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Field-dependent transport properties in paramagnon systems

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We examine, theoretically, the low-field dependence of certain transport properties in nearly magnetic itinerant-fermion systems which exhibit strong spin fluctuations describable by the paramagnon theory. We study more precisely the viscosity $\eta(H)$ of partially polarized normal liquid ^{3}He at low field H and low temperature, and also the magnetoconductivity $\sigma(H)$ of exchange-enhanced metals. We find analytical expressions relating these transport properties to the field dependence of the magnetization $M=\chi(H)H$. We find that $\eta(H)$ and $\sigma(H)$ increase or decrease when H increases, depending upon whether $\chi(H)$ becomes smaller or larger than $\chi(0)$. Comparison of our results with up-to-date experiments for $\eta(H)$ in liquid ^{3}He is quite satisfactory as far as the sign and magnitude of the effect are concerned.

I. INTRODUCTION

There has recently been some controversy about the experimentally observed variation of the viscosity η of normal liquid ^{3}He, as a function of the polarization, in a finite magnetic field. The experiments of Ref. 1, together with the nearly metamagnetic picture of Ref. 2, show that, for increasing polarization, η first decreases, reaches a minimum, and then increases again. In contrast, Ref. 3 exhibits an initial *increase* of η instead of a *decrease.*

The authors of Ref. 3 suggest that their observed increase of η would be, on physical grounds, in qualitative agreement with the paramagnon description of the magnetization in liquid ^{3}He. In the nearly magnetic picture of the paramagnon model, the magnetization has been shown[4] to deviate from the initial linear field dependence from *below,* in absence of band-structure effects, when the applied magnetic field increases. In other words, the relative variation of the field-dependent susceptibility is negative, i.e., $[\chi(H)-\chi(0)]/\chi(0)<0$. Instead, in the nearly metamagnetic model[5] of the nearly localized picture[6] for liquid ^{3}He, the magnetization deviates from *above* the initial linear law, i.e., $[\chi(H)-\chi(0)]/\chi(0)>0$, and so strongly that a metamagnetic transition is expected. The results of Ref. 3, $\delta\eta/\eta>0$, is qualitatively consistent with $\delta\chi/\chi<0$ of the paramagnon theory, while the one of Ref. 1, $\delta\eta/\eta<0$, is rather compatible with $\delta\chi/\chi>0$, of the nearly metamagnetic picture. Further experiments will have to definitely settle which one of the above experimental results is correct.

Motivated by these observations, we have computed the viscosity $\eta(H)$ as a function of the field-dependent dynamical spin-correlation functions, starting from the Boltzmann equation and examined more specifically the paramagnon model. As the mathematics will be shown to be quite similar, we compute also the magnetoconductivity $\sigma(H)$ in a nearly magnetic metal.[7] We show that $\eta(H)$ and $\sigma(H)$ can be explicitly expressed in terms of the spin-dependent static susceptibility $\chi(H)$. We compare our results with previous theoretical approaches. We compare also more particularly our formula for $\eta(H)$ with the experimental observations of Refs. 1 and 3 in ^{3}He. We point out that while the nearly metamagnetic nearly localized result of Refs. 2, 5, and 6 support consistently the *strong decrease* of η when H increases, observed in Ref. 1, the paramagnon approach considered here, agrees with the *small increase* with H found in the experiments of Ref. 3. Thus, if a clear-cut conclusion could be reached on the experimental side, one would thus, as well, know which one of the theoretical approaches could better describe the field-dependent properties of normal liquid ^{3}He.

II. PREVIOUS ZERO- AND FINITE-FIELD THEORIES FOR TRANSPORT PROPERTIES IN A FERMI LIQUID, RECALLED

A. The zero-field expressions of $\eta(0)$ and $\sigma(0)$ in the paramagnon model

The zero-field expression for $\eta(0)$ has been computed long ago[8] in the paramagnon model[9] for a nearly magnetic Fermi liquid. It may be expressed as follows [see, in particular, formula (B 10) in Ref. 8(b)]:

$$\eta^{-1}(0)\propto T^{-1}\int_0^1 \bar{q}^{\,3}d\bar{q}(1-\bar{q}^{\,2}) \times \int_0^\infty \mathrm{Im}\chi(\bar{q},\omega)n(1+n)\omega\, d\omega\ . \tag{1}$$

n is the Bose factor

$$n=(e^{\omega/T}-1)^{-1};\mathrm{Im}\chi(\bar{q},\omega)$$

is the imaginary part of the paramagnon propagator[9]

$$\chi(\bar{q},\omega)=\chi^0(\bar{q},\omega)/[1-I\chi^0(\bar{q},\omega)];\chi^0(q,\omega)$$

is the bare-particle–hole spin-correlation function in absence of interaction, whose small frequency ω and small momentum $q(=2k_F\bar{q},\ k_F$ the Fermi momentum) expansion is well known for a parabolic band (we use atomic units throughout the paper):

$$\chi^0(\bar{q}\to 0,\omega\to 0)\simeq N(\varepsilon_F)[1-\bar{q}^2/3+i(\pi\omega)/(8\varepsilon_F\bar{q})]\ldots \quad (2)$$

$N(\varepsilon_F)$ is the density of states for one spin direction at the Fermi energy ε_F; I is the Hubbard-type strong contact repulsion between opposite spins, supposedly strong enough, in a nearly magnetic Fermi liquid, so that

$$IN(\varepsilon_F)\equiv\bar{I}=(3I)/(4\varepsilon_F)\sim 1 \ . \quad (3)$$

The Stoner enhancement of the $T=0$ mean-field static susceptibility is

$$\frac{\chi(\bar{q}=0,\omega=0,H=0)}{\chi^0(\bar{q}=0,\omega=0,H=0)}\equiv S=\frac{1}{1-\bar{I}}\gg 1 \ . \quad (4)$$

$\bar{q}$ is related to the scattering angle θ between the particle and the hole by $\bar{q}=\sin(\theta/2)$. The mathematical expression (1) for $\eta(0)$ is very similar (apart from a different $\bar{q}$ integral) to the one for the conductivity $\sigma(0)$ due to the scattering of s electrons on the paramagnons formed in the d band of nearly magnetic transition metals, as computed in Ref. 10. If only one (parabolic) band would be responsible both for the scattered electrons and the scattering paramagnons, the magnetoresistivity $\rho(0)=\sigma^{-1}(0)$ would read, analogously to (1):

$$\rho(0)\propto T^{-1}\int_0^1\bar{q}^3 d\bar{q}\int_0^\infty \mathrm{Im}\chi(\bar{q},\omega)n(1+n)\omega\, d\omega \ . \quad (5)$$

One writes: $\omega n(1+n)/T\equiv T\partial n/\partial T$; then with (2) and changing ω into $\omega=Tx$ and using[11]

$$\int_0^\infty x\,dx/(e^x-1)=\pi^2/6$$

one gets for the lowest T dependence of either $\eta^{-1}(0)$ or $\rho(0)$:

$$(\eta^{-1}(0),\rho(0))\propto T^2\int_0^1 F(\bar{q})\frac{d\bar{q}}{\bar{q}}\frac{1}{(1-\bar{I}+\bar{I}\bar{q}^2/3)^2} \ , \quad (6)$$

where

$$F(\bar{q})\equiv\begin{cases}[\bar{q}^3(1-\bar{q}^2)] \text{ in } \eta^{-1}(0),\\ \bar{q}^3 \text{ in } \rho(0).\end{cases} \quad (7)$$

Then, the coefficient of T^2 in (6) depends on whether, in the denominator of (6), the $\bar{q}^2$ is negligible or not compared to $(1-\bar{I})$. This was discussed in Ref. 12; more precisely:

(a) If $S=(1-\bar{I})^{-1}$ is moderate ($\bar{I}\lesssim 0.75$). Then the $\bar{q}^2$ term is negligible since $(\bar{I}\bar{q}^2/3)<1-\bar{I}$ in the whole $\bar{q}$ range. In that case,

$$(\eta^{-1}(0),\rho(0))\propto\frac{T^2}{(1-\bar{I})^2}\propto\chi^2(T=0,H=0)T^2 \ , \quad \bar{I}\lesssim 0.75 \ . \quad (8)$$

Note however that (8) holds for moderate S (say for $0.5<\bar{I}\lesssim 0.75$) but does not apply for vanishing $\bar{I}\to 0$. Indeed, as usual in the paramagnon calculations, one only retains the leading terms in $S=(1-\bar{I})^{-1}$ and one drops less divergent ones which would be important when $S\sim 1$, i.e., $\bar{I}\to 0$;

(b) Or if S is large $0.75<\bar{I}\lesssim 1$, then the $\bar{q}^2$ term in the denominator of (6) must be taken into account. In that case one gets

$$(\eta^{-1}(0),\rho(0))\propto\frac{T^2}{\sqrt{1-\bar{I}}}\propto\chi^{1/2}(T=0,H=0)T^2 \ , \quad \bar{I}\lesssim 1 \ . \quad (9)$$

In either case, it is tempting to suggest that, in a finite field ($H\neq 0$), formulas (8) and (9) might still hold if one replaces $\chi(H=0)$ by $\chi(H\neq 0)$. This was already conjectured[13] for $\rho(H)$. We will show, later in this paper, that this is indeed so.

B. Previously found field dependences for $\eta(H)$ and $\sigma(H)$ in Fermi liquids

(a) On general grounds, the theory of transport properties in metals is well known and has been much reviewed in the past,[14] in particular, for conduction electrons scattered by localized impurity spins.

(b) As far as the scattering in finite fields is concerned, $\rho(H)$, due to scattering of electrons on local paramagnons formed on nearly magnetic impurities in metallic alloys has been computed in Ref. 15. $\rho(H)$ due to scattering on uniform paramagnons (the ones of interest for us here), in uniformly enhanced nearly magnetic metals, was considered in Ref. 16. The result, however, was given under the form of formal integrals so that the H dependence of $\rho(H)$ was not clearly extracted. Besides, only the mean-field "effective" field SH was mentioned and has been misleading for various experimentalists, while the effective field, including fluctuation effects, is rather[13,4] $(S^{3/2}H)$. In other words (in appropriate units), the characteristic field is $H_{sf}=\varepsilon_F/S^{3/2}$ and not ε_F/S. This remark can be quite important in cases involving large S values. For these reasons we rederive $\rho(H)$ explicitly in the following.

(c) As far as the viscosity in an applied field is concerned, Ref. 17 derived the transport coefficients of dilute ^{3}He-^{4}He solutions in finite fields. Such solutions imply very small interactions between the ^{3}He spins. The study of Ref. 17 appears complementary to ours since it thus corresponds to $\bar{I}\to 0$, a case which is excluded from our present calculation for the reasons explained in case (a) of Sec. II A. Reference 17 gives, without much detail, however, a formula for η as a function of the polarization whose expansion is easy to perform yielding an *increase* of η with H. Therefore, Ref. 17 for vanishing $\bar{I}$ finds the same qualitative tendency that we are going to derive here for moderate or strong $\bar{I}$.

(d) Finally Ref. 2 (already mentioned in the Introduction), using the field-dependent Landau parameters of Ref. 5, in the framework of the almost localized model of Ref. 6 for liquid ^{3}He, provides with little detail a result for η which first *decreases* when H increases at low polarizations. Reference 2, however, recalls that, for fully polarized ^{3}He, the authors of Ref. 18 calculated that the viscosity *is increased* compared to $\eta(0)$. Consistently with this last finding, the authors of Ref. 2 finally find that $\eta(H)$, after initially decreasing, passes through a minimum at finite polarization and increases at large polarizations. The question to answer is, thus, whether at low polarizations, η decreases as in Ref. 2 or increases when the polarization increases.

III. CALCULATION OF $\sigma(H)$ AND $\eta(H)$: APPLICATION TO THE PARAMAGNON MODEL

A. Comparison of the formal expressions for $\sigma(H)$ and $\eta(H)$

We follow the usual procedure of Refs. 14(a) and 14(b) writing separately the general transport equations, for spins up $(+)$ and down $(-)$ of the scattered particles. We treat on the same footing the magnetoconductivity $\sigma(H)$ and the viscosity $\eta(H)$. The scattered particles, in one case, are the conduction electrons of a metal, while they are the ^{3}He nuclei in the other case. In both cases the Boltzmann equation follows from the general condition that the fermion distribution functions for up and down spins $f_\pm$, under the influence of the appropriate perturbation, are determined in a steady-state situation:

$$\left[\frac{\partial f_\pm}{\partial t}\right]_d+\left[\frac{\partial f_\pm}{\partial t}\right]_c=0 . \tag{10}$$

The first term in (10) is the "drift" term, the second is the "collision" one (t is the time).

For the magnetoconductivity in metals, the whole Fermi distributions are shifted under the influence of an electric field F_x supposedly applied parallel to the x axis; then one gets (in atomic units)

$$\left[\frac{\partial f_\pm}{\partial t}\right]_d=\left[\frac{\partial f_\pm}{\partial t}\right]_{\text{el. field}}=-F_x k_x\frac{\partial f_{0\pm}}{\partial \varepsilon_\pm} \tag{11}$$

with the Fermi distribution function $f_{0\pm}(\varepsilon_\pm)$ given by

$$f_{0\pm}(\varepsilon_\pm)=\{\exp[(\varepsilon_\pm-\varepsilon_F)/T]+1\}^{-1} . \tag{12}$$

T is the temperature and

$$\varepsilon_\pm=k^2/2\pm\Delta E . \tag{13}$$

k_x in (11) is the projection of the momentum $\mathbf{k}$ on the x axis. ΔE in (13) is the "Zeeman energy" $\Delta E=B$. B identifies with the applied field H in the case where localized impurity spins are the scattering centers;[14(c),19] but B may be a more complicated object containing both H and the interaction in the paramagnon case,[20,4] as will be seen later in Sec. III B, formula (32).

For the viscosity in ^{3}He, we suppose (see for instance the Steinberg's paper in Ref. 8) that the fermion system is in a shear motion with a constant local velocity v_x in the x direction and a uniform velocity gradient $\partial v_x/\partial y=a$ in the y direction; then,

$$\left[\frac{\partial f_\pm}{\partial t}\right]_d=\left[\frac{\partial f_\pm}{\partial t}\right]_{\text{shear}}=-ak_xk_y\frac{\partial f_{0\pm}}{\partial \varepsilon_\pm} . \tag{14}$$

In both cases one supposes, as usual,[14] that the scattering processes entering in the collision term of (10), may be described by relaxation times $\tau_\pm$ so that

$$\left[\frac{\partial f}{\partial t}\right]_c=-\frac{f_\pm-f_{0\pm}}{\tau_\pm} . \tag{15}$$

Then the magnetoconductivity is $\sigma(H)=\sigma_+(H)+\sigma_-(H)$ with $\sigma_\pm$ given by the ratios of the current densities to the applied field F_x:

$$\sigma_\pm(H)=\frac{1}{F_x}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k_x(f_\pm-f_{0\pm}) . \tag{16}$$

Similarly the shear viscosity is $\eta(H)=\eta_+(H)+\eta_-(H)$, with $\eta_\pm$ given by the ratios of the kinetic shear stress due the velocity gradient of the fermion flow, to the transverse flow velocity gradient a, with

$$\eta_\pm(H)=\frac{1}{a}\int\frac{d^3\mathbf{k}}{(2\pi)^3}k_xk_y(f_\pm-f_{0\pm}) . \tag{17}$$

As is clear from the comparison of (16) and (17), the mathematical difference between the expression for $\sigma(H)$ and $\eta(H)$ will arise from the different angular integrals involving k_x in one case and k_xk_y in the other, which will be simply related to the scattering angle θ as will be seen shortly.

If one calls $P(k_\alpha,k'_\beta)$ the probability per unit time that a particle in state $\mathbf{k}$ with a spin $\alpha(=\pm\frac{1}{2})$ makes, a transition (due to collisions with the scattering agent) to a state $\mathbf{k}'$ with spin $\beta(=\pm\alpha)$, one has, separating the processes without spin flip $(\beta=\alpha)$ from the processes with spin flip $(\beta=-\alpha)$,

$$\left[\frac{\partial f_\pm}{\partial t}\right]_c=\frac{1}{8\pi^3}\int d^3\mathbf{k}'_\pm\{-P(k_\pm\to k'_\pm)f(k_\pm)[1-f(k'_\pm)]+P(k'_\pm\to k_\pm)f(k'_\pm)[1-f(k_\pm)]\}$$
$$+\frac{1}{8\pi^3}\int d^3\mathbf{k}'_\mp\{-P(k_\pm\to k'_\mp)f(k_\pm)[1-f(k'_\mp)]+P(k'_\mp\to k_\pm)f(k'_\mp)[1-f(k_\pm)]\} . \tag{18}$$

At equilibrium, in absence of external perturbation, a detailed balance condition imposes that

$$P(k'_\beta,k_\alpha)f_0(k'_\beta)[1-f_0(k_\alpha)]=P(k_\alpha,k'_\beta)f_0(k_\alpha)[1-f^0(k'_\beta)] \tag{19}$$

in spin-flip and nonspin-flip cases.

Putting altogether (10), (11), or (14), (15), and (18), one gets for the relaxation times $\tau_\pm$:

$$\frac{1}{\tau_\pm(k)}=\frac{1}{8\pi^3}\int d^3\mathbf{k}'_\pm P(k_\pm\to k'_\pm)F(k,k')\left\{1-f_0(k)\left[1-\exp\left[-\frac{\varepsilon(k'_\pm)-\varepsilon(k_\pm)}{T}\right]\right]\right\}^{-1}$$
$$+\frac{1}{8\pi^3}\int d^3\mathbf{k}'_\mp P(k_\pm\to k'_\mp)G(k,k')\left\{1-f_0(k)\left[1-\exp\left[-\frac{\varepsilon(k'_\mp)-\varepsilon(k_\pm)}{T}\right]\right]\right\}^{-1} \quad (20)$$

where, in the magnetoresistivity case,

$$F_{\text{el. field}}(k,k')=1-\frac{k'_x}{k_x}=1-\cos\theta \ ,$$
$$G_{\text{el. field}}(k,k')=1-\frac{\tau_\mp}{\tau_\pm}\frac{k'_x}{k_x}=1-\frac{\tau_\mp}{\tau_\pm}\cos\theta \ , \quad (21)$$

while for the viscosity,

$$F_{\text{shear}}(k,k')=1-\frac{k'_xk'_y}{k_xk_y}=\tfrac{3}{2}\sin^2\theta \ ,$$
$$G_{\text{shear}}(k,k')=1-\frac{\tau_\mp}{\tau_\pm}\frac{k'_xk'_y}{k_xk_y}=1-\frac{\tau_\mp}{\tau_\pm}(1-\tfrac{3}{2}\sin^2\theta) \ , \quad (22)$$

with θ the scattering angle between $\mathbf{k}$ and $\mathbf{k}'$; the momentum transfer $\mathbf{q}$ (with $q=|\mathbf{q}|$) between $\mathbf{k}$ and $\mathbf{k}'$ is related to θ, as already noted, by $\bar{q}=q/(2k_F)=\sin[\theta/2]$. The relations between k'_x,k'_y and θ may be extracted from the Steinberg's paper.[8] We will use later

$$F_{\text{el. field}}(k,k')\sin\theta\, d\theta=\frac{q^3dq}{2k_F^4}=8\bar{q}^{\,3}d\bar{q} \ , \quad (23a)$$

$$F_{\text{shear}}(k,k')\sin\theta\, d\theta=\frac{3}{2}\frac{q^3dq}{k_F^4}\left[1-\frac{q^2}{4k_F^2}\right]$$
$$=24\bar{q}^{\,3}d\bar{q}(1-\bar{q}^{\,2}) \ . \quad (23b)$$

We now turn to the transition probabilities $P(k_\alpha,k'_\beta)$:

$$P(k_\alpha\to k'_\beta)=2\pi\sum_{\text{in,fin.}}[\,|M(k_\alpha\to k'_\beta)|^2\delta(E_{\text{final}}-E_{\text{initial}})] \ . \quad (24)$$

$E_{\text{final (initial)}}$ is the sum of all the energies involved after (before) the collision, $M(k_\alpha\to k'_\beta)$ is the matrix element describing the transition from the initial to the final state, and the sum $\sum$ has to be done on all the states. All the above is known, but we find it pedagogical to recall it here, first to compare the mathematics of η and σ and also to follow more easily what comes next.

B. Expression for $\sigma(H)$ and $\eta(H)$ in the paramagnon model

We now examine the transition probabilities $P(k_\alpha\to k'_\beta)$ and thus the matrix elements $M(k_\alpha\to k'_\beta)$ in the specific case of the paramagnon model. We only give one of them; the others follow similarly (such calculations are found in Refs. 8 and 10):

$$|M(k_+\to k'_-)|^2=|\langle\text{final}|S_+(\mathbf{k}-\mathbf{k}')|\text{initial}\rangle|^2 \ , \quad (25)$$

where a fermion $\mathbf{k}$ with spin up and energy ε_+ is scattered by a particle of spin $\mathbf{S}$ and finds itself in a final state $\mathbf{k}'$ with spin down and energy ε_-, while the scattering fermion goes from a spin state down ($S_z=-\frac{1}{2}$) to a final spin-up state ($S'_z=+\frac{1}{2}$). The energy conservation involved in (24) thus implies $\delta(\varepsilon_{S'_z=+1/2}+\varepsilon'_- -\varepsilon_{S_z=-1/2}-\varepsilon_+)$. Calling $\varepsilon'_- -\varepsilon_+=\omega$ and using the definition of the function $\delta(\varepsilon)=\int e^{i\varepsilon t}dt/(2\pi)$ we get

$$P(k_+\to k'_-)=\int dt\, e^{i\omega t}e^{(i\varepsilon_{1/2}t-i\varepsilon_{-1/2}t)}|\langle\text{final}|S_+(\mathbf{k}-\mathbf{k}')|\text{initial}\rangle|^2 \ . \quad (26)$$

If $\mathcal{H}_S$ is the Hamiltonian describing the scattering particles, we have by definition (and with $\mathbf{q}=\mathbf{k}'-\mathbf{k}$),

$$S_+(\mathbf{q},t)=e^{i\mathcal{H}_St}S_+(\mathbf{q})e^{-i\mathcal{H}_St}$$

and $S_-(\mathbf{q},0)=S_-(q)$. Finally,

$$P(k_+\to k'_-)=\int dt\, e^{i\omega t}\langle S_-(\mathbf{q},0)S_+(\mathbf{q},t)\rangle=S^{-+}(q,\omega)=\frac{2}{1-e^{-\omega/T}}\text{Im}\chi^{-+}(q,\omega)=2[1+n(\omega)]\text{Im}\chi^{-+}(q,\omega) \ . \quad (27)$$

$S^{-+}(q,\omega)$ is a dynamical structure factor of the scattering fermions (which interact among themselves to form paramagnons); $n(\omega)$ is the Bose factor defined after formula (1); $\text{Im}\chi^{-+}(q,\omega)$ is the imaginary part of the transverse spin-correlation function (transverse paramagnon) of the system of interacting scattering particles. Transverse paramagnons are involved in spin-flip processes. Similarly we have also nonspin-flip processes which will imply, instead, longitudinal paramagnons, $\chi^{++}(q,\omega)$ or $\chi^{--}(q,\omega)$ (their expressions are given below). [Note that the multiplicative constant entering in the previous formulas will play no role in the relative variation that we wish to compute $\delta\sigma(H)/\sigma(0)$ or $\delta\eta(H)/\eta(0)$. Therefore, in the following, we do not mention them anymore.]

Putting all the previous ingredients together, we get, gathering nonspin-flip as well as spin-flip processes,

$$\frac{1}{\tau_{\pm}(k)} \propto \int\int k'_{\pm} d\varepsilon'_{\pm} \frac{\sin\theta\, d\theta\, F(k,k')}{1-f(\varepsilon)(1-e^{\omega/T})}\bigg|_{\omega=\varepsilon'_{\pm}-\varepsilon_{\pm}} [1+n(\omega)]\mathrm{Im}\chi^{\mp\mp}(q,\omega)$$
$$+\int\int k'_{\mp} d\varepsilon'_{\mp} \frac{\sin\theta\, d\theta\, G(k,k')}{1-f(\varepsilon)(1-e^{\omega/T})}\bigg|_{\omega=\varepsilon'_{\mp}-\varepsilon_{\pm}} [1+n(\omega)]\mathrm{Im}\chi^{\mp\pm}(q,\omega) . \quad (28)$$

The $F(k,k')$ and $G(k,k')$ are given in (21)–(23), depending on the physical property that one is interested in.

The longitudinal and transverse paramagnon propagators are, respectively,[20,4]

$$\chi^{\mp\mp}(q,\omega) \propto \frac{1}{2}\frac{I}{1-I^2\chi^{0\mp\mp}(q,\omega)\chi^{0\pm\pm}(q,\omega)} ,$$
$$\chi^{\mp\pm}(q,\omega) \propto \frac{I}{1-I\chi^{0\mp\pm}(q,\omega)} . \quad (29)$$

The χ^0's are the field dependent noninteracting spin-correlation functions. One could, in principle, as usual,[20] take care of not counting the first terms twice, ($\propto I$), in the expansions of (29); however, the subtracted terms will not matter as they will not be enhanced when $\bar{I}\to 1$. We indeed, at this stage, make an approximation which will greatly simplify all further calculations, at least which render them tractable analytically.

We suppose that we are only interested by the cases of moderate or strong enhancements, so that only leading terms in $(1-\bar{I})^{-1}$ will be retained in the following. Thus, only the field dependences of the Imχ's in (28) will be enhanced and we may drop the field dependence of $k'_{\pm}$ and $\varepsilon'_{\pm}$, both in $d\varepsilon'_{\pm}$ which will reduce to $d\omega$, and in the denominators in (28) involving the Fermi functions. This will yield the important consequence that we will be able, at the end, to express $\eta(H)$ and $\sigma(H)$ in terms of the field-dependent overall static susceptibility $\chi(H)$. This would not be so for a very weak interaction,[17] ($\bar{I}\to 0$) in which case all the field dependences appearing in (28) would be equally important. This is not so either if the scattering centers would not be paramagnons formed from itinerant interacting fermions but localized moments in dilute alloys of noble hosts with magnetic impurities. There the moments are well defined, no "enhancement" is involved and all field dependences matter equally. In such a case the expression for the magnetoconductivity, in first[14(c)] and second[19] order in the Born approximation, including the well-known Kondo contribution, is not just a function of the magnetization but implies a mathematical mixture of the magnetization and some other functions of the ratio H/T arising from the field dependences of $f_{0\pm}$ [corresponding to the denominators in (28)].

From (29) we have

$$\mathrm{Im}\frac{I}{1-I^2\chi^{0\mp\mp}\chi^{0\pm\pm}} = I\frac{\mathrm{Im}(I^2\chi^{0\mp\mp}\chi^{0\pm\pm})}{[1-I^2\mathrm{Re}(\chi^{0\mp\mp}\chi^{0\pm\pm})]^2+[I^2\mathrm{Im}(\chi^{0\mp\mp}\chi^{0\pm\pm})]^2} ,$$
$$\mathrm{Im}\frac{I}{1-I\chi^{0\mp\pm}} = I\frac{\mathrm{Im}(I\chi^{0\mp\pm})}{[1-I\,\mathrm{Re}\chi^{0\mp\pm}]^2+[I\,\mathrm{Im}\chi^{0\mp\pm}]^2} . \quad (30)$$

We are, in this paper, interested by the modifications brought by a finite magnetic field to the zero-field dependences[8,10] of $\eta^{-1}(0)$ or $\rho(0)$ recalled in (8) and (9), which is proportional to T^2 to lowest order in T. Therefore, in (30) we neglect ω in the denominators and only retain the ω dependences of the numerators, responsible for the T^2 dependence of (8) and (9). We thus write

$$\mathrm{Im}\frac{I}{1-I^2\chi^{0\mp\mp}\chi^{0\pm\pm}} \frac{I\,\mathrm{Im}(I^2\chi^{0\mp\mp}\chi^{0\pm\pm})}{[1-I^2\mathrm{Re}(\chi^{0\mp\mp}\chi^{0\pm\pm})]^2_{\omega=0}} ,$$
$$\mathrm{Im}\frac{I}{1-I\chi^{0\mp\pm}} \sim \frac{I\,\mathrm{Im}(I\chi^{0\mp\pm})}{[1-I\,\mathrm{Re}\chi^{0\mp\pm}]^2_{\omega=0}} . \quad (31)$$

The remaining calculations are still intricated. We extensively use the procedure, ingredients, and tricks of Ref. 4. In particular, for any function $L(q,\omega,h)$, we write the identity

$$L(q,\omega,h)=L(q,\omega,0)+[L(q,\omega,h)-L(q,\omega,0)] ,$$

which, although trivial, allows the avoidance of a number of complications when integrals over q and ω, in the various ω and q ranges, will be performed later.

Recall[19,4] that, in appropriate units,

$$h=(B/T_F)=-(H+IM/2)/T_F . \quad (32)$$

T_F is the Fermi temperature and M is the magnetization. As announced after (13), B is a "dressed" Zeeman field containing the applied field H and the fluctuations contained in M. The sign in (32) follows from the definition in Refs. 19 and 4 and will play no role since, to lowest order at least, only h^2 will appear. Also, as well as in Ref. 4, the most important terms turn out to involve small q and ω values.

C. The particular case of strong enhancement $\bar{I} \lesssim 1$

In this paragraph we study explicitly the case where $\bar{I}$ is very close to 1, which was the one considered in Ref. 4. Let us recall here the small $\bar{q}$, ω, and h expansions[4] of some relevant quantities:

$$\mathrm{Im}(\chi^{0\mp\mp}\chi^{0\pm\pm}) \sim N(\varepsilon_F)\frac{\omega}{k_F q}(\mathrm{Re}\chi^{0\mp\mp}+\mathrm{Re}\chi^{0\pm\pm})_{\omega=0} \sim 2N^2(\varepsilon_F)\frac{\omega}{k_F q}\left[1-\frac{\bar{q}^2}{3}-\frac{h^2}{8}(1+\bar{q}^2)+\cdots\right], \tag{33}$$

$$\mathrm{Re}(\chi^{0\mp\mp}\chi^{0\pm\pm})_{\omega=0} = (\mathrm{Re}\chi^{0\mp\mp})_{\omega=0}(\mathrm{Re}\chi^{0\pm\pm})_{\omega=0} \sim N^2(\varepsilon_F)\left[1-\frac{2\bar{q}^2}{3}-\frac{h^2}{2}\left[1+\frac{2\bar{q}^2}{3}\right]+\cdots\right], \tag{34}$$

$$(\mathrm{Re}\chi^{0+-})_{\omega=0} = (\mathrm{Re}\chi^{0-+})_{\omega=0} \sim N(\varepsilon_F)\left[1-\frac{\bar{q}^2}{3}-\frac{h^2}{8}\left[\frac{1}{3}+\frac{\bar{q}^2}{5}\right]+\cdots\right]. \tag{35}$$

We thus obtain

$$\begin{aligned}\tfrac{1}{2}\mathrm{Im}\frac{I}{1-I^2\chi^{0\mp\mp}\chi^{0\pm\pm}} &\simeq I\bar{I}^2\frac{\omega}{k_F q}\left[\frac{1}{\{1-\bar{I}^2[1-(2\bar{q}^2/3)]\}^2}-\frac{\bar{I}^2h^2}{\{1-\bar{I}^2[1-(2\bar{q}^2/3)]\}^3}\right],\\ \mathrm{Im}\frac{I}{1-I\chi^{0\mp\pm}} &\simeq I\bar{I}\frac{\omega}{k_F q}\left[\frac{1}{\{1-\bar{I}[1-(\bar{q}^2/3)]\}^2}-\frac{\bar{I}^2h^2}{\{1-\bar{I}[1-(\bar{q}^2/3)]\}^3}\right].\end{aligned} \tag{36}$$

As announced earlier, it is clear that the h dependences of (36) are strongly enhanced when $\bar{I}\sim 1$ and $\bar{q}\sim 0$, which justifies our approximation of retaining only the field dependences of the χ's in (28) and neglecting all the others.

Putting all the above formulas back in (28) we obtain

$$\frac{1}{\tau_\pm(k)} \simeq \frac{1}{\tau_0(k)} - h^2\delta\left[\frac{1}{\tau}\right] \tag{37}$$

to leading order in $(1-\bar{I})^{-1}$ and to lowest order in h, $\tau_+^{-1}=\tau_-^{-1}$; then G reduces to F in (21) and (22), and (37) reads

$$\frac{1}{\tau_0(k)} \propto I\bar{I}k_F\int\int\frac{\sin\theta\, d\theta\, F(k,k')}{1-f(\varepsilon)(1-e^{\omega/T})}[1+n(\omega)]\frac{\omega d\omega}{\bar{q}}\left[\frac{\bar{I}}{[1-\bar{I}^2+2(\bar{I}^2/3)\bar{q}^2]^2}+\frac{1}{[1-\bar{I}+\bar{I}(\bar{q}^2/3)]^2}\right], \tag{38}$$

$$\delta\left[\frac{1}{\tau}\right] \propto I\bar{I}^3k_F\int\int\frac{\sin\theta\, d\theta\, F(k,k')}{1-f(\varepsilon)(1-e^{\omega/T})}[1+n(\omega)]\frac{\omega d\omega}{\bar{q}}\left[\frac{\bar{I}}{[1-\bar{I}^2+2(\bar{I}^2/3)\bar{q}^2]^3}+\frac{1}{12[1-\bar{I}+\bar{I}(\bar{q}^2/3)]^3}\right]. \tag{39}$$

With the forms (23) put into (38) and (39), one verifies that the most important terms in the viscosity, when $(1-\bar{I})^{-1}>1$, will arise from the $\bar{q}^3$ term of (23b), while the $\bar{q}^5$ term will be negligible in comparison and may be dropped (this can easily be seen by comparing

$$\int_0^1\frac{\bar{q}^2d\bar{q}}{(1-\bar{I}+\bar{q}^2)^n} = \int_0^1\frac{d\bar{q}}{(1-\bar{I}+\bar{q}^2)^{n-1}} - (1-\bar{I})\int_0^1\frac{d\bar{q}}{(1-\bar{I}+\bar{q}^2)^n}$$

and

$$\int_0^1\frac{\bar{q}^4d\bar{q}}{(1-\bar{I}+\bar{q}^2)^n} = \int_0^1\frac{d\bar{q}}{(1-\bar{I}+\bar{q}^2)^{n-2}} - 2(1-\bar{I})\int_0^1\frac{d\bar{q}}{(1-\bar{I}+\bar{q}^2)^{n-1}} + (1-\bar{I})^2\int_0^1\frac{d\bar{q}}{(1-\bar{I}+\bar{q}^2)^n}$$

with $n=2$ and $n=3$). Therefore, the condition $(1-\bar{I})^{-1}>1$ will allow to reduce F_{shear} to $3F_{\text{el.field}}$ in (23) and one will have, for the magnetoconductivity as well as for the viscosity, the same forms of $\tau_0^{-1}(k)$ and $\delta(\tau^{-1})$, namely (with $\omega = Tx$),

$$\begin{aligned}\frac{1}{\tau_0(k)} &\propto T^2\int_0^\infty\frac{x[1+n(x)]dx}{1-f(\varepsilon)[1-e^x]}\int_0^1\bar{q}^2d\bar{q}\left[\frac{\bar{I}}{[1-\bar{I}^2+2(\bar{I}^2/3)\bar{q}^2]^2}+\frac{1}{[1-\bar{I}+I(\bar{q}^2/3)]^2}\right]\\ &\sim\frac{15\pi\sqrt{3}}{8}\frac{T^2}{(1-\bar{I})^{1/2}}\int_0^\infty\frac{x[1+n(x)]dx}{1-f(\varepsilon)[1-e^x]},\quad \bar{I}\to 1,\end{aligned} \tag{40}$$

and

$$\delta\left[\frac{1}{\tau}\right]\propto T^2\int_0^\infty \frac{x[1+n(x)]dx}{1-f(\varepsilon)[1-e^x]}\int_0^1 \bar{q}^{\,2}d\bar{q}\left[\frac{\bar{I}}{[1-\bar{I}^{\,2}+(2\bar{I}^{\,2}\bar{q}^{\,2}/3)]^3}+\frac{1}{12[1-\bar{I}+\bar{I}(\bar{q}^{\,2}/3)]^3}\right]$$

$$\sim\frac{5\pi\sqrt{3}}{32}\frac{T^2}{(1-\bar{I})^{3/2}}\int_0^\infty\frac{x\,dx[1+n(x)]}{1-f(\varepsilon)[1-e^x]}\ . \tag{41}$$

It then remains to compute the integral over x which we write

$$Y=\int_0^\infty\frac{x[1+n(x)]dx}{1-f+fe^x}$$
$$=\frac{1-f}{f}\int_0^\infty\frac{x\,dx}{(1-f)/f+e^x}+\int_0^\infty\frac{x\,dx}{e^x-1}\ . \tag{42}$$

Ultimately in (16) and (17) with (15) and (10), one will have to consider essentially the various quantities at the Fermi level through (11) and (14). Keeping that in mind, we write

$$\frac{1-f}{f}=1+\left[\frac{1-f}{f}-1\right]=1+\varepsilon_Y \tag{43}$$

and we consider ε_Y as a small quantity compared to 1. Then one expands Y in ε_Y:

$$Y\simeq(1+\varepsilon_Y)\left[\int_0^\infty\frac{x\,dx}{e^x+1}-\varepsilon_Y\int_0^\infty\frac{x\,dx}{(e^x+1)^2}\right]$$
$$+\int_0^\infty\frac{x\,dx}{e^x-1}\ . \tag{44}$$

The various integrals in (44) are known[11] and one gets

$$Y\sim\frac{\pi^2}{4}+\varepsilon_Y\ln2=\frac{\pi^2}{4}+\left[\frac{1}{f}-2\right]\ln2\ . \tag{45}$$

Then (37) with (40)–(42) reads

$$\frac{1}{\tau_\pm(k)}=\frac{15\pi\sqrt{3}}{8}\frac{T^2}{(1-\bar{I})^{1/2}}Y\left[1-\frac{1}{12}\frac{h^2}{1-\bar{I}}\cdots\right],$$
$$\bar{I}\sim1\ , \tag{46}$$

which, combined with (32) is the key result of this paper. Indeed it remains to invert (46) and integrate over the energies

$$[\sigma(H),\eta(H)]\propto-\tfrac{1}{2}\left[\int\tau_+\frac{\partial f_+}{\partial\varepsilon_+}d\varepsilon_+\right.$$
$$\left.+\int\tau_-\frac{\partial f_-}{\partial\varepsilon_-}d\varepsilon_-\right] \tag{47a}$$
$$\propto\left[\frac{15\pi\sqrt{3}}{8}\frac{T^2}{\sqrt{1-\bar{I}}}\left[1-\frac{1}{12}\frac{h^2}{(1-\bar{I})}\right]\right]^{-1}$$
$$\times\int_0\frac{df}{Y(f)}\ , \tag{47b}$$

where $Y(f)$ is given by (45). The integral over f is thus a pure number of no importance for our purpose but we give it here anyway:

$$\int_0\frac{df}{Y(f)}=\frac{1}{(\pi^2/4)-2\ln2}\left[1-\frac{\ln2}{(\pi^2/4)-2\ln2}\right.$$
$$\left.+\ln\left[\frac{\pi^2}{4\ln2}-1\right]\right]$$
$$=1.202\ . \tag{48}$$

Let us consider $\rho(H)$ and $\eta^{-1}(H)$ rather than $\sigma(H)$ and $\eta(H)$. The essential ingredient is given in the bracket of (47b):

$$\left[\rho(H),\frac{1}{\eta(H)}\right]\propto\frac{T^2}{\sqrt{1-\bar{I}}}\left[1-\frac{1}{2}\frac{h^2}{1-\bar{I}}+\cdots\right],\quad \bar{I}\sim1\ . \tag{49}$$

To simplify the reading we recall here that, from (32),

$$h^2=B^2/T_F^2=(H+IM/2)^2/T_F^2\ ,$$

we recall also for completeness the result obtained in Ref. 4 for the magnetization M, where we separate, on purpose, the zero-temperature contribution, i.e., the mean-field Stoner result, and the T-dependent one containing paramagnon effects.[21] Note that the result below has been obtained for a *parabolic band:*

$$M(T,H)=S\chi_{\text{Pauli}}H\left[\left[1-\tfrac{1}{6}S^3\frac{H^3}{T_F^2}\right]-S^2\frac{T^2}{T_F^2}\left[\alpha_1-(\beta_1+\tfrac{2}{3}\alpha_1)S^3\frac{H^2}{T_F^2}\right]+\cdots\right],H<(T_F/S^{3/2}),\quad T<(T_F/S) \tag{50}$$

with

$$\chi_{\text{Pauli}} = 2N(\varepsilon_F) \tag{51}$$

and S the Stoner enhancement given in (4), $S = (1-\bar{I})^{-1}$. α_1 and β_1 are pure numbers, slightly different depending whether $H < T$ or $H > T$ as indicated in Ref. 4. We emphasize here again the existence of the characteristic field $H_{sf} = S^{-3/2} T_F = (1-\bar{I})^{3/2} T_F$. Then to *lowest* order in T and H in (49), M reduces to its first zero-temperature Stoner result:

$$M(T=0, H \to 0) \frac{2N(\varepsilon_F) H}{1-\bar{I}} \tag{52}$$

so that

$$\frac{\langle h^2 \rangle}{1-\bar{I}}(T=0, H \to 0) \simeq \frac{H^2}{(1-\bar{I})^3} \ . \tag{53}$$

Putting this back into (49), it turns out that the $(T=0, H \neq 0)$ coefficient of T^2 is just

$$[M(T=0, H \neq 0)/H]^{1/2} = \chi^{1/2}(T=0, H \neq 0) \ ,$$

i.e., involves the $T=0$, lowest H expansion of the square root of (50). We cannot, at this stage, consider the T dependence of M since then one should take into account equally the ω (and thus T) dependences in the denominators of (30) which we have dropped. This would increase considerably the degree of difficulty in the calculation. However, we believe that the above analytical result concerning the $(T=0, H \neq 0)$ coefficient of T^2 in $\rho(H)$ and $\eta^{-1}(H)$ proportional to $\chi^{1/2}(T=0, H \neq 0)$, together with the previously found[10,8] $(T=0, H=0)$ coefficient of T^2 in $\rho(0)$ and $\eta^{-1}(0)$ proportional to $\chi^{1/2}(T=0, H=0)$ and recalled in formula (9) allows us to write, as conjectured after formula (9) [and in Ref. 13 for $\rho(H)$]:

$$\left[\rho(H), \frac{1}{\eta(H)}\right] \propto \chi^{1/2}(T,H) T^2, \quad \bar{I} \lesssim 1 \tag{54}$$

with

$$\chi(T,H) = M(T,H)/H$$

the low-field, low-temperature dependence of the static susceptibility, including paramagnon effects, where $M(T,H)$, computed in Ref. 4 is recalled above in formula (50) (for a parabolic band of fermion).

Formula (54) is the key result of this paper applicable for strong enhancements $S \gg 1$ or $\bar{I} \sim 1$.

D. The case of moderate enhancements, $0.5 < \bar{I} \lesssim 0.75$

We just showed that (54) generalizes the zero-field result (9) of Sec. II A, case (b) valid for strong enhancements. It would be interesting to compute the generalization of the other zero-field result (8) of Sec. II A, case (a) when $\bar{I}$ is moderate, i.e., to provide the analog of (49) where $\bar{I} \sim 0.75$, for instance. We do not perform such a calculation here, it would be much more complicated since we would not be able to retain only the most divergent terms in $(1-\bar{I})^{-1}$. Furthermore, even if we would do it, one would also have to recompute $M(T,H)$ for moderate enhancements since Ref. 4 only considered the cases when $\bar{I} \sim 1$. However, encouraged by the generalization obtained above from (9) to (54), we assume that similarly the generalization of (8) to finite H would read

$$\left[\rho(H), \frac{1}{\eta(H)}\right] \propto \chi^2(T,H) T^2, \quad 0.5 < \bar{I} \lesssim 0.75 \ , \tag{55}$$

i.e., when $(1-\bar{I})^{-1} > 1$.

IV. DISCUSSION AND COMPARISON WITH EXPERIMENTS

A. Discussion of the theoretical formulas (54) and (55)

It was shown that, depending on the strength of $\bar{I}$, (54) or (55) read

$$[\rho(H), \eta^{-1}(H)] \propto \chi^n(T,H) T^2$$

where n is a pure number either $\frac{1}{2}$ of 2. Let us consider here the relative variations:

$$\left.\begin{array}{l} \dfrac{\Delta\rho(H)}{\rho(0)} = \dfrac{\rho(H)-\rho(0)}{\rho(0)} \\[2ex] \dfrac{\Delta\eta^{-1}(H)}{\eta^{-1}(0)} \simeq -\dfrac{\eta(H)-\eta(0)}{\eta(0)} \end{array}\right\} \simeq n \frac{\chi(T,H)-\chi(T,0)}{\chi(T,0)} = n \frac{\Delta\chi(H)}{\chi(0)} \ . \tag{56}$$

From (56) and whatever is n, it is clear that

(a) the resistivity will increases or decrease, when H increases, depending whether χ itself increases or decreases as conjectured previously;[13]

(b) the viscosity will increase or decrease, when H increase, depending whether χ decreases or increases.

In this paper, we have only considered a unique parabolic band of fermions (responsible both for the scattered and scattering particles). In particular, the variation with T and H of M in formula (50), i.e., the sign of the various terms, have been obtained under such an hypothesis. However, it has been emphasized elsewhere[13] that on general grounds the various coefficients entering into M, or $\chi(T,H)$, *depend on the band structure* and involve combinations of the density of states at the Fermi level and its successive derivative $N(\varepsilon_F)$, $N'(\varepsilon_F)$, $N''(\varepsilon_F)$ (in particular the sign of the band curvature at ε_F is very important). That is why, more generally, depending on the band structure, $\Delta\chi(H)$ may be positive or negative, and thus according to (56), $\Delta\rho(H)$ and $[-\Delta\eta(H)]$ will consequently be positive or negative.

Remark (b) is qualitatively well obeyed by both the opposite theoretical tendencies of the nearly metamagnetic nearly localized picture for liquid ^{3}He of Refs. 5 and 6, and the paramagnon picture of Ref. 4. In the former case indeed, $\chi(T,H)$ increases above $\chi(T,0)$ when H increases (although *not* for band-structure reasons) and so strongly that a metamagnetic transition could occur, while in the latter case, ^{3}He is regarded as remaining paramagnetic (although strongly exchange enhanced) but $\chi(T,H)$ decreases compared to $\chi(T,0)$ when H increases. Consequently, in the former case, Ref. 2 calculates a decrease of $\eta(H)$ compared to $\eta(0)$ in agreement with (56) (and also with the experimental finding of Ref. 1), while in the latter case, that we consider here, we expect $\eta(H)$ to increase with H following (56) together with (50) (in agreement with the experiments of Ref. 3 as will be discussed in detail below).

As far as remark (a) is concerned, a discussion of some experiments was presented in Ref. 13 where the result (56) for $\Delta\rho(H)$ was conjectured.

Let us now go back to the particular case of a parabolic band for which (50) is valid.

Let us consider (56) and $\Delta\chi(H)/\chi(0)$. From (50) this expression reads, at $T=0$ (the Stoner result):

$$\Delta\chi(H)/\chi(H=0)_{T=0}=-S^3H^2/(6T_F^2)$$

which may be replaced as well [still using (50) and with $\chi_{\text{Pauli}}=2N(\varepsilon_F)=3/(2T_F)$] by

$$\left[\frac{\Delta\chi(H)}{\chi(H=0)}\right]_{T=0}=-\tfrac{1}{6}S(\tfrac{2}{3}M_{\text{Stoner}})^2=-\tfrac{2}{27}SM_{\text{Stoner}}^2 \;. \tag{57}$$

Therefore, in *low fields and at* $T=0$,

$$\left[\frac{\Delta\rho(H)}{\rho(0)}\right]_{T=0}\propto-\left[\frac{\Delta\eta(H)}{\eta(0)}\right]_{T=0}\propto-M_{\text{Stoner}}^2 \;. \tag{58}$$

[Note that the coefficient of proportionality in the last term of (58) contains the enhancement $S=(1-\bar{I})^{-1}$.] This is very similar to the theoretical result of Ref. 17 for dilute ^{3}He-^{4}He solutions where the lowest field expansion of their Ref. 5.3 also gives $\Delta\eta/\eta\propto M^2$. This has also to be compared with the lowest field expansion of the magnetoresistivity in the localized moment case of dilute alloys[19] where

$$\Delta\rho/\rho\propto-[g\mu_BH/T]^2\propto-M_{\text{loc. spin}}^2$$

(g the g factor of the impurity spin, μ_B the Bohr magneton). In that case though, $M_{\text{loc. spin}}$ obeys to a Curie law adequate for local moments, while M_{Stoner} above is a Stoner law characteristic of itinerant fermions. In either case (local moments or fermions in a band) the *lowest* field relative variation of $\rho(H)$ or $\eta(H)$ can be expressed in terms of the square of the appropriate magnetization. However, as already emphasized, in our present strongly enhanced itinerant fermion system, $\rho(H)$ or $\eta^{-1}(H)$ *themselves* are proportional to some power of $\chi(H)$, while in the Kondo alloy case of Ref. 19, $\rho(H)$ is not itself directly proportional to a power of the impurity spin magnetization, for the reasons explained just before formula (30).

B. Comparison of the theoretical variation $\Delta\eta/\eta$ with the experiments of Ref. 3(a)

We wish now to compare the results of our formulas (50) and (56) [which include (54) and (55)] with the latest observations of $\eta(H)$ in Ref. 3(a), measuring the viscosity of partially polarized normal liquid ^{3}He. The result of that experiment was that at $T=45$ mK and under a pressure of $p=30$ bars, with a frequency $\nu=317.528\times10^6$ Hz which produced a partial polarization $\bar{m}=3.9\%$, a *small* relative *increase* of viscosity was observed of $\Delta\eta/\eta=(3\pm1.5)\times10^{-3}$ in disagreement with the *strong* initial *decreases* of Ref. 1, and this in disagreement with the theoretical prediction[2,5,6] for the occurrence of a metamagnetic transition in liquid ^{3}He.

Let us first note that our formula (50) applies as such to liquid ^{3}He since, in that system, a parabolic band is strictly valid, i.e., there is no band-structure effect to expect. For a meaningful comparison with experiments, we use, to render our formulas quantitative, the same liquid ^{3}He parameters[22] as those used in Ref. 3(a). From the value of ν used in Ref. 3(a) one gets $H=97.798$ KG. On the other hand, the theoretical ratio H/T is actually equal to $(2\pi\nu\hbar)/(2k_BT)$, so that, at $T=0.045$ K we get $H/T\simeq0.17$. Therefore, we take in (50) the values of α_1 and β_1 derived in Ref. 4 for $H<T$, i.e., $\alpha_1=\pi^2/6$ and $\beta_1=23\pi^2/(24)^2$. The bare Fermi temperature is computed from $p_F^2/(k_B2m)$ with p_F given in Ref. 22 at 30 bars, yielding $T_F=6.271$ K. Then there is the correspondence between the characteristic temperature of Ref. 22, T^*, and our $(1-\bar{I})T_F$ which is[20] $(1-\bar{I})^{-1}T_F^{-1}=2/(3T^*)$ with T^* in Ref. 22 to be $T^*=0.185$ K at 30 bars. This yields $S=(1-\bar{I})^{-1}\sim22.598$ and $\bar{I}\sim0.956$. Such a value, strictly speaking, is closer to $\bar{I}\sim1$ (strong enhancements) than to $\bar{I}\sim0.75$ (moderate enhancements). However, we find it reasonable to still compute $\Delta\eta/\eta$ from (56) from both cases (i.e., $n=\frac{1}{2}$ and $n=2$) to examine whether the experimental value lies in such a *range* or not. [Note also that there is a number of uncertainties: for instance the measured η contains the full T dependence while we restrict here only to the lowest T dependence; moreover we use (50) (derived for strong enhancement) also in the moderate enhancement case but, this, we believe, introduces only a minor error.]

Putting the above quantities in (50) gives us a polarization $\bar{m}$:

$$[\bar{m}(H,T)]_{\text{theor}}\simeq3.957\times10^{-2} \tag{59}$$

in good agreement with

$$[\bar{m}(H,T)]_{\text{expt}}\simeq3.9\times10^{-2} \;. \tag{60}$$

Secondly we find from (54) and (55):

$$1.15\times10^{-3}<\left[\frac{\Delta\eta}{\eta}\right]_{\text{theor}}<4.62\times10^{-3} \tag{61}$$

and the experimental result of Ref. 3(a) being

$$\left.\frac{\Delta\eta}{\eta}\right|_{\mathrm{expt}}=(3\pm1.5)\times10^{-3} \quad (62)$$

lies in the range (61) of possible theoretical values.

Therefore, we believe that the agreement between the present paramagnon approach and the experimental result of Ref. 3(a) is quite satisfactory. It would remain to have more experimental data in order to allow a plot of η versus $\chi(H)$ and decide whether the exponent n is closer to $\frac{1}{2}$ or to 2, but at present this is not possible.

Even within the experimental and theoretical uncertainties, both Ref. 3(a) and our present result cannot be compatible with the closeness of a metamagnetic transition at least for the involved polarization which imposes that $\chi(H)<\chi(0)$. It is not excluded though, that, at much higher fields, $\chi(H)$ crosses $\chi(0)$ and becomes larger, but this seems very unlikely or at least would require a physical explanation which is not obvious. Indeed in metals[23] the metamagnetic transition is announced by a $\chi(H)>\chi(0)$ which reflects a band-structure effect. But in liquid ^{3}He such an effect cannot be invoked; therefore, if more experiments confirm the results of Ref. 3(a) the idea of a metamagnetic transition in liquid ^{3}He ought to be dropped or the proposed field dependence of the Landau parameters in Ref. 5 ought to be revisited.

Nonetheless, the fact that the paramagnon approach accounts rather well for the spin properties of liquid ^{3}He is not in conflict with its nearly solid (or nearly localized) behavior as emphasized in Ref. 24. It so happens that for the time being, the paramagnon model better describes the *spin-dependent* properties of ^{3}He while the nearly localized model better describes the *spin-independent* properties. This is quite reasonable given the ingredients put into each one of these models to start with (for instance, the Hamiltonian of the paramagnon model only contains a spin-dependent interaction and *no* spin-independent one). At present only the Landau phenomenological theory accounts for both types of interactions. It would be most fruitful to be able to find as well a unified (and simple to handle) microscopic theory, since we believe that the nearly magnetic (paramagnon) character of liquid ^{3}He and its nearly localized one do *not* exclude each other but, on the contrary, are quite *complementary*.[24]

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[1]P. Kopietz, A. Dutta, and C. N. Archie, Phys. Rev. Lett. **57**, 1231 (1986).

[2]D. W. Hess and K. F. Quader, Phys. Rev. B **36**, 756 (1987).

[3](a) G. A. Vermeulen, A. Schuhl, F. B. Rasmussen, J. Joffrin, G. Frossati, and M. Chapellier, Phys. Rev. Lett. **60**, 2315 (1988); (b) see also, C. C. Kranenburg, S. A. I. Wiegers, G. A. Vermeulen, R. Jochemsen, and G. Frossati, Jpn. J. Appl. Phys. **26-3**, 215 (1987).

[4]M. T. Béal-Monod and E. Daniel, Phys. Rev. B **27**, 4467 (1983).

[5]K. S. Bedell and C. Sanchez-Castro, Phys. Rev. Lett. **57**, 854 (1986).

[6]D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984).

[7]A short version concerning only $\eta(H)$, and without any details, has been submitted elsewhere, for a rapid comparison with the experiments of Ref. 3(a).

[8](a) M. J. Rice, Phys. Rev. **159**, 153 (1967); **162**, 189 (1967); (b) D. S. Betts and M. J. Rice, *ibid.* **166**, 159 (1968); both papers make use of M. S. Steinberg, *ibid.* **109**, 1486 (1958).

[9]N. F. Berk and J. R. Schrieffer, Phys. Rev. Lett. **17**, 433 (1966); S. Doniach and S. Engelsberg, *ibid.* **17**, 750 (1966).

[10]D. L. Mills and P. Lederer, J. Phys. Chem. Solids **27**, 1805 (1966); A. I. Schindler and M. J. Rice, Phys. Rev. **164**, 759 (1967).

[11]See, for instance, I. S. Gardshteyn and I. W. Ryshik, *Table of Integrals, Series and Products* (Academic, New York, 1965).

[12]D. L. Mills, J. Phys. Chem. Solids **34**, 679 (1973).

[13]See, for instance, M. T. Béal-Monod, Physica **109-110B**, 1837 (1982); Phys. Rev. B **24**, 261 (1981).

[14]See, for instance, the reviews (a) H. Jones, in *Theory of Electrical and Thermal Conductivity in Metals*, Vol. 19 of *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), p. 227; (b) F. J. Blatt, in *Theory of Mobility of Electrons in Solids*, Vol. 4 of *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, London, 1957), p. 200; (c) for the magnetoresistivity in dilute alloys containing localized spins, see, in particular, the first Born result of K. Yosida, Phys. Rev. **107**, 396 (1957); T. Van Peski-Tinbergen and A. J. Dekker, Physica **29**, 917 (1963); for the second Born result see Ref. 20 below.

[15]H. Schulz, Z. Phys. **242**, 381 (1971).

[16]P. Hertel, J. Appl, and D. Fay, Phys. Rev. B **22**, 534 (1980).

[17]E. P. Bashkin and A. E. Meierovich, Zh. Eksp. Teor. Fiz. **74**, 1904 (1978) [Sov. Phys.—JETP **47**, 992 (1978)].

[18]K. S. Bedell and K. F. Quader, Phys. Lett. **96A**, 91 (1983); D. Hess, D. Pines, and K. F. Quader (unpublished).

[19]See, for instance, M. T. Béal-Monod and R. A. Weiner, Phys. Rev. **170**, 552 (1968), in particular, formula (23).

[20]See M. T. Béal-Monod, S. K. Ma, and D. R. Fredkin, Phys. Rev. Lett. **20**, 929 (1968), and also Ref. 4 above.

[21]The paramagnons do not modify the $T=0$ Stoner result when $\bar{I}\sim1$, but modify strongly the finite-temperature contributions as shown in Ref 19 above the extensively studied in M. T. Béal-Monod and K. Maki, Phys. Rev. Lett. **34**, 1461 (1975); J. Hertz, Phys. Rev. B **14**, 1165 (1976).

[22]J. C. Wheatley, Rev. Mod. Phys. **47**, 415 (1975).

[23]E. P. Wohlfarth and P. Rhodes, Philos. Mag. **7**, 1817 (1981).

[24]M. T. Béal-Monod, Phys. Rev. B **31**, 1647 (1985).