Fermi-liquid theory for the periodic Anderson model: Response functions

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We present here a reformulation of the Fermi-liquid theory for the periodic Anderson model (for the particular case of the simplest type of Anderson Hamiltonian) in a basis in which c and f electrons are explicitly distinguished. We show that provided the system is normal, i.e., the f-electron self-energy is analytic in momentum and frequency near the Fermi wave vectors and Fermi energy, respectively, all quasiparticle parts of the response functions of the system reduce to the form as expected from our usual understanding of a Fermi liquid. In doing so we also show the validity of a kinetic equation, and are able to obtain formal formulas for the physical quantities that a quasiparticle carries when the corresponding quantities for the original electrons are given. We point out however that the value of these quantities, and also the nonquasiparticle part of the response, can in general be rather different from a too naive understanding to a single-component Fermi liquid.

I. INTRODUCTION

There is a lot of interest recently in the so-called heavy-fermion compounds.[1,2] On the phenomenological side, it is quite often assumed, for sufficiently low temperatures, long wavelengths, and low frequencies, that the Landau Fermi-liquid theory (FLT) applies so long as the systems remain normal;[1,3] with perhaps the complications of (i) many branches cutting the Fermi surface, (ii) nonspherical Fermi surface, or (iii) possible (pseudo-) spin-orbit interactions. On the microscopic side, the essential ingredient is widely accepted as the periodic Anderson model, describing a conduction (c) electron band(s), localized f electrons (f), a hybridization (V) among these, and an interaction (U) among the f electrons.

Historically, Fermi-liquid theory was put forward for a genuine liquidlike ^{3}He by Landau first phenomenologically,[4(a)(b)] and then justified microscopically[4(c)] by demonstrating the applicability of the Landau-Boltzmann kinetic equation for the description of collective modes of the system, by considering the divergence of the two-particle Green's function (see also Refs. 5–7).

The theory was subsequently extended to and justified in other more general cases, especially the case of interacting fermions (electrons) in a periodic lattice. Luttinger[8] has emphasized the role of the Fermi surface and has discussed static properties, and his work is valid even for the case of multibands of electrons in the solid. Dynamics of the system was also discussed by making an explicit definition of quasiparticle distribution function in terms of the Green's function, first introduced by Kadanoff and Baym[9] and subsequently developed by various people (e.g., Ref. 10), initially in the case of a liquid, and finally extended to the case of a lattice by Jones and McClure.[11] This latter work is valid for the multiband case, with the identity among electrons of various bands taken into account.

The dynamics of the Fermi liquid can also be discussed by considering the response function of the systems.[7,12,13] It was, in particular, shown that the response function can be separated into a quasiparticle part and a nonquasiparticle part. The former is precisely the form one would get by the use of the Landau kinetic equation, and the latter, in general, always exists unless due to conservation laws in the system. All these works are valid in the presence of the lattice but are done explicitly only for the case of a single band of electrons or when the identity between the core electrons and the conduction electrons can be ignored.

Let us now go back to the periodic Anderson model. Consider for definiteness the simplest periodic Anderson model (this in no way restricts the generalities of the discussion below)

$$H=\sum_{k,\sigma}\varepsilon_{c,k}c^{\dagger}_{k\sigma}c_{k\sigma}+\sum_{i,\sigma}\varepsilon_f f^{\dagger}_{i\sigma}f_{i\sigma}+\sum_{i,\sigma}V(e^{ik\cdot r_i}f^{\dagger}_{i\sigma}c_{i\sigma}+\text{c.c.})+U\sum_i f^{\dagger}_{i\uparrow}f^{\dagger}_{i\downarrow}f_{i\downarrow}f_{i\uparrow}\ , \tag{1.1}$$

with only one c, one f band, a hybridization V among c and f with the same spin and a local interaction U among f's of opposite spins. $\varepsilon_{c,k}$ the conduction electron energies and ε_f the "bare" f level. There has been various discussions in the relevant regime (e.g., Refs. 14–17). We shall assume that the system is "normal" at zero temperature, and as a result we have two hybridized bands (defined as poles of single-particle Green's function) and for simplicity we assume that the Fermi level crosses only one of the bands, as depicted qualitatively in Fig. 1. Notably in (1.1) is the explicit introduction of two "types" of electrons (in contrast to all the above work on Fermi liquid mentioned in the previous paragraphs). One may ask whether this model will reduce to the Fermi liquid already discussed in the relevant regime, and it would be valuable to demonstrate this by starting directly from the Anderson model itself (rather than arguing that the Anderson model can be deduced from a starting point of a single "type" of electron and then resorting to the exist-

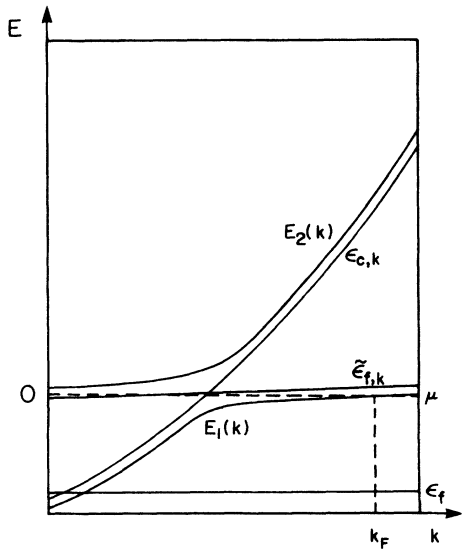

FIG. 1. Schematic representation of the two "hybridized bands." Fermi level is at zero energy.

ing work in Fermi liquid to argue the validity of the Fermi-liquid picture). To this end we may still adopt two strategies: (a) find the relevant operators for the "band" electrons in Fig. 1, and then directly resort to the existing works, in particular those of Luttinger[8] and Jones and McClure,[11] or (b) stay within the basis of explicit c and f electrons and directly demonstrate the Fermi-liquid behavior from here. The former approach has the advantage of being rather economical, for not much work is required. In this paper we shall, however, adopt the second approach because we would like to have explicit formulas in terms of the c and f electrons, which may be convenient for future approximate calculations starting from (1.1) itself. The equilibrium properties, in particular the Luttinger theorems, has been discussed by Ohkawa[14] and shall not be repeated here. We shall simply concentrate on the dynamics. We shall in particular consider the response functions, the procedure of which is particularly presented rather systematically in Leggett's work on superfluid Fermi liquid,[12] partly because of its simplicity if we want to stay within the c-f basis. The use of the Kadanoff and Baym method starting from the c-f basis requires us to make unitary transformations, which would automatically lead us to the band electron basis and reduce us to the first strategy and is inconvenient for our present purposes. However, we do make a brief discussion along these lines in Sec. V, to make connection with existing work on the Fermi liquid, in particular, that of Jones and McClure.

Before we begin the investigation we would like to warn against some simple-minded, intuitively appealing, but incorrect statements. One would tend to think that we can take a unitary transformation from c,f, to α,β such that the G matrix is diagonalized, $G_{\alpha\beta}=G_{\beta\alpha}=0$, in a way that $G_{\alpha\alpha}$ and $G_{\beta\beta}$ will have poles only at the lower and upper bands, respectively, and then the problem can be trivially solved. However, this is *not* true (perhaps with the $U=0$ or $V=0$ cases as the only exceptions). If we do that then *both* $G_{\alpha\alpha}$ and $G_{\beta\beta}$ will have poles at the lower band, one still has to show various contributions to the response function to add up to the form of the single-component Fermi liquid (SCFL). One convenient form, perhaps, is the form introduced in Sec. V, which with $G_{\alpha\alpha}$ as the only element has poles at the lower band, but the matrix G is not diagonalized. This is the basis which diagonalizes the $\tilde{g}_{\text{near}}$ introduced in Sec. III, but not $\tilde{g}$ itself. This difficulty, precisely speaking, should arise whenever there is hybridization among the interacting electrons and not necessarily confined to the periodic Anderson model (e.g., when the identity among the core and conduction electron is taken into account). The large hybridization here just prevents us from making an expansion and regarding the corrections as small. Therefore, if one starts from the Hamiltonian (1.1), it is actually not *a priori* obvious that the result will reduce to a single-component Fermi liquid.[18]

More precisely, perhaps, the questions that we shall address can be stated as follows. [We shall ignore the possible complications (i) and (iii) listed previously to avoid complications in the discussion—i.e., we shall assume that the Fermi surface has only one piece, but not necessarily spherical]. In Landau's single-component Fermi-liquid theory one can describe the Fermi liquid by the quasiparticles occupation numbers $\delta n_{p\sigma}$ at momentum p and spin σ, which obeys a kinetic equation (linearized) (in the collisionless regime)[19]

$$\frac{\partial\delta n_p}{\partial t}+\left[\frac{\partial\varepsilon_p}{\partial p}\frac{\partial\delta n_p}{\partial r}-\frac{\partial\delta n_p}{\partial p}\frac{\partial\varepsilon_p}{\partial r}\right]=0\ , \tag{1.2}$$

where ε_p is itself a function of δn_p

$$\varepsilon_{p\sigma}=\varepsilon^0_{p\sigma}+\sum_{p'} f_{pp'\sigma\sigma'}\delta n_{p'\sigma'}\ . \tag{1.3}$$

All physical quantities (more precisely the deviation from their equilibrium value) δQ can be expressed as

$$\delta Q=\sum_p Q_{p\sigma}\delta n_{p\sigma}\ . \tag{1.4}$$

An often discussed quantity which is also directly related to experiment is the (linear) response function of the system.[7,12,13] In this case one is interested in perturbing the system by an external probe which adds a term $\hat{A}h(q,\omega)$ to the Hamiltonian, where $\hat{A}$ is an operator on the system given by $\hat{A}=A_p a_p^\dagger a_p$ (see Sec. III for more precise subscript labels) and h an external field. One is then interested in measuring a quantity $\hat{B}$ (another operator on the system). The response function is, roughly speaking, the B (now a quantity) one would measure per unit of h. It usually consists of a nonquasiparticle part R^{BA}_{nqp}, not within the Landau theory, and a quasiparticle part R^{BA}_{qp}.[7,12] The latter is of the form (we shall drop the spin indices when convenient)

$$R^{BA}_{\text{qp}}=\sum_p B'_p\frac{v_p q}{\Omega-v_p q}\left[1-F_{pp'}\frac{v_{p'}q}{\Omega-v_{p'}q}\right]^{-1}A'_{p'}\ , \tag{1.5}$$

where B' and A' are "renormalized vertices," and the denominator gives the many-body renormalization expressible in terms of Landau parameters. For example, for the static magnetic susceptibility of ^{3}He, i.e., the spin-spin response function, $B'_p = A'_p = B_p = A_p = \mu_N$ is the ^{3}He nuclear magnetic moment, and the denominator is $(1+F_0^a)^{-1}\delta_{pp'}$. For nonconserved quantities, e.g., the spin-current, $A'_p = (1+F_1^a/3)(p/m^*)\sigma \neq A_p = (p/m)\sigma$.[7,12,13]

The questions in the present case are the following.

(a) Given that we start with Eq. (1.1) with c and f electrons, can one find a distribution function only for a *single* kind of quasiparticle which describes the system [like (1.4)] and obeys the kinetic equation as in a single-component Fermi liquid [(1.2) and (1.3)]?

(b) When we look at the response function of the system, which is now a bilinear form in A_c and A_f on the one hand and B_c and B_f on the other, can we separate a quasiparticle part and non-quasiparticle part as discussed above, and for the former, can they be put in the form of (1.5) as expected for *single*-component FL with well-defined effective vertices B_p^*, A_p^* despite the fact that the external fields usually couple to two types of electrons;[20] in particular, we note the following questions. (i) Can effective vertices B_p^* and A_p^* be defined for given operators in terms of c and f electrons—that, e.g., B^* is independent of which particular external probe A^* we apply and vice versa.[21] (ii) Given that the many-body interaction itself determines the "composition" of the "hybridized particles" in terms of the original c and f electrons, will, so to speak, the external field change the constituent of these hybridized particles (cf. Ref. 21) so that the hybridization depends on q,Ω of the external perturbation such that the effective vertices B^*, A^* above are q,Ω dependent? [Note that it is not sufficient to argue that $q/p_F \ll 1$ and $\Omega/E_F \ll 1$, where $p_F E_F$ are the Fermi momentum and energy, since we also have to worry about possible pole of the form $1/\Omega - \mathbf{v}_p^* \cdot \mathbf{q}$, where $\mathbf{v}_p^*$ is the group velocity $\partial E_p^*/\partial \mathbf{p}$, of the "quasiparticles"]. (iii) What are the effective vertices in terms of the known bare interaction in terms of the c and f electrons? (Equivalently what is $Q_{p\sigma}$ when the corresponding physical quantities in terms of c and f electrons are given?)

Our aim is to answer the above questions. In this paper we shall not find the correspondence between $\delta n_{p\sigma}$ and the matrix distribution function in the c-f space, but we shall show that the collisionless collective modes are indeed determined by solving equations of the form (1.2) and (1.3). Equation (1.4) is trivially valid (if we can identify $\delta n_{p\sigma}$) since it is just a variational equation, but since we are not identifying explicitly $\delta n_{p\sigma}$ we shall not find and show the well-definedness of $Q_{p\sigma}$ directly [nor can we show explicitly that $\delta n_{p\sigma}$ always satisfy the usual modified form of (1.2) and (1.3) when an external field is present]. However, we shall concentrate on the (in principle physically measurable) response functions. We shall show that, under the special case of (1.1) and assuming that the Fermi level cuts across only one of the two quasiparticle eigenvalues (one of the two "bands") that our result can be cast into a single-component Fermi-liquid theory (provided the system remains normal: more precise conditions are given below). We find that the A_p^*, etc. (and hence indirectly $Q_{p\sigma}$) are well defined and we shall give explicit expressions for them in terms of the c and f electron vertices. Specifically, for a physical quantity $\hat{B}$ whose operator form is $\hat{B} = \sum_p (B_{c,p} c_p^\dagger c_p + B_{f,p} f_p^\dagger f_p)$ [for precise definition see (3.2)] we find

$$B_p^* = a_c B_{c,p} + a_f (R_{21} B_{c,p} + R_{22} B_{f,p}) , \tag{1.6}$$

where R_{ij} are renormalizations *operators;* a_c and a_f are the weight of the pole of the c and f electron Green's function at the Fermi energy (quantitative statements given below). We shall then identify (indirectly) the $Q_{p\sigma}$'s by comparing the response functions with those of SCFL. It is of interest to compare these results with the naive idea of hybridization, where we say that the quasiparticle α_p at the Fermi level is given as a linear combination of the original electrons [e.g., obtained by diagonalization of an effective mean-field Hamiltonian of the form (2.8) below]

$$\alpha_p = \tilde{u}_p c_p + \tilde{v}_p f_p , \tag{1.7}$$

which would give the physical quantity B^* carried by a quasiparticle as

$$B_p^* = \tilde{u}_p^2 B_{c,p} + \tilde{v}_p^2 B_{f,p} . \tag{1.8}$$

Since R_{ij} in (1.6) are *operators,* (1.6) and (1.8) will never agree on all physical quantities B, whatever $\tilde{u}$ and $\tilde{v}$ we choose (as shall be shown explicitly in examples given below). Too naive an interpretation of the linear combination found from the effective noninteracting Hamiltonian of the effective mean field theories (e.g., $1/N$ expansion at the $1/N^0$ level or Gutzwiller ansatz) will be entirely misleading.

The difference is, in retrospect, not surprising since we are dealing with a true many-body system. A quasiparticle excitation consists of excited c or f electrons, as well as two c electrons and one f hole, etc. This fact is not reflected in (1.7).

The organization of the paper is as follows: In Sec. II we shall review the zero-temperature Green's function for the model (1.1), which shall be used extensively below. In Sec. III we handle these Green's functions in matrix form and consider the low q,Ω limit. Our procedure here is closely parallel to that done by Leggett[12] for superfluid Fermi liquid (see also Refs. 7 and 13) and the intermediate procedure looks, in many cases, rather similar. We shall show there that the collective modes of the system is describable by the kinetic equation as in SCFL. This fact can actually be demonstrated more easily by extending Landau's argument[4(c)] for SCFL. This is shown in Appendix A. We shall also show there explicitly how the quasiparticle part of the response is reduced to the form of the SCFLT in static as well as dynamic cases, except appearances of "effective quantities" (vertices). In Sec. IV we shall apply this formulation to calculate some quasiparticle responses in the static limit, calculate the effective quantities, and identify $Q_{p\sigma}$ of (1.4) in some cases, where we shall also compare our results with simple-minded hybridization results. Our ability to calculate these responses, despite the fact that R are opera-

tors [see (1.6)], relies on the existence of some Ward type of identities, which we discuss in Appendix B. We present a discussion on the periodic Anderson model with the Kadanoff and Baym method in Sec. V, and finally summarize in Sec. VI. In Appendix C we discuss the physical current, in particular how its SCFLT result is restored.

II. REVIEW OF GREEN'S FUNCTION RESULTS

Our discussions below are in terms of Green's function, which has been discussed in some detail by Yamada and Yosida.[17] We briefly summarize their result and define our notations. In this paper we shall exclusively deal with reactive phenomena and all dissipative terms shall be ignored. (In accordance to this we confine ourselves to $T=0$.) We shall *not* assume spherical symmetry, except occasionally when we want to compare our results with the more familiar situation.

First we consider the case without magnetic field. Since the basic Hamiltonian (1.1) conserves the spin all Green's functions are spin scalars and we shall omit the spin indices. Since there are two kinds of electrons, we define four Green's function G_c, G_{cf}, G_{fc}, G_f, where, for example, $G_c=-i\langle Tcc^\dagger\rangle$ and $G_{cf}\equiv -i\langle Tcf^\dagger\rangle$. We obtain easily

$$G_c(k,\omega)=\left[\omega-\varepsilon_k-\frac{V^2}{\omega-\varepsilon_f-\Sigma(k,\omega)}\right]^{-1}, \tag{2.1}$$

$$G_f(k,\omega)=\left[\omega-\varepsilon_f-\Sigma(k,\omega)-\frac{V^2}{\omega-\varepsilon_k}\right]^{-1}, \tag{2.2}$$

and

$$G_{cf}(k,\omega)=G_{fc}(k,\omega)=V(\omega-\varepsilon_k)^{-1}\left[\omega-\varepsilon_f-\Sigma(k,\omega)-\frac{V^2}{\omega-\varepsilon_k}\right]^{-1}. \tag{2.3}$$

$\Sigma(k,\omega)$ is the "true" f electron self-energy, i.e., all f self-energy diagrams which are not one particle (neither c nor f) reducible.

The quasiparticle energies are given by the poles of these Green's functions, i.e.,

$$[\omega-\varepsilon_f-\Sigma(k,\omega)](\omega-\varepsilon_k)-V^2=0\ . \tag{2.4}$$

We shall be interested only in the eigenvalues near the Fermi level, chosen to be the zero of the energies. We expand $\Sigma(k,\omega)$ around $\omega=0$ (Refs. 22 and 23)

$$\Sigma(k,\omega)=\Sigma(k,0)+\omega\left.\frac{\partial\Sigma(k,\omega)}{\partial\omega}\right|_{\omega=0}. \tag{2.5}$$

$|\partial\Sigma/\partial\omega|$ is expected to be a large number from our knowledge of the single-ion Kondo effect, due to the many-body nature of the problem. If we use (2.5) for all ω then we can rewrite (2.6) as

$$(\omega-\tilde{\varepsilon}_{f,k})(\omega-\varepsilon_k)-\tilde{V}_k^2=0\ , \tag{2.6}$$

where we have defined

$$\tilde{\varepsilon}_{f,k}\equiv[\varepsilon_f+\Sigma(k,0)]\Big/\left[1-\frac{\partial\Sigma}{\partial\omega}\right], \tag{2.7}$$

$$\tilde{V}_k^2\equiv V^2\left[1-\frac{\partial\Sigma}{\partial\omega}\right]^{-1}. \tag{2.8}$$

The roots of (2.6), or the quasiparticle energies, are

$$E_{1,2}(k)=\frac{\varepsilon_k+\tilde{\varepsilon}_{f,k}}{2}\mp\left[\left[\frac{\varepsilon_k-\tilde{\varepsilon}_{f,k}}{2}\right]^2+\tilde{V}_k^2\right], \tag{2.9}$$

and is thus the same form as the Gutzwiller ansatz result[15] or the mean-field level result of the $1/N$ expression,[16] with $\tilde{\varepsilon}_{f,k}$, $\tilde{V}_k$ playing the role of effective f level and effective hybridization of the "equivalent (noninteracting) mean-field Hamiltonian"

$$H_{\text{eff}}=\sum_k[\varepsilon_k c_k^\dagger c_k+\tilde{\varepsilon}_{f,k}\tilde{f}_k^\dagger\tilde{f}_k+\tilde{V}_k(c_k^\dagger\tilde{f}_k+\tilde{f}_k^\dagger c_k)]\ . \tag{2.10}$$

Equation (2.10) can be trivially diagonalized with the "quasiparticle operators"[24]

$$\tilde{\alpha}_k=\tilde{u}_k\tilde{f}_k+\tilde{v}_k c_k,\quad \tilde{\beta}_k=-\tilde{v}_k\tilde{f}_k+\tilde{u}_k c_k\ , \tag{2.11a}$$

where

$$\frac{\tilde{v}_k}{\tilde{u}_k}=\frac{E_{1,k}-\tilde{\varepsilon}_{f,k}}{\tilde{V}_k}\ . \tag{2.11b}$$

The quasiparticle energies are as sketched in Fig. 1. [Equations (2.6)–(2.11) shall *not* be used below, whereas (2.5) shall be used only near the Fermi level: hence we are not making any approximations. Here we also define new operators $\tilde{f}$, to which we do not attach physical meaning at present, for the convenience of discussions below.]

We assume that the Fermi level is at one "band" of (2.7), for definiteness taken to be the lower band 1 and shall simply write $E_1(k)=E^*(k)$. We are thus assuming that $\Sigma(k,\omega)$ analytic in k and ω for $k\approx k_F$ and $\omega\approx 0$ (which is our definition of the systems being "normal;" these assumptions are used implicitly throughout this paper). Here $k_F=k_F(\hat{\mathbf{k}})$ is the Fermi wave vector which by definition is

$$E^*(\mathbf{k}_F)=0\ . \tag{2.12}$$

But for convenience we often keep $E^*(k_F)$ explicitly below so that the formulas shall be of more familiar form.

Of particular relevance below are the residues of the various Green's functions at E^* ($\mathbf{k}\approx\mathbf{k}_F$). They can easily be found from (2.1)–(2.3) by noting that the energies are the zeros of the denominators. The results are[17] for $\omega\approx E^*(k_F)$

$$G_f=\frac{a_{f,k}}{\omega-E^*(k)}+G_f^{\text{inc}}\ ,$$

$$G_c=\frac{a_{c,k}}{\omega-E^*(k)}+G_c^{\text{inc}}\ , \tag{2.13}$$

$$G_{cf}=G_{fc}=\frac{(a_c a_f)^{1/2}}{\omega-E^*(k)}+G_{fc}^{\text{inc}}\ ,$$

with

$$a_{f,k}=\left[1-\frac{\partial}{\partial\omega}\Sigma(k,\omega)+\frac{V^2}{(\omega-\varepsilon_k)^2}\right]^{-1}\Bigg|_{\omega=E^*(k)},$$
$$a_{c,k}=\left[\frac{V}{\omega-\varepsilon_k}\right]^2 a_{f,k}\ , \tag{2.14}$$

and $G^{\rm inc}$ as the "incoherent" parts.

Note the useful identity

$$a_c+a_f\left[1-\frac{\partial\Sigma}{\partial\omega}\right]=1\ . \tag{2.15}$$

[We shall omit the $\pm i\delta$ convergent factors in (2.13). They shall be added below and be clear from the context.] The quasiparticle velocity v_k^*, is by definition

$$v_k^*=\frac{\partial E_k^*}{\partial k}\ , \tag{2.16}$$

and can be found by differentiating the equation for E_k^* [or more conveniently (2.4) after divided by $(E_k^*-\varepsilon_k)$]

$$v_k^*=a_{c,k}v_{c,k}+a_{f,k}\frac{\partial}{\partial k}\Sigma(k,\omega)\Big|_{\omega=E_k^*}\ , \tag{2.17}$$

where the last derivative is on the k variable only. On the other hand, the effective f level $\tilde{\varepsilon}_{f,k}$, given in (2.7), has the velocity [ignoring second derivatives of Σ, in contrast to (2.17), which is exact]

$$\frac{\partial\tilde{\varepsilon}_{f,k}}{\partial k}=\left[1-\frac{\partial\Sigma}{\partial\omega}\right]^{-1}\left[\frac{\partial\Sigma}{\partial k}\right]\ , \tag{2.18}$$

so

$$v_k^*\approx a_c v_c+\left[1-\frac{\partial\Sigma}{\partial\omega}\right]a_f\frac{\partial\tilde{\varepsilon}_{f,k}}{\partial k}\ . \tag{2.19}$$

The density of states of the c and f electrons are discussed in Ref. 17 (defined from $\mathrm{Im}G_c$ and $\mathrm{Im}G_f$), and obeys

$$\rho_{c,f}(k,\omega)=a_{c,f,k}\delta(\omega-E^*(k))\ ,$$
$$\rho_{c,f}(\omega)=\sum_k\rho_{c,f}(k,\omega)\ , \tag{2.20}$$
$$\rho_c+\sum_{|k|}\left[1-\frac{\partial\Sigma(k,\omega)}{\partial\omega}\right]\rho_f(k,\omega)=\nu_{\hat{\mathbf{k}}}(\omega)/2\ ,$$

where $\nu_{\hat{\mathbf{k}}}(0)$ is the density of states (for both spins) that (or more precisely its angular average) enters the specific heat. This is of the form

(quasiparticle quantity)

$$=(c\text{-electron contribution})$$
$$+\left[1-\frac{\partial\Sigma}{\partial\omega}\right]\times(f\text{-electron contribution})\ , \tag{2.21}$$

Eq. (2.21) (in particular its validity) shall be discussed repeatedly below [cf. (1.8)]. Note that the approximate equation (2.19) [but not (2.17)] has this form.

III. RESPONSE FUNCTIONS: FORMULATION

We shall formally discuss here the response function (at zero temperature it can be easily extended to finite temperature as in Ref. 12). More precisely we consider an external perturbation, of the form

$$H'(t)=\hat{A}(t)e^{-i\Omega t}h(q,\Omega)\ ,$$
$$\hat{A}(t)=\sum_p(A_{c,p\sigma_+\sigma_-}c^\dagger_{p_+\sigma_+}c_{p_-\sigma_-}+A_{f,p\sigma_+\sigma_-}f^\dagger_{p_+\sigma_+}f_{p_-\sigma_-})\ , \tag{3.1}$$

where $p_\pm=p\pm q/2$, and q and Ω are the external wave vector and frequency, respectively.

We shall be interested in "measuring" the quantities B, of the form

$$\hat{B}(t)=\sum_p(B_{c,p\sigma_-\sigma_+}c^\dagger_{p_-\sigma_-}c_{p_+\sigma_+}+B_{f,p\sigma_-\sigma_+}f^\dagger_{p_-\sigma_-}f_{p_+\sigma_+})(t) \tag{3.2}$$

per unit h.[25] As is well known, this (Fourier-transformed) retarded response function is obtained by evaluating the Fourier transformed of the commutator of B and A,

$$\langle\langle B:A\rangle\rangle(q,\Omega)=-i\int_{-\infty}^{\infty}\langle\hat{B}(t)\hat{A}(t')-\hat{A}(t')\hat{B}(t)\rangle\times\theta(t-t')e^{i\Omega(t-t')}d(t-t')\ , \tag{3.3}$$

where θ is the step function. Equation (3.3) is most conveniently evaluated by considering instead the Fourier transform of the time-ordered product $\langle T\hat{B}(t)\hat{A}(t')\rangle$ at $\Omega>0$ and then put $\Omega\to\Omega+i\delta$. We are then led to evaluate the various "bubbles" as shown in Fig. 2, where the solid lines for electrons can either be c, f, or mixed (cf or fc). The bare vertex A (where solid lines meet wavy lines) is A_c or A_f simply according to whether these solid lines are c or f (note both lines must be of the same electron type[25]). Similar discussion hold for the vertex B. We shall use the simple notation of $\bar{p}\equiv(p,\omega)$ and $(GG)_{\bar{p}}$ as the pair $G(p_+,\omega+\Omega)G(p_-,\omega)$ for any Green's functions [i.e., $(G_{cf}G_{fc})_{\bar{p}}=G_{cf}(p_+,\omega+\Omega)G_{fc}(p_-,\omega)$] and

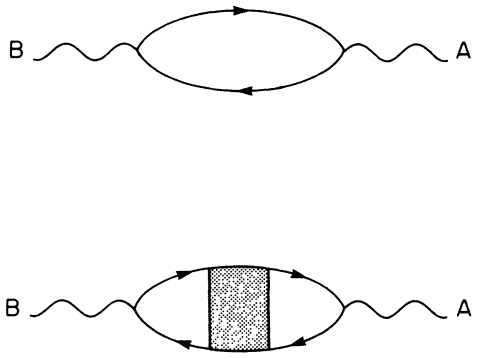

FIG. 2. Diagrammatic representation of response functions. Intermediate lines can be either G_f, G_c, G_{fc}, or G_{cf} except it must end as f electrons in the f electron interaction vertex Γ_f denoted by the solid square.

shall simply use summation (integration) over repeated dummy momentum, frequency, and spin indices when no confusion arises. The "empty bubble" is then

$$\int d\omega \frac{1}{2\pi i}[B_{c,p}(G_cG_c)_{\bar{p}}A_{c,p}+B_{c,p}(G_{cf}G_{fc})_{\bar{p}}A_{f,p} +B_{f,p}(G_{fc}G_{cf})_{\bar{p}}A_{c,p}+B_{f,p}(G_fG_f)_{\bar{p}}A_{f,p}] . \tag{3.4}$$

(Throughout this paper we suitably reabsorb i in the frequency integrals so that they do not appear elsewhere explicitly and allow easier comparison with Ref. 12.)

For the bubble with the interaction vertex, we notice that in our model only f electrons interact. Therefore, all possible four-legged diagrams that cannot be cut into two parts by cutting a single line must have all external legs being f electrons, i.e., the two f-electron vertex Γ_f. Therefore, all diagrams of the type shown in Fig. 2(b) are automatically included by including only Γ_f, and letting the solid lines be of various types (including G_{cf} and G_{fc}), except it must end as an f electron on Γ_f. Thus the contribution is

$$B_{c,p}(G_{cf}G_{fc})_{\bar{p}}\Gamma_{f,\bar{p}\bar{p}'}(G_{fc}G_{cf})_{\bar{p}'}A_{c,p'} +B_{c,p}(G_{cf}G_{fc})_{\bar{p}}\Gamma_{f,\bar{p}\bar{p}'}(G_fG_f)_{\bar{p}'}A_{f,p'} +(\text{terms with } B_f\text{'s}) , \tag{3.5}$$

where $\Gamma_{f,pp'}$ is defined as in Fig. 3 and the spin and frequency sum is as in the following example:

$$\int d\omega \frac{1}{2\pi i}\int d\omega' \frac{1}{2\pi i}B_{c,p;\sigma_-\sigma_+}(G_{cf}G_{fc})_{\bar{p}}\Gamma_{f,\bar{p}\bar{p}'\sigma_+\sigma_-\sigma'_+\sigma'_-} (G_fG_f)_{\bar{p}'}A_{f,p';\sigma'_+\sigma'_-} , \tag{3.6}$$

similarly for (3.3). (The integration over intermediate frequencies $\int d\omega/2\pi i$ shall be implicit, as for summation over p and σ.) The above can be collected in simple notation by defining the following matrices [we shall name the space as c-f space: note all quantities are matrices in spin-space and spin-matrix multiplication as in (3.6) is implied]:

$$\tilde{g}_{\bar{p}}=\begin{pmatrix}(G_cG_c)_{\bar{p}} & (G_{cf}G_{fc})_{\bar{p}}\\ (G_{fc}G_{cf})_{\bar{p}} & (G_fG_f)_{\bar{p}}\end{pmatrix} , \tag{3.7}$$

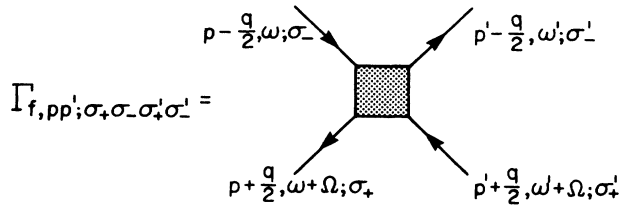

FIG. 3. Variables entering the f-electron interaction vertex Γ_f.

$$\tilde{A}_p=\begin{pmatrix}A_{c,p} & 0\\ 0 & A_{f,p}\end{pmatrix} , \tag{3.8}$$

as similarly for $\tilde{B}_p$,

$$\tilde{\Gamma}_{\bar{p}\bar{p}'}=\begin{pmatrix}0 & 0\\ 0 & \Gamma_{f\bar{p}\bar{p}'}\end{pmatrix} . \tag{3.9}$$

The above response function is then

$$K^{BA}(q,\Omega)=\mathrm{Tr}[\tilde{B}(\tilde{g}+\tilde{g}\tilde{\Gamma}\tilde{g})\tilde{A}\tilde{P}] , \tag{3.10}$$

where

$$\tilde{P}\equiv\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} .$$

Here Tr is trace over the matrix c-f space, spin-space [as in (3.6)], momentum space $\int d^3p[1/(2\pi)^3]$, and frequency space $\int d\omega(1/2\pi i)$. We shall use implicit matrix notation as far as no confusion arises.

The crucial step in making connection with a Landau theory is the separation of the response K into quasiparticle part K_{qp} and nonquasiparticle part K_{nqp}. This is done elegantly by Leggett[12] for the superfluid Fermi liquid by decomposing the Green's function and vertex function into parts near and far away from the Fermi surface. Our procedure below is an adaptation of his procedure into our case.

One starts with writing down the Bethe-Salpeter equation for Γ_f and subsequently for $\tilde{\Gamma}$ [see also Ref. 4(c)]. The equation for Γ_f is as shown in Fig. 4, which is quite analogous to the case of SCFL. Here $\Gamma^{(1)}$ denotes the vertex part irreducible with respect to pairs of particle-hole lines of total momentum q and frequency Ω. From the discussions preceding (3.5), one easily convinces oneself that in the second term of the right-hand side, one only needs to include diagrams with intermediate f electron lines. Thus in simplified notation

$$\Gamma_f=\Gamma_f^{(1)}+\Gamma_f^{(1)}(G_fG_f)\Gamma_f , \tag{3.11}$$

where we suppress all momentum, spin-matrix indices. In our c-f matrix space

$$\tilde{\Gamma}=\tilde{\Gamma}^{(1)}+\tilde{\Gamma}^{(1)}\tilde{g}\tilde{\Gamma} , \tag{3.12}$$

where

$$\tilde{\Gamma}^{(1)}\equiv\begin{pmatrix}0 & 0\\ 0 & \Gamma_f^{(1)}\end{pmatrix} .$$

One then, for any pair of Green's function (GG),

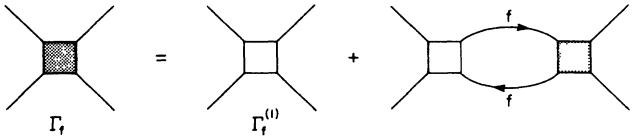

FIG. 4. Bethe-Salpeter equation for $\Gamma_f\bar{p}=(p,\omega)$. The open square, $\Gamma_f^{(1)}$ is the particle-hole irreducible part of Γ_f (with respect to q,Ω).

separates the "near" and "far" parts. By standard arguments as in Ref. 4(c), or Sec. 17 of Ref. 5, Sec. 18 of Ref. 6, or Sec. 6.4 of Ref. 7(b) (the last reference in particular points out that we have not assumed spherical symmetry), one obtains, for example,

$$(G_c G_c)=(G_c G_c)_{\text{near}}+(G_c G_c)_{\text{far}} , \tag{3.13}$$

where

$$(G_c(p+q/2,\omega+\Omega)G_c(p-q/2,\omega))_{\text{near}} = \frac{2\pi i a_{c,\hat{\mathbf{p}}}^2 \mathbf{v}_{Fp}^* \cdot \mathbf{q}\delta(\omega)\delta(E_p)}{\Omega-\mathbf{v}_{F,p}^*\cdot\mathbf{q}} . \tag{3.14}$$

$(G_c G_c)_{\text{far}}$ is regular as $\mathbf{q}$ and/or Ω approaches zero, and can be regarded as a constant (independent of q and Ω). a_c is defined in (2.14) [and evaluated from now on at $p_F=p_F(\hat{\mathbf{p}})$], and a useful relation is

$$(GG)_{\text{far}}=(GG)^{\omega} , \tag{3.15}$$

where the superscript ω means the "ω limit" ($\lim_{\Omega\to 0}\lim_{q\to 0}$). (We use standard notation, but note it is not ω, but Ω, that approaches zero.) We use a similar notation as for Γ. Standard arguments then yield from (3.11)

$$\Gamma_f^{\omega}=\Gamma_f^{(1)}+\Gamma_f^{(1)}(G_f G_f)_{\text{far}}\Gamma_f^{\omega} , \tag{3.16}$$

$$\Gamma_f=\Gamma_f^{\omega}+\Gamma_f^{\omega}(G_f G_f)_{\text{near}}\Gamma_f , \tag{3.17}$$

or, in matrix space

$$\tilde{\Gamma}^{\omega}=\tilde{\Gamma}^{(1)}+\tilde{\Gamma}^{(1)}\tilde{g}_{\text{far}}\tilde{\Gamma}^{\omega} , \tag{3.18}$$

$$\tilde{\Gamma}=\tilde{\Gamma}^{\omega}+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{near}}\tilde{\Gamma} , \tag{3.19}$$

where $\tilde{\Gamma}^{\omega}$ is of the form

$$\begin{pmatrix} 0 & 0 \\ 0 & \Gamma_f^{\omega} \end{pmatrix} .$$

One then notes the matrix identity[12,26]

$$\tilde{g}+\tilde{g}\tilde{\Gamma}\tilde{g}=\tilde{g}_{\text{far}}(1+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{far}}) +(1+\tilde{g}_{\text{far}}\tilde{\Gamma}^{\omega})\tilde{g}_{\text{near}} \times(1-\tilde{\Gamma}^{\omega}\tilde{g}_{\text{near}})^{-1}(1+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{far}}) . \tag{3.20}$$

Substitution into (3.10) yields the desired separation, where

$$K_{\text{nqp}}^{BA}=\text{Tr}[\tilde{B}\tilde{g}_{\text{far}}(1+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{far}})\tilde{A}\tilde{P}] , \tag{3.21}$$

$$K_{\text{qp}}^{BA}=\text{Tr}[\tilde{B}(1+\tilde{g}_{\text{far}}\tilde{\Gamma}^{\omega})\tilde{g}_{\text{near}}(1-\Gamma^{\omega}\tilde{g}_{\text{near}})^{-1} \times(1+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{far}})\tilde{A}\tilde{P}] . \tag{3.22}$$

Or, defining the matrix

$$\tilde{R}\equiv(1+\tilde{\Gamma}^{\omega}\tilde{g}_{\text{far}}) \tag{3.23}$$

and

$$\tilde{A}''=\tilde{R}\tilde{A} , \tag{3.24}$$

and so for $\tilde{B}''$, we get

$$K_{\text{qp}}^{BA}=\text{Tr}[\tilde{B}''^{t}\tilde{g}_{\text{near}}(1-\tilde{\Gamma}^{\omega}\tilde{g}_{\text{near}})^{-1}\tilde{A}''\tilde{P}] , \tag{3.25}$$

where the superscript t denotes the transpose, the precise meaning of which is just that the various indices should be in the order depicted in (3.22). (It turns out that by a manipulation below we can be rather careless about the transpose in c-f space.) Equation (3.24) represents the renormalization of the bare vertices by what occurs in the system far away from the Fermi surface. Consistent with the low-frequency long-wavelength limit Γ^{ω}, $\tilde{g}_{\text{far}}$ are taken to be constant (i.e., independent of external probe, q,Ω, and temperature). Using the form of $\tilde{\Gamma}^{\omega}$ one sees that $\tilde{R}$ has the form

$$\tilde{R}=\begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix} , \tag{3.26}$$

explicitly

$$\begin{aligned} R_{11}&=1 , \\ R_{21}&=\Gamma_f^{\omega}(G_{fc}G_{cf})^{\omega} , \\ R_{22}&=1+\Gamma_f^{\omega}(G_f G_f)^{\omega} . \end{aligned} \tag{3.27}$$

Thus

$$\tilde{A}''\tilde{P}=\begin{pmatrix} R_{11}A_c & 0 \\ R_{21}A_c & R_{22}A_f \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} =\begin{pmatrix} A'_{11} & A'_{11} \\ A'_{22} & A'_{22} \end{pmatrix}\equiv\tilde{A}'\tilde{P} , \tag{3.28}$$

where

$$\tilde{A}'\equiv\begin{pmatrix} A'_{11} & 0 \\ 0 & A'_{22} \end{pmatrix} \tag{3.29}$$

and

$$\begin{aligned} A'_{11}&\equiv R_{11}A_c , \\ A'_{22}&=R_{21}A_c+R_{22}A_f . \end{aligned} \tag{3.30}$$

Substituting (3.28) back and using the cyclic invariance of the trace, we move $\tilde{P}$ to the front and note

$$\tilde{P}(\tilde{R}\tilde{B})^t=(\tilde{R}\tilde{B}\tilde{P})^t=(\tilde{B}'\tilde{P})^t=\tilde{P}\tilde{B}' ,$$

where we defined $\tilde{B}'$ as in (3.29). Note $\tilde{A}'$ and $\tilde{B}'$ are diagonal in c-f space. We again move $\tilde{P}$ back to the end of the trace and finally obtain

$$K_{\text{qp}}^{BA}=\text{Tr}[\tilde{B}'\tilde{g}_{\text{near}}(1-\tilde{\Gamma}^{\omega}\tilde{g}_{\text{near}})^{-1}\tilde{A}'\tilde{P}] . \tag{3.31}$$

We need to evaluate the inverse operator in (3.31). With the help of (3.14) we can evaluate the intermediate frequency sums. We also sum over the magnitude of p (i.e., E_p for fixed $\hat{\mathbf{p}}$) by using

$$\int d^3p\frac{1}{(2\pi)^3}=\tfrac{1}{2}\int d\Omega_{\hat{\mathbf{p}}}\frac{1}{4\pi}\nu_{\hat{\mathbf{p}}}\int dE .$$

Here $\nu_{\hat{\mathbf{p}}}=[p_f(\hat{\mathbf{p}})]^2/\pi^2 v_{F,p}^*$. Note $\nu_{\hat{\mathbf{p}}}=\nu(0)$, the density of states if we assume spherical symmetry. One simplified (3.31) to

$$K^{BA}_{qp}=\tfrac{1}{2}\mathrm{Tr}_{\hat{p}\sigma}\left[\tilde{B}'_{\hat{p}}\frac{v_{\hat{p}}\mathbf{v}^*_{\hat{p}}\cdot\mathbf{q}}{\Omega-\mathbf{v}^*_{\hat{p}}\cdot\mathbf{q}}\times\tilde{a}_{\hat{p}}\left[\delta_{\hat{p}\hat{p}'}-\Gamma^{\omega}_{\hat{p}\hat{p}'}\frac{v_{\hat{p}'}\mathbf{v}^*_{\hat{p}'}\cdot\mathbf{q}}{\Omega-\mathbf{v}^*_{\hat{p}'}\cdot\mathbf{q}}\tilde{a}_{\hat{p}'}\right]^{-1}\tilde{A}'_{\hat{p}'}\tilde{P}\right], \tag{3.32}$$

where only trace over c-f space, integration over momentum direction $\int d\Omega_{\hat{p}}/4\pi$ and spin sums are left. Here[27] from Ref. 27,

$$\tilde{a}\equiv\begin{bmatrix}a_c^2 & a_c a_f\\ a_f a_c & a_f^2\end{bmatrix}.$$

We showed the momentum indices explicitly, and due to the $\delta(E)$ factor in (3.14), all momenta are to be evaluated at the Fermi surface and we have therefore shown the momenta directions only. $\mathbf{v}^*_{\hat{p}}\equiv\mathbf{v}^*_{p_F}$ is the group velocity of the excitations. One is therefore led to evaluate $\tilde{\zeta}$ defined by

$$\left[\delta_{\hat{p}\hat{p}'}-\tilde{\Gamma}^{\omega}_{pp'}\frac{v_{\hat{p}'}\mathbf{v}^*_{p'}\cdot\mathbf{q}}{\Omega-\mathbf{v}^*_{\hat{p}'}\cdot\mathbf{q}}\tilde{a}_{\hat{p}'}\right]\tilde{\zeta}_{\hat{p}'}=\tilde{A}'_p\ . \tag{3.33}$$

Explicitly

$$\begin{aligned}&\zeta_{11,\hat{p}}=A'_{11,\hat{p}}\ ,\\ &\zeta_{12}=0\ ,\\ &-\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f\hat{p}'}a_{c\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{p'}\cdot\mathbf{q}}{\Omega-\mathbf{v}_{p'}\cdot\mathbf{q}}A'_{11,\hat{p}}+\left[1-\Gamma^{\omega}_{\hat{p}\hat{p}'}a^2_{f,\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{p'}\cdot\mathbf{q}}{\omega-\mathbf{v}_{p'}\cdot\mathbf{q}}\right]\zeta_{21,\hat{p}'}=0\ ,\\ &\left[1-\Gamma^{\omega}_{\hat{p}\hat{p}'}a^2_{f,\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{p'}\cdot\mathbf{q}}{\Omega-\mathbf{v}_{p'}\cdot\mathbf{q}}\right]\zeta_{22,\hat{p}'}=A'_{22,\hat{p}}\ .\end{aligned} \tag{3.34}$$

Before we solve (3.34) for $\tilde{\zeta}$, we note that we can obtain the equation for the collective modes of the system: we simply look for divergences in the response functions, or solve (3.34) for finite $\tilde{\zeta}$ with $\tilde{A}=0$. Hence we are solving

$$\left[1-\Gamma^{\omega}_{\hat{p}\hat{p}'}a^2_{f\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{p'}\cdot\mathbf{q}}{\Omega-\mathbf{v}_{p'}\cdot\mathbf{q}}\right]h_{\hat{p}'}=0\ ,$$

or, with $g_{\hat{p}}\equiv a_{f,\hat{p}}(\Omega-\mathbf{v}_{\hat{p}}\cdot\mathbf{q})^{-1}h_{\hat{p}}$

$$(\Omega-\mathbf{v}^*_p\cdot\mathbf{q})\nu_{\hat{p}}g_{\hat{p}}-\nu_p(a_{f,\hat{p}}\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f,\hat{p}'})\nu_{\hat{p}'}g_{\hat{p}'}=0\ , \tag{3.35}$$

which is just the collisionless kinetic equation for a single-component Fermi liquid (1.2) when we notice $\delta n_p=\delta(E)g_{\hat{p}}$ and integrate over the momentum magnitude. We thus have shown that a SCFLT kinetic equation does describe the collective modes of the system, and we identify v^*_p as the group velocity for the quasiparticles (a very natural conclusion) and $a_{f,\hat{p}}\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f,\hat{p}'}$ as the Landau parameter $f_{\hat{p}\hat{p}'}$. (Note our choice of $g_{\hat{p}}$ above makes this symmetric as it must be. If we have spherical symmetry then $F_{\hat{p}\hat{p}'}=\nu(0)a_f^2\Gamma^{\omega}_{\hat{p}\hat{p}'}$.) One can obtain the above result perhaps more directly by considering the divergence in Γ using the Bethe-Salpeter equation (3.17). [This is more analogous to Landau's original approach[4(c)] and shall be discussed in Appendix A (our procedure is essentially the same thing due to the use of Eq. (3.19))]. Note that we have not identified what $g_{\hat{p}}$ in (3.34) is in terms of the c and f electrons, and questions (b) and (c) raised in the Introduction still need to be answered. This shall be done below.

With the $\tilde{\zeta}$ defined in (3.34) the quasiparticle response function is

$$K^{BA}_{qp}=\tfrac{1}{2}\mathrm{Tr}\left[\tilde{B}'\frac{v_{\hat{p}}\mathbf{v}_p\cdot\mathbf{q}}{\Omega-\mathbf{v}_p\cdot\mathbf{q}}\tilde{a}\tilde{\zeta}\right],$$

which simplified to (with $\zeta_{12}=0$)

$$K^{BA}_{qp}=\frac{\nu_{\hat{p}}}{2}(a_cB'_{11}+a_fB'_{22})\times\frac{\mathbf{v}_p\cdot\mathbf{q}}{\Omega-\mathbf{v}_p\cdot\mathbf{q}}[a_c\zeta_{11}+a_f(\zeta_{21}+\zeta_{22})]\ , \tag{3.36}$$

where we have performed the trace in the c-f space.

We directly obtain by multiplying suitable factors to (3.34) and adding the equations, to get

$$\left[1-a_{f\hat{p}}\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f,\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{\hat{p}'}\cdot\mathbf{q}}{\Omega-\mathbf{v}_{\hat{p}'}\cdot\mathbf{q}}\right](a_c\zeta_{11}+a_f\zeta_{21}+a_f\zeta_{22})_{\hat{p}'}=(a_cA'_{11}+a_fA'_{22})_{\hat{p}}\ . \tag{3.37}$$

(Note the labels on the a_c and a_f's.) Thus

$$K^{BA}_{qp}=\frac{\nu_{\hat{p}}}{2}B^*_p\frac{\mathbf{v}_p\cdot\mathbf{q}}{\Omega-\mathbf{v}_p\cdot\mathbf{q}}\left[1-a_{f,\hat{p}}\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f,\hat{p}'}v_{\hat{p}'}\frac{\mathbf{v}_{\hat{p}'}\cdot\mathbf{q}}{\Omega-\mathbf{v}_{\hat{p}'}\cdot\mathbf{q}}\right]^{-1}A^*_{p'}\ , \tag{3.38}$$

where

$$\begin{aligned}A^*&=a_cA'_{11}+a_fA'_{22}\\ &=a_cA_c+a_f(R_{21}A_c+R_{22}A_f)\ ,\end{aligned} \tag{3.39}$$

with an *identical* formula for B^*.[28,29] The result (3.38) is exactly the same form as a SCFL [(3.32) when all c-f matrices reduce to scalars], with the Landau parameter $f_{\hat{p}\hat{p}'}$ as $a_{f\hat{p}}\Gamma^{\omega}_{\hat{p}\hat{p}'}a_{f,\hat{p}'}$ as discussed previously, and moreover we

have shown that we have well-defined effective vertices (in the sense discussed in the Introduction) $B_{\mathbf{p}}^*$ and $A_{\mathbf{p}}^*$, the expression for which is as in (3.39) (Note that there are still intermediate integrations due to the operators R which involve both kinds of electrons even in the α,β basis). We have thus achieved one of our main aims of this paper [the answering of questions (a), (b i) and (b ii) in the Introduction]. The low-frequency limit of the Hamiltonian (1.1), provided $\Sigma(k,\omega)$ has the desired analytical properties, does behave as a SCFL. What remains to be done is to answer question (b iii). We shall do this in the next section.

IV. EXAMPLES OF STATIC RESPONSES AND EFFECTIVE VERTICES

We have just seen that the quasiparticle part of the response function does look like a SCFL. We shall now evaluate the various effective vertices and identify the physical quantities $Q_{p\sigma}$ a quasiparticle carries [in the sense of Eq. (1.4)] is this section. It turns out that in some cases the *value* of $Q_{p\sigma}$ *is* surprising at first sight from a simple-minded understanding of formation of the Fermi-liquid state through hybridization, but is in fact rather natural in a more microscopic picture, for example that discussed here. Since as shown in the last section the effective vertices (and hence $Q_{p\sigma}$) are independent of q,Ω, we shall mostly confine our discussions to the static limit. Since the calculation involves Green's function far away from the Fermi surface, calculation of the vertices are feasible only when Ward's identities can be found. We shall discuss these identities in Appendix B, but only the result here.

A. Density-density response

The external field couples to the system via $\hat{A}\equiv(c^\dagger_{p_+\sigma}c_{p_-\sigma}+f^\dagger_{p_+\sigma}f_{p_-\sigma})$ and we are measuring the same quantity (except $q\rightarrow -q$). Thus

$$A_c=A_f=B_c=B_f=1\ . \tag{4.1}$$

The identity (B7) then shows that the corresponding quantities (3.29) should be given by $A'_{11}=B'_{11}=1$, $A'_{22}=B'_{22}=1-(\partial\Sigma/\partial\omega)$, the renormalized vertices are then, by (3.39) and (2.15)

$$A^*=B^*=1\ , \tag{4.2}$$

(4.2) just reflects the fact that we are considering a conserved quantity, each quasiparticle carries one particle. The static density-density response function is then of the familiar form

$$\begin{aligned}K^{\rho\rho}_{\rm qp}&=-\nu_{\hat{\mathbf{p}}}(1+a_{f\hat{\mathbf{p}}}\Gamma^{\omega,s}_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}a_{f\hat{\mathbf{p}}'}\nu_{\hat{\mathbf{p}}'})^{-1}\\&=-\nu_{\hat{\mathbf{p}}}(1+f^s_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}\nu_{\hat{\mathbf{p}}'})^{-1}\ ,\end{aligned} \tag{4.3a}$$

which reduces to

$$K^{\rho\rho}_{\rm qp}=-\frac{\nu(0)}{1+F^s_0} \tag{4.3b}$$

for the case of spherical symmetry. $K^{\rho\rho}_{\rm nqp}$, corresponding to the $q=0$, $\Omega\rightarrow 0$ limit of the response of a conserved quantity, is identically zero, as can be shown directly from (3.3).[7]

B. Spin-density–spin-density response

We consider then $\hat{A}\equiv c^\dagger_{p_+\uparrow}c_{p_-\uparrow}+f^\dagger_{p_+\uparrow}f_{p_-\uparrow}-(\uparrow\leftrightarrow\downarrow)$ and similarly for B (again with $q\rightarrow -q$). We have, therefore,

$$A_{c,p\sigma}=A_{f,p\sigma}=B_{c,p\sigma}=B_{f,p\sigma}=\sigma\ . \tag{4.4}$$

Note that spin is conserved in our system, and a Ward identity (B8) enables us to get results similar to the last case, in particular,

$$A^*_{p\sigma}=B^*_{p\sigma}=\sigma\ . \tag{4.5}$$

The static spin-spin response is thus (for spherical symmetry)

$$K^{\sigma\sigma}_{\rm qp}=-\frac{\nu(0)}{1+F^a_0}\ . \tag{4.6}$$

Again $K^{\sigma\sigma}_{\rm nqp}=0$.[30] Similar results apply for any auto-response functions for any conserved quantities. We now turn to some (in general) nonconserved quantities.

C. Current-current response (related to the conductivity)

The external field (in the real case this is related to the vector potential) couple to the system via $\hat{A}=\hat{J}=v_{c,p}c^\dagger_p c_{p-}+v_{f,p}f^\dagger_p f_{p-}$, where $v_{c,p},v_{f,p}$ are the bare particle velocities, i.e., $A_c=v_{c,p}=\partial\varepsilon_{c,p}/\partial p$ and $A_f=v_{f,p}=0$ in our case, see Eq. (1). Although the current is not conserved, we can find a Ward identity by the analogous procedure of the SCFLT. We find (B12)

$$(R_{21}v_c)_k=\frac{\partial\Sigma}{\partial k}+\Gamma^{\omega,s}_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}a_{f,k'}\nu_{\hat{\mathbf{k}}'}v^*_{k'}\ , \tag{4.7}$$

where v^*_k is the quasiparticle velocity as defined in (2.16). Thus

$$\begin{aligned}A^*_k&=a_c v_c+a_f\left[\frac{\partial\Sigma}{\partial k}+\Gamma^{\omega,s}_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}a_{f,\hat{\mathbf{k}}'}\nu_{\hat{\mathbf{k}}'}v^*_{k'}\right]\\&=[1+(a_{f,\hat{\mathbf{k}}}\Gamma^{\omega,s}_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}a_{f,\hat{\mathbf{k}}'})\nu_{\hat{\mathbf{k}}'}]v^*_{\hat{\mathbf{k}}}\ .\end{aligned} \tag{4.8}$$

The quasiparticle response is

$$K^{JJ}_{\rm qp}=A^*_{\hat{\mathbf{p}}}\nu_{\hat{\mathbf{p}}}\frac{v^*_{\hat{\mathbf{p}}}\cdot\mathbf{q}}{\Omega-v^*_{\hat{\mathbf{p}}}\cdot\mathbf{q}}\left[1-f^s_{\hat{\mathbf{p}}\hat{\mathbf{p}}'}\nu_{\hat{\mathbf{p}}'}\frac{v^*_{\hat{\mathbf{p}}}\cdot\mathbf{q}}{\Omega-v^*_{\hat{\mathbf{p}}}\cdot\mathbf{q}}\right]^{-1}A^*_{\hat{\mathbf{p}}'}\ . \tag{4.9}$$

The result for A^* is completely the analogous to that of a SCFL. For spherical symmetric case $A^*=(1+F^s_1/3)v^*$.

The current that a quasiparticle carries is the Fermi-liquid correction factor times its group velocity, as in a SCFL. Note that the physical current also consists of the diamagnetic current, and the nonconservation of current implies that $K^{JJ}_{\rm nqp}\neq 0$. We shall discuss these in detail in Appendix C.

D. Spin-current–spin-current response

We consider external field coupling to the system via

$$\hat{M} \equiv \hat{A} = (v_{c,p} c^{\dagger}_{p_+\sigma} c_{p_-\sigma} + v_{f,p} f^{\dagger}_{p_+\sigma} f_{p_-\sigma})\sigma \ ,$$

i.e., $A_{c,p\sigma} = v_{c,p}\sigma$, $A_{f,p\sigma} = 0$. Analogous Ward identity can be found as in the last case except with σ indices at appropriate places and we find [for spherical symmetry; it is obvious that the result for general case is (4.8) and (4.9) with $\Gamma^{\omega,a}$ instead of $\Gamma^{\omega,s}$]

$$A^*_{k\sigma} = (1 + F^a_1/3) v^*_k \sigma \tag{4.10}$$

thus each quasiparticle carries a spin-current of $v^*_k\sigma$ with the Fermi-liquid interaction correction of $(1+F^a_1/3)$, as in a SCFL.

E. Magnetic-moment–magnetic moment response (magnetic susceptibility)

The external (magnetic field) couples to the system via $\hat{A} = (\mu_c c^{\dagger}_{p_+\sigma} c_{p_-\sigma} + \mu_f f^{\dagger}_{p_+\sigma} f_{p_-\sigma})\sigma$, and in general $\mu_c \neq \mu_f$. First we consider μ_c, μ_f independent of p. Thus

$$A_{c,p\sigma} = \mu_c \sigma, \quad A_{f,p\sigma} = \mu_f \sigma \ . \tag{4.11}$$

When $\mu_c \neq \mu_f$, the magnetic moment is not conserved. We do not have the Ward's identity of either kind discussed, and therefore have no general quantitative result. But generally we expect (by symmetry) $A^* = \mu^* \sigma$ which defines an effective moment for the quasiparticles (by comparing the resulting response with SCFLT), where

$$\mu^* \sigma = a_c \mu_c \sigma + a_f (R_{21} \mu_c \sigma + R_{22} \mu_f \sigma) \ , \tag{4.12}$$

and thus μ^* is a function of both μ_c and μ_f

$$\mu^* = \mu^*(\mu_c, \mu_f) \ . \tag{4.13}$$

Moreover, the non-quasiparticle part of the response is expected to be nonzero. $K^{\mathrm{MM}}_{\mathrm{qp}}$ has the Fermi-liquid form, whereas $K^{\mathrm{MM}}_{\mathrm{nqp}}$ is q,Ω-independent by definition. In noninteracting systems and $\Omega = 0$ $q \to 0$ they correspond to the (static) Pauli paramagnetism and Van Vleck paramagnetism, respectively. The results above thus give the natural generalization of the Van Vleck contribution to the interacting case (and $q, \Omega \neq 0$): it is simply the non-quasiparticle part of the magnetic susceptibility, and arises only when the magnetic moment is a nonconserved quantity (as in the noninteracting case): otherwise $K^{\mathrm{MM}}_{\mathrm{nqp}}$ would vanish by argument as noted below Eq. (4.3). This non-quasiparticle response of course, has no analog in a SCFL.

To avoid a misconception we make one more comment. For a SCFL if $\mu = \mu(\hat{\mathbf{p}})$, then the magnetic moment does not commute with the interaction term of the Hamiltonian of the system and thus $K^{\mathrm{MM}}_{\mathrm{nqp}} \neq 0$. In our case if μ_f depends on $\hat{\mathbf{p}}$, this very same contribution will arise (but not for μ_c). This contribution is *not* within the conventional Van Vleck paramagnetism since it is purely due to the interaction. Our $K^{\mathrm{MM}}_{\mathrm{nqp}}$ include both this and the generalized Van Vleck contribution, and is not easily separated [cf. Ref. 16(b)].

F. Magnetic-moment-current self-response function

This response function does not seem to be directly measurable, but may be relevant indirectly, since we are able to infer from this the magnetic-moment-current a quasiparticle carries. We imagine an external field coupling to the system, via

$$\hat{A} \equiv \hat{J}_M = (\mu_c v_c c^{\dagger} c + \mu_f v_f f^{\dagger} f)\sigma = \mu_c v_c c^{\dagger} c \sigma$$

since $v_f = 0$. Since μ_c is just a numerical factor, our desired result can be directly read off from example (4), e.g., for spherical symmetry

$$A^*_{k\sigma} = (1 + F^a_1/3) v^*_k \mu_c \sigma \ , \tag{4.14}$$

i.e., a quasiparticle carries a magnetic moment of $\mu_c\sigma$ [with Fermi-liquid correction factor of $(1+F^a_1/3)$, which we shall omit below]. Note that it is *not* $\mu^* v^*$, as one would at first sight expect. Our result is physically reasonable, as all electrons that move and transport the magnetic moment are the c electrons: μ_f simply will not play a role in J_M. (It is clearest when we consider the particular case of $\mu_c = 0$ but $\mu_f \neq 0$. It is obvious, especially when we consider the diagrams as in Fig. 2, that the system has no response, since then $\mu_c v_c = \mu_f v_f = 0$. The naive guess is obviously incorrect since $\mu^* \neq 0$ and $v^* \neq 0$, in general.)

The above example nicely illustrates a point. For any quantity $A^{(1)}$, the quasiparticle response vertex $A^{(1)*}$ is a functional of $A^{(1)}_c$ and $A^{(1)}_f$: $A^{(1)*} = A^{(1)*}(A^{(1)}_c, A^{(1)}_f)$, and similarly for $A^{(2)}$. When, however, we are interested in the physical quantity $A^{(1)}A^{(2)} \equiv A^{(3)}$ then we should consider the renormalization of $\hat{A}^{(3)} \equiv A^{(1)}_c A^{(2)}_c c^{\dagger} c + A^{(1)}_f A^{(2)}_f f^{\dagger} f$, and we have $A^{(3)*} \equiv A^{(3)*}(A^{(1)}_c A^{(2)}_c, A^{(1)}_f A^{(2)}_f)$, and in general we do *not* expect $A^{(3)*} = A^{(1)*} A^{(2)*}$. In the language of physical quantities, if a quasiparticle carries the physical quantities $Q^{(1)}_{p\sigma}$, and $Q^{(2)}_{p\sigma}$, for the physical quantity which corresponds to $Q^{(1)} \times Q^{(2)}$ when expressed in terms of the c and f electrons, is in general *not* given by $Q^{(1)}_{p\sigma} Q^{(2)}_{p\sigma}$.

One naturally asks if there is any other (directly or indirectly) physically relevant quantity that we should also take this particular care of. In particular, one may be interested in the energy or momentum current that a quasiparticle carries since these occur in thermal conductivity and viscosity. Since neither of these quantities are conserved, nor is it obvious which variation of the self-energy we should consider, we cannot find any Ward identities to assist us to give explicit expressions for the renormalized energy current or momentum current using this method. However, once we accept that a Landau kinetic equation (1.2) describes the quasiparticles, and each quasiparticle carries an energy E^*_k and momentum (in the ith direction) k_i, we can consider the time derivative of energy and momentum density using this equation as in a SCFL, the forms of which (a continuity equation) allow us to identify the energy current and momentum current of the quasiparticles as $E^*_k v^*_k$ and $k_i v^*_j$, respectively, as in SCFL

(with appropriate Fermi-liquid parameters, if any). Similar argument holds for the transport of all conserved quantities. Thus among the transports, the "surprises" mentioned are to be found in transport of nonconserved quantities only. (Compare the results for the spin-current and magnetic-moment current.)

V. RELATION TO BAND FORMULATION

We here discuss the Hamiltonian (1.1) with the method of Kadanoff and Baym, and make connection with the paper by Jones and McClure. We write the Green's function in matrix c-f space [cf. (3.7)]

$$\begin{aligned}\tilde{G}(k,\omega)&=\begin{bmatrix} G_{cc} & G_{cf} \\ G_{fc} & G_{ff} \end{bmatrix} \\ &=\frac{1}{[(\omega-\varepsilon_k)(\omega-\varepsilon_f-\Sigma_f)-V^2]}\begin{bmatrix} \omega-\varepsilon_f-\Sigma_f(\omega) & V \\ V & \omega-\varepsilon_k \end{bmatrix} \\ &\approx\frac{1}{\left[1-\frac{\partial\Sigma}{\partial\omega}\right][\omega-E_1(k)][\omega-E_2(k)]}\begin{bmatrix} \left[1-\frac{\partial\Sigma}{\partial\omega}\right](\omega-\tilde{\varepsilon}_f) & V \\ V & \omega-\varepsilon_k \end{bmatrix},\end{aligned} \tag{5.1}$$

where we have expanded around $\omega\approx 0$. By partial fraction we find

$$\tilde{G}(k,\omega)=\frac{1}{\left[1-\frac{\partial\Sigma}{\partial\omega}\right]}\frac{1}{E_1(k)-E_2(k)}\left\{\left[\frac{1}{\omega-E_1(k)}\begin{bmatrix} \left[1-\frac{\partial\Sigma}{\partial\omega}\right][E_1(k)-\tilde{\varepsilon}_f] & V \\ V & E_1(k)-\varepsilon_k \end{bmatrix}\right]-[1\leftrightarrow 2]\right\}, \tag{5.2}$$

and so the spectral function is

$$\tilde{A}(k,\omega)=\frac{2\pi}{\left[1-\frac{\partial\Sigma}{\partial\omega}\right]}\frac{1}{E_1(k)-E_2(k)}\delta[\omega-E_1(k)]\begin{bmatrix} \left[1-\frac{\partial\Sigma}{\partial\omega}\right][E_1(k)-\tilde{\varepsilon}_f] & V \\ V & E_1(k)-\varepsilon_k \end{bmatrix}+1\leftrightarrow 2\ . \tag{5.3}$$

We diagonalize the matrix proportional to $\delta[\omega-E_1(k)]$, the part that have poles near the Fermi level, by a unitary transformation, with the resulting matrix proportional to $\left(\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right)$. This transformation is equivalent to go to the new operators defined by

$$\begin{bmatrix}\beta_k\\ \alpha_k\end{bmatrix}=\begin{bmatrix} u_k & v_k \\ -v_k & u_k \end{bmatrix}\begin{bmatrix}c_k\\ f_k\end{bmatrix}, \tag{5.4}$$

where u_k, v_k can be chosen real and positive with

$$\frac{v_k}{u_k}=-\left[1-\frac{\partial\Sigma}{\partial\omega}\right]^{1/2}\frac{E_k^*-\tilde{\varepsilon}_f}{\tilde{V}}, \tag{5.5}$$

$$u_k^2+v_k^2=1\ , \tag{5.6}$$

[compare (5.5) with (2.11)].[31,32] Now only $G_{\alpha\alpha}$ has poles near the Fermi level. Note however that $\tilde{G}$ is *not* diagonalized (cf. Ref. 20). By first writing down the matrix Green's function $\tilde{G}^<$ of Kadanoff and Baym in c-f basis and then going to the new α_k, β_k basis, one can easily see that the new $G^<$ matrix has exactly the same properties as that of Jones and McClure[11] in their Bloch wave multiband $G^<$ matrix. Thus α_k, β_k are legitimate electronic operators for the bands.[32] All the results in Jones and McClure can be taken to our case trivially, in particular, (i) in general it is necessary to define *four* (16 if counting also spin) distribution functions, however, (ii) at equilibrium only $G^<_{\alpha\alpha}$ is nonzero (only "lower band" is occupied), and (iii) in nonequilibrium the distribution function defined from $\delta G^<_{\alpha\alpha}$ does form a complete representation of the low-lying states of our system (i.e., the distribution functions defined from $\delta G^<_{\alpha\beta}$, etc., can be eliminated in favor of that defined from $\delta G^<_{\alpha\alpha}$: but they are in general nonzero even in the $q,\Omega\to 0$ limit). Physically the nonzero $\delta G^<_{\alpha\beta}, \delta G^<_{\beta\alpha}$ reflects the changing "composition" of the quasiparticles (cf. above and Ref. 21). Note that u_k, v_k involves U implicitly.

We can, of course, use these band operators instead of the c and f electrons in the previous sections. Since we would not produce new understanding of the physics we shall not do that (cf. Ref. 27).

VI. CONCLUSION

We summarize our results. We have shown that, within the simple version of the periodic Anderson Ham-

iltonian (1.1), provided the true f-electron self-energy $\Sigma(k,\omega)$ is analytic (in k and ω, but it can be nonanalytic in V or U) at least near k_F determined by the Luttinger theorem and ω is the chemical potential, and the Fermi surface only intersects one "band" of the energy eigenvalues, at $q,\Omega\to 0$ we indeed have a (almost) usual Fermi liquid. We have, however, seen that due to "the second band," or more precisely that we have two types of electrons to start with, that the renormalization of the vertices and hence the physical quantities associated with a quasiparticle is more subtle than one would naively expect.

The way that the original particles hybridize (in the sense of a linear combination [like (5.4)]) to form the quasiparticles actually depends on the external perturbation, but they do so in a way that these only appear in the non-quasiparticle part of the response and the quantities that the quasiparticle carries (the effective vertices), but without changing the form of the quasiparticle response[33] (cf. Ref. 21).

Our discussion can be regarded to some extent, as a particular case of the Landau theory in which we take into account the identity of the core electrons and the conduction electrons.

The directions along which we would like to generalize our results are numerous. Of particular interest include the case where c electrons also interact, spin-orbit couplings, different multiplicity of c or f levels, finite temperature, and scatterings.

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APPENDIX A

Here we discuss the derivation of the kinetic equation for the collective oscillations of the system analogous to Landau's original argument.[4(c)] The collective excitations are by definition the divergence of the two-particle interaction vertex. There are various kinds of these vertices according to the type of electrons involved. From the property of these vertices as discussed below Eq. (3.4), we know that any of these divergences either correspond to the divergence of Γ_f or single-particle excitations. Therefore, we need only consider the divergence of (3.17) at q, $\Omega\to 0$. Substituting (3.14) into (3.17) yields an equation rather similar to that of Landau.[4(c)] Carrying out the intermediate frequency and momentum magnitude integration

$$\Gamma_{f,\hat{\mathbf{p}}\hat{\mathbf{p}}'}=\Gamma^{\omega}_{f,\hat{\mathbf{p}}\hat{\mathbf{p}}'}+\Gamma^{\omega}_{f,\hat{\mathbf{p}}\hat{\mathbf{k}}}a^2_{f,\hat{\mathbf{k}}}\nu_{\hat{\mathbf{k}}}\frac{\mathbf{v}^*_{\mathbf{k}}\cdot\mathbf{q}}{\Omega-\mathbf{v}^*_{\hat{\mathbf{k}}}\cdot\mathbf{q}}\Gamma_{f,\hat{\mathbf{k}}\hat{\mathbf{p}}'}\ ,$$

where we are left with only integration over $\hat{\mathbf{k}}$ and summation over spin: all momentum magnitudes are at Fermi surface. By the standard argument of noting $\hat{\mathbf{p}}'$ is the dummy, ignoring the inhomogeneous term, and $\Gamma_{f,\hat{\mathbf{p}}\hat{\mathbf{p}}'}\to h(\hat{\mathbf{p}})$ give the two equations before (3.35) and hence the kinetic equation.

APPENDIX B

Here we show the various Ward identities used in Sec. IV.

(1) Number conservation. We consider an external perturbation of

$$H(t)=\sum_k \delta u(t)(c^\dagger_k c_k+f^\dagger_k f_k)\ ,\qquad \delta u(t)=\delta u e^{-i\Omega t},\quad \Omega\to 0\ . \tag{B1}$$

Since the total number is conserved, the Hamiltonian can be diagonalized simultaneously with (B1) when $\Omega\to 0$. The change in any Green's function is

$$\delta G=-\frac{\partial G}{\partial\omega}\delta u\ , \tag{B2}$$

which can also be directly evaluated from perturbation theory, e.g., for G_f

$$\delta G_f(t,t')=-\langle Tf(t)f^\dagger(t')[c^\dagger_{k''}(t'')c_{k''}(t'')+f^\dagger_{k''}(t'')f_{k''}(t'')]\rangle\delta u(t'') \tag{B3}$$

Passing to the limit $\Omega\to 0$, we have from (B2) and (2.1)–(2.3)

$$\delta G_f=\left[\left[1-\frac{\partial\Sigma}{\partial\omega}\right](G_fG_f)^\omega+(G_{fc}G_{cf})^\omega\right]\delta u\ , \tag{B4}$$

whereas (B3) yields

$$\delta G_f=[(G_fG_f)^\omega+(G_{fc}G_{fc})^\omega+(G_fG_f)^\omega\Gamma^\omega(G_{fc}G_{cf}+G_fG_f)^\omega]\delta u \tag{B5}$$

[note again that we have absorbed the $(-i)$ factor in the term involving Γ in the frequency integral $\int d\omega(1/2\pi i)$ as in text] and thus by comparison [note both (B4) and (B5) are matrix multiplications in momentum and spin]

$$\left[1-\frac{\partial\Sigma}{\partial\omega}\right]=[1+\Gamma^\omega(G_{fc}G_{cf}+G_fG_f)^\omega]1 \tag{B6}$$

$$=(R_{21}+R_{22})1\ . \tag{B7}$$

(2) Spin conservation: We just carry over our results for case (1) by considering instead the external perturbation $\delta u(t)(c^\dagger_{k\sigma}c_{k\sigma}+f^\dagger_{k\sigma}f_{k\sigma})\sigma$. Hence, we have

$$\left[1-\frac{\partial\Sigma}{\partial\omega}\right]\sigma=[1+\Gamma^\omega_{\sigma\sigma'}(G_{fc}G_{cf}+G_fG_f)^\omega_{\sigma'}]\sigma' \tag{B8}$$

(where we have shown the spin indices explicitly to avoid confusion).

The above argument can be extended to any conserved

quantities by considering the appropriate external perturbation, i.e., for any conserved quantity A (necessarily $A_c = A_f = A$)

$$\left[1-\frac{\partial\Sigma}{\partial\omega}\right]A=(R_{21}+R_{22})A \ ,$$

and

$$A^*=a_c A+a_f(R_{21}+R_{22})A$$
$$=\left[a_c+a_f\left[1-\frac{\partial\Sigma}{\partial\omega}\right]\right]A=A \ . \tag{B9}$$

A quasiparticle carries the same (conserved) quantity as the bare particles themselves.

(3) Current vertex[34]—The Ward identity for this is of a different type. We consider the momentum derivative of the self-energy Σ_f, or $\delta\Sigma=\Sigma_{f,k+q}-\Sigma_{f,k}$. Our method follows Sec. 6.5 of Ref. 7(b) closely. Noting the property of the interaction vertices as discussed following Eq. (3.4), we find

$$\delta\Sigma_{f,k}=\Gamma_f^{(1)}\delta G_f \ , \tag{B10}$$

where $\Gamma_f^{(1)}$ is again the f-electron interaction vertex irreducible with respect to the particle-hole lines of momentum q (frequency $\Omega=0$). Direct evaluation of $\delta G_f = G_{f,k+q}-G_{f,k}$ yields

$$\delta k\left[(G_{fc}G_{cf})v_c+(G_fG_f)\frac{\partial\Sigma}{\partial k}\right]$$

and passing to the $k=0$ limit yields

$$\frac{\partial\Sigma}{\partial k}=\Gamma^{(1)}\left[(G_{fc}G_{cf})^k v_c+(G_fG_f)^k\frac{\partial\Sigma}{\partial k}\right] , \tag{B11}$$

where the k superscript denotes the k limit. Using (3.13) and (3.14), we get, e.g.,

$$(G_fG_f)^k=(G_fG_f)^\omega-2\pi i a_f^2\delta(\omega)\delta(E_p) \ ,$$

we can rewrite (B11) as

$$[1-\Gamma^{(1)}(G_fG_f)^\omega]\frac{\partial\Sigma}{\partial k}$$
$$=\Gamma^{(1)}(G_{fc}G_{cf})^\omega v_c$$
$$-\Gamma^{(1)}_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}a^2_{f,\hat{\mathbf{k}}'}\nu_{\hat{\mathbf{k}}'}\left[\frac{\partial\Sigma}{\partial k'}+\frac{V^2}{(\omega-\varepsilon_{k'})^2}v_c\right] ,$$

where in the last term but not the rest, we have performed the magnitude of momentum sum and frequency integration: we have shown explicitly the $\hat{\mathbf{k}}'$ to remind us of this fact. We invert the operator on the left-hand side and use (3.11) to resum the series to yield

$$\frac{\partial\Sigma}{\partial k}=\Gamma^\omega(G_{fc}G_{cf})^\omega v_c-\Gamma^\omega_{\hat{\mathbf{k}}\hat{\mathbf{k}}'}a_{f,\hat{\mathbf{k}}'}\nu_{\hat{\mathbf{k}}'}(v^*_{\hat{\mathbf{k}}'}) \ , \tag{B12}$$

where we have also used (2.17) for the quasiparticle velocity with the help of (2.14). Equation (B12) is then (4.7) in text.

Alternatively we can invert (B11) to yield $\partial\Sigma/\partial k = \Gamma^k(G_{fc}G_{cf})^k v_c$ and then obtain the relation between the k limit and ω limit as done for SCFL in Ref. 7(b).

Almost identical treatment can be made for the spin current by considering a shift in the diagram of Σ_k as $k\to k+q$ for all up spins and $k\to k-q$ for down spins. Note this is legitimate since both spin and momentum are conserved.

APPENDIX C

In this appendix we discuss the physical current. The current operator for the unperturbed system has been given in the text, namely

$$\hat{J}_q=v_{c,k}c^\dagger_{k_-}c_{k_+} \tag{C1}$$

which satisfies the continuity equation

$$i\dot{\rho}_q=[\rho_q,H]=\mathbf{q}\cdot\mathbf{J}_q \ , \tag{C2}$$

where $\rho_q=c^+_{k_-}c_{k_+}+f^\dagger_{k_-}f_{k_+}$ is the destruction operator for a density of wave vector q, and H is our basic Hamiltonian (1.1). Under the perturbation $H_1=\hat{\mathbf{J}}^\dagger_q\cdot\mathbf{A}$ [here we restore the usual notation of $\mathbf{A}$ as the vector potential, (except a sign and proportionality factors): all vertices concerned are shown explicitly] the correct definition for the current $\hat{\mathbf{J}}^{\rm tot}$ should be such that $i\dot{\rho}_q=[\rho_q,H+H_1]=\mathbf{q}\cdot\mathbf{J}^{\rm tot}_q$. One may easily verify that the operator form should be

$$\hat{\mathbf{J}}^{\rm tot}_q=\hat{\mathbf{J}}_q+\hat{\mathbf{J}}^d \ , \tag{C3}$$

where $\hat{\mathbf{J}}^d$, the diamagnetic current, is given by the operator

$$J^d_{q,i}=\left[\frac{\partial}{\partial k_i}\mathbf{v}_{c,k}\right]\cdot\mathbf{A}(c^\dagger_k c_k) \ . \tag{C4}$$

According to the fact that we only need linear response, $c^\dagger_k c_k$ can be replaced by the equilibrium expectation value.

For simplicity of discussion we confine ourselves to the case of spherical symmetry. If we further assume $\varepsilon_{c,k}=k^2/2m_c$, (C4) reduces to

$$\mathbf{J}^d=K^d\mathbf{A} \tag{C5}$$

with

$$K^d=\frac{N_c}{m_c} \ . \tag{C6}$$

Here N_c is the equilibrium expectation value of the number of c electrons. Equation (C6) can in fact be obtained also easily by writing the c electron kinetic energy term in (1.1) in real space, i.e., $1/2m_c\,|\,(\nabla+iA)\psi_c\,|^2$ and noting no other gradients are involved. The gauge current is then trivially

$$\frac{1}{m_c}\psi^\dagger_c\psi_c\,\mathbf{A}=\frac{N_c}{m_c}\mathbf{A} \ .$$

We, therefore, have

$$J_i^{\rm tot}=K_{ij}^{\rm tot}A_j \ , \tag{C7}$$

where

$$K_{ij}^{\rm tot}=K_{ij}^{JJ}+K^d \ . \tag{C8}$$

Explicitly, we have[7,12]

$$K_{ij}^{JJ}=\sum_n \left[\frac{(J_{q,i})_{0n}(J_{q,j}^\dagger)_{n0}}{\Omega-\omega_{n0}+i\eta}-\frac{(J_{q,j}^\dagger)_{0n}(J_{q,j})_{n0}}{\Omega+\omega_{n0}+i\eta} \right] , \tag{C9}$$

where n is the complete set of exact many-body states, ω_{n0} is the energy of that state over the ground state 0. Note that in general $K_{ij}^{JJ}\neq K^{JJ}\delta_{ij}$.[7(a)] However, for either $qv_F \gg \Omega$ or $\Omega \gg qv_F$, $K_{ij}^{JJ}=K^{JJ}\delta_{ij}$: the former reduces to a gauge transformation and the latter is the response to a uniform field. In these cases one can then calculate K^{JJ} easily by considering the longitudinal response. In particular using (C2) one gets, with $\hat{\mathbf{q}}$ along i,

$$K^{JJ}(q,0)=\sum_n \frac{|(J_{q,i}^\dagger)_{n0}|^2 2\omega_{n0}}{-\omega_{n0}^2}$$

$$=-\frac{1}{q^2}\sum_n |(\rho_q^\dagger)_{n0}|^2 2\omega_{n0} \ , \tag{C10}$$

which can be evaluated by an argument parallel to the derivation of the f-sum rule (Sec. 2.2 of Ref. 13) except noting that now ρ_q involves both c and f electrons whereas J_q only involves the former. One easily gets, for the case of quadratic energy spectrum of the c electrons,

$$K^{JJ}(q,0)=-\frac{N_c}{m_c} \ ,$$

where N_c is the total number of c electrons. Thus

$$K_{ij}^{\rm tot}(q,\Omega)=K_{ij}^{JJ}(q,\Omega)-K^{JJ}(q,0)\delta_{ij} \ . \tag{C11}$$

This form is expected since as $\Omega=0$ the effect of the vector potential on the system reduces to a gauge transformation and thus $J^{\rm tot}=0$ (and hence the validity of this cancellation is independent of the assumption on $\varepsilon_{c,k}$).

For $q=0$ the quasiparticle response vanishes: $K_{\rm qp}^{JJ}(q=0,\Omega)=0$. Applying (C9) with the help of (C2) again and using the corresponding spectral decomposition of $K^{\rho\rho}(q,\Omega)$ [cf. (C9)] we easily verify (i not summed)

$$K_{ii}^{JJ}(q,\Omega)-K^{JJ}(q,\Omega=0)=\frac{\Omega^2}{q^2}K^{\rho\rho}(q,\Omega) \ , \tag{C12}$$

where $K^{\rho\rho}$ is the density-density response. Evaluating the $q\to 0$ limit of both sides yields[12] [recall $K_{\rm nqp}^{JJ}$ is q,Ω independent, thus $K_{{\rm nqp},ij}^{JJ}=K_{\rm nqp}^{JJ}\delta_{ij}$ by the argument below (C9)]

$$K_{\rm nqp}^{JJ}-K^{JJ}(q\to 0,\Omega=0)=\frac{N}{m^*}(1+F_1^s/3) \ . \tag{C13}$$

Thus

$$K_{ij}^{\rm tot}=K_{{\rm qp},ij}^{JJ}+\frac{N}{m^*}(1+F_1^s/3)\delta_{ij} \ . \tag{C14}$$

Note that this result is entirely in terms of parameters which specify the quasiparticles. Here N is the total number of c and f electrons in the conduction band determined by the generalized Luttinger sum rule (Ref. 14).

To illustrate the significance of this, we recall that as discussed in the Introduction we can consider an alternative coupling of the vector potential to the system $(\mathbf{k}/m)(c_{k+}^\dagger c_{k-}+f_{k+}^\dagger f_{k-})\cdot\mathbf{A}$, imagine both the original c and f are free, and it is the result of lattice potential that results in the effective Hamiltonian of (1.1). This changes both $K_{\rm nqp}^{JJ}$ and K^d, but the physical result (C14) is left unchanged. The change in $K_{\rm nqp}^{JJ}$ and K^d cancels exactly due to particle conservation. Therefore, one can have the freedom of choosing the starting points for the "bare particles" (i.e., the "high energy" physics), the "low energy" physics, i.e., the Fermi liquid, can nevertheless be the same provided we yield the same set of quasiparticle properties (e.g., our method directly adopted to the SCFL case would yield, e.g., $K^d=N/m_c$, and renormalization R_J for the current are all different from that of Leggett,[12] but all physical results, in particular $K_{ij}^{\rm tot}, R_J J$ are the same). This is because of the relation (C13) (which is implicit in Ref. 12) (cf., however, the magnetic moment auto-response discussed in Sec. IV of text).

Equation (C14), for example, gives us the uniform conductivity $\sigma(\Omega)$ and zero-temperature Landau penetration depth λ_L, respectively, as (both related to $K_{ij}^{\rm tot}$ by setting $K_{\rm qp}^{JJ}=0$)

$$\sigma(\Omega)=-\frac{N}{i\Omega m^*}(1+F_1^s/3)$$

and

$$\lambda_L^{-2}=\frac{4\pi N}{m^*}(1+F_1^s/3) \ ,$$

which are the usual SCFLT results (with the unit where electronic charge and velocity of light equal 1) [despite the fact that our intermediate result (C6) is different from the SCFL (e.g., Ref. 12)].

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[17]K. Yamada and K. Yosida, Prog. Theor. Phys. **76**, 621 (1986).

[18]E. W. Fenton, in Solid State Commun. **60**, 351 (1986) has also raised doubts on the validity of FLT for the periodic Anderson model, though reasoning along rather different lines. He argued that the noninteracting system is to be taken as $V=0$, and from our knowledge of single-site Kondo problem, one does not expect analyticity in V, and there is no one-to-one correspondence between his noninteracting system and the real system. We shall see that analyticity in the parameters V or U is at least not directly relevant, in contrast to what is often stated in the literature.

[19]We shall drop the occasionally spin indices and momentum indices when there is no danger of confusion.

[20]The question is nontrivial by going to a new α and β for in general all $G_{\alpha\alpha}, G_{\alpha\beta}, \ldots$ has poles at the lower "band," "bubble" diagrams involving them are still needed. This is still true for the more sophisticated procedure of forming a 2×2 matrix G in terms of the two types of electrons, and then diagonalize it by an ω dependent unitary transformation: both diagonal elements still have poles at the "lower band." If we use the transformation in Sec. V, then only $G_{\alpha\alpha}$ has poles at the lower band and contributes to g_{near}, but G is not diagonalized.

[21]This is not *a priori* obvious: consider adding extra particles to the system. As discussed in Ref. 15 the effective f level $\tilde{\varepsilon}_f$ in Fig. 1. rises in such a way to keep the number of f-electron almost constant. As a result, (i) the whole "band" in Fig. 1 changes completely, *even for states quite far away from the Fermi surface,* and due to this (ii) the "composition" of the quasiparticles is also changing. At first sight at least this casts doubt on a SCFLT description. The resolution of this "paradox" is simply that the "band" (new band) is just the energy difference between the $N\pm 1$ ($N'\pm 1$) particle states over the N (N') particle ground state. The change of the shape of this curve has nothing to do with the validity of FLT. But this example does, however, speak against naive "justifications." Other examples can be easily found. Consider two noninteracting bands α and β as in Fig. 1 (as the $U=0$ limit). If the original c and f electrons have nonequal magnetic moment then a magnetic field introduces mixing among them (Van Vleck paramagnetism): the "hybridization" changes. Similar argument applies to other external perturbations of the system, or internal interaction among electrons within the system. The interaction U and the fact that Fig. 1 itself is a result of solving the Hamiltonian (1.1) make it not *a priori* obvious why the response of the system is just like a SCFLT, especially at finite q and Ω.

[22]We are ignoring the damping and collisions of the quasiparticles, and thus have taken $\Sigma(k,\omega)$ real. This is justified for the quasiparticles near the Fermi surface at sufficiently low temperature and q, Ω. The failure of this assumption for the quasiparticles far away from the Fermi surface is of no consequence in the discussions below.

[23]Precisely speaking, (2.5) is invalid for $E_{1,2}(k)$ far from zero for then the approximation (2.5) breaks down (also the imaginary parts become large). Our discussion below for the FLT shall precisely avoid talking about those states; the fact that (2.7) is invalid there shall therefore not concern us.

[24]This hybridization is used to obtain the quasiparticle interactions in the $1/N$-expansion interactions from the interaction vertex of the c and f electrons [Refs. 16(c) and (e)]. It is not clear to the author whether this "effective hybridization" can be so taken seriously. Both the Gutzwiller ansatz and the $1/N^0$ order in the $1/N$ expansion are minimization and saddle point evaluation of the energy of the *ground state* (or the statistical average of the free energy at finite temperature) of the system as a *whole.* The methods are not designed in any aspect to reflect the *structure* of *excited states* or *quasiparticles* of the system (cf. Sec. V).

[25]One may consider also terms of the form $c^\dagger f$ or $f^\dagger c$. Since this greatly complicates the treatment below and not necessary for most quantities within the possible FLT we shall simply not consider them.

[26]This relation is perhaps much clearer if we directly consider the response (3.9) in terms of diagrams. One expands (3.9) in terms of $\Gamma^{(1)}$, $\bar{g}_{\text{near}}$, and $\bar{g}_{\text{far}}$, and recollect terms with the help of (3.18) (or rather its series form). The geometric series involving $\tilde{\Gamma}^{\omega}\bar{g}_{\text{near}}$ is resummed to give the inverse operator in (3.20).

[27]$\bar{a}$ and hence $\bar{g}_{\text{near}}$ can be diagonalized by a unitary transformation so that the only nonzero matrix element occurs at the lower right corner. This transformation is the same as the one discussed in Sec. V. In fact, using this property and the observations in the last reference, one easily sees that the final result of the response functions can be put in the form (1.5), thus partly answering the question (b) raised in the paragraph below that equation.

[28]Evaluating the inverse operator in (3.32) is equivalent to solving the (matrix) kinetic equation for the c and f electrons. We are thus showing indirectly that this matrix kinetic equation can be effectively replaced by a scalar one, even when we consider the response to an external field. [Compare the procedure here with (2.104)–(2.111) of Ref. 7(a).]

[29]This factorization must work because the formula above (3.36) has the same form as a $U=0$ system, except with external perturbation $\tilde{\zeta}$. Since Landau theory is trivial for this system, B^* must be well defined. By symmetry, this also holds for A^*. This observation allows us to see many trivial generalizations of the present justification for FLT to many other slightly more complicated periodic Anderson Hamiltonians.

[30]Yamada and Yosida (Ref. 17) have also calculated the spin

susceptibility. The result, however, is of quite different form and we have not been able to find Green's functions identities to relate the results.

[31]The above is, of course, equivalent to the procedure of diagonalizing (3.8) of Jones and McClure.

[32]It can be easily checked that the part proportional to $\delta[\omega - E_2(k)]$ is not diagonalized, and also the lower right element is not zero. Thus only the lower band operator is "well behaved," in contrast to (3.8) in Ref. 11. Their work, however, is not affected since this property is only needed for the bands intersecting the Fermi surface.

[33]It is easy to find examples of nonlinear response of the system where the reduction to a simple-minded SCFLT fails. In text we have seen that in the linear regime responses are of the form $B^* A^*$ (we shall leave out the intermediate "quasiparticle propagator," i.e., terms involving $\tilde{g}_{\text{near}}$, for simplicity of writing), with B^* and A^* independent of each other, despite the fact that it is a linear combination of $B_c A_c$, $B_c A_f$, $B_f A_c$, and $B_f A_f$. A simple extension of this result to the nonlinear case is obviously false. As an example consider the system with $\mu_c \neq \mu_f$ under a static magnetic field h along $\hat{z}$ axis. By conservation of spin it is obvious that we just need to add spin indices in Sec. III. K_{qp}^{BA} is then of the form (3.38) with appropriate spin indices, with A^*, B^* in (3.39). Both A^*, B^* depends on h through (i) a_c, a_f and (ii) Γ and g_{far} in the renormalization factor R_{ij}. This dependence is through μ_c and μ_f, and cannot be described by μ^* alone. [To make the example even more clear consider $\hat{A}$ being also a magnetic moment, thus we are measuring a quantity $\hat{B}$ to second order in h. Then B^* in (3.38) is a function of μ_c, μ_f. If for some other hypothetical magnetic field h' coupling to the system through some other hypothetical magnetic moment μ_c', μ_f', then B^* for this measurement will be different from that of the last.]

[34]Yamada and Yoshida have also considered the current vertex. They have shown the correct relation in their calculation. However, they have not shown the various limits of $\Gamma_{kk'}$ (which of course is crucial) in their formulas explicitly.

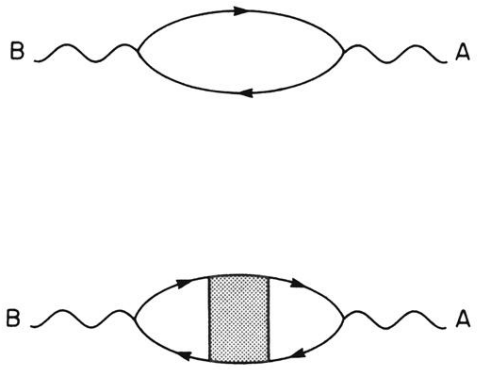

FIG. 2. Diagrammatic representation of response functions. Intermediate lines can be either G_f, G_c, G_{fc}, or G_{cf} except it must end as f electrons in the f electron interaction vertex Γ_f denoted by the solid square.

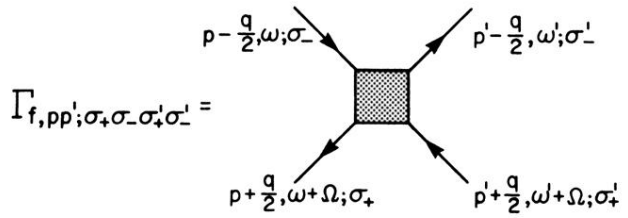

FIG. 3. Variables entering the f-electron interaction vertex Γ_f.

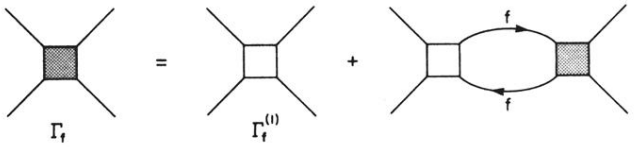

FIG. 4. Bethe-Salpeter equation for $\Gamma_f \bar{p} = (p, \omega)$. The open square, $\Gamma_f^{(1)}$ is the particle-hole irreducible part of Γ_f (with respect to q, Ω).