Current-voltage characteristics in a two-dimensional model for flux flow in type-II superconductors

H. J. Jensen,* A. Brass, Y. Brechet,† and A. J. Berlinsky
Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 7 July 1988)

It is argued that flux motion near threshold in a macroscopic, two-dimensional lattice with a random potential, under the influence of an external driving force, always involves plastic flow of moving portions of the lattice past regions that are pinned. Convincing evidence for this picture is obtained from a comparison of experimental measurements to computer simulations of the nonlinear curent-voltage characteristics of finite systems.

Over the past year we have undertaken an extensive series of numerical simulations of a system that models flux pinning and flux flow in two dimensions. As a result of these studies it has become apparent that two-dimensional flux flow near threshold in macroscopic systems always involves plastic motion in which some regions of the flux lattice move with respect to other regions which remain pinned. We have described this motion in an earlier work[1] as pulsating flow through channels.

This qualitative picture is quite different from the well-known analytic theory of Larkin and Ovchinnikov[2] (LO) which was thought to apply at least for the case of weak pinning. According to LO theory, correlated regions of the flux lattice arrange themselves in local minima of the random potential. The bulk pinning force is then given by the sum of the fluctuations of the random force over each of the correlated regions. Their analysis is based on elasticity theory and is inapplicable when the mechanical properties of the lattice are affected by dislocations or other defects. Above threshold, LO theory corresponds to the situation in which the lattice jumps from one metastable elastic minimum to another, without plastic deformation (i.e., maintaining the association of nearest neighbors). We refer to this type of flow as the "elastic instability regime," and our simulations show that such behavior occurs only for systems of finite size. Specifically we have found that, for a system of size L, there is a critical strength of the random potential, $A_{\text{plas}} \sim 1/(\ln L)$, above which dislocations and higher order defects such as disclinations appear. For pinning strengths larger than A_{plas} and, presumably, for arbitrary pinning strength in infinite systems, flux flow proceeds by plastic deformation of the disordered lattice.

It is by no means surprising that a two-dimensional lattice in a random potential should be unstable against the formation of dislocations and disclinations. In fact the disclinations are required to destroy long-range bond orientational order since, according to a theorem of Imry and Ma,[3] long-range order corresponding to the breaking of a continuous symmetry is destroyed by a random potential. Our observation that the strength of the random potential that nucleates lattice defects scales as $1/(\ln L)$ implies, by finite-size scaling, that the average distance between defects has the form

$$\xi(A) = \xi_0 \exp(A_0/A) ,$$

where A_0 is the nucleation energy for a defect and A is the strength of the random potential. A_{plas} is then given by

$$\xi(A_{\text{plas}}) = L .$$

For a two-dimensional (2D) crystal with no random potential at finite temperature, the shear restoring force vanishes at the Kosterlitz-Thouless temperature at which dislocation pairs unbind. For the 2D lattice in a random potential at $T=0$, we expect that unbound dislocations are always present and that they dominate the mechanical properties and the nature of the flow when the lattice is moved by an external force.

This simple and self-contained description that emerged from our early work is quite appealing, but it is still necessary to make convincing contact with experiment. This has now been accomplished through simulations of the nonlinear current-voltage relations for the 2D system in a random potential. We find that for weak random potentials for finite systems, i.e., in the elastic instability regime, the voltage above threshold is linear in the current, unlike the experimental results, while for $A > A_{\text{plas}}$, the nonlinear behavior above threshold is strikingly similar to the experimental data.[4,5] We interpret the nonlinear current as resulting from a fluidlike flow of the flux lines. This motion is associated with the depinning of more and more flux lines as the current is increased until eventually most or all of the flux becomes unstuck. The analytic form of the current above threshold is difficult to determine, but our data are consistent with $I \sim |V - V_c|^{\zeta}$ with the value of ζ about $\frac{3}{2}$.

Our system consists of N_v vortices with positions $\mathbf{r}_i(t) = (x_i(t), y_i(t))$ and N_p attractive pins with random fixed positions $\mathbf{r}_i^p$ in an area $A = L_x L_y$ with periodic boundary conditions.[1,6,7] The potential energy is given by

$$U = \frac{1}{2} \sum_{i \neq j} V_v(|\mathbf{r}_i - \mathbf{r}_j|) + \sum_{i,j} V_p(|\mathbf{r}_i - \mathbf{r}_j^p|) - \sum_i r_i \cdot \mathbf{F}_{\text{dr}} ,$$
$$V_v(r) = A_v v(r/R_v), \quad V_p(r) = -A_p v(r/R_p) , \tag{1}$$

where $v(\rho)$ is a Gaussian-type potential and $\mathbf{F}_{\text{dr}}$ is an homogeneous external driving force. The units are fixed by the choice $A_v = 1$ and the ideal triangular vortex lattice spacing $a_0 = 1$. Our simulations are made on a system consisting of 1020 vortices and 219 or 438 pins. The range of the vortex-vortex interaction is $R_v = 0.6$, corresponding to a shear modulus $C_{66}=0.2695$ and a compression modulus $C_{11}=1.9943$. The range of the vortex-pin interaction is $R_p=0.25$. The vortices follow a diffusive equation of motion

$$\eta \frac{d\mathbf{r}_i}{dt} = -\frac{\partial U}{\partial \mathbf{r}_i} . \qquad (2)$$

The time scale is set by choosing the friction coefficient $\eta=1$.

The simulations are run as follows: We first set the driving force to zero and relax an ideal triangular lattice to the random potential by using molecular-dynamics annealing.[1] We then apply a constant homogeneous driving force larger than the threshold force F_T and follow the vortex system as it evolves according to Eq. (2). The system is followed until the center of mass (c.m.) has shifted at least three lattice spacings . The c.m. velocity $\langle v \rangle$ time averaged over the run, is then measured. This corresponds to measuring the I-V characteristic of a type-II superconductor in which case the average center-of-mass velocity of the flux lines is proportional to the voltage and the driving force is proportional to the current.[8]

We have summarized our results in Fig. 1. The velocity versus driving force are shown for a range of values of the amplitude of the pinning centers. One can clearly see how a nonlinear region starts to develop for $A_p \gtrsim A_{cr} \simeq 0.04$. The value of A_{cr} is indistinguishable from A_{plas}, the value of A_p at which the random potential becomes able to induce plastic deformations in the vortex lattice.[1] Above A_{plas}, some vortices remain trapped on pins as the rest of the vortex system begins to flow. The crossover at A_{cr} also coincides with a steep increase in the threshold pinning force.[7] In the linear regions of the $\langle v \rangle - F_{dr}$ curves, i.e., for all driving forces for $A_p \lesssim A_{cr}$ and for large driving forces in the regime $A_p \gtrsim A_{cr}$, the moving vortices form a well-defined lattice. A fluidlike form of the radial distribution function $G(r)$ is observed in the nonlinear regions of the $\langle v \rangle - F_{dr}$ curves, whereas $G(r)$ is always solidlike in the linear regions. Figure 2 shows an example of the flow pattern in the nonlinear region. These findings agree well with neutron scattering experiments on moving *FL* lattices in type-II superconductors.[9] All this strongly indicates that the random potential induces defects in the vortex lattice when the potential reaches a certain strength, $A_{cr} \simeq A_{plas}$. It should be emphasized that the amplitude A_{cr} is much smaller than the amplitude needed for a single pin to be able to induce elastic instabilities or trapping of vortices.[7]

It is worth noticing the similarity between the curves in Fig. 1 for which $A_p \gtrsim A_{cr}$ and the I-V characteristics for type-II superconductors. An example of the latter (from Huebener[4]) is shown in Fig. 3. The field dependence of the I-V characteristics is consistent with the A_p dependence of the curves in Fig. 1. The pinning potential in a type-II superconductor scales with magnetic field as $1-b$, where $b=B/B_{c_2}$ is the reduced field, B the internal field, and B_{c_2} the upper critical field.[10] The shear modulus C_{66} of the flux line lattice depends on the magnetic field as $b(1-b)^2$ (see Ref. 11). So the pinning strength becomes relatively weaker as b is increased as long as b is not too high.

It has recently been suggested by Fisher[12] that the nonlinear behavior connected with the depinning phenomena in the case of weak pinning, at least in the case of charge-density waves (CDW), might be an example of a dynamical critical phenomenon, and experiments on nonlinear I-V characteristics in type-II superconductors have been interpreted along these lines.[5] If such an interpretation is correct one would expect the correlation length of the velocity-velocity correlation function to diverge as the threshold force is approached from above.[12] We have not been able to find any sign of an increase in the velocity-

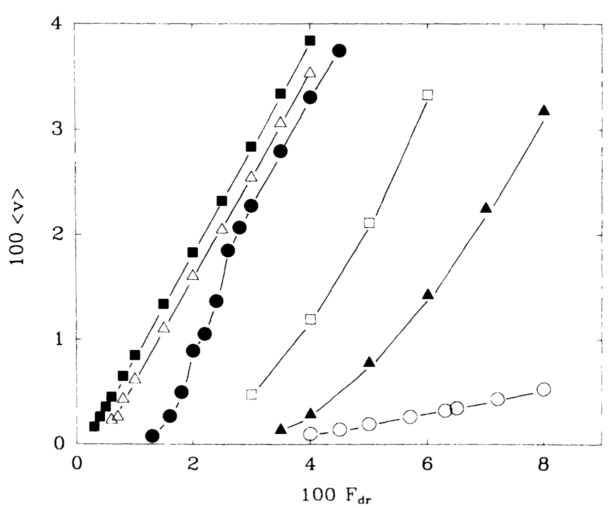

FIG. 1. Center-of-mass velocity vs applied driving force for different values of the pinning strength: ■, $A_p=0.025$; △, $A_p=0.04$; ●, $A_p=0.05$; □, $A_p=0.07$; ▲, $A_p=0.1$; and ○, $A_p=0.5$. The system parameters are $N_v=1020$, $R_v=0.6$, $N_p=438$, and $R_p=0.25$.

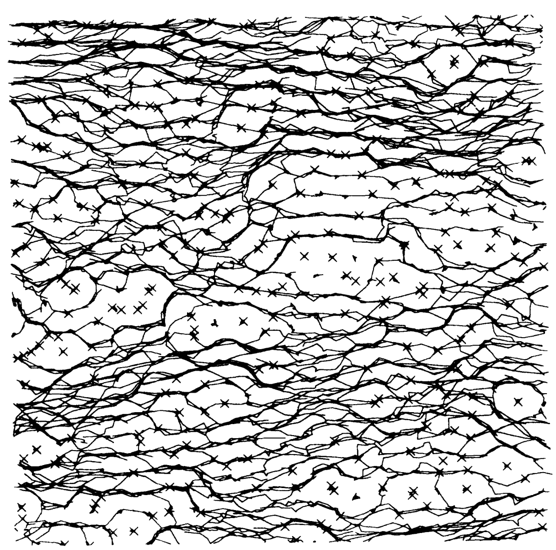

FIG. 2. A portion of the flow pattern in the nonlinear part of the $\langle v \rangle - F_{dr}$ curve. The lines show the trajectories of the vortices as they respond to the driving force. Short lines, resembling dots, occur in trapped regions. Crosses are the pinning centers. The system parameters are as in Fig. 1, and $A_p=0.07$ and $F_{dr}=0.04$.

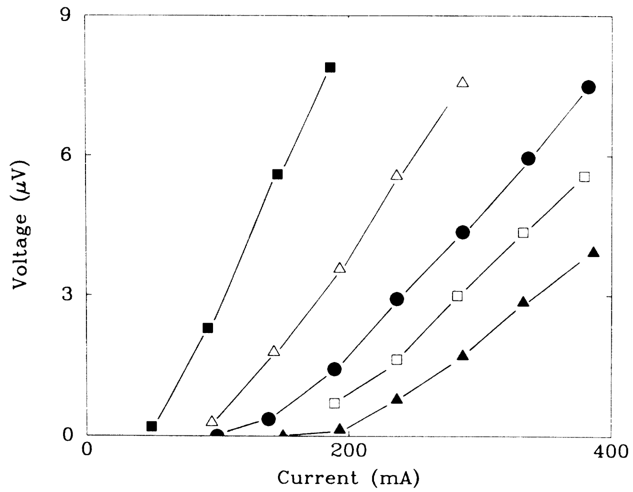

FIG. 3. Experimental I-V characteristics for different values of the magnetic field. The signature is as follows: ■, $b=0.74$; △, $b=0.55$; ●, $b=0.46$; □, $b=0.41$; and ▲, $b=0.37$.

velocity correlation length. The absence of any critical behavior in this regime might not be surprising, given the fluidlike flow in the nonlinear regions.[13] Nor did we find any critical behavior in the weak pinning regime below A_{cr}; which is again not surprising, as there are no nonlinear regions in the $\langle v\rangle - F_{dr}$ curves in this regime. We would therefore rather interpret the experimentally observed nonlinear I-V characteristics as an indication of the presence of pinning which is strong in the sense that the flux line lattice is plastically deformed and flows in a fluidlike manner with some vortices remaining pinned.

The situation may be different in the charge-density-wave case,[13] where the order parameter, the phase ϕ, has a single component, compared to our two-component order parameter, the positions, $\mathbf{r}_i$. In addition, the CDW systems which have been studied so far experimentally involve bulk three-dimensional crystals, rather than two-dimensional films which would be the analogs of the system we have considered. It is worth mentioning, however, that plastic flow has been observed in CDW systems,[14] and the importance of "phase vortices" for plastic flow in CDW conductors has been emphasized by Maki and coworkers.[15]

It is clear from Fig. 1 that the curves flatten out and that the width of the nonlinear region increases with stronger pinning. However, it is difficult to analyze the form of the $\langle v\rangle - F_{dr}$ curves near the threshold. The mean-field calculation in Ref. 12 suggests that near threshold $\langle v\rangle = B(F_{dr}-F_T)^\zeta$ with the mean-field value of ζ equal to $\frac{3}{2}$ and B depending on the random pinning potential. We have tried to fit our data to this form and also to the parabolic shape $a(F_{dr}-F_T)+b(F_{dr}-F_T)^2$. Both forms fit equally well, as was also found experimentally.[5] Consistent fits can be made to the power law, with B decreasing with increasing pinning strength and ζ about 1.5, or to the parabola with a and b decreasing with increasing A_p. It should be noted though, that there is a considerable uncertainty in the fitted parameters, B, F_T, ζ, and a, b, F_T, respectively, for the present data.

In summary we have studied a model which contains all the relevant qualitative physics of two-dimensional viscous flow of flux lines in type-II superconducting films. It is found that a nonlinear I-V characteristic is always caused by a fluidlike flow of the vortex system. This happens when the random pin potential is strong enough to induce defects in the vortex system which destroy the vortex lattice. We infer that the observation of nonlinear I-V characteristics in superconducting thin films is evidence for plastic flow in these systems.

We gratefully acknowledge useful comments on the manuscript by Catherine Kallin and Per Bak. One of us (H.J.J.) is partially supported by the Danish Natural Science Research Council, and another (A.B.) by North Atlantic Treaty Organization (NATO). This work was supported by the Natural Sciences and Engineering Research Council of Canada.

*Present address: Nordisk Institute for Teoretisk Atomfysik (NORDITA), Belgdamsvej 17, DK-2100 Copenhagenø, Denmark.

†Permanent address: Laboratoire de Thermodynamique et Physicochimie Metallurgique, Ecole Nationale Supérieure d'Electronique et d'Electromécanique de Grenoble, Domaine Universitaire de Grenoble, Boite Postal 75, 38402 Saint-Martin-d'Hères, France.

[1]H. J. Jensen, A. Brass, and A. J. Berlinsky, Phys. Rev. Lett. **60**, 1676 (1988).

[2]A. I. Larkin and Yu.N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[3]Y. Imry and S.-k. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[4]R. P. Huebener, Phys. Rep. **13C**, 143 (1974); A. M. Campbell and J. E. Evetts, *Critical Currents in Superconductors* (Taylor & Francis, London, 1972).

[5]F. de la Cruz, J. Luzuriaga, E. N. Martinez, and E. J. Osquiguil, Phys. Rev. B **36**, 6850 (1987).

[6]E. H. Brandt, J. Low Temp. Phys. **53**, 41 (1983); **53**, 71 (1983); Phys. Rev. Lett. **50**, 1599 (1983).

[7]A. Brass, H. J. Jensen, and A. J. Berlinsky, Phys. Rev. B (to be published).

[8]Y. B. Kim and M. J. Stephen, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2.

[9]P. Thorel, R. Kahn, Y. Simon, and D. Cribier, J. Phys. (Paris) **34**, 447 (1973).

[10]E. V. Thuneberg, J. Low Temp. Phys. **57**, 415 (1984).

[11]E. H. Brandt, Phys. Rev. B **34**, 6514 (1986).

[12]D. S. Fisher, Phys. Rev. Lett. **50**, 1486 (1983); Phys. Rev. B **31**, 1396 (1985).

[13]D. S. Fisher, in *Nonlinearity in Condensed Matter,* edited by A. R. Bishop, D. K. Campbell, P. Kumar, and S. E. Trullinger (Springer-Verlag, Berlin, 1987).

[14]R. P. Hall, M. F. Hundley, and A. Zettl, Phys. Rev. Lett. **56**, 2399 (1986).

[15]See K. Maki and X. Z. Huang, Phys. Rev. B **37**, 8668 (1988); and references therein.