Static critical behavior of the two-dimensional Ising spin glass $Rb_2Cu_{1-x}Co_xF_4$

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(Received 20 May 1988)

The temperature and field dependences of the nonlinear susceptibility are examined in $Rb_2Cu_{1-x}Co_xF_4$ with $x = 0.218$, which is an experimental realization of the two-dimensional Edwards-Anderson Ising spin glass with random nearest-neighbor bonds. From the divergence of the nonlinear terms in the expansion of the magnetization in the field as well as from static scaling, the spin-glass transition is shown to occur at zero temperature, which implies the lower critical dimensionality for Ising spin glasses to exceed 2. The critical exponents $\gamma = 4.5 \pm 0.2$ and $\beta = 0.0 \pm 0.1$ agree with recent theoretical estimates for the $\pm J$ Ising spin glass in two dimensions.

I. INTRODUCTION

In the last few years, major progress has been made in elucidating whether the peculiar characteristics of spin glasses[1,2] (SG) indicate an equilibrium phase transition. In particular, experiments on the singular behavior of the *nonlinear* susceptibility have played a decisive role in providing evidence for a transition at a finite critical temperature T_c. In the framework of the mean-field theory,[3] the nonlinear susceptibility had earlier been shown to scale with the SG order-parameter susceptibility,[4,5] and therefore to diverge according to a power law. Such a critical behavior has at present been established in a variety of SG compounds, all of which are three-dimensional ($d = 3$).[6-9] In the case of $d = 2$ systems, by contrast, theoretical arguments inclusive of extensive numerical simulations have inferred that $T_c = 0$, despite a freezing at a finite temperature.[10-17] Experimental verification of this important result however, is lacking.

In this paper, we investigate the temperature and field dependences of the nonlinear susceptibility in the short-range $d = 2$ Ising SG $Rb_2Cu_{0.782}Co_{0.218}F_4$,[18] with the primary aim to determine whether T_c is zero or finite from the static critical behavior. The results evidence a power-law divergence of the leading nonlinear terms in an expansion of the susceptibility at zero temperature, culminating in an excellent static $T_c = 0$ scaling of the total nonlinear susceptibility. This implies that the lower critical dimensionality exceeds 2. The critical exponents γ and β further appear to compare to theoretical estimates.

II. MAGNETIC PROPERTIES OF $Rb_2Cu_{1-x}Co_xF_4$

In this section, we discuss the magnetic properties of $Rb_2Cu_{1-x}Co_xF_4$ in relation to the $d = 2$ Ising Edwards-Anderson model for short-range SG.[19] The crystal structure of $Rb_2Cu_{1-x}Co_xF_4$ is, as that of the pure end members, of the K_2NiF_4 type. The magnetic ions are situated on a simple square lattice within widely separated $Cu_{1-x}Co_xF_2$ layers. The intralayer exchange is at least 3 orders of magnitude stronger than the interlayer exchange. Rb_2CoF_4 is an archetypal $d = 2$ Ising antiferromagnet, with the spins ordering along the tetragonal

axis below a Néel temperature $T_N = 103.0$ K.[20,21] Rb_2CuF_4 is a $d = 2$ ferromagnet with predominant Heisenberg exchange and small ($\sim 1\%$) XY anisotropy leaving the spins within the plane.[22] The Curie temperature is $T_c = 6.05 \pm 0.09$ K. The magnetic interactions in $Rb_2Cu_{1-x}Co_xF_4$, primarily originating from exchange, are to a high degree between nearest neighbors only, with next-nearest-neighbor interactions down to $\sim 1\%$.

Randomizing Co and Cu spins over the square lattice results in a distribution of interactions that simulates bond randomness. In Cu compounds of the K_2NiF_4 structure, the Cu^{2+} $d_{z^2-x^2}$ and $d_{z^2-y^2}$ ground-state orbitals are arranged such that they alternate along the crystallographic [100] and [010] axes, leading to a ferromagnetic Cu-Cu exchange.[23] As illustrated in Fig. 1(a), substitution of Co for Cu results in two distinct Cu-Co interactions, dependent on the orientation of the Cu^{2+} orbital relative to the Cu-Co bond. The exchange is antiferromagnetic in case the d-orbital lobe points towards the Co^{2+} ion, and ferromagnetic otherwise.[24] In $Rb_2Cu_{1-x}Co_xF_4$, we thus have two ferromagnetic (Cu-Cu and Cu-Co) and two antiferromagnetic interactions (Cu-Co and Co-Co). Both Cu and Co have, the latter effectively, a spin value $S = \frac{1}{2}$. The nearest-neighbor bond strength thus is the only random variable of relevance, making $Rb_2Cu_{1-x}Co_xF_4$ an almost ideal representative of a $d = 2$ random-bond magnetic system. The huge uniaxial crystalline anisotropy of the Co^{2+} ion further makes the Co-Co and the two Cu-Co interactions strongly Ising-like in character. As it turns out, SG freezing is achieved for $0.18 \leq x \leq 0.40$.[22] Summing up, therefore, $Rb_2Cu_{1-x}Co_xF_4$ constitutes an ideal realization of the $d = 2$ Ising Edwards-Anderson model, except for minor details in the bond distribution.

In more detail, the explicit form of the bond probability $P(J)$ is, on the assumption of site randomness, given by

$$P(J) = x^2\delta(J - J_{Co-Co}) + x(1-x)\delta(J - J^{AF}_{Cu-Co})$$
$$+ x(1-x)\delta(J - J^F_{Cu-Co})$$
$$+ (1-x)^2\delta(J - J_{Cu-Cu}) , \qquad (1)$$

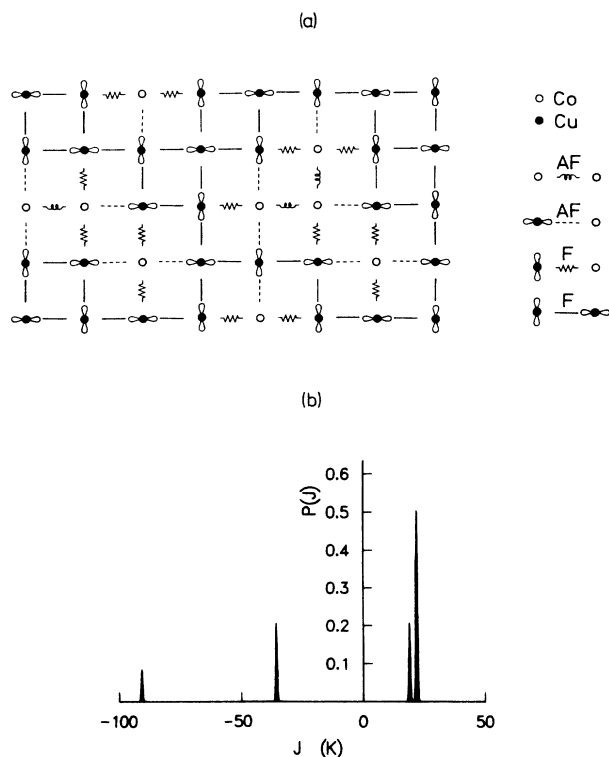where J_{A-B} is the exchange constant between A and B,

(a)

(b)

FIG. 1. (a) Representative distribution of ferromagnetic (F) Cu-Cu and Cu-Co and antiferromagnetic (AF) Co-Co and Cu-Co exchange interactions in $Rb_2Cu_{1-x}Co_xF_4$ for $x = 0.22$. Also indicated are the in-layer d-orbital lobes of Cu^{2+}. (b) Schematic bond probability function $P(J)$ according to Eq. (1) for $Rb_2Cu_{1-x}Co_xF_4$ with $x = 0.29$.

and the superscripts AF and F distinguish between the antiferromagnetic and ferromagnetic Cu-Co bonds, respectively. We have $J_{Co-Co} = -90.8$ K, as derived from T_N in Rb_2CoF_4 with Onsager's rigorous result[25] for the $d = 2$ Ising system, and $J_{Cu-Cu} = 22.0$ K, as estimated from J_{Cu-Cu} in K_2CuF_4 (Ref. 26) scaled with the Curie temperatures of the Rb and K compounds. To assess the Co-Cu interactions, we make the reasonable assumption that at the midpoint of the SG range of concentrations, i.e., $x = 0.29$, the average bond strength $\bar{J} = 0$. By use of Eq. (1) and

$$\bar{J} = \sum_{i=1}^{4} P(J_i)J_i , \qquad (2)$$

with $i = 1, 2, 3$, and 4 indexing the four bonds, it then follows that $J_{Cu-Co}^{AF} + J_{Cu-Co}^{F} = -17$ K. From this in relation to J_{Co-Co} and J_{Cu-Cu}, we estimate $J_{Cu-Co}^{F} = 20 \pm 10$ K and $J_{Cu-Co}^{AF} = -37 \pm 10$ K. The resultant bond distribution is sketched in Fig. 1(b). Its discrete nature indicates that $Rb_2Cu_{1-x}Co_xF_4$ compares to the $\pm J$ Edwards-Anderson SG model, which has

$$P(J') = \tfrac{1}{2}\delta(J' - J) + \tfrac{1}{2}\delta(J' + J) ,$$

better than to the Gaussian counterpart, which is characterized by a continuous distribution of J, symmetric

about zero. An additional argument in favor of the $\pm J$ model is provided by the nonvanishing number of spins in $Rb_2Cu_{1-x}Co_xF_4$ that can be flipped without any energy cost because of completely balancing exchange fields.[22] Such spins are indeed abundant in the $\pm J$ model, but virtually absent in the Gaussian model. Although in the case of $d = 2$ both models have $T_c = 0$, the critical exponents differ. The models thus seem to represent different universality classes,[13,16] presumably because the Gaussian model has a nondegenerate ground state for $T_c = 0$, whereas the $\pm J$ model has a finite ground-state entropy.

The typical spread of the distribution of J in $Rb_2Cu_{1-x}Co_xF_4$, defined by the second moment

$$\Delta J = \left[\sum_{i=1}^{4} P(J_i)(J_i - \bar{J})^2 \right]^{1/2} , \qquad (3)$$

is, together with $\bar{J}$, shown versus x in Fig. 2. It is seen that ΔJ is as large as 30–40 K in the SG regime. The fact that SG freezing is only observed near 3 K ($\approx 0.1 \Delta J$) thus is in itself already indicative of a zero critical temperature for the present $d = 2$ SG model compound. In the mean-field theory of SG,[3] a SG phase is found for $|\bar{J}| < \Delta J$, whereas for $\bar{J} > \Delta J$ the system orders ferromagnetically, and similarly antiferromagnetically for $\bar{J} < -\Delta J$. From the inset of Fig. 2, which shows the ratio $\Delta J / |\bar{J}|$ as calculated from Eqs. (2) and (3), it is seen that the boundaries of the SG regime actually found in $Rb_2Cu_{1-x}Co_xF_4$, $x = 0.18 \pm 0.02$ and 0.40 ± 0.01, correspond to $\Delta J / |\bar{J}| \approx 3$.
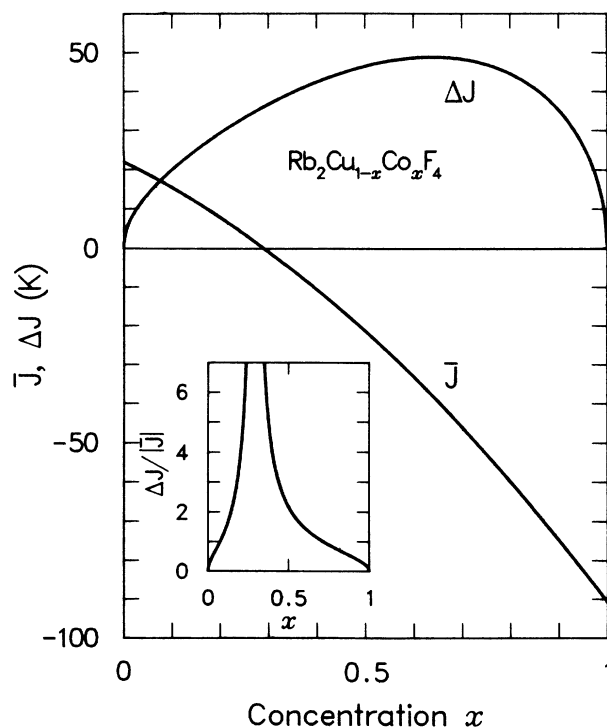

FIG. 2. Average bond strength $\bar{J}$ and the spread ΔJ vs the concentration x for $Rb_2Cu_{1-x}Co_xF_4$. Inset shows $\Delta J / |\bar{J}|$ vs x.

III. EXPERIMENTAL DETAILS AND RESULTS

The experiments were performed on a good-quality Czochralski-grown single crystal of $Rb_2Cu_{1-x}Co_xF_4$ with $x = 0.218 \pm 0.004$, as determined with inductively coupled plasma spectroscopy. The sample was shaped roughly to the form of an ellipsoid with dimensions $7.0 \times 3.9 \times 1.3$ mm.[3] Magnetic moments were measured by use of a vibrating-sample magnetometer with a sensitivity of 10^{-5} emu. External fields ranging from 1.0 G to 12.5 kG were applied along the c axis, and the earth magnetic field was compensated. All fields and susceptibilities were corrected for demagnetization. The sample was immersed in liquid helium, the temperature of which was stabilized to within 0.01 K by controlled pumping. All data of the magnetization $M(T,H)$ were collected under field-cooling conditions, i.e., the field H was switched on at a high temperature (≈ 30 K), and kept constant during subsequent slow cooling (typically at a rate of 5 mK/s) to the temperature of interest.

The results for the static susceptibility $X = M/H$ versus the temperature are plotted in Fig. 3 for a selection of fields between 1.0 G and 10 kG. It is seen that the susceptibility rises continuously with decreasing temperature through the freezing temperature, which at the time scale
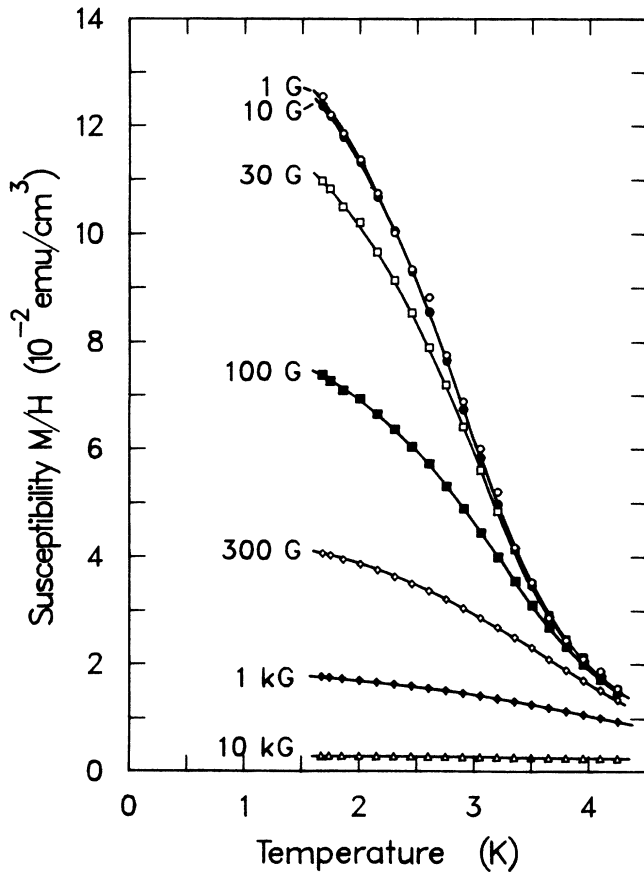

FIG. 3. Static susceptibility M/H vs the temperature for a selection of external magnetic fields. Solid lines are guides to the eye.

typical of the present experiment ($\sim 10^2$ s) amounts to $T_f \approx 2.97$ K.[27] Generally for a $d = 2$ SG, the susceptibility, when observed at a finite time scale, is anticipated to reach a plateau below T_f (cf. results of Monte Carlo simulations[11]), despite the fact that the equilibrium linear susceptibility keeps rising with decreasing temperature. That such a plateau does not occur in $Rb_2Cu_{1-x}Co_xF_4$ is plausible on the grounds that, even in a frozen domain structure, the thermal decrements of the magnetizations residing at individual ions, in particular those of the abundant Cu ions, have already become significant at temperatures as low as a few Kelvin. Nonequilibrium of the system below approximately T_f is manifest from a cooling-rate dependence of the susceptibility in this temperature range. The susceptibility is observed to be quite large, as expected for a concentrated SG at these low temperatures. The susceptibility is further enhanced by residual short-range ferromagnetic correlations, which are apparent from data taken at higher temperatures. In fact, up to about 20 K the susceptibility follows an effective Curie-Weiss law, i.e., $X(T) = C/(T-\theta)$, with $\theta = 5.1 \pm 0.2$ K.[28]

The prominent feature contained in the data of Fig. 3 is that the magnetization becomes increasingly nonlinear with the field upon lowering the temperature. In order to examine this in detail, we have measured $M(T,H)$ at a set of discrete temperatures for as many as 35 fields ranging from 1.0 G to 12.5 kG. From these data, $M(H)$ isotherms were deduced.

IV. NONLINEAR-SUSCEPTIBILITY ANALYSIS

In the presentation of the $M(H)$ isotherms, we eliminate the linear part X_0 of the susceptibility in order to find the dimensionless nonlinear part

$$X_{nl}(T,H) \equiv 1 - \frac{M}{X_0 H} . \qquad (4)$$

Here, X_0, the susceptibility in the linear regime, is inserted as deduced from the experiments in small fields. The results for $X_{nl}(H)$, given in Fig. 4 for a selection of temperatures, clearly manifest a dramatic increase of the nonlinearity with decreasing temperature. To parametrize this, we start from the common $T > T_c$ expansion of the magnetization in odd powers of the field,

$$M = a_1 \left[\frac{H}{T}\right] - a_3 \left[\frac{H}{T}\right]^3 + a_5 \left[\frac{H}{T}\right]^5 - a_7 \left[\frac{H}{T}\right]^7 + \cdots ,$$

$$(5)$$

defined such that for a paramagnet the coefficients $a_1, a_3, \ldots$ are constants. In the case of a SG, however, $a_3, a_5, \ldots$ are related to the SG order-parameter susceptibility, which exhibits conventional critical behavior at T_c.[4,5] Note that the linear part of the susceptibility a_1/T, remains nonsingular at T_c. From expansion of the free energy,[4,7] it appears that a_{2n+1} diverges according to $(T-T_c)^{-n(\gamma+\beta)+\beta}$ for $n \geq 1$, i.e.,

$$a_3 \propto (T - T_c)^{-\gamma} , \qquad (6a)$$

$$a_5 \propto (T - T_c)^{-(2\gamma + \beta)} , \qquad (6b)$$

$$\vdots$$

An examination of the temperature dependences of $a_3, a_5, \ldots$ thus allows to determine T_c as well as the critical exponents γ and β.

Prior to carrying out this analysis, we have to deal, in one way or another, with the problem that in a real SG the interactions do not perfectly average out to zero, as opposed to Eq. (5), where $\mathcal{X}_0$ is assumed to obey the simple Curie law $\mathcal{X}_0 \propto 1/T$ appropriate to a symmetric distribution of exchange interactions. As is commonly done, we resort to the intuitive procedure, introduced by Omari et al.,[7] of expanding M in powers of $\mathcal{X}_0 H$ in place of H/T. Equation (5) then becomes

$$M = \mathcal{X}_0 H - a_3 (\mathcal{X}_0 H)^3 + a_5 (\mathcal{X}_0 H)^5 - a_7 (\mathcal{X}_0 H)^7 + \cdots , \qquad (7)$$

or, equivalently,

$$\mathcal{X}_{nl} = a_3 (\mathcal{X}_0 H)^2 - a_5 (\mathcal{X}_0 H)^4 + a_7 (\mathcal{X}_0 H)^6 - \cdots . \qquad (8)$$

It has recently been argued on the basis of an effective-mean-field model[29] that this approach, although not entirely rigorous in the case of SG, yields the dominant correction.

Equation (7), cut off after the first three terms, has been fitted to the $M(H)$ isotherms with $\mathcal{X}_0$, a_3, and a_5 as adjustable parameters. The fitting has been restricted to the range of fields where $a_7 (\mathcal{X}_0 H)^7$ is still negligible. Fits of good quality $(\mathcal{X}^2 \approx 1)$ are then found, as is apparent from the solid lines in Fig. 4, which represent Eq. (8) with the resultant fitting parameters inserted. The output values for $\mathcal{X}_0$ and a_3 further appear to coincide within errors with the results from the fits of Eq. (7) limited to the first two terms. As regards the result for $\mathcal{X}_0$, its temperature dependence essentially equals that of M/H given in Fig. 3 for the fields up to 10 G. The resultant temperature dependences of a_3 and a_5 are presented in Fig. 5. The rapid increase of both these coefficients is seen to level off at a point somewhat below T_f, obviously because the slow but finite cooling rate inhibits their equilibrium values to be attained.

Above approximately T_f, the results for a_3 and a_5 are both well accounted for by a power law "series" diverging at $T_c = 0$ K (solid lines in Fig. 5). This, in fact, comprises a major conclusion of this work. The exponents obtained from fitting of Eqs. (6) are $\gamma = 4.5 \pm 0.2$ and $2\gamma + \beta = 9.0 \pm 0.5$. Combination of these results in turn yields $\beta = 0.0 \pm 0.7$. In Sec. VI, we will compare the exponents with various theoretical estimates. We note that additional data above 4.25 K (not presented here),
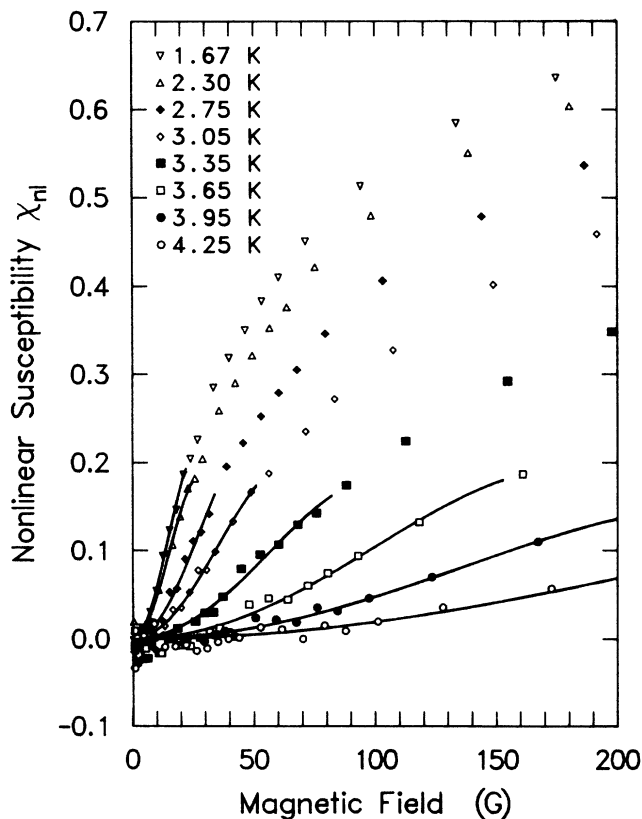

FIG. 4. Nonlinear susceptibility $\mathcal{X}_{nl} = 1 - M/\mathcal{X}_0 H$ vs the internal field for a selection of temperatures. Solid lines are according to Eq. (8).
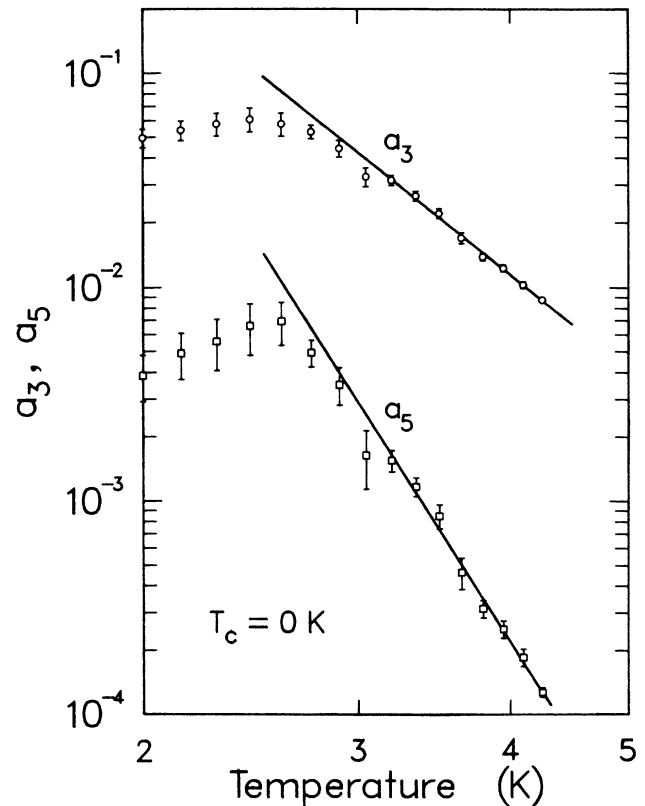

FIG. 5. Nonlinear susceptibility coefficients a_3 and a_5 (in emu) vs the temperature. Solid lines denote power-law divergences towards $T_c = 0$ K, Eqs. (6).
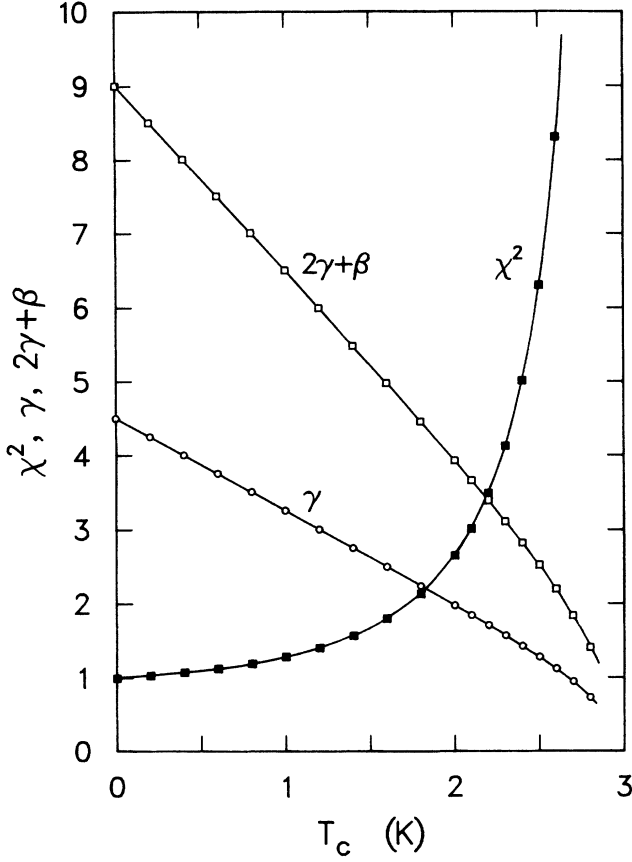
FIG. 6. Results for χ^2, γ, and $2\gamma+\beta$ vs T_c, obtained from fitting Eqs. (6) with a finite T_c inserted. Solid lines are guides to the eye.

which suffer from increased scatter, are consistent with the power-law dependences found for a_3 and a_5.

In order to establish more firmly that the divergence of the nonlinear susceptibility conforms to $T_c = 0$, it is expedient to show that a finite T_c of order T_f is not reconcilable with the experimental findings. To this end, we have additionally adjusted Eqs. (6) to the results for a_3 and a_5 above T_f, but with T_c fixed at values ranging from 0 to T_f. The results for γ and $2\gamma+\beta$, as well as the corresponding χ^2, are given in Fig. 6 as a function of T_c. For values of T_c just below T_f (2.5–2.9 K), the fits turn out to be markedly inferior ($\chi^2 = 5$–40), while χ^2 becomes minimum at $T_c = 0$ K. The results of Fig. 6 thus rule out $T_c \approx T_f$, but, of course, would be consistent with any T_c below about 1 K, such as could be due to residual interlayer coupling. Since the critical behavior is accessible to experiment at feasible time scales only above approximately 3 K, such a small T_c would however not detract from $Rb_2Cu_{1-x}Co_xF_4$ to realize the $d = 2$ SG with $T_c = 0$.

V. STATIC SCALING ANALYSIS

The dependence of the nonlinear susceptibility on the field and the temperature has in addition been examined in the context of static scaling, an approach that has

flourished in the field of SG.[6–8,30–32] Within the scaling picture, all critical physical quantities are determined by the correlation length ξ. The underlying ansatz is that in the critical regime the dependences of ξ on H and T can be cast into a scaling form, the relevant argument being H scaled to some power of the temperature. That is, for $T_c = 0$,[33]

$$\xi = T^{-\nu} f \left[\frac{H}{T^\Delta} \right] = H^{-\nu/\Delta} \tilde{f} \left[\frac{T}{H^{1/\Delta}} \right] , \qquad (9)$$

with f and $\tilde{f}$ scaling functions, and ν and $\Delta = 1 + \frac{1}{2}(\gamma+\beta)$ critical exponents. Note that this scaling relation implies $\xi \propto T^{-\nu}$ for $H = 0$, and $\xi \propto H^{-\nu/\Delta}$ for $T = 0$.

Similarly to Eq. (9), the scaling relation for the non-linear susceptibility reads $\chi_{nl} = T^\beta g (H/T^\Delta)$.[4,6] In order to account for deviations of χ_0 from strictly Curie behavior, we replace, as in Sec. IV, H/T by $\chi_0 H$, to arrive at

$$\chi_{nl} = T^\beta \mathcal{F} \left[\frac{\chi_0 H}{T^{\Delta-1}} \right] . \qquad (10)$$

In the limit $x \to 0$ the scaling function $\mathcal{F}(x)$ is given by $c_0 x^2$, while for $x \to \infty$ $\mathcal{F}(x) = 1 - c_\infty x^{-1/\Delta}$, with c_0 and c_∞ constants.

In Fig. 7, the nonlinear susceptibility of $Rb_2Cu_{0.782}Co_{0.218}F_4$ scaled according to Eq. (10), i.e., $\chi_{nl} T^{-\beta}$ versus $\chi_0 H/T^{\Delta-1}$, is shown for the entire range
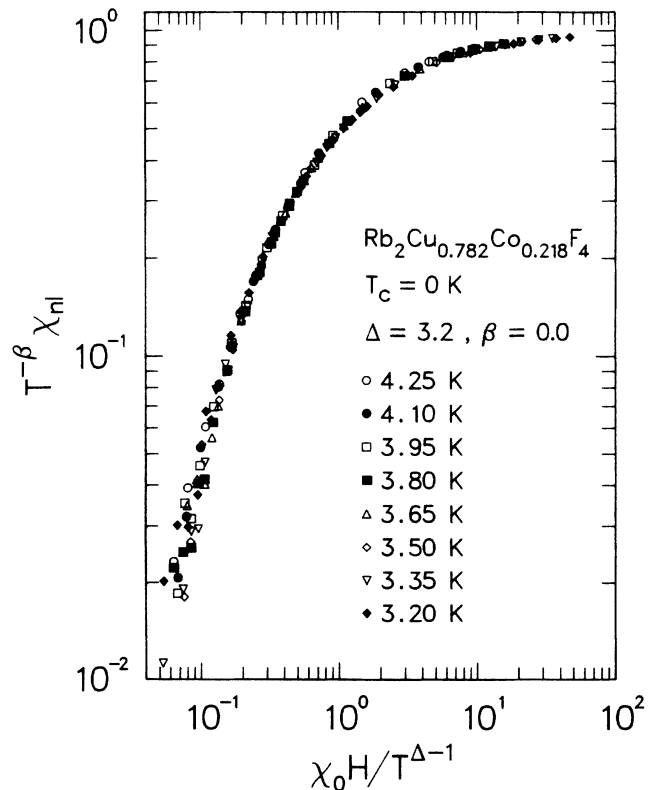

FIG. 7. $T_c = 0$ scaling plot of the nonlinear-susceptibility data. External fields range from 1.0 G to 12.5 kG.

of fields considered (1.0 G to 12.5 kG) and temperatures from 4.25 to 3.20 K. It is seen that truly excellent conformity with the $T_c=0$ scaling prediction is achieved after adjustment of Δ and β for optimum coincidence of the data on a universal curve. The critical exponents obtained are $\beta=0.0\pm0.1$ and $\Delta=3.2\pm0.2$, which implies $\gamma=4.4\pm0.4$. It has been noticed that critical exponents deduced from scaling may suffer from systematic deviations when using too large a field or temperature range.[8] In the present case, however, essentially the same results are obtained upon restricting the data set to fields so small that the nonlinear part of $M(H)$ is less than 10% of χ_0. In Sec. VI, the critical exponents will be discussed in some more detail. Inclusion of temperatures below 3.1 K leads to a deterioration of the fit, obviously because the system has fallen out of equilibrium at these temperatures. The scaling of χ_{nl} in $Rb_2Cu_{0.782}Co_{0.218}F_4$ thus further establishes $T_c=0$, with essentially the same exponents as derived in the previous section. We gain a more accurate determination of β, and, more importantly, we are able to extend the analysis to much higher fields. The latter is feasible because Eq. (10) encompasses *all* singular contributions contained in Eq. (8), whereas the dominant nonlinear terms a_3 and a_5 suffice at low fields only.

Our results show an overall agreement with the findings from Monte Carlo simulations of the $d=2$ Edwards-Anderson Ising SG. This includes $T_c=0$ scaling of the nonlinear susceptibility and the critical exponents, for which the simulations yielded $\beta=0$ and $\Delta=3.5\pm0.5$.[11] Although ample evidence is available that $T_c=0$ for the $d=2$ Edwards-Anderson SG model, a remarkable feature of the Monte Carlo simulation data is that they could equally well be scaled to a $d=3$ scaling expression, distinguished from $d=2$ by a nonzero transition temperature. To show that the present data for $Rb_2Cu_{0.782}Co_{0.218}F_4$ exhibit an entirely similar behavior, we have fitted them to the $d=3$ analogue of Eq. (10),

$$\chi_{nl}=t^{\beta}\mathcal{G}\left[\frac{\chi_0 H}{t^{(\gamma+\beta)/2}}\right], \quad (11)$$

in which $t=(T-T_c)/T$. The scaling function $\mathcal{G}(x)$ behaves as $c_0 x^2$ for small x, and as $c_\infty x^{2/\delta}$ in the high x limit, with δ an additional critical exponent. For the range of T_c from 3.0 down to 0.5 K, fits of a quality more or less comparable to that of the fit of Eq. (10) are obtained. We find $\delta=21.0\pm0.2$, and, with decreasing T_c, γ varying from 0.8 to 23, and β from 0.04 to 1.1. As in the previous section (cf. Fig. 6), there appears to be a definite trend, albeit a small one, towards better fits with lower T_c, which is indicative of $T_c=0$ K.

VI. DISCUSSION AND CONCLUSIONS

We have successfully examined the nonlinear part of the susceptibility of $Rb_2Cu_{0.782}Co_{0.218}F_4$ on the basis of (i)

the power-law divergences of $a_3(T)$ and $a_5(T)$ towards $T_c=0$ and (ii) a static $T_c=0$ scaling analysis. This verifies the theoretical evidence for a vanishing T_c, as obtained for $d=2$ SG models with different random-bond distributions. The two analyses have yielded essentially equal exponents. Summing up, we find $\beta=0.0\pm0.1$ and $\gamma=4.5\pm0.2$. The result for γ agrees within errors with the result recently obtained from dynamic scaling of the ac susceptibility, $\gamma=4.2\pm0.6$.[34]

In discussing the exponents, we first note that the theoretical results seem to differ for the two best-studied Ising square-lattice SG model systems, viz., the Gaussian and the $\pm J$ model. For the Gaussian model $\beta=0$, while various methods of calculation have resulted in the remarkably high value of $\gamma\approx7$.[15] Early work on the $\pm J$ model, involving large-scale Monte Carlo simulations and transfer-matrix methods, yielded $\gamma\approx4$.[10] More recently, a promising new replica Monte Carlo method permitted lower temperatures to be achieved, resulting in $\gamma=5.3$.[12] The same value, 5.3 ± 0.3, was found from high-temperature series expansions,[17] while a finite-size scaling study gave $\gamma=4.5\pm0.5$.[15] As regards β in the case of the $\pm J$ model, a very small, but finite, value may be estimated from the hyperscaling relation $2\beta=\nu(d-2+\eta)$.[15] Upon comparing the theoretical results with the present experimental ones, we see that our result for γ, 4.5 ± 0.2, seems to be somewhat smaller than the mean of the more recent theoretical estimates for the $d=2$ $\pm J$ model. The agreement must nevertheless be considered quite satisfactory.

Our results have been derived in a temperature range corresponding to about 0.1 or 0.2 times ΔJ, with ΔJ characterizing the width of the bond distribution. For comparison, the results from the Monte Carlo simulations and finite-size scaling pertain to temperatures down to $\sim0.5\Delta J$. The present γ, therefore, is more sure to refer to the true asymptotic critical regime. We further note that the error given for γ may be underestimated somewhat because of possible systematic uncertainties contained in the correction for the residual ferromagnetic correlations. To assess the effects of the latter, it would be desirable to similarly determine γ in a $Rb_2Cu_{1-x}Co_xF_4$ SG with larger x, say $x=0.35$.

In summary, the scaling and the power-law divergence of the static nonlinear susceptibility evidence that the SG transition in the $d=2$ Ising SG model compound $Rb_2Cu_{1-x}Co_xF_4$ occurs at zero temperature, with the critical exponents in conformity with theory. Experimental verification is thus provided that the lower critical dimensionality for short-range Ising SG exceeds 2.

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