Noise of junction devices

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The conduction and noise mechanisms and the relevant models are quite different and more complex for bipolar than for unipolar media and devices. A new general approach to *p-n* junction noise, which ascribes its origin to the charge fluctuations of the defect centers, is proposed. For the single defect the relaxation time, the Langevin noise sources, and the modulation of the generation-recombination (GR) current across the neighboring defects, are computed according to a previous model by means of the Shockley-Read-Hall theory, the Schottky theorem, and the Poisson equation. The interactions of the charge and current fluctuations of the single defect with the output short-circuit currents are then expressed by means of proper charge and current coupling coefficients. In their turn, these are computed in closed general form for a long junction, for both neutral and space-charge regions, by means of continuity and Poisson equations and using a new method which reduces the noise coupling problems from three to one dimension. In this way, a general expression of the noise spectrum of the junction, which holds good for any doping and bias voltage and for any frequency up to the reciprocal of the lifetime of the carrier, is obtained. It contains two contributions. One of them, for reverse bias and high frequency, leads to two-thirds shot noise whereas, in most other cases, it gives a full shot noise. The other excess term, deriving from the GR current modulation, for equal relaxation time τ of the defects, produces a GR noise, whereas, for dispersed τ's, it yields a $1/f^{\gamma}$ noise with $\gamma \simeq 1$ down to the lowest measurable frequency f. For zero-bias voltage, according to Nyquist's theorem, the two terms give the thermal noise. According to a recent model, even for the *p-n* junctions the $1/f^{\gamma}$ noise originates from the fact that the excess term is the superimposition of Lorentzian spectra proportional to $\tau^{1+\nu}$ with $\nu > 0$ so that a very small fraction of defects with dispersed τ may be sufficient to generate itself. Finally, the new charge coupling coefficients and the continuity equations allow us to compute the fluctuation of the output current in the time domain due to the random burst charge fluctuations of each single-energy-level defect and, thus, to account for the burst noise, too. By accounting for all types of noise in the junction devices, the new unified approach appears to be a general and exhaustive model.

I. INTRODUCTION

Since the semiconductor junction devices have been practically realized, their noise, owing to its technical and scientific importance, has been deeply analyzed, both theoretically and experimentally, by many researchers[1–19] and most of its main properties have been well established.

However, some fundamental aspects of the flicker and burst noises and, as also shown by a recent work of van der Ziel *et al.*,[19] even some basic elements of the shot noise itself, i.e., the main noise sources of the bipolar electron devices (BED's) are still subject for discussion and for theoretical and experimental research. That especially happens for the present almost ideal junctions[20] and submicrometer devices to which, being characterized by few defect centers generating noise, previous collective and also corpuscular models cannot be directly applied.

Such main models are the following.

By means of a corpuscular model, i.e., the Shockley-Read-Hall (SRH) theory[21,22] of carrier generation-recombination (GR) in the defect centers, and using the random-telegraph-signal statistical approach,[1] Lauritzen[6] computed the noise associated with GR phenomena in the space-charge region (SCR) of *p-n* junctions. In particular he showed that such a noise may vary from two-thirds and three-fourths to full shot noise, depending upon frequency and bias conditions.

van Vliet,[8,10,14] by means of a collective approach based on kinetic and transport equations for the carriers, supplemented by Langevin noise sources as given by the SRH and GR noise theories, computed the noise both in the SCR (by obtaining the same results of Lauritzen[10,14]) and in the quasineutral regions (QNR)[8] of the junction on the basis of the two opposite adiabatic approximations that the free (trapped) carrier densities in SCR (QNR) adjust fast compared the trapped (free) one.

van der Ziel alone[2,15] and, recently, with other authors,[19] proposed a theory of the shot noise in *p-n* junctions and Schottky diodes which, being based upon a transmission-line analogy that takes into account the diffusion and GR noises by means of properly distributed sources, holds good solely in the QNR (outside SCR) where only the diffusion transport mechanism of the carriers, necessary for the line analogy, prevails.

Therefore a complete and unified theory of the shot noise of the junction devices appears to be still lacking.

On the other hand, Kleinpenning has studied the flicker noise of p-n diodes[17] and interpreted his experimental results upon the basis of mobility fluctuation alone, which, according to Pellegrini,[23] cannot exist without carrier density fluctuation.

On the basis of a preceding general corpuscular-collective approach of the BED noise, here we propose a new model of p-n junction noises which removes such difficulties and limits.

Such a model allows us to account, in a unified way, for the thermal, shot, flicker, GR, and burst noises, i.e., all the junction noises, in both SCR and QNR, for any bias conditions and any frequency below the reciprocal of the lifetime of the minority carriers.

The approach is corpuscular in that it ascribes the origin of noise to the single independent defects and, at the same time, it is collective in that it solves the coupling problem between the fluctuations of each defect and those of the output current by means of Poisson and continuity equations.[24]

Its general bases, analysis methods, and results are the following. The SRH theory[21,22] and Schottky theorem, applied in a corpuscular way to each defect center, allow us to compute the relaxation time τ, the Langevin noise sources, the charge fluctuation spectrum, and the modulation of the GR current across the other neighboring defects.[24]

Then the coupling coefficients Γ between the stochastic currents, which are injected from the conduction and valence bands into the defect and which produce its charge fluctuations, and the fluctuations induced in the short-circuit currents at the devices terminals, are computed, through a collective approach, by means of the continuity and Poisson equations alone, without any adiabatic approximations. Such equations, indeed, allow us to express Γ through other coupling coefficients α between the defect charge fluctuations, on the one hand, and those induced in the carrier densities and in the GR current, on the other.[24]

Indeed, such coupling current coefficients Γ, as is shown here, are sufficient in themselves to compute completely the entire noise spectrum of the short-circuit current in long p-n junction diodes. On the other hand, the charge coupling coefficients α, necessary to obtain such a result, are computed by means of Poisson's equation which, especially in SCR, is solved according a new method[24] that reduces the noise coupling problem from three dimensions to one.

The total noise spectrum so obtained contains two contributions which for zero-bias voltage, according to Nyquist's theorem, correctly give the thermal noise.

Apart from the allocation, in SCR and QNR, and from the dispersion of the defect parameters, the first contribution—which in SCR coincides with the result obtained by Lauritzen[6] and van Vliet,[10,14] through other approaches—leads to a two-thirds shot noise for reverse-bias voltage and high frequency, whereas in most other cases, up to frequencies equal to the reciprocal of the lifetime of the minority carriers, it yields full shot noise. The trapping effects at SCR edges, indeed, as shown here for the first time, for small forward bias and high frequencies may lead to a noise greater than the full shot noise.

The second, excess contribution, originating from the GR current modulation produced by the defect charge fluctuations, yields a GR noise for defects with equal relaxation times τ, whereas for dispersed τ's, according also to the McWhorter model[25] applied by Fonger[26] and Hsu[7] to junction devices, it gives a $1/f^{\gamma}$ noise with $\gamma \simeq 1$ down to the lowest measurable frequency f.

According to a recent model,[27,28] the $1/f^{\gamma}$ noise origin is also a consequence of the fact that for the junction devices, too, the excess contribution is a superimposition of Lorentzian spectra proportional to $\tau^{1+\nu}$, with $\nu > 0$, so that even ratios [(number of defects with dispersed τ's)/(number of defects with equal τ's)] so low as $10^{-6}-10^{-10}$ may be sufficient for it to be generated.[27]

In QNR the excess contribution, for both GR and $1/f^{\gamma}$ noise, is proportional to the square diffusion current and is inversely proportional to the square density of the majority carriers, whereas in SCR the dependence on the current and impurity concentrations, except for reverse bias conditions, is much more complex.

The new current and charge coupling coefficients also allow us to compute, in a straightforward way, the fluctuations, in the time domain, of the output current induced by the random-telegraph-signal variation of the charge of a single defect and, thus, to account for the eventual burst noise, too.

Finally, as a necessary basis for noise analysis, the conduction mechanisms and currents in QNR and SCR of the junction are evaluated in a general form by means of the SRH model corpuscularly applied to each defect center, by taking into account in this way even the dispersion of the energy, capture probabilities, and relaxation time of the defects, unlike what happens in the case of classical theory of the p-n junction current put forward by Shockley, Sah, and Noyce.[29,30]

Experimental data of the literature, which fit well with the results of the new model, are reported.

II. CURRENT

Since the objective is to compute current and, especially, noise of p-n junction devices by means of the general model proposed in Ref. 24, we must first of all recall certain of its results which will be used here.

According to such a model, the GR current and the noise, apart from that produced by scattering phenomena, are due to the SRH defect centers characterized by single or many energy levels. Here we refer to the more frequent and meaningful case of single-energy-level defects.[27]

In recalling the results of Ref. 24 and henceforth, for sake of simplicity, we shall indicate the average value of a quantity $y(t)$, which in general depends on time t, with the same symbol y, while we shall use $\Delta y(t)$ and $\delta y(j\omega)$ to indicate its total fluctuation and the phasor of the component of Δy itself at the frequency $f = \omega/2\pi$, respectively.

Let us begin by recalling the expression of the defect relaxation time τ given by

$$\tau^{-1}=\tau_n^{-1}+\tau_p^{-1} , \tag{2.1}$$

$$\tau_n^{-1}=c_n(n+n_1), \quad \tau_p^{-1}=c_p(p+p_1) , \tag{2.2}$$

where c_n (c_p) is the electron (hole) capture probability, $n(p)$ is the electron (hole) density, and n_1 (p_1) is the same density when the quasi-Fermi-level coincides with the energy E of the defect.

Such concentrations and the intrinsic one n_i, for nondegenerate semiconductors, are given by

$$n=N_C \exp\left[\frac{F_n-E_C}{kT}\right], \quad p=N_V \exp\left[\frac{E_V-F_p}{kT}\right] , \tag{2.3}$$

$$n_1=N_C \exp\left[\frac{E-E_C}{kT}\right], \quad p_1=N_V \exp\left[\frac{E_V-E}{kT}\right] , \tag{2.4}$$

$$n_i^2=n_1p_1=n_0p_0=N_CN_V \exp(-E_G/kT) , \tag{2.5}$$

in which T is the absolute temperature, k is the Boltzmann constant, N_C (N_V) and E_C (E_V) are the effective state density and the edge energy, respectively, of the conduction (valence) band, n_0 (p_0) is the electron (hole) density at the thermal equilibrium, $E_G=E_C-E_V$ is the forbidden energy gap, and F_n, F_p, and F_T are the quasi-Fermi-levels for the free electrons and holes and for the electrons trapped by the defect being considered, respectively.

In its turn, the occupation factor ϕ (ϕ_h) of the island energy level E by an electron (hole) is given by

$$\phi=1-\phi_h=\left[1+\exp\left[\frac{E-F_T}{kT}\right]\right]^{-1}=\tau(c_nn+c_pp_1) , \tag{2.6}$$

whereas the electron-hole recombination rate per volume unit U becomes

$$U=\frac{np-n_i^2}{p\tau_N}=\frac{np-n_i^2}{n\tau_P}=\frac{np-n_i^2}{n_i\tau_i} , \tag{2.7}$$

where the electron (hole) lifetime τ_N (τ_P) and τ_i are defined by

$$\frac{1}{p\tau_N}=\frac{1}{n\tau_P}=\frac{1}{n_i\tau_i}=N_S\int \tau c_pc_nD_S\, d\Phi , \tag{2.8}$$

in which N_S is the density of the defects at $\mathbf{r}$ and $D_S(\mathbf{r},E,c_n,c_p)$ is their distribution in the space (E,c_n,c_p) of which $d\Phi=dE\,dc_p\,dc_n$ is the volume element.

Now let us compute the current $i=-i_1=i_2$, necessary to get the noise, versus the bias voltage v of a one-dimensional abrupt p-n junction, as shown in Fig. 1 where x_1 and x_2 represent the edges of the SCR and x_1-X_1 and X_2-x_2 are the widths of the p and n QNR, respectively. We consider the case of low injection,

$$n<p=p_0=N_A, \quad x\le x_1 \tag{2.9}$$

$$p<n=n_0=N_D, \quad x\ge x_2 \tag{2.10}$$

and of a long diode[19]

$$x_1-X_1>4L_N, \quad X_2-x_2>4L_P , \tag{2.11}$$

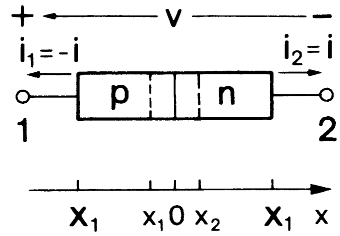

FIG. 1. Sketch of an abrupt p-n junction.

where N_A (N_D) and L_N (L_P) are the acceptor (donor) concentration and the electron (hole) diffusion length, respectively, in the p (n) side.

In conditions (2.11) very few electrons (holes) reach the terminal 1 (2) so that for the electron (hole) current density J_n (J_p) we have

$$J_n(X_1)=J_p(X_2)=0 , \tag{2.12}$$

$$\Delta J_n(X_1,t)=\Delta J_p(X_2,t)=0 . \tag{2.13}$$

Therefore, from the continuity equations

$$\frac{\partial J_n}{\partial x}=-\frac{\partial J_p}{\partial x}=qU , \tag{2.14}$$

and from (2.12), the current $i=AJ_p(X_1)=AJ_n(X_2)$ of the junction, of cross section A, becomes

$$i=i_n+i_p+i_r , \tag{2.15}$$

where

$$i_n=qA\int_{X_1}^{x_1}U\,dx , \quad i_p=qA\int_{x_2}^{X_2}U\,dx , \quad i_r=qA\int_{x_1}^{x_2}U\,dx , \tag{2.16}$$

are the contributions of electron diffusion in p QNR, of hole diffusion in n QNR, and of the GR processes in SCR, respectively; q is the electron charge.

Equations (2.15) and (2.16), for long diodes, are totally general, that is, they are also true for high injection, i.e., when (2.9) and (2.10) do not hold good.

Vice versa, if (2.9) and (2.10) hold good, from (2.5), (2.7), and (2.8) we get

$$U=\frac{n-n_0}{\tau_N}, \quad x<x_1 ; \qquad U=\frac{p-p_0}{\tau_P}, \quad x>x_2 \tag{2.17}$$

where, according to (2.1), (2.2), and (2.8)–(2.10), τ_N and τ_P become independent of x if, together with (2.9) and (2.10), we have

$$c_n n \ll c_p p, \quad x < x_1 \tag{2.18}$$

$$c_p p \ll c_n n, \quad x > x_2 \ . \tag{2.19}$$

In this case from the current equations

$$J_n = qD_n \frac{\partial n}{\partial x}, \quad x < x_1; \quad J_p = -qD_p \frac{\partial p}{\partial x}, \quad x > x_2 \tag{2.20}$$

where D_n (D_p) is the electron (hole) diffusion constant in the p (n) region, and from (2.14) and (2.17) we obtain

$$n - n_0 = n_0 \left[\exp\left[\frac{qv}{kT} \right] - 1 \right] \exp\left[\frac{x - x_1}{L_N} \right], \quad x \le x_1 \tag{2.21}$$

$$p - p_0 = p_0 \left[\exp\left[\frac{qv}{kT} \right] - 1 \right] \exp\left[\frac{x_2 - x}{L_P} \right], \quad x \ge x_2 \tag{2.22}$$

where the diffusion lengths are given by $L_N = (D_n \tau_N)^{1/2}$ and $L_P = (D_p \tau_P)^{1/2}$. Equations (2.21) and (2.22) also originate from the fact that, according to (2.9) and (2.10), the quasi-Fermi-levels F_p and F_n are constant for $x < x_1$ and $x > x_2$, respectively, and that, on the assumption made by Shockley of a small GR current across SCR,[29,30] they also maintain such constant values in SCR itself across which we therefore also have

$$F_n(x) - F_p(x) = qv, \quad x_1 < x < x_2 \ . \tag{2.23}$$

Finally, by setting

$$I_n = \frac{qAL_N n_i^2}{N_A \tau_N}, \quad I_p = \frac{qAL_P n_i^2}{N_D \tau_P} , \tag{2.24}$$

from (2.5), (2.9), (2.10), (2.16), (2.17), (2.21), and (2.22) we get the classical equations of Shockley,[29]

$$i_n = I_n \left[\exp\left[\frac{qv}{kT} \right] - 1 \right] , \qquad i_p = I_p \left[\exp\left[\frac{qv}{kT} \right] - 1 \right] , \tag{2.25}$$

which, however, in virtue of (2.1), (2.2), (2.8), and (2.24), are also extended to the case in which the defect parameters are dispersed, in any way at all, in the space E, c_p, c_n.

In SCR, where both n and p, as a function of x, change over a very large range, (2.9), (2.10), (2.18), and (2.19) are no longer true and, accordingly, the computation of i_r becomes much more difficult than that of the diffusion currents i_n and i_p.

From (2.3), (2.5), (2.7), (2.8), (2.16), and (2.23), in SCR we have

$$i_r = I_r(v)[\exp(qv/kT) - 1] , \tag{2.26}$$

where, unlike I_n and I_p,

$$I_r(v) = qA \int_{x_1}^{x_2} \left[\int \tau(v) c_p c_n n_i^2 N_S D_S d\Phi \right] dx , \tag{2.27}$$

depends on v through x_1, x_2, and, especially, $\tau(v)$ which, according to (2.1)–(2.5) and by setting

$$\tau_M = (1/n_i)(c_n c_p)^{1/2} , \tag{2.28}$$

may also be written in the form

$$\tau = \frac{\tau_M}{2} \left[\exp\left[\frac{qv}{kT} \right] \cosh\left[\frac{V - V_0}{kT} \right] + \cosh\left[\frac{E - E_0}{kT} \right] \right] , \tag{2.29}$$

where E_0 is the value of E for which $c_n n_1 = c_p p_1$ and $V_0 = V(x_0)$ is the value of the electron potential energy $V(x)$ at the abscissa x_0, if it exists, where $c_n n(x_0) = c_p p(x_0)$.

Indeed, we encounter two major difficulties in computing the integral (2.27), as well as, on the other hand, (2.8) and the following similar ones involving D_S. One, of a physical nature, springs from the fact that the distribution $D_S(\mathbf{r}, E, c_p, c_n)$, depending on several microscopic physical quantities and technological parameters, is generally not known, even from an experimental standpoint. The other is the intrinsic mathematical difficulty encountered in computing the integrals in closed form.

However, in spite of that, general meaningful results, as shown by (2.24) and (2.25) for i_n and i_p, and as also shown in the following parts, may be inferred for current and noise.

In particular for reverse bias conditions, when in SCR we may assume $n \simeq p \simeq 0$, from (2.1) and (2.2), we have

$$\tau = (e_n + e_p)^{-1} , \tag{2.30}$$

where $e_n = c_n n_1$ and $e_p = c_p p_1$ are the emission probability of electrons and holes, respectively, so that, on the assumption that N_S and D_S are independent of x, from (2.5), (2.27), and (2.30) we get

$$I_r = qwAN_S \int \frac{D_S}{e_n^{-1} + e_p^{-1}} d\Phi , \tag{2.31}$$

where $w = x_2 - x_1$ is the SCR width (Fig. 1).

As a conclusion to this section, we wish to point out that the new technologies make it possible to obtain "ideal" Shockley p-n junctions[20] in which, indeed, i_r is negligible, especially for forward-bias conditions, as regard i_n and i_p.

III. TOTAL NOISE SPECTRUM

Let us now compute the power spectral density σ_I of the short-circuit current fluctuations $\Delta i_2 = -\Delta i_1 = \Delta i$ at the device terminals (Fig. 1) produced by a single independent defect and, then, by all of them together.

For this purpose, too, we must recall certain general results to be found in Ref. 24.

First of all, it must be remembered that the stochastic currents η_n and η_p which feed the defect being considered from the conduction and valence bands, respectively, are characterized, according to the Schottky theorem and the van Vliet results,[8,10] by the relevant shot noises, whose respective spectra σ_{η_n} and σ_{η_p} and $\sigma_n = \sigma_{\eta_n} + \sigma_{\eta_n}$ are given by[24]

$$\sigma_{\eta_n}=2q^2\tau c_n[c_p(np+n_i^2)+2c_n nn_1] , \quad (3.1)$$

$$\sigma_{\eta_p}=2q^2\tau c_p[c_n(np+n_i^2)+2c_p pp_1] , \quad (3.2)$$

$$\sigma_{\eta}=4q^2\tau^{-1}\phi\phi_h . \quad (3.3)$$

The problem is now to find the coupling mechanism and coefficients between η_n, η_p, and the output currents Δi_1 and Δi_2.

For this we have defined the current coupling coefficients $\Gamma_{Bdh}\equiv\delta i_{dh}/\delta i_B$ as the ratio between the current δi_{dh} of type d coming out the terminal h ($d=n,p,e$ for electron, hole, and displacement current, respectively) and the (small) current δi_B, inducing δi_{dh} itself, which is injected into the defect in question at **r** from the conduction ($B=C$) or the valence ($B=V$) band.[24]

Since the displacement current is null in the neutral regions, and, according to (2.13), the electron and hole currents are also null at terminals 1 and 2 (Fig. 1), respectively, from the previous definition and from the continuity and Poisson equations, without any adiabatic approximation or any other assumption, we get[24]

$$\Gamma_{Be1}=\Gamma_{Be2}=\Gamma_{Bn1}=\Gamma_{Bp2}=0 , \quad (3.4)$$

$$\Gamma_{Bn2}=-\Gamma_{Bp1}=\Gamma_B , \quad (3.5)$$

$$\Gamma_C=\frac{\alpha_U-\tau_p^{-1}+j\omega\alpha_p}{\tau^{-1}+j\omega} , \quad (3.6)$$

$$\Gamma_V=\frac{\alpha_U+\tau_n^{-1}+j\omega\alpha_n}{\tau^{-1}+j\omega} , \quad (3.7)$$

in which the charge coupling coefficients α_y are defined by

$$\alpha_p=\frac{q}{\delta Q}\int_\Omega \delta p\, d^3x ,$$
$$\alpha_n=\frac{q}{\delta Q}\int_\Omega \delta n\, d^3x , \quad (3.8)$$
$$\alpha_U=\frac{q}{\delta Q}\int_\Omega \delta U\, d^3x ,$$

δn, δp, and δU being the variations of $n(t)$, $p(t)$, and $U(t)$, respectively, induced by charge variation δQ produced by δI_B in the defect being considered; Ω is the sample volume.

Then, according to (3.4) and (3.5), σ_I becomes

$$\sigma_I=|\Gamma_C|^2\sigma_{\eta_n}+|\Gamma_V|^2\sigma_{\eta_p} . \quad (3.9)$$

Since, as will be shown in Sec. V E, α_p, α_n, and α_U are real up to frequencies of the order of τ_P^{-1} and τ_N^{-1}, up to such frequencies, from (3.1)–(3.3), (3.6), (3.7), and (3.9), we have

$$\sigma_I=\frac{(\sigma_L+\tau^2\omega^2\sigma_H)+\sigma_U}{1+\tau^2\omega^2} , \quad (3.10)$$

where we have put

$$\sigma_L=\left[\frac{\tau}{\tau_p}\right]^2\sigma_{\eta_n}+\left[\frac{\tau}{\tau_n}\right]^2\sigma_{\eta_n} , \quad (3.11)$$

$$\sigma_H=\alpha_p^2\sigma_{\eta_n}+\alpha_n^2\sigma_{\eta_p} , \quad (3.12)$$

$$\sigma_U=\tau^2\alpha_U^2\sigma_\eta+2\tau^2\alpha_U(\tau_n^{-1}\sigma_{\eta_p}-\tau_p^{-1}\sigma_{\eta_n}) . \quad (3.13)$$

Then from (2.1), (2.2), (3.1), (3.2), and (3.11) we obtain[8]

$$\sigma_L=2q^2\tau[c_pc_n(np+n_i^2)-2\tau^2c_p^2c_n^2(np-n_i^2)^2] , \quad (3.14)$$

and from (3.1), (3.2), and (3.12) we get

$$\sigma_H=2q^2\tau[(\alpha_n^2+\alpha_p^2)c_nc_p(np+n_i^2)+2\alpha_p^2c_n^2nn_1+2\alpha_n^2c_p^2pp_1] , \quad (3.15)$$

and, finally, from (2.1), (2.2), (2.5), (2.6), (3.1)–(3.3), and (3.13) we get the relationship

$$\sigma_U=4q^2\tau\alpha_U[\alpha_U\phi\phi_h+\tau c_pc_n(np-n_i^2)(\phi-\phi_h)] , \quad (3.16)$$

in which, if necessary, ϕ and ϕ_h and hence σ_U may be expressed—as happens for σ_L and σ_H, according to (3.14) and (3.15)—through the capture probabilities and the densities of the carriers by using (2.6).

Finally, the power spectral density S_i of the current fluctuations due to all the defects, on the assumption that their charge fluctuations are independent,[24] becomes

$$S_i=A\int_{X_1}^{X_2}\left[\int\sigma_I N_S D_S d\Phi\right]dx=S_S+S_F , \quad (3.17)$$

where, according to (3.10), we have put

$$S_S(\omega,v)=A\int_{X_1}^{X_2}\left[\int\left[\frac{\sigma_L+\tau^2\omega^2\sigma_H}{1+\tau^2\omega^2}\right]N_SD_Sd\Phi\right]dx , \quad (3.18)$$

$$S_F(\omega,v)=A\int_{X_1}^{x_2}\left[\int\frac{\sigma_U}{1+\tau^2\omega^2}N_SD_Sd\Phi\right]dx . \quad (3.19)$$

Therefore, the general model in Ref. 24 allows us to determine completely the noise spectrum of a long p-n junction by reducing the coupling problem to that of computing the charge coupling coefficients α_n, α_p, and α_U. This computation will be performed in the next section.

IV. CHARGE COUPLING COEFFICIENTS

A. General properties and neutral regions

According to the conclusions drawn in the preceding section, the coupling problem is reduced to computing the charge coupling coefficients α_n, α_p, and α_U by means of (3.8), or, with regard to any quantity y and in the case of frequency independence of its fluctuation, by means of the relationship

$$\alpha_y(x_I)=\frac{q}{\Delta Q}\int_\Omega \Delta y\, d^3x , \quad (4.1)$$

where Δy is the variation produced by the fluctuation ΔQ of the charge of the defect in question at $x=x_I$.

First of all, since, according to (3.4), the variation in the electric field has been assumed to be negligible on the device surface, from the Gauss theorem we get

$$q\int_{\Omega}(\Delta p-\Delta n)d^3x+\Delta Q=0\ , \tag{4.2}$$

so that from (3.8) we obtain the relationship

$$\alpha_n-\alpha_p=1\ , \tag{4.3}$$

which reduces the computation of α_n and α_p to that of a single coefficient α_n or α_p.

On the assumption, which, as shown in Sec. V E, is true for $f<\tau_N^{-1},\tau_P^{-1}$, that $y[V(\mathbf{r})+\Delta V(\mathbf{r},t)]=y(V)+\Delta V(\partial y/\partial V)$ depends on $\mathbf{r}$ and t only through the potential energy $V(\mathbf{r})+\Delta V(\mathbf{r},t)$ (we have set $\partial y/\partial V=[\partial y/\partial(V+\Delta V)]_{\Delta V=0}$), (4.1) may also be written in the form

$$\alpha_y=\frac{q}{\Delta Q}\int_{\Omega}\Delta V\frac{\partial y}{\partial V}d^3x\ . \tag{4.4}$$

From (2.1)–(2.5), (2.7), (2.8) (taking into account that $\Delta E_C=\Delta E_V=\Delta V$), (4.3), and (4.4) and from Poisson's equation, for QNR we obtain in a straightforward way[24]

$$\alpha_n=\frac{n}{n+p},\quad \alpha_p=-\frac{p}{n+p}\ , \tag{4.5}$$

$$\alpha_U=\frac{np-n_i^2}{n_i^2\tau_U}=\frac{U}{n_i}\frac{\tau_i}{\tau_U}\ , \tag{4.6}$$

$$\tau_U^{-1}=\frac{n_i^2N_S}{n+p}\int c_pc_n\tau^2(c_pp-c_nn)D_S\,d\Phi\ . \tag{4.7}$$

B. Space-charge region

By means of (4.1) or (4.4), the direct computation of α_y in SCR, in three-dimensional space, should be much more difficult than in the previous case of neutral regions.

However, according to the method proposed in Ref. 24, the problem may be simplified from three to one dimension by writing (4.1) and (4.4) in the form

$$\alpha_y=\frac{q}{\mu\,\Delta Q}\int_{X_1}^{X_2}\Delta y'\,dx=\frac{q}{\mu\,\Delta Q}\int_{X_1}^{X_2}\Delta V'\frac{\partial y}{\partial V}dx=\frac{1}{\mu\,\Delta Q}\int_{X_1}^{X_2}\frac{\Delta V'}{\xi}\frac{\partial y}{\partial x}dx\ , \tag{4.8}$$

where $\Delta y'$ and $\Delta V'$ are the variations of y and of the potential energy, respectively, produced by a fictitious charge $\mu\,\Delta Q\,\delta(x-x_I)$ uniformly distributed on the plane $x=x_I$ with an arbitrary surface density $\mu\,\Delta Q$, while

$$\xi=\frac{1}{q}\frac{\partial V}{\partial x} \tag{4.9}$$

is the steady-state electric field.

By integrating (4.8) by parts we obtain

$$\alpha_y=\alpha_y'+\alpha_y''\ , \tag{4.10}$$

where we have set

$$\alpha_y'=\frac{1}{\mu\,\Delta Q}\left[y\frac{\Delta V'}{\xi}\right]\Bigg|_{X_1}^{X_2}\ ,\qquad \alpha_y''=-\frac{1}{\mu\,\Delta Q}\int_{X_1}^{X_2}y\frac{\partial}{\partial x}\left[\frac{\Delta V'}{\xi}\right]dx\ . \tag{4.11}$$

Therefore, the problem is now to compute ξ and $\Delta V'$ by means of Poisson's equation which, for SCR, gives the usual relationships

$$V=-q^2N_A(x-x_1)^2/2\epsilon+q(v_b-v),\quad x_1\le x\le 0 \tag{4.12}$$

$$V=q^2N_D(x-x_2)^2/2\epsilon,\quad 0\le x\le x_2 \tag{4.13}$$

$$w=[2\epsilon(N_A+N_D)(v_b-v)/qN_AN_D]^{1/2}\ , \tag{4.14}$$

$$x_1=-wN_D/(N_A+N_D),\quad x_2=wN_A/(N_A+N_D)\ , \tag{4.15}$$

where ϵ is the permittivity and v_b is the junction built-in potential.

Again from Poisson's equation concerning the charge $\mu\,\Delta Q\,\delta(x-x_I)$ alone we get

$$\Delta V'=-\mu q\,\Delta Q(x_1-x)(x_I-x_2)/\epsilon w,\quad x_1\le x\le x_I \tag{4.16}$$

$$\Delta V'=-\mu q\,\Delta Q(x_2-x)(x_I-x_1)/\epsilon w,\quad x_I\le x\le x_2 \tag{4.17}$$

whereas $\Delta V'=0$ elsewhere.

Therefore from (4.9), (4.11)–(4.13), (4.16), and (4.17) we obtain

$$\alpha_y'=\frac{(x_I-x_1)y(x_2)}{wN_D}-\frac{(x_2-x_I)y(x_1)}{wN_A}\ , \tag{4.18}$$

$$\alpha_y''=\begin{cases}\dfrac{x_I-x_1}{N_A}\displaystyle\int_{x_I}^{0}\frac{y}{(x-x_1)^2}dx, & x_I<0\\[2ex] -\dfrac{x_2-x_I}{N_D}\displaystyle\int_{0}^{x_I}\frac{y}{(x_2-x)^2}dx, & x_I>0\ .\end{cases} \tag{4.19}$$

On the usual assumption

$$n=p=0,\quad x_1<x<x_2\ , \tag{4.20}$$

used to determine (4.12) and (4.13), from (2.9), (2.10), (4.10), (4.18), and (4.19) we obtain

$$\alpha_n\simeq\alpha_n'=\frac{x_I-x_1}{w}\equiv\frac{q_p}{q},\quad \alpha_p\simeq\alpha_p'=-\frac{(x_2-x_I)}{w}\equiv-\frac{q_n}{q}\ , \tag{4.21}$$

where q_n and q_p, according to Lauritzen[6] and van Vliet,[10,14] should be the charge transferred in the external circuit due to electron and hole migration, respectively.

On the assumption (4.20), α_n and α_p can be obtained directly from a recent extension[31] of the Ramo-Shockley theorem which removes the limits and difficulties of such a theorem pointed out by van Vliet[10] and, concerning the required independence of the moving carriers, by Price.[32]

Finally, from (2.3), (2.5), (2.7), (2.8), (2.23), (4.10), (4.18), and (4.19) we again obtain (4.6) where we now have

$$1/\tau_U=1/\tau_U'+1/\tau_U''\ , \tag{4.22}$$

$$\frac{1}{\tau'_U}=\frac{n_i}{w}\left[\frac{x_I-x_1}{\tau_i(x_2)N_D}-\frac{x_2-x_I}{\tau_i(x_1)N_A}\right], \tag{4.23}$$

$$\frac{1}{\tau''_U}=\begin{cases}\dfrac{n_i(x_I-x_1)}{N_A}\displaystyle\int_{x_I}^{0}\frac{1}{\tau_i(x-x_1)^2}dx, & x_I<0\\ -\dfrac{n_i(x_2-x_I)}{N_D}\displaystyle\int_{0}^{x_I}\frac{1}{\tau_i(x_2-x)^2}dx, & x_I>0\ .\end{cases} \tag{4.24}$$

When one uses (4.6) and (4.21)–(4.24) in (3.15)–(3.19), x_I has to be replaced by x.

Further analytical developments of the computation of τ''_U by means of (2.8), (2.29), (4.12), (4.13), and (4.24) are very complex. On the other hand, approximate or numerical evaluations of τ''_U are beyond the scope of this work.

V. SHOT AND THERMAL NOISE

A. Comparison with other models

Before further computations, some comparisons have to be made between the proposed model and the previous ones.

From (4.21) we see that, in SCR and for equal defects, the expressions (3.14) and (3.15) are in perfect agreement with the results obtained by Lauritzen,[6] by means of a corpuscular theory and of a random-telegraph-signal statistical approach, as well as by van Vliet[10,14] according to a collective model based upon kinetic and transport equations, supplemented by Langevin noise sources as given by the GR noise theory. In both cases an *a priori* adiabatic approximation for the free carriers is performed.

Apart from the fact that this agreement provides a check on the soundness of the model and of the calculations made with it, it is also remarkable in that the three approaches are radically different.

However, vice versa, it is to be observed that such an agreement has been obtained on the assumption (4.20), i.e., ultimately, also in this case on an adiabatic assumption for the free carriers in SCR which, nevertheless, here becomes an *a posteriori* approximation. Indeed, the new nonapproximated model through (4.18) and (4.19), by avoiding the unnecessary assumption (4.20), should yield exact and more complete coefficients $\alpha_n=q_p/q+\alpha_n^*$ and $\alpha_p=-q_n/q+\alpha_p^*$ which, therefore, should lead to adjunctive parts α_n^* and α_p^* and results which cannot be obtained by the models of Lauritzen and van Vliet owing to their *a priori* adiabatic assumptions.

As matter of fact, it is such an approximation lack that allows the new model to give a general unified approach which, in addition to SCR, holds good, unlike the previous ones, for QNR too.

The other important difference between the new model and any other previous ones[2–19] is that it only takes into account the modulation produced by each defect upon the GR current of the others and thus, as will be shown in the next section, through the corresponding spectrum contribution S_F [see (3.17) and (3.19)] it allows us to account for the GR and flicker noises, as well as for the burst one, of the junction electron devices.

Furthermore, unlike in the collective model of van Vliet and in the line analogy of van der Ziel, the proposed approach, owing to its corpuscular-collective nature, may easily take into account the dispersion of the parameters of the defects, both in QNR and in SCR, and any of their number, even only one.

More generally speaking, as will be shown in the following sections, the new model, unlike the previous ones, allows us to compute, for any bias voltage and any frequency, both in SCR and QNR, all types of noise of the p-n junctions, i.e., thermal, shot, flicker, GR, and burst noises.

B. Zero bias: Nyquist's theorem, thermal noise

At zero bias voltage $v=0$, i.e., at thermal equilibrium, we have $F_n=F_p=$const throughout the sample, so that, from (2.3), (2.5), (2.7), (3.16), (3.19), and (4.6), we obtain

$$np=n_i^2\ , \tag{5.1}$$

$$S_F(\omega,0)=0\ , \tag{5.2}$$

that is, the "excess noise" S_F for $v=0$ is null at any frequency.

For $\omega=0$, i.e., at low frequency, from (2.5), (3.14), (3.17), (3.18), (5.1), and (5.2) we have also

$$S_i(0,0)=4qI_0\ , \tag{5.3}$$

in which $I_0=I(0)$ is the value, at $v=0$, of the current

$$I(v)=qA\int_{X_1}^{X_2}\left[\int \tau(v)c_nc_pn_i^2N_SD_Sd\Phi\right]dx$$
$$\simeq I_n+I_p+I_r(v)\ , \tag{5.4}$$

where, according to (2.1)–(2.3) and (2.29), $\tau(v)$ is a function of v and the second equality, according to (2.1), (2.2), (2.9), (2.10), (2.24), (2.27), and (4.20), follows from the fact that in SCR $\tau(0)$ is much greater than elsewhere.

Moreover, the junction conductance $G_0=(di/dv)\,|_{v=0}$ at $v=0$, from (2.1)–(2.5), (2.7)–(2.10), (2.15), (2.16), (2.21)–(2.23), (2.27), and (5.1) becomes $G_0=qI_0/kT$ so that (5.3) and (5.4) yield

$$S_i(0,0)=4kTG_0\ , \tag{5.5}$$

i.e., in agreement with Nyquist's theorem, the model correctly gives the thermal noise in conditions of equilibrium.

On the other hand, since, as will be shown in the following section, we have $\sigma_L\simeq\sigma_H$, from (3.18) and (5.2) we find that (5.5), which is also a further check of soundness of the model, holds good for any frequency at which the model itself is valid.

C. Forward bias: Full shot noise

The objective is now to show that the contribution S_S, given by (3.18), of the noise spectrum, except for reverse bias and high frequencies, in most cases produces a full shot noise.

For this purpose we set

$$\sigma'_L = 2q^2 \tau c_p c_n (np + n_i^2) , \quad (5.6)$$

and we observe that, according to (2.1), (2.2), and (5.6), we have

$$\sigma'_L / q^2 \tau \ll 1/\tau^2 , \quad (5.7)$$

except on the surface L_1 of the space $(E, c_p, c_n, \mathbf{r})$ on which $c_p p = c_n n$ or, for $np \ll n_i^2$, on the surface L_2 on which $c_p p_1 = c_n n_1$. However, on L_1 and L_2, too, we have, at least, $\sigma'_L / q^2 \tau < 1/2\tau^2$.

Therefore, except in the very special case in which the defect-center states are situated on the locus L_1 or L_2, or near them, from (3.14), (5.6), and (5.7), especially in the integral (3.18), we can consider for any v

$$\sigma_L = \sigma'_L , \quad (5.8)$$

so that from (2.7), (2.8), (2.15), (2.16), (3.18), (5.4), (5.6), and (5.8) for $\omega = 0$ we have

$$S_S(0, v) = 2q(i + 2I) , \quad (5.9)$$

i.e., at low frequency the contribution of S_S consists in a full shot noise for any bias condition.

However, in special cases the term proportional to τ^3 of (3.14), which according to (5.7) has been neglected, may lead to a reduction $2qI_r/4$ of $S_S(0,v)$ given by (5.9). By following Lauritzen[6] and van Vliet,[10,14] indeed, one may obtain such a result if, as it may happen in SCR for intermediate forward bias, a region of the locus L_1 exists where $np \gg n_i^2$, $p \gg p_1$, and $n \gg n_1$.[10,14] Such a reduction may occur when the GR current in SCR prevails on diffusion ones in QNR's (see Sec. VIII).

At high frequency, according to (3.18), we have to also consider the term σ_H which, from (3.15), (4.3), and (5.6), may be also written in the form

$$\sigma_H = \sigma'_L + \sigma'_H + \sigma_{Hp} + \sigma_{Hn} , \quad (5.10)$$

where

$$\sigma'_H = 2\alpha_n \alpha_p \sigma'_L , \quad (5.11)$$

$$\sigma_{Hp} = 4\tau q^2 \alpha_n^2 c_p^2 p p_1 , \quad \sigma_{Hn} = 4\tau q^2 \alpha_p^2 c_n^2 n n_1 . \quad (5.12)$$

In QNR from (2.9), (2.10), and (4.5) we have $2|\alpha_n \alpha_p| \ll 1$ for any v, so that, according to (5.10) and (5.11), for any bias condition we can consider

$$\sigma'_H = 0, \quad x < x_1, \quad x > x_2 . \quad (5.13)$$

In the QNR $x < x_1$, from (2.3), (2.4), (2.9), (4.5), (5.6), and (5.12) for any v and

$$E < E_C - (F_p - E_V) + kT \ln(c_p N_V / 2c_n N_C) , \quad (5.14)$$

$$E > E_V + 2(F_p - F_V) - (E_C - F_n) + kT \ln(2cpN_C / c_n N_V) , \quad (5.15)$$

we have, respectively,

$$\sigma_{Hn} \ll \sigma'_L, \quad \sigma_{Hp} \ll \sigma'_L . \quad (5.16)$$

Therefore, by repeating the same considerations for $x > x_2$, from (5.10), (5.13), and (5.16) we can conclude that in QSR's and in large zones of the space $(E, c_p, c_n, \mathbf{r})$ we have, for any v,

$$\sigma_H = \sigma'_L, \quad x < x_1, \quad x > x_2 , \quad (5.17)$$

whereas in SCR, according to (4.21) the terms of σ_H given by (5.10)–(5.12) may become comparable among themselves.

Now, in order to simplify the computation of S_S by means of (3.18), in (5.11), for SCR, we shall replace $\alpha_n \alpha_p$ with its mean value which, according to (4.21), becomes

$$\frac{1}{w} \int_{x_1}^{x_2} \alpha_n \alpha_p \, dx = -\tfrac{1}{6} . \quad (5.18)$$

According to (2.23), (3.18), (5.6), and (5.11), such a substitution is equivalent to considering τ as a constant in x_1 and x_2. Indeed, in virtue of (2.30), this occurs for the reverse-bias condition.

Then, for $\omega = \infty$, that is, for

$$\omega \gg \tau_m^{-1} , \quad (5.19)$$

τ_m being the minimum value of τ in SCR, from (2.7), (2.8), (2.15), (2.16), (2.27), (3.18), (5.4), (5.6), (5.8), (5.9), (5.11), (5.17), and (5.18) for any v we get

$$S_S(\infty, v) = S_S(0, v) - 2q(i_r + 2I_r)/3 + S_E , \quad (5.20)$$

where

$$S_E = A \int_{x_1}^{x_2} \left[\int (\sigma_{Hn} + \sigma_{Hp}) N_S D_S \, d\Phi \right] dx , \quad (5.21)$$

as shown in the Appendix, is the contribution due to the trapping effects at the SCR edges, given by the relationship

$$S_E = \frac{4(2\epsilon kT)^{3/2}}{3qw^2} A N_S \times \int D_S \left\{ e_p \left[\frac{1}{N_A} \ln \left[\frac{c_p N_A}{e_n + e_p} \right] \right]^{3/2} + e_n \left[\frac{1}{N_D} \ln \left[\frac{c_n N_D}{e_n + e_p} \right] \right]^{3/2} \right\} d\Phi . \quad (5.22)$$

Since, according to (2.15), (2.25)–(2.27), (4.14), (5.9), and (5.22), S_E is a slowly varying function of v in comparison with $S_S(0,v)$ and $2q(i_r + 2I_r)/3$, its contribution, for small forward voltage, may become detectable and greater than the reduction $2q(i_r + 2I_r)/3$.

However, when the defect density N_S is small, as happens in modern junctions,[20] the last two terms of (5.20) may become negligible, at least for bias voltages above proper values, so that for these (5.9) and (5.20) give the full shot noise

$$S_S(\omega, v) = 2q(i + 2I) , \quad (5.23)$$

at any frequency for which the model is valid and contribution S_F does not intervene.

D. Reverse bias: Two-thirds shot noise

For reverse-bias conditions, according to (2.30), τ is independent of x also in SCR so that the substitution of

$\alpha_n \alpha_p$ with its mean value given by (5.18) is perfectly correct and thus (5.20) becomes exact, as well as the value of S_E given by (5.22) becomes accurate (see Appendix).

Therefore, from (2.15), (2.25), (2.26), (5.4), (5.9), and (5.20), for $v < -4kT/q$, we get

$$S_S(0,v) = 2q(I_n + I_p + I_r) , \tag{5.24}$$

$$S_S(\infty,v) = 2q(I_n + I_p + 2I_r/3) + S_E , \tag{5.25}$$

where I_n, I_p, and I_r given by (2.24) and (2.31), respectively.

On the other hand, according to (2.1), (2.2), (2.9), (2.10), and (4.20), τ is much greater in SCR than in QNR so that from (2.8), (2.24), and (2.27) we have $I_r \gg I_n, I_p$ and, accordingly, (5.24) and (5.25) give

$$S_S(\infty,v) = \tfrac{2}{3}S_S(0,v) + S_E = \tfrac{2}{3}2qI_r + S_E . \tag{5.26}$$

Moreover, for instance, for equal defects with $e_n \gg e_p$ and for $N_D \gg N_A$, from (2.31) and (5.22) we get

$$S_E = (\tfrac{2}{3}\chi^3)2qI_r , \tag{5.27}$$

where, with quantities referred to $x < x_1$,

$$\chi = [(E_C - E) - (F_p - E_V) + kT \ln(c_p N_V / c_n N_C)]/q(v_b - v) . \tag{5.28}$$

Therefore, according to (5.26)–(5.28), for high reverse bias we have $S_S(\infty,v) = \frac{2}{3}S_S(0,v) = \frac{2}{3}2qI_r$, i.e., we find that at high frequency S_S becomes two-thirds of the full shot noise at low frequency.

This result agrees with that obtained by Lauritzen[6] and van Vliet,[10,14] who, in any case, neglect the trapping effects at the SCR edges. [They, rather, compute $S_S(0,v) = 2qI_r/2$, i.e., they obtain a half-full shot noise, in the extremely unlikely case that all the defects have $e_n = e_p$, i.e., they lie on the locus L_2. In that case from (2.27), (3.14), and (3.18) we obtain, of course, the same result.]

E. Upper frequency limit

Now let us determine the upper frequency limit up at which the previous results hold good.

For this purpose we wish to recall that the SRH GR theory and Schottky's theorem, used to determine relaxation time, the stochastic equation, GR current modulation, and Langevin noise sources of each defect, and especially the method employed to determine the charge coupling coefficients through the continuity and Poisson equations alone, without utilizing transport equations, require the carrier densities to respond instantaneously to the variations in the potential at any point.

Such an assumption, in its turn, needs the mean transit time T_t across SCR to be negligible in relation to ω^{-1}, i.e., we must have

$$\omega \ll T_t^{-1}, \quad x_1 < x < x_2 \tag{5.29}$$

whereas, according to the diffusion theory,[2] in QNR we must have

$$\begin{aligned} &\omega \ll (\tau_N + T_t)^{-1}, \quad x < x_1 \\ &\omega \ll (\tau_P + T_t)^{-1}, \quad x > x_2 . \end{aligned} \tag{5.30}$$

Indeed, relationships (5.29) and (5.30) establish the upper frequency limit up to which the model and its results hold good. For higher frequencies we must take into account that in (3.6) and (3.7) α_U, α_n, and α_p are complex and they must be computed using the transport equations too.[24]

The reduction $2q(I_r + 2I_r)/3$ given by (5.20) of the shot noise of SCR, in the frequency interval established by (5.19) and (5.29), is due to the fact that, according to the extension of the Ramo-Shockley theorem[31] and (4.21), the charges induced at the device terminals by a free carrier during its flight across SCR, when it approaches and when, after its storage, it leaves the defect, are fractions of the electron charge, and the relevant current pulses, for $\omega \gg \tau_m^{-1}$, appear to be statistically independent.[6,14]

At low frequency, rather, the two current pulses appear as a single event to which one unit charge q crossing the entire junction corresponds. That accounts for the full shot noise for $\omega \to 0$.

For the forward-bias condition each of the minority electrons (holes), except those which are stored by the defects during the longest mean times τ and which contribute to the flicker noise through S_F [see (3.18) and (3.19) and following section], during the mean time $T_t + \tau_N$ ($T_t + \tau_p$), by crossing the depletion region and by recombining itself in the p (n) QNR, crosses the entire junction and, at device terminals, induces a current pulse[31] carrying one unit charge q. That explains in physical terms the origin of full shot noise for $v > 0$ and any frequency satisfying (5.30).

VI. $1/f^{\gamma}$ NOISE

A. Noise spectrum versus current

It remains for us to compute the contribution S_F of the noise spectrum due to the GR current modulation produced by the defect charge fluctuation.

First of all let us determine its current dependence. For this, let us write (3.19) in the form

$$S_F = S_{Fn} + S_{Fp} + S_{Fr} , \tag{6.1}$$

where S_{Fn}, S_{Fp}, and S_{Fr} are the S_F contributions relevant to the p QNR, n QNR, and SCR, respectively.

From (2.28), (3.16), and (4.6) we obtain

$$\sigma_U = 4q^2 \tau \alpha_U^2 R , \tag{6.2}$$

where we have set

$$R = \phi\phi_h + (\tau\tau_U/\tau_M^2)(\phi - \phi_h) . \tag{6.3}$$

Moreover, let us also set

$$\beta = \frac{\int c_p c_n \tau^2 (c_p p - c_n n) D_S d\Phi}{\int c_p c_n \tau D_S d\Phi} , \tag{6.4}$$

so that from (2.8) and (4.7) we have

$$\frac{p\tau_N}{n_i^2\tau_U}=\frac{n\tau_P}{n_i^2\tau_U}=\frac{\beta}{n+p} . \tag{6.5}$$

In QNR's and in the low injection case defined by (2.9), (2.10), (2.18), and (2.19), the quantities τ, τ_U, and β, according to (2.1), (2.2), (4.7), and (6.4) do not depend on x. On the other hand, again in the low injection case, in large zones of the space $(E,c_p,c_n,\mathbf{r})$, we have also

$$c_n n \ll c_p p_1, \quad x<x_1 \tag{6.6}$$

$$c_p p \ll c_n n_1, \quad x>x_2 \tag{6.7}$$

so that in them, according to (2.6) and (6.3), ϕ, ϕ_h, and R are also independent of x.

Therefore, from (2.8)–(2.10), (2.18), (2.19), (2.21), (2.22), (2.24), (2.25), (3.19), (4.6), (4.7), (6.2), and (6.5)–(6.7) we get

$$S_{Fn}=\frac{2i_n^2\beta_n^2N_S}{AL_NN_A^2}\int\frac{\tau R}{1+\tau^2\omega^2}D_S d\Phi, \quad x<x_1 \tag{6.8}$$

$$S_{Fp}=\frac{2i_p^2\beta_p^2N_S}{AL_pN_D^2}\int\frac{\tau R}{1+\tau^2\omega^2}D_S d\Phi, \quad x>x_2 \tag{6.9}$$

where β_n and β_p are the values of β in the p and n QNR, respectively.

Since in large zones of the space $(E,c_p,c_n,\mathbf{r})$ we also have $c_n n_1 \ll c_p p$ for $x<x_1$ and $c_p p_1 \ll c_n n$ for $x>x_2$, from (2.1)–(2.4), (2.6), (2.18), (2.19), and (6.4) we get

$$\beta_n=\int c_n(1-\phi_p)^2 D_S d\Phi \Big/ \int c_n(1-\phi_p)D_S d\Phi, \quad x<x_1 \tag{6.10}$$

$$\beta_p=-\int c_p\phi_n^2 D_S d\Phi \Big/ \int c_p\phi_n D_S d\Phi, \quad x>x_2 \tag{6.11}$$

where ϕ_p (ϕ_n) is given by (2.6) where F_p (F_n) replaces F_T. Therefore, according to (6.10) and (6.11), we have $|\beta|\le 1$, where the equality sign holds good when the defects are located above (below) the Fermi level F_p (F_n) of the holes (electrons) for the p (n) region.

In SCR, where the quantities greatly depend on x, at least as far as forward bias is concerned, we cannot perform the integration in a closed form with regard to x so that, from (2.26), (3.19), (4.6), and (6.2), we obtain

$$S_{Fr}=4q^2A\left[\frac{i_r}{I_r}\right]^2\int_{x_1}^{x_2}\left[\int\frac{\tau R N_S D_S}{(1+\tau^2\omega^2)\tau_U^2}d\Phi\right]dx , \tag{6.12}$$

where τ, τ_U, and R, according to (2.29), (4.12), (4.13), (4.22)–(4.24), and (6.3), are very complex functions of x and of the other parameters. In particular, apart from across $i_r(v)$, S_{Fr} depends on v, in a complex way, through I_r, τ, and τ_U, too.

Therefore, in general, according to (6.1), (6.8), (6.9), and (6.12) we do not have a simple dependence of S_F on the current.

However, when the GR current in the space-charge region is negligible,[20] i.e., $i_r=0$, from (2.15), (2.25), (6.1), (6.8), (6.9), and (6.12) we find that, as happens for the flicker and GR noise in unipolar devices, the excess noise S_F becomes proportional to the square total current i^2 while, unlike these, S_F is inversely proportional to the square density of the majority carriers. This last result, in particular, explains why the flicker noise of bipolar devices, in general, is smaller than that of the unipolar ones for the same carrier concentration.

B. Reverse bias

The integral of x of (6.12) can be computed for reverse bias when (2.30) and (4.20) hold good so that, according (2.6), (2.8), (4.22)–(4.24), and (6.3), τ, τ_i, $\phi=\tau e_p$, $\phi_h=\tau e_n$, and R become independent of x. Indeed, from (2.8), (2.30), (2.31), and (4.22)–(4.24), taking into account that $\tau_i^{-1}(x)\gg\tau_i^{-1}(x_1),\tau_i^{-1}(x_2)$, for τ_U we obtain

$$\frac{1}{\tau_U}=-\frac{I_r x}{wQ} , \tag{6.13}$$

where

$$Q=-qAN_Ax_1=qAN_Dx_2 \tag{6.14}$$

is the charge in the two parts of SCR (Fig. 1).

Therefore, from (2.26), (2.28), (2.30), (6.3), and (6.12)–(6.14) we obtain

$$S_{Fr}=\frac{2I_rN_S}{N_AN_D}\int\frac{\tau^3e_pe_n}{1+\tau^2\omega^2}\times\left[\frac{2}{3}\frac{I_r}{wA}\left[\frac{N_D}{N_A}+\frac{N_A}{N_D}-1\right]+q(e_p-e_n)(N_D-N_A)\right]D_S d\Phi . \tag{6.15}$$

Therefore for reverse bias, from (2.15), (2.24)–(2.27), (6.1), (6.8), (6.9), and (6.15), by recalling that $I_r\gg I_n,I_p$, we find that the excess noise S_F is reduced to S_{Fr}, i.e.,

$$S_F=S_{Fr} , \tag{6.16}$$

where S_{Fr} contains one term that is proportional to the current $I_r=-i$ and another that is proportional to its square $I_r^2=i^2$.

C. Frequency exponent

Owing to the analytical complexity of the integrals and expressions which, according to (3.19), (6.1)–(6.3), (6.8), (6.9), (6.12), and (6.15), give $S_F(\omega)$ and owing to the fact itself that the distribution D_S is usually unknown, the analytical computation, in a closed form, of the dependence of S_F on the frequency is generally not possible.

However, certain general properties of this frequency dependence of S_F, and more generally speaking of S_i, may be inferred by means of a recent method[27,28] for computing the frequency exponent of any noise spectrum.

To this end the total noise spectrum, according to (3.17)–(3.19), may also be written in the form

$$S_i = S_F' + S_H \ , \tag{6.17}$$

in which we have set

$$S_H = \int \int \sigma_H N_S D_S d\Phi \, d\mathbf{r} \ , \tag{6.18}$$

$$S_F' = \int \int \frac{(\sigma_L + \sigma_U - \sigma_H)}{1+\tau^2\omega^2} N_S D_S d\Phi \, d\mathbf{r} = \int \int \frac{\tau \Lambda(\tau, E, \boldsymbol{\rho})}{1+\tau^2\omega^2} dE \, d\boldsymbol{\rho} \ , \tag{6.19}$$

where S_H is independent of ω and $\Lambda(\tau,E,\boldsymbol{\rho})$ is a function of τ, E, and $\boldsymbol{\rho} \equiv (c_p, c_n, \mathbf{r})$.

When, as happens, in the case of forward bias, we can disregard the contribution of $(\sigma_H' + \sigma_{Hp} + \sigma_{Hn})$ relevant to SCR, from (5.8), (5.10), and (5.17) we get $\sigma_H = \sigma_L$ so that from (3.18), (3.19), (6.18), and (6.19) we also get $S_H = S_S$ and $S_F' = S_F$.

By solving the quadratic equation of $\zeta = \exp(E/kT)$ which we obtain from (2.1), (2.2), and (2.4), we can express $E(\tau,\boldsymbol{\rho})$ through τ and $\boldsymbol{\rho}$ in (6.19) which, therefore, become[27,28]

$$S_F' = \int \int \frac{\tau W(\tau,\boldsymbol{\rho})}{1+\tau^2\omega^2} d\boldsymbol{\rho} \, d\tau = B^2 N \int \frac{\tau(\tau/\tau_r)^{\nu}}{1+\tau^2\omega^2} D_\tau(\tau) d\tau \ , \tag{6.20}$$

where $W(\tau,\boldsymbol{\rho}) = \Lambda[\tau, E(\tau,\boldsymbol{\rho}), \boldsymbol{\rho}](\partial E/\partial \tau)$, $B^2 N(\tau/\tau_r)^{\nu(\tau)} D_\tau(\tau) = \int W d\boldsymbol{\rho}$, B is a constant which has a current dimension, $\tau_r = 1/2\pi f_r$ is an arbitrary reference relaxation time, $N = \int_\Omega N_S d\mathbf{r}$ is the total defect number, and D_τ is the distribution of τ over the whole sample.

By making the further variable changes $\theta = \ln(\tau/\tau_r)$ and $\theta_\omega = \ln(f/f_r)$, (6.20) also becomes

$$S_F' = \exp(-\theta_\omega) \int \int \frac{W_\theta(\theta,\boldsymbol{\rho})}{\cosh(\theta+\theta_\omega)} d\boldsymbol{\rho} \, d\theta = S_0 \exp(-\theta_\omega) \int \frac{F(\theta)}{\cosh(\theta+\theta_\omega)} d\theta \ , \tag{6.21}$$

where $W_\theta = 2^{-1}\tau_r^2 \exp(\theta) W[\tau(\theta),\boldsymbol{\rho}]$, $S_0 = B^2 N \tau_r$, and $F(\theta) = 2^{-1}\tau_r \exp\{\theta + \theta\nu[\tau(\theta)]\} D_\tau[\tau(\theta)] = S_0^{-1} \int W_\theta d\boldsymbol{\rho}$.

Now the spectrum $S_i(f)$ may be expressed, in a proper band around any given frequency f_0, in the power form[27,28]

$$S_i = S(f_0) \left[\frac{f_0}{f} \right]^{\gamma(f_0)} \ , \tag{6.22}$$

where the frequency exponent $\gamma(f)$ is given by

$$\gamma = -\frac{\partial \ln(S_i/S_0)}{\partial \theta_\omega} \ . \tag{6.23}$$

In recent works,[27,28] starting from the third member of (6.21) we computed γ in a general form and we showed that the fact that $\nu > 0$ leads to an easier explanation of the paramount characteristics of flicker noise and that, in particular, a very small fraction of the N defects with dispersed τ, e.g., 10^{-6}–10^{-10}, is sufficient for it to be generated.

Now let us compute γ again, starting, this time, from the second member of (6.21) which, according to (6.23), leads to[27]

$$\gamma = \frac{1+\delta}{1+S_H/S_F'} \ , \tag{6.24}$$

where now

$$\delta = \int \int \tanh(\theta+\theta_\omega) \psi(\theta,\boldsymbol{\rho}) d\boldsymbol{\rho} \, d\theta \ , \tag{6.25}$$

in which the distributionlike function Ψ is given by

$$\Psi = \frac{W_\theta(\theta,\boldsymbol{\rho})}{\cosh(\theta+\theta_\omega)} \left[\int \int \frac{W_\theta(\theta,\boldsymbol{\rho})}{\cosh(\theta+\theta_\omega)} d\boldsymbol{\rho} \, d\theta \right]^{-1} . \tag{6.26}$$

From (6.25) and (6.26) we get $|\delta| \le 1$ so that (6.24) gives $0 \le \gamma \le 2$, as must happen in the case of superimposition of Lorentzian spectra.

When the defects have the same relaxation time $\tau_b = \tau_r \exp(\theta_b)$, that is for $W_\theta \propto \delta(\theta - \theta_b)$, from (6.25) and (6.26) we get

$$\delta = \tanh(\theta_b + \theta_\omega) = (\tau_b^2\omega^2 - 1)/(\tau_b^2\omega^2 + 1) \ , \tag{6.27}$$

i.e., we obtain the value of δ for a single Lorentzian which, according to (6.17) and (6.19), we have in this case.

If the values of τ are dispersed but in such a way that W_θ is different from zero only in the region $|\theta + \theta_\omega| > 1.55$ of the space $\theta, \boldsymbol{\rho}$, from (6.25) and (6.26) we find that (6.27), with $\theta_b = \theta$, again holds good, i.e., we have $|\delta| = 1$ with a negligible error.

In all the other cases of dispersed relaxation times τ, owing to the hyperbolic cosine of (6.26), ψ tends to become essentially different from zero and an even function around $-\theta_\omega$, around which $\tanh(\theta+\theta_\omega)$ is an odd function, so that from (6.25) we get $|\delta| \to 0$.

This result and (6.24), for $S_F' \gg S_H$, explain how we can have $\gamma \simeq 1$ for many decades down to the lowest measurable frequency, that is how the flicker noise of bipolar devices may originate from defects with dispersed values of τ which with their charge fluctuations modulate the GR current of neighboring defects, that, nevertheless, may not have dispersed values of c_p, c_n, and τ, too.

The same conclusion, since $\nu > 0$, may be reached starting from the third member of (6.20) and (6.21) according to the previous models.[27,28]

D. Notes

The $1/f^\gamma$ noise model proposed above is based on the dispersion of defect relaxation times τ, the shift towards the greatest values of their distribution due to $\nu > 0$, the modulation of GR current produced by the fluctuations of the defect charges, the use of new methods to solve the coupling problems, and to compute the frequency exponent of the power spectrum.

Such bases and tools should overcome the limits of the $1/f$ noise model of p-n junctions of Kleinpenning[17] which is based on the mobility fluctuations of the carriers.

One such limit is the use of an empirical nonfundamental formula, the Hooge one, as a basis of a theory.

Other difficulties derive from the fact that mobility fluctuations cannot exist without those of the carrier density[23] and that the empirical formula, owing to the small density of free carriers in SCR, leads to mobility fluctuations so great that they cannot be physically explained.[23]

Finally, for QNR, as it is physically difficult to account for the fluctuations of a mean statistical quantity, the mobility, so it is even more difficult to justify those of the diffusion constant on the basis of Einstein's equation, i.e., a relationship holding true between average steady quantities.

On the other hand, the model of the hemimicroscopic-mobility fluctuations, which takes into account the trapping effects on the free-carrier velocity due to the defects, and which is quite equivalent to the approach of the carrier-number fluctuations,[23] cannot directly be transferred from the unipolar devices to the bipolar ones because it only takes into account the interaction between each defect and the carriers of an alone band.

VII. GENERATION-RECOMBINATION AND BURST NOISE

A. Generation-recombination noise

If all the N defects, or their group N_b, are characterized by an equal relaxation time $\tau=\tau_b$, according to (3.19) or (6.27) they contribute to the noise S_F by means of a Lorentzian spectrum S_{Fb} that, like the shot noise to which it is added, is independent of the frequency up to about $f_b=1/2\pi\tau_b$.

Such a contribution S_{Fb}, which, according to (6.1), (6.8), and (6.9), may be proportional to the square current, may be considered as a GR noise of the junction. It has never been dealt with in any previous model.

The remaining defects $(N-N_b)$ with dispersed values of τ may generate a spectrum $S_{Ff}\propto 1/f^{\gamma}$, with $\gamma\simeq 1$ down to lowest possible frequency, which emerges over S_{Fb} even if, according to a previous approach,[27] their fraction $(N-N_b)/N$ is extremely low, such as, for instance, 10^{-8}. However, for S_{Ff} to become measurable, it has to become greater than $(S_{Fb}+S_S)$ in which we may have $S_S \gg S_{Fb}$.

B. Burst noise

The present general approach also allows us to account for the burst noise.

In fact, according to the definition of the current coupling coefficients Γ_{Bdh} and to (2.1), (3.4)–(3.7), and (4.3), we find that the current fluctuation δi, in the frequency domain, produced by the fluctuations of a single defect, is given by

$$\delta i=\Gamma_C\,\delta\eta_n+\Gamma_V\,\delta\eta_p = \left[\alpha_U-\frac{\alpha_p}{\tau_p}-\frac{\alpha_p}{\tau_n}\right]\delta Q+\alpha_p\,\delta\eta_n+\alpha_n\,\delta\eta_p\ , \qquad (7.1)$$

where we also have taken into account that from the charge conservation equation of the defect we obtain

$$\delta Q=(\delta\eta_n+\delta\eta_p)/(\tau^{-1}+j\omega)\ . \qquad (7.2)$$

For the frequencies which satisfy (5.29) and (5.30), the coefficients α_U, α_n, and α_p are independent of the frequency so that (7.1) holds good also in the time domain, i.e., we have the equation

$$\Delta i=\left[\alpha_U-\frac{\alpha_n}{\tau_p}-\frac{\alpha_p}{\tau_n}\right]\Delta Q+\alpha_p\eta_n+\alpha_n\eta_p\ , \qquad (7.3)$$

which may also be obtained directly from the charge continuity equations for the electrons (or holes) and for the defect and from (2.13), (4.1), and (4.3).

In (7.3), $\eta_n(t)$ and $\eta_p(t)$ consist of pulses, while, between consecutive pulses, ΔQ is a random telegraph signal whose amplitude, for a single-energy defect, is the electron charge q.

Therefore, if the coefficient of ΔQ in (7.3) for the defect being considered and its relaxation time are much greater than in the case the other defects, the corresponding burst current fluctuations emerge over the whole fluctuation due to all the other defects themselves.

The amplitude of such bursts, as has been found experimentally,[18] according to (2.2), (2.16), (2.17), (2.21), (2.22), (2.26), (2.27), (4.5)–(4.7), (4.21)–(4.24), and (7.3), depends on current and temperature.

VIII. SOME EXPERIMENTAL DATA

Our direct experimental verifications of the proposed model, which, however, are in progress, as well as a detailed analysis of data of other authors, are beyond the scope of the present work. This in order also to contain the length of the work.

However, some comparisons between experimental data existing in literature and the model may be quickly performed.

The noise measurements of p-n junctions in most cases have been made on bipolar transistors in the common-base configuration.

After Chenette and van der Ziel[3] had detected no shot noise reduction at low frequency and at room temperature, Wade and van der Ziel,[11] together with van Vliet and Chennette,[12] at low temperature when the recombination in emitter SCR prevails, obtained a reduction factor of the shot noise falling between 0.8 and 0.85.

The same value, somewhat larger than the 0.75 of the theoretical models, has been obtained by Blasquez,[16] at room temperature, for the shot noise directly associated with the recombination current in the emitter SCR.

According to the present model such values greater than 0.75 are due to the dispersion of the defect parameters.

Moreover, Wade, van der Ziel, Chennette, and Roig,[13] at low temperature, found an excess noise which, according to the proposed model, can be ascribed to GR processes.

For reverse bias the factor of two-thirds shot noise was verified by Scott and Strutt[5] on large-area diodes at frequencies between 20 and 50 kHz, when (5.19) is satisfied.

Kleimpenning[17] has measured the flicker noise of several diodes. Among them the most apt devices to be compared with the proposed model are the long p-n junctions with the largest area (2.5×10^{-6} m^2), this in order to avoid edge effects, in the current range where the ideality factor is equal to 1 so that both the GR processes in SCR and the ohmic effects are negligible. For them, according to the proposed model, the flicker noise is about proportional to the square current.

Also the measurements, versus temperature and bias current, of the burst noise in bipolar transistors[18] agree with the model.

Therefore, even if further proper experiments could be performed to verify directly the main results of the proposed model, we can conclude that the existing experimental data largely validate it.

IX. CONCLUSIONS

A corpuscular-collective model of noise of junction devices has been proposed which, through a single unified approach, accounts for all noise sources of p-n junctions in any region and bias condition.

Indeed, it takes into account thermal, shot, flicker generation-recombination, and burst noise in both neutral and space-charge regions, for forward, zero, and reverse bias voltage.

The model is developed through a detailed and complete analysis. It utilizes the SRH model and Schottky theorem which, corpuscularly applied to each single-energy-level defect, make it possible to compute its relaxation time and Langevin noise sources and the modulation of GR current across the other neighboring defects.

The coupling coefficients between the stochastic currents—which, from conduction and valence bands, supply the defect and produce its charge fluctuations—and the variations of the output short-circuit currents are computed, through a collective approach, in a new and simple way by means of continuity equations alone; in this way they may be expressed by means of other coupling coefficients between the defect charge fluctuations and those of the carrier concentrations and of GR current. Such charge coupling coefficients, in their turn, are evaluated by means of Poisson and transport equations and of a new method which reduces the noise coupling problem, especially for the space-charge region, from three dimensions to one. Moreover, at the frequencies below which the transit time across SCR and the diffusion time in QNR are negligible, they may be computed directly from the Poisson equation alone.

The total noise spectrum thus obtained, which for zero-bias voltage, according to Nyquist's theorem, gives the thermal noise, is made up of two contributions. One of them originates from the variations, induced by defect charge fluctuations, in carrier densities and in currents directly supplying each defect. Such a contribution in the space-charge region tallies with the result obtained by Lauritzen and van Vliet using other methods.

When these types of contributions relevant to all the sample defects are summed together, both in space-charge and neutral regions, they give two-thirds shot noise in the case of high frequency and reverse bias, whereas in most other cases, up to frequencies of the order of the reciprocal of the minority-carrier lifetime, they produce a full shot noise. This result is independent of the properties of the defects, i.e., of their energy, carrier capture probabilities, allocation, and relaxation times, and it does not even depend on their distributions.

Indeed, according to a recent extension of the Ramo-Shockley theorem, the full shot noise is a direct consequence of the fact that each carrier, during its complete flight from one electrode to the other, induces an output current pulse carrying one electron charge and of the fact that the flight time is of the order of the carrier lifetime.

However, for small bias voltage and high frequency, the trapping effects at the SCR edges may lead to a noise greater than the full shot noise.

The second "excess" contribution, which has not been taken into account by any previous model, originates from the modulation, produced by each single defect, of the GR current of all the other neighboring defects.

This second term gives contributions which, for the neutral regions, are proportional to the square diffusion currents, whereas, for the space-charge region, the dependence on its GR current is more complex.

When the defects have equal relaxation times, the excess contribution leads to a Lorentzian spectrum which, as in unipolar devices, we can describe as a GR noise source.

When, however, the defects, or even a fraction of them, have different energy, capture probabilities, and position, their relaxation time τ can assume very dispersed values.

Since the Lorentzian spectrum of each defect is proportional to $\tau^{1+\nu}$ with $\nu>1$, according to a new method for computing the frequency exponent γ, in this case of dispersed τ the excess contribution tends to give a $1/f^{\gamma}$ noise with $\gamma\simeq1$ down to lowest measurable frequency.

Such a noise can be generated by a fraction of defects with dispersed parameters as low as $10^{-6}-10^{-10}$.[27]

Finally, the new coupling coefficients between the defect charge fluctuations and the fluctuations of the carrier and GR current densities, together with the continuity equations, have allowed us to compute, in a new general form, the fluctuations in the time domain of the output current due to the burst charge fluctuation of each single defect. In this way, therefore, burst noise is also included and accounted for by the new model.

Even if further experiments, especially on the present almost ideal junctions,[20] would be opportune in order to better verify the model, however, the existing experimental data of literature largely validate it.

The new theory developed for long abrupt p-n junctions, which in particular contains previous models,[6,8,10] can be applied or easily extended to other junction types and devices such as Schottky diodes, heterojunctions, bipolar junction transistors, junction field-effect transistors, junction photodevices, and so on.

So the solution of transport equations by means of the new method of reduction of the noise coupling problems from three to one dimension[24] should make it possible to easily determine the frequency dependence of the charge coupling coefficients and, hence, to extend the frequency upper limit of the model beyond the reciprocal of the minority carrier lifetime too.

In conclusion, the proposed noise model appears to be a general and effective method for analyzing the conduction and fluctuation phenomena of the bipolar media and devices.

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APPENDIX

Let us compute S_E according to (5.21). For this let us define the abscissae x_0, x_1^*, and x_2^* of SCR by means of the relationships $c_p p(x_0)=c_n n(x_0)$ and $c_p p(x_1^*)=c_p p_1+c_n n_1=c_n n(x_2^*)$, and let v^* be the value of bias voltage v for which $x_{1\max}^*=x_0=x_{2\min}^*$.

For $v<v^*$ and $x_1<x<x_2^*$, from (2.1)–(2.4) and (5.12), we have

$$\sigma_{Hp}\simeq\frac{4q^2\alpha_n^2c_p^2pp_1}{c_pp+c_pp_1+c_nn_1}=\frac{4q^2\alpha_n^2c_pp_1}{1+\exp\left[\dfrac{V'-V_p'}{kT}\right]}, \tag{A1}$$

where

$$V'=q(v_b-v)-V,\quad V_p'=kT\ln\left[\frac{c_pN_A}{c_pp_1+c_nn_1}\right], \tag{A2}$$

being $V_p'=V'(x_1^*)$.

For $x=x_I<0$, from (4.12) and (4.21) we also have

$$x-x_1=\left[\frac{2\epsilon V'}{q^2N_A}\right]^{1/2},\quad \alpha_n^2=\frac{2\epsilon V'}{q^2N_Aw^2}. \tag{A3}$$

Since, unlike for $x<x_1^*$ for which, according to (A1) and (A2), σ_{Hp} is a constant, σ_{Hp} itself for $x_1^*<x<x_2$ decreases exponentially in the case both of (5.12) and (A1), in (5.21) we can assume that (A1) and (A3) hold good in all the SCR $x_1<x<x_2$. In particular such an assumption gives accurate results for reverse bias.

Therefore, for $V_p'\gg kT$, the integral in x of σ_{Hp}, and the analogous one of σ_{Hn}, according (5.21), (A1), and (A3), leads to (5.22).

It is worth noting that, according to (A1) and (5.21), S_E is a contribution due to the trapping effects at the SCR edges.

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