formalism.

Note that the conductivity obtained above is a minimum at zero flux, as would be predicted from the theory of weak localization. Imry and Shiren[23] found instead a maximum at zero flux in the absence of any spin-orbit scattering, which they attributed to the energy denominator effect described in the weak scattering regime. In view of the above analysis, they got a maximum at zero flux because only the interband term was included in their calculation. They also claimed, in contrast to our results, that no change was observed when the chemical potential was shifted.

With increasing temperature, the conductivity is aver-

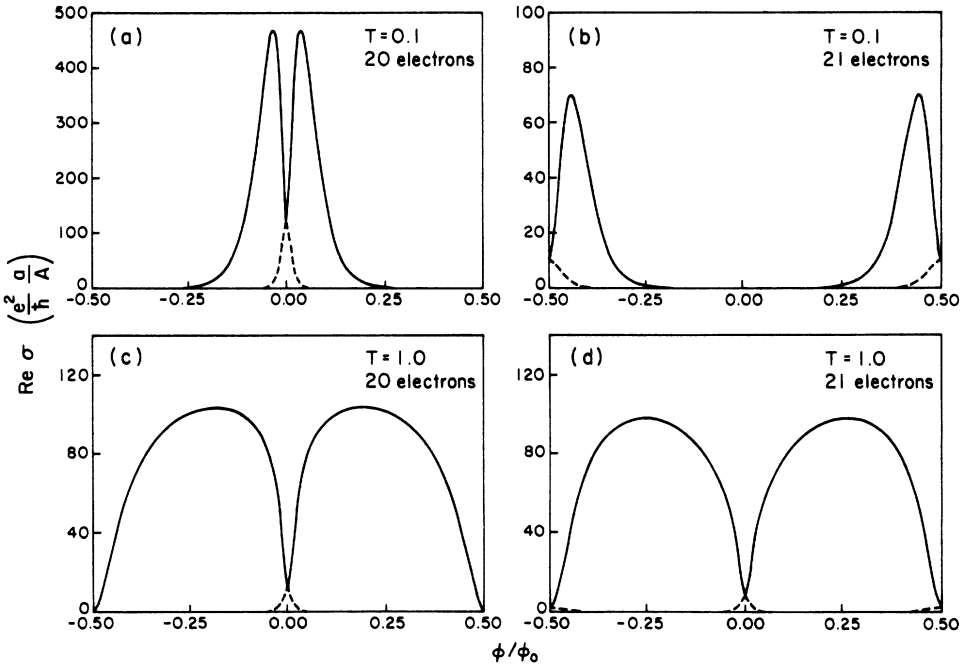

FIG. 7. dc conductivity as a function of the flux for two different temperatures. The figures are arranged in the same fashion as Fig. 4, where the corresponding reactive response is shown. The dotted curve is the interband contribution and the solid curve is the total dissipative response from (4.1) including both intraband and interband contributions.

aged over energies of order $k_B T$ which washes out the quantum-size effects. The sensitivity of σ to the position of the chemical potential is lost. However, as seen in the behavior of σ with ϕ at two different temperatures in Fig. 7, the periodicity of σ changes[20,23] to $hc/2e$ as the temperature becomes of order δE. The inelastic scattering shows qualitatively different behavior in that if the temperature is low, increasing γ does not produce sufficient averaging of the conductivity and the dominant periodicity continues to be hc/e. This is shown in Fig. 8, and is due in part to the fact that here we have ignored the level mixing caused by the coupling to the environment. This effect is not seen in Ref. 23 because in that calculation the temperature is high enough to produce the averaging by

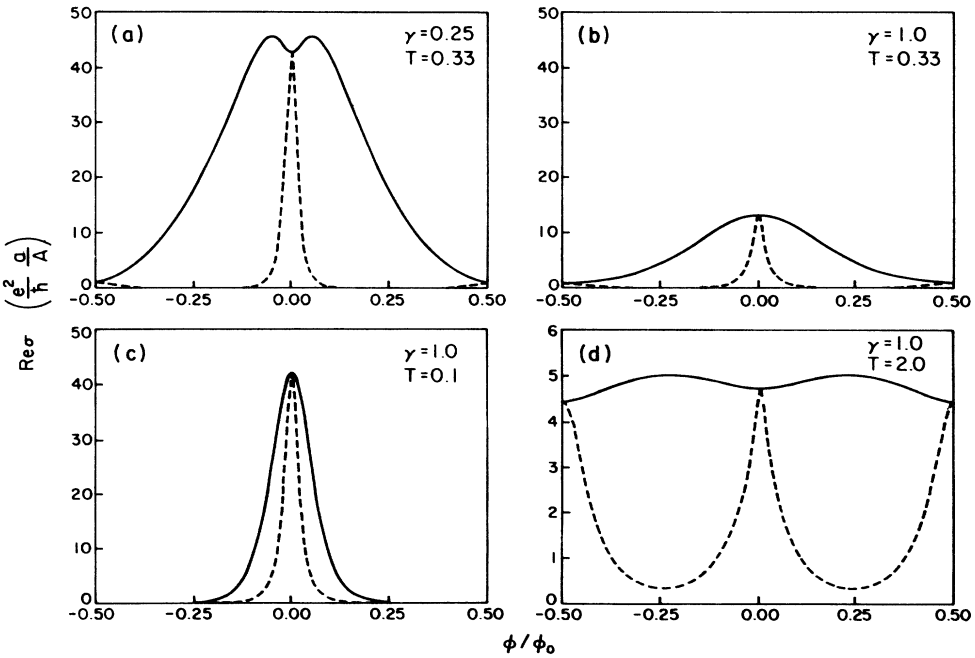

FIG. 8. dc conductivity as a function of the enclosed magnetic flux as the magnitudes of the inelastic scattering and temperature are changed. Only the case of 20 electrons on the 40 site ring is shown, since the case of an odd number of electrons is similar.

increasing the inelastic broadening. Inelastic scattering has a greater effect on the intraband conductivity compared to the interband term. We also find that the manner in which one averages changes the results one sees; averaging via a thermal distribution is in general much more effective than averaging via inelastic broadening of the electron states. Of course, in a real system both processes occur simultaneously and the inelastic effects are temperature dependent.

The effects arising above from the sensitivity to the flux is a measure of the conductance of a loop.[31] It is clear that the effects we are describing become negligible as the disorder is increased and localization sets in.[32] If the states are localized, by applying a gauge transformation all the vector potential can be put on a link where the wave function is zero. The electron then no longer "sees" the magnetic field.

V. DISCUSSION AND CONCLUSIONS

We have given a description of electrons in a normal metallic ring threaded by a magnetic flux using a tight-binding model. We have generalized the Kubo linear-response formalism to include two important effects pertinent to a calculation of response functions: (a) diamagnetic contribution arising from the special multiply connected geometry of the ring and (b) phenomenological description of inelastic scattering included in a way to ensure that probability is conserved. In order to separate the intrinsic properties of the ring from the perturbation caused by leads attached to it for measurement purposes, we have considered here an isolated ring. The response functions of such an isolated ring can be measured by a resonant-cavity experiment.

The dissipative response has been studied in considerable detail over the past couple of years by the Landauer transmission matrix method.[13,15,14] Here we have shown that similar results can be obtained via the Kubo formulation if the diamagnetic effects that lead to intraband scattering are retained. An important consequence of retaining the diamagnetic contribution is that $\mathrm{Re}\sigma$ has a minimum at $\phi=0$ in agreement with the predictions of weak localization. Previous approaches[23] that neglected such a term found a maximum instead.

We find that if the temperature and inelastic broadening are much smaller than the typical level spacing, the response of the system is very sensitive to the number of electrons in the band. This is the response that might be expected for a very small system that could almost be described as "molecular." In this regime the dissipative part of the conductivity shows a periodicity in flux of ϕ_0. As the inelastic broadening is increased or the temperature is raised, these "quantum-size effects" disappear and the response is no longer sensitive to the number of electrons. Within a single ring the periodicity changes to $\phi_0/2$. The actual averaging of the conductivity does not, however, appear until $k_B T$ or γ are much larger than the typical level spacing. Also, the energy averaging produced by a thermal distribution of electrons is found to be much more effective than that produced by inelastic broadening.

We have also studied the inductive response of the ring. If the inelastic length is longer than the size of the ring, the wave functions of the electron are coherent and lead to a persistent current in the ring. We suggest that it may be simpler to see the *change* in the persistent current in a ring via a phase-sensitive technique than to measure the current itself. The reactive response of a *single* ring continues to show a periodicity of ϕ_0 even when the temperature or the inelastic scattering rate are comparable to the spacing between levels, though the amplitude of the response goes down dramatically. In an *array* of rings the situation is different, and we expect the periodicity of ϕ_0 to be replaced by $\phi_0/2$. However, it is not so much the average over the different impurity configurations in each ring that produces a doubling of the period, but an average over even and odd number of electrons, (or average over positions of the chemical potential), that has the effect of changing the periodicity to $\phi_0/2$. Thus to see the doubled periodicity in the inductive response, it is, in fact, necessary to average over the number of electrons, while for the absorptive part this appears without the additional averaging.

Further work is needed to calculate the effect of multiple channels in the ring and also to understand the role of disorder on the reactive response. It would also be useful to develop a microscopic description of dissipation by coupling the ring to a bath of oscillators in a less *ad hoc* fashion than that used here and to study its effect on persistent currents and Zener tunneling.

Note added in proof. After this paper was submitted, we received a copy of a paper by H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih [Phys. Rev. B **37**, 6050 (1988)] which also discusses the question of persistent currents in a one-dimensional ring.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we discuss some of the subtleties associated with the relaxation time approximation and the linear-response formulation. We clarify the conditions under which the different forms for the conductivity, including those found by other workers[23,30] are obtained. We derive the correct expression for the conductivity in the case of a wire with open ends where the diamagnetic effects are unimportant. We also show that, while for the sake of purity, the chemical potential shift should be included, it has negligible effect on the dc conductivity of a loop.

The equation of motion of the density operator is given by

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}+\hat{H}_1(t),\rho] + \left[\frac{d\hat{\rho}}{dt}\right]_{\mathrm{col}}, \tag{A1}$$

where $\hat{H}$ is the unperturbed Hamiltonian with $\hat{H}\,|\,\alpha\rangle=\epsilon_\alpha\,|\,\alpha\rangle$. The second (collision) term on the right-hand side of (A1) is the change in ρ because of coupling to an external bath and is typically written within a relaxation time approximation as $-\gamma(\hat{\rho}-\hat{\rho}_{\rm QE})$. Here, γ is the relaxation rate and $\hat{\rho}_{\rm QE}$ is the quasiequilibrium distribution that is discussed in some detail below. In the spirit of linear response, we assume that $\hat{H}_1$ is a small perturbation and expand $\hat{\rho}=\hat{\rho}_0+\delta\hat{\rho}$, where $\hat{\rho}_0$ is the global equilibrium described by the Fermi distribution function. We also assume that the quasiequilibrium distribution can be expanded in powers of H_1 and $\hat{\rho}_{\rm QE}=\hat{\rho}_0+\delta\hat{\rho}_{\rm QE}$. We find

$$\langle\alpha\,|\,\delta\hat{\rho}\,|\,\beta\rangle=\frac{(f_\alpha-f_\beta)\langle\alpha\,|\,\hat{H}_1\,|\,\beta\rangle-i\gamma\langle\alpha\,|\,\delta\hat{\rho}_{\rm QE}\,|\,\beta\rangle}{\epsilon_{\alpha\beta}-\omega-i\gamma} \tag{A2}$$

for the off-diagonal components of the change in the density matrix, where $\hat{\rho}_0\,|\,\alpha\rangle=f_\alpha\,|\,\alpha\rangle$ and $\epsilon_{\alpha\beta}=\epsilon_\alpha-\epsilon_\beta$. The diagonal components are given by

$$\langle\alpha\,|\,\delta\hat{\rho}\,|\,\alpha\rangle=\frac{i\gamma}{\omega+i\gamma}\langle\alpha\,|\,\delta\hat{\rho}_{\rm QE}\,|\,\alpha\rangle\ . \tag{A3}$$

We now discuss three different choices for $\delta\hat{\rho}_{\rm QE}$ and derive the corresponding expressions for the conductivity. Note that in each case if one takes the limit $\gamma\to0$, the real part of the conductivity is given by the Kubo-Greenwood[12] formula (4.3) and also obeys the sum rule given in (4.2).

1. Case I: $\delta\hat{\rho}_{\rm QE}=0$

This situation arises when the conservation of particles is ignored entirely. In the same spirit we will assume that the f-sum rule is invoked to cancel the diamagnetic term. One then finds that the off-diagonal elements of the density matrix are given by

$$\langle\alpha\,|\,\delta\hat{\rho}\,|\,\beta\rangle=\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}-\omega-i\gamma}\langle\alpha\,|\,\hat{H}_1\,|\,\beta\rangle \tag{A4}$$

and correspondingly, the conductivity which only has an interband component to it is

$$\sigma(\omega)=\frac{i}{\omega\mathcal{V}}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\frac{\omega+i\gamma}{\epsilon_{\alpha\beta}-\omega-i\gamma}\,|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2\ . \tag{A5}$$

The real part of the conductivity is given as

$$\mathrm{Re}\sigma(\omega)=\frac{1}{\mathcal{V}}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\beta\alpha}}\,|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2\times\frac{\gamma}{(\epsilon_{\alpha\beta}-\omega)^2+\gamma^2}\left[\frac{2\epsilon_{\alpha\beta}^2}{(\epsilon_{\alpha\beta}+\omega)^2+\gamma^2}\right]\ . \tag{A6}$$

This is the correct generalization of (4.3) that includes the effect of inelastic scattering on a system assumed to relax to a state of *global* equilibrium.

2. Case II: $\langle\alpha\,|\,\delta\hat{\rho}_{\rm QE}\,|\,\alpha\rangle=0$

This situation is obtained for a wire with open ends, since the current operator then has no diagonal elements. From (A3) we see that probability is trivially conserved since the diagonal components of the density matrix are unchanged by the external perturbation. Then the change in the density operator involves only off-diagonal components. The quasiequilibrium density matrix is given by

$$\langle\alpha\,|\,\delta\hat{\rho}_{\rm QE}\,|\,\beta\rangle=\frac{(f_\alpha-f_\beta)\langle\alpha\,|\,\hat{H}_1\,|\,\beta\rangle}{\epsilon_{\alpha\beta}}\ , \tag{A7}$$

where note that case II differs from case I in that the perturbation H_1 has been included in the description of the equilibrium state. The off-diagonal elements of $\delta\rho$ are the same as in (2.5) and differs from case I by a multiplicative factor $(\epsilon_{\alpha\beta}-i\gamma)/\epsilon_{\alpha\beta}$. Once again, evaluating the diamagnetic and paramagnetic contributions to the induced current, we find

$$\sigma(\omega)=\frac{i}{\mathcal{V}}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\alpha\beta}}\frac{1}{\epsilon_{\alpha\beta}-\omega-i\gamma}\,|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2\ , \tag{A8}$$

where the diamagnetic contribution is completely canceled by a part of the paramagnetic term since $\langle\alpha\,|\,\delta\rho\,|\,\alpha\rangle$ vanishes. It is obvious that the results for the wire are quite different from the situation discussed in Sec. II, in that there is no intraband term.

The dissipative part of the conductivity in (A8) is then given by

$$\mathrm{Re}\sigma(\omega)=\frac{1}{\mathcal{V}}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\frac{f_\alpha-f_\beta}{\epsilon_{\beta\alpha}}\,|\,\langle\alpha\,|\,\hat{J}_p\,|\,\beta\rangle\,|^2\frac{\gamma}{(\epsilon_{\alpha\beta}-\omega)^2+\gamma^2} \tag{A9}$$

which is the form obtained by Thouless and Kirkpatrick[30] for the interband term in a wire. Here we have shown that this modification of the Kubo-Greenwood conductivity in the presence of inelastic scattering can be justified if the system is allowed to relax to a quasiequilibrium state that includes the effects of perturbation. The imaginary part of the conductivity in (A8) has no divergence at zero frequency because of the different boundary conditions in a wire compared to the loop considered in this paper.

3. Case III: $\langle\alpha\,|\,\delta\hat{\rho}\,|\,\alpha\rangle\neq0$

This is the situation encountered for a ring geometry. While the form of the interband scattering is the same as in a wire given in (A9), there is an additional intraband term. Also as discussed in Sec. II the system must be allowed to relax to a state of quasiequilibrium with a time-dependent chemical potential in order to conserve proba-

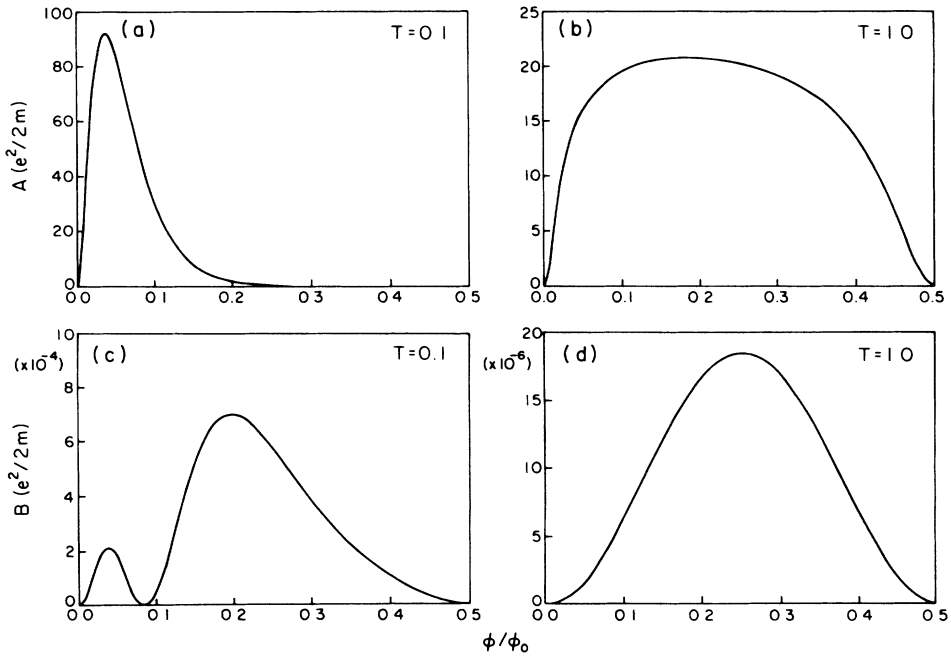

FIG. 9. The behavior of the two terms A, given in (A10), and B, given by (A11), for two different temperatures. Both terms contribute to the change in the diagonal components of the density matrix (2.6). The B term arising from the change in the chemical potential with flux is indeed seen to be negligible compared with A. Note the different scales for A and B.

bility. We wish to show that the chemical potential shift is quite small. Consider the two competing terms given in (2.9). Let us define

$$A=\sum_{\alpha}\frac{\partial f_{\alpha}}{\partial\epsilon_{\alpha}}(\langle\alpha\,|\,\hat{J}_p\,|\,\alpha\rangle)^2 \tag{A10}$$

and the term arising from the shift in the chemical potential by

$$B=\sum_{\alpha}\frac{\partial f_{\alpha}}{\partial\epsilon_{\alpha}}\langle\alpha\,|\,\hat{J}_p\,|\,\alpha\rangle\langle J_p\rangle\ , \tag{A11}$$

where $\langle J_p\rangle$ is defined in (2.7). In Fig. 9 we show the behavior of the terms A and B at two different temperatures. The correction introduced by the chemical potential shift is indeed negligible. Thus our conclusion is that it is not essential to include a chemical potential shift in the quasiequilibrium distribution function, however, it is crucial to include the perturbation H_1 in $\langle\alpha\,|\,\delta\rho_{\rm QE}\,|\,\beta\rangle$.

APPENDIX B

We obtain conditions under which self-inductance processes in the loop can be neglected. In the discussion in Sec. II we obtained the current induced in the loop by applying a time-dependent magnetic flux through it. In general, the induced current will also produce a flux, $\phi_{\rm ind}=\mathcal{L}I_{\rm ind}$, where $\mathcal{L}$ is the self-inductance of the loop and $I_{\rm ind}$ is the induced current in the loop. Thus

$$I_{\rm ind}=(g_1+ig_2)\frac{i\omega}{c}\phi_{\rm tot}\ , \tag{B1}$$

where the total flux through the loop is the sum of the external and the induced flux and $G=g_1+ig_2$ is the conductance of the ring. Substituting for the induced flux in terms of the self-inductance of the loop, we get

$$I_{\rm ind}=\frac{g_1+ig_2}{1+(i\omega/c)\mathcal{L}(g_1+ig_2)}(i\omega/c)\phi_{\rm ext} \tag{B2}$$

which reduces to

$$I_{\rm ind}=[g_1+i(g_2-\omega\mathcal{L}g_1^2)/c](i\omega/c)\phi_{\rm ext} \tag{B3}$$

if it is assumed that the induced effects are small. In general, g_1 is also renormalized by the self-inductive effects, but we focus here on the reactive part. The condition for the reactive response of the ring not to be masked by self-inductive effects is

$$\frac{g_2}{g_1}\gg\frac{\omega}{c}\mathcal{L}g_1\ . \tag{B4}$$

If we assume the thickness of the wire, δ, is much smaller than its length, we find the inductance to be given approximately by[33]

$$\mathcal{L}=\frac{2L}{c}(-\ln\eta)\ , \tag{B5}$$

where $\eta=2\pi\delta/L$. Consider typical numbers for a 1-μm loop having a square cross-sectional area of 0.01×0.01 μm^2. If the resistivity is taken as 2 $\mu\Omega$ cm, then $g_1\approx20e^2/\hbar$. Using the expression for the inductance of a loop in (B5), we find $\mathcal{L}g_1/c\approx0.5\times10^{-12}$ sec. This can be combined with (B4) to set limits on the frequency used to study the response functions. To estimate g_2 note that it is of the order of $(c/\omega L)(ev_F/\phi_0)$ as can be seen from (3.1). This implies that $\omega\ll10^{12}$ sec^{-1} if intrinsic response is to be seen, unmasked by ordinary inductive effects.

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