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Linear temperature dependence of the resistivity in the new superconductors

L. Civale and E. N. Martinez
Centro Atómico Bariloche, 8400 Bariloche, Río Negro, Argentina
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We present an elementary proof that, irrespective of dimensionality, the electrical resistivity of a simple conductor should be linear with temperature for $T > 2p_F s/k_B \ll \Theta_D$ if $p_F \ll h\pi/a$, where p_F is the Fermi momentum, s the speed of sound, a the lattice constant, and Θ_D the Debye temperature.

The properties of the new high-T_c superconductors[1] have attracted widespread interest over the last year. While their extremely high superconducting transition temperatures are their most striking features, their properties in the normal state (and those of nonsuperconducting related copper oxides[2]) are also anomalous, and it is widely expected[3-5] that they might offer valuable clues to the nature of the new phenomenon. In particular, it has been felt by many researchers that the electrical resistivity in the normal state in some samples is linear over too wide a range of temperature.[6] Indeed, in well-oxygenated samples,[5] this linear dependence starts right at the transition temperature and extends, with a constant slope, to the highest temperatures that have been measured, 1100 K. The absence of saturation at high temperatures means that the electron mean free path is still longer than the lattice constant at those temperatures, and at low temperatures should be much longer than the extremely short coherence lengths which have been deduced[7] for these materials, which should accordingly be treated as clean superconductors.[8] The linear temperature behavior of the electrical resistivity is to be expected for temperatures larger than Θ_D, the Debye temperature of the solid (if it is due to electron-phonon interaction; see Ref. 3 for another view). What is deemed unusual is that in these materials it can start at temperatures markedly lower than Θ_D. The situation is specially striking in first generation high-T_c superconductors, with their lower transition temperatures: in Fig. 1, we show a typical resistivity curve for $La_{1.8}Sr_{0.2}CuO_4$ taken from Ref. 5. The Debye temperature, as obtained from low-temperature specific-heat measurements,[9] or independently, from ir and Raman measurements[10] is shown on the T axis. The slight upturn at high temperatures is attributed[11] to loss of oxygen over the finite time taken by the resistivity measurements.

Recently, Micnas, Ranninger, and Robaszkiewicz[6] have addressed this problem: they find that this behavior might be linked to the two-dimensional (2D) character of electrical conduction (along Cu—O planes) in these materials,[12] provided that the Fermi momentum, p_F, is much less than $\pi\hbar/a$, where a is the lattice constant. They find that under these conditions, whenever the temperature is higher than $T^* = 2sp_F/k_B \ll \Theta_D$, where s is the speed of sound in the medium, a linear temperature dependence of the resistivity should be observed.

We shall show in this note, using the simplest arguments, that the above defined T^* should replace Θ_D in determining the different temperature regimes for the electrical resistivity, in agreement with Micnas *et al.*[6] It will be evident from our arguments that the results need not be restricted to two-dimensional systems, and that dimensionality is, in fact, irrelevant in this context.

For the sake of simplicity, we shall work with the same assumptions as Micnas *et al*:[6] quasi free electrons with a parabolic dispersion relation and Fermi velocity V_F; and Debye phonons with speed of sound s. That $V_F \gg s$ is well known.[9,10,13]

Electron-phonon interactions cause the electric current carried by electrons with momentum p and energy $\varepsilon(p)$ to decay at a rate given by[14]

$$\Gamma(\varepsilon_p) = \int \frac{d^D q}{(2\pi)^{D-1}} |v(p-q)|^2 \left(1 - \frac{\mathbf{p}\cdot\mathbf{q}}{p^2}\right) \{[N(\omega_{p-q}) + 1 - f(\varepsilon_q)]\delta(\varepsilon_q - \varepsilon_p + \omega_{p-q}) + [N(\omega_{p-q}) + f(\varepsilon_q)]\delta(\varepsilon_q - \varepsilon_p - \omega_{p-q})\} , \quad (1)$$

where ω_q is the energy of a phonon with momentum q, $N(\omega)$ and $f(\varepsilon)$ are equilibrium Bose and Fermi distribution functions at temperature T, respectively, and $v(k)$ is the electron-phonon scattering matrix element. D is the dimensionality of the system.

In normal situations, the only temperature scale in expression (1) is provided by the Debye temperature Θ_D: There are no phonons with energies larger than $k_B\Theta_D$. The second energy entering (1), the chemical potential of the electrons ε_F, is so high as to be irrelevant at ordinary temperatures. Consequently, there are two distinct regimes in $\Gamma(\varepsilon_p)$ [and also in the electrical resistivity $\rho(T)$, as its calculation from $\Gamma(\varepsilon_p)$ entails no energy constants other than ε_F]: For $T \ll \Theta_D$, as T is raised, new, more energetic phonon modes come into play which cause wider-angle electron scatterings, thus increasing their efficiency to relax the electric current. The result is the well known T^5 (in 3D, or T^4 in 2D) dependence of $\rho(T)$ at low temperatures. For $T \gg \Theta_D$ all phonon modes are classically excited, electron scattering by phonons is as wide angle as

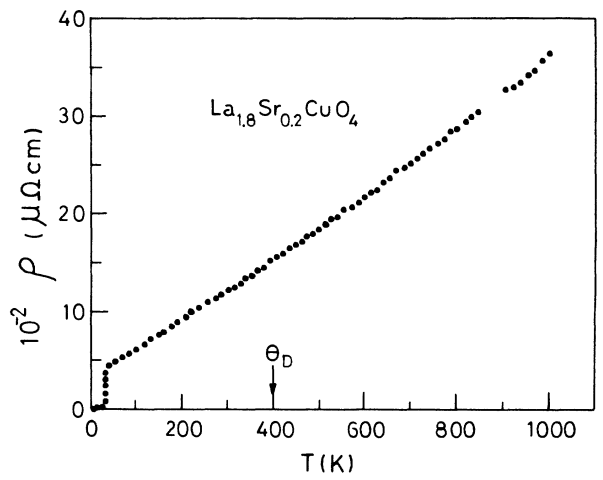

FIG. 1. Electrical resistivity vs temperature for optimally oxygenated $La_{1.8}Sr_{0.2}CuO_4$ sample. The Debye temperature Θ_D is marked on the T axis.

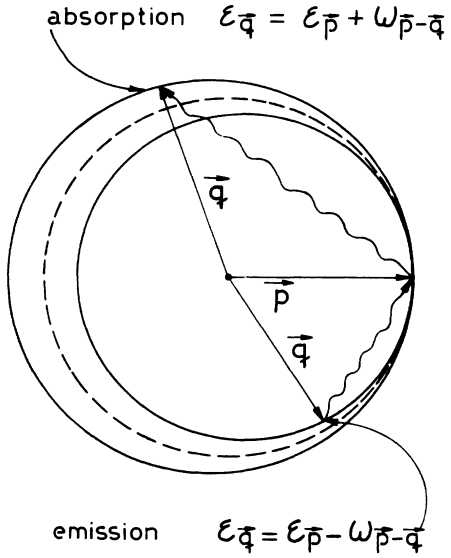

FIG. 2. One-phonon scattering of electron with initial momentum p. The tips (tails) of allowed phonons lie on the full outer (inner) energy surface for phonon absorption (emission). The ε_p constant energy surface is shown by a dotted line. The ratio s/v_F is vastly exaggerated to 0.1.

it can be, and raising the temperature only produces more processes of the same kind. The number of processes increases linearly with temperature, and the linear regime in $\rho(T)$ is obtained.

The time-proven Grüneisen-Bloch relation[15]

$$\rho(T) = C\left(\frac{T}{\Theta_R}\right)^5 \int_0^{\Theta_R/T} \frac{x^5 dx}{(e^x-1)(1-e^{-x})} \qquad (2)$$

gives a useful interpolation between these two regimes in terms of only one parameter (C is a scale factor), the temperature Θ_R. For most metallic elements, this Θ_R coincides with Θ_D within a factor of 2 (even within 20% in most cases).[16]

Consider now what happens if we assume that $p_F \ll h\pi/a$, i.e., that the Fermi surface is much smaller than the Brillouin zone. The conservation relations that electron-phonon collisions must satisfy should be studied now. Energy conservation is explicitly contained in the δ functions in (1). In normal processes, momentum is also conserved. Figure 2 shows which phonons may be either absorbed or emitted by an electron with initial momentum p.[17] Neglecting terms of order $s/V_F \ll 1$, we see that phonons with momenta larger than $2p$ cannot interact with such an electron.

At temperatures much lower than the Fermi temperature ε_F/k_B, the only electrons which contribute to the transport of electrical current are in a narrow energy region around ε_F. This implies that phonons with momenta larger than $2p_F$ do not contribute to normal scattering processes. Although such phonons lead to valid umklapp processes, these will be exponentially scarce if $T < \Theta_D$, as discussed by Micnas *et al.*,[6] and can be safely neglected.

If phonons with momenta larger than $2p_F$, or energies larger than $2p_F s$, cannot take part in electron scattering, the electrical resistivity should behave as if they were not there: Its temperature dependence will be that of a solid with $\Theta_D \simeq 2p_F s/k_B$. This was obtained in Ref. 6; we can see that dimensionality is not relevant. It should be noted, however, that dimensionality might be of importance in a less direct way. In effect, as the volume density of electrons n and p_F are related—always for free electrons—by $n \sim p_F^p$, for a given n the two-dimensional relation yields the smaller p_F: Two-dimensional behavior would thereby contribute to the fulfillment of the condition $p_F \ll h\pi/a$.

The Grüneisen-Bloch temperature should then be given, within a factor of 2, by

$$\Theta_R \simeq \min(\Theta_D, 2p_F s/k_B)$$

and as $\Theta_D \simeq h\pi s/k_B a$, we have

$$\Theta_R \simeq \Theta_D \min(1, p_F a/h) \ . \qquad (3)$$

This result is easily generalized to nonspherical Fermi surfaces: $p_F a/h$ in (3) is basically the ratio between the "diameters" of the Fermi surface and the first Brillouin zone.

Does (3) allow us to derive any conclusion on the size of the Fermi surface in, say, La-Sr-Cu-O? Not really. Although it would be tempting to conclude from the fact that La-Sr-Cu-O shows a linear resistivity for $T \geq 40$ K and a $\Theta_D \simeq 400$ K (Refs. 9 and 10) that $p_F a/h \leq 0.1$, the conclusion would be unwarranted (albeit probably true). The fact is that in normal conductors the linear regime extends very often to temperatures rather lower than Θ_D. This is shown in the standard Grüneisen-Borelius relation:[18] $\rho(T)/\rho(\Theta_R)$ is linear in T/Θ_R for $T/\Theta_R \geq 0.2$ for isotropic metals. Deviations do occur. In addition to this, Θ_R can be markedly lower than Θ_D. Some examples contained in Meaden's monograph[16] should suffice: for calcium $\Theta_R/\Theta_D \simeq 0.65$; for uranium 0.62, for Zn 0.71. A linear behavior starting at $0.15\Theta_D$ does not constitute an exception, and for none of these metals can we draw the above conclusion.

There is another factor that should be taken into account: The Debye temperature of La-Sr-Cu-O is taken as

400 K in order to obtain the right specific heat at high temperatures. It has to account for all 21 phonon branches. Not all phonons are equally effective in scattering long-wavelength electrons, however, and a "Debye temperature" which took into account only acoustical phonons would perhaps be more adequate for comparison in (3). This temperature would be nearer 200 K, as obtained from low-temperature specific heat.[9]

We conclude that, while the existence of a small Fermi surface in the center of the first Brillouin zone should extend downwards, due to the conservation rules for electron-phonon processes, the range of linear temperature dependence of the resistivity, the available resistivity data for ceramic La-Sr-Cu-O do not allow one to draw useful conclusions, either about the size of its Fermi surface, or about the dimensionality of the system that effectively conducts the electricity in this material. For Y-Ba-Cu-O, with its lower Θ_D and higher T_c, the same conclusion applies. Other data are needed. For instance, the strong anisotropy of resistivity in Y-Ba-Cu-O single crystals[19] provides convincing evidence for 2D transport in this case.

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