Semiconductor noise

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A corpuscular-collective model of noise generated by the Shockley-Read-Hall (SRH) defect centers is proposed for the semiconductors. The interactions of the single-defect centers with the charge carriers both of the conduction and valence energy bands, with their currents and with generation-recombination (GR) current are taken into account. With regard to the unipolar conduction media and devices, such interactions make the analysis and its results much more complex. By starting from the SRH theory, we first determine the average occupation factor and GR current of the single multiple-energy-level defect and the relevant shot noise associated to the GR transitions. Then, again according to such a corpuscular approach, by taking into account the fluctuations of the quasi-Fermi-levels and of the electric potential, we compute the relaxation time and the Langevin equation of the single defect and the modulation which its charge fluctuations induce in the electron, hole, and GR currents. Finally the coupling problem between the defect and the currents of the device terminals is solved, by means of collective approach, by defining proper coupling coefficients between such output currents and the currents and charge injected into the defect itself from the conduction and valence bands. Then the relationships between such coefficients, for any shape and terminal number of the device, are computed in a general form by means of the transport, continuity, and Poisson equations and of a new method which allows us to evaluate the three- and two-dimensional effects of a single defect through a one-dimensional approach. The new model should be able to account for the thermal, shot, flicker, burst, and GR noises of the semiconductor devices, for any bias condition and for any defect number and allocation in the neutral and space-charge regions.

I. INTRODUCTION

As is well known, both experimental and theoretical analyses of the noise and conduction mechanisms, which are closely correlated, are much more difficult for semiconductor bipolar electron devices (BED's) than for unipolar ones (UED) owing to the greater complexity and multiplicity of the phenomena, conditions, and quantities involved.

Indeed BED's have two charge carriers, electrons and holes, which, as carriers, may be created and annihilated, or captured, stored, and released by defect centers that, from their energy levels in the forbidden energy gap, can exchange charges with both the conduction and valence bands. The carrier densities, in their turn, may vary by many orders of magnitude from one region to another of the same sample.

On the other hand, several conduction and generation mechanisms—electric drift, diffusion, generation-recombination (GR) processes, tunnel and thermoionic emissions, and so on—interest the carriers both in the neutral and space-charge regions of the BED, where low and high electric fields, respectively, act and make the analysis different.

Furthermore, the various phenomena quantities are very sensitive to the work and boundary conditions, such as sample temperature, bias voltages, and electromagnetic irradiations.

All these elements greatly affect the thermal shot, flicker, burst, and GR noises, i.e., the chief BED noises, in as far as they produce complexity and dispersion of the experimental data and difficulties in their theoretical analysis and explanation which, indeed, are much greater than in the case of the UED.

In the past three decades the BED noise, owing to its technical and scientific importance, has been extensively studied, both experimentally and theoretically, by many researchers[1–18] so that most of its features have been found, while some others are still to be clarified or made utilizable for the new submicrometer devices.

The main theoretical results for BED are due to van der Ziel,[1,15,18] Lauritzen,[6] and, especially, van Vliet.[4,8–10,14]

The collective noise models of van der Ziel, concerning the quasineutral regions (QNR), is based upon a transmission-line analogy.

Lauritzen, rather, by means of a corpuscular model, i.e., the Shockley-Read-Hall (SRH) theory[19,20] of carrier generation-recombination in the defect centers, and of a probabilistic approach, computes the noise associated with GR phenomena in the space-charge regions (SCR).

Finally van Vliet computes the BED noise, both in QNR and SCR, by means of some general collective approaches, i.e., a statistical method valid for GR processes not involving spatial coordinates,[4] Green's-function formulation of transport theory of noise[9] holding true for symmetric SRH recombination, and the model of the

transport equations supplemented by Langevin noise sources as given by the SRH and GR noise theories.[2,10,14]

The last more effective approach, which for SCR gives the same results as Lauritzen's method, is developed (i) on the two apposite "adiabatic" approximations that the trapped- (free-) carrier densities in QNR (SCR) adjust fast compared to the free- (trapped-) carrier concentrations, and (ii) on the assumption of using the steady-state concentrations of the free carriers in computing the (dominant) relaxation time of the defect and the noise sources.[3,6,8,10]

Moreover, the previous models do not take into account the parameter dispersion of SRH defect centers, assumed as equal, and their interaction.

On the other hand, Kleinpenning[17] studies the $1/f$ noise of the p-n junctions on the basis of the carrier mobility fluctuations and Hooge's empirical formula[21] whose theoretical bases deserve further research.[22]

The above-mentioned problems, the renewed interest for some foundations of the BED noise, as shown also by a recent work of van der Ziel *et al.*,[18] the unsolved questions relevant to the $1/f$ noise, the new submicrometer devices characterized by very few defects which cannot be dealt with by means of collective approaches, induce us to study the BED noise by means of a previous model, developed for the UED,[23] which corpuscularly puts the noise sources in the single defects, fed by the shot noise associated to the electron transitions into and from the defect itself, and which collectively deals with the coupling problem between defect and outputs by means of the impedance field method.

The corpuscular-collective model which we are proposing, by retaining the only assumption (ii) of van Vliet, overcomes the other previous approximations' limits and problems and it allows us to take into account the above-mentioned manifold physical aspects in studying the noise of the semiconductor devices in their bipolar operations.

According to such an approach,[22–25] and other previous models,[1,3,4,6–10,14] the sources of the various types of noise, except thermal noise, are again identified in the defects of the conducting medium, of any nature and allocation whatsoever, and in particular in the SRH defect centers, which in the case of the semiconductors, interact in various ways with the carriers both of the conduction and the valence bands and among themselves too.

Such multifold interactions of the defects, the existence of two types of carriers and bands, and all the other above-mentioned physical processes greatly change the previous model developed for the UED,[23] making it more complex.

Here the model is carried out according the following lines.

By starting from the SRH model[19,20] applied to the single defects, i.e., by following a corpuscular approach, we first determine the average value of the occupation factor and of the GR current of each defect and, through them, the shot noise associated with the GR transitions.[4,8,10]

Then, by taking into account the fluctuation of the quasi-Fermi-levels and of the electric potential, as well as the interaction between the defects, we compute their (dominant) relaxation time and Langevin's equation. [Such a time and equation, as well as the previous shot noise, indeed, are determined according to the steady-state assumption (ii) of van Vliet.]

The defect interaction allows us to show that the defect charge fluctuation, through the potential fluctuation which it generates around itself, besides the electron and hole flows in the respective energy bands, also modulates the GR current crossing the other neighboring defects.

Any other effect of such an interaction, on the other hand, is neglected because it leads to a system of as many coupled Langevin equations as the sample defects, which is not possible to solve. However, such an approximation gives correct results for the neutral regions where, as in the UED, the defects are screened from each other by the free carrier, whereas it may lead to approximate or insufficient results for the depleted zones where such a screening does not exist.

Finally, the problem of the coupling between each defect and the device terminals, which, of course, must be dealt with in order to determine the output noise spectra, is solved by means of a collective approach, i.e., by defining proper coupling coefficients between the output currents and the current and charge injected into the defects itself from the valence and conduction bands. Such coupling coefficients and their general relationships are then computed for any terminal number and shape of the device by means of current, continuity, and Poisson equations, making no adiabatic assumptions, taking into account the GR modulation, and using a new method which allows us to evaluate any three- and two-dimensional effects of each single defect through a much simpler one-dimensional approach.

Owing to its corpuscular-collective nature, accuracy, and generality the new model should be able to account for the shot, flicker, burst, and GR noises of the semiconductor devices, whatever the bias conditions, the shape, size, and the terminal number of the sample, and the number and allocation of the defects, both in neutral and space-charge regions, may be.

However, in order to limit the length of the work, any application of the proposed theoretical tool to specific devices will be described elsewhere.[24]

II. GENERATION-RECOMBINATION CURRENT AND SHOT NOISE OF A SINGLE DEFECT

A. Currents

According to preceding models,[1,3,4,6–10,14,22,23,25] the low-frequency noises are generated by defects in the conducting medium, of whatever nature, size, and allocation, which are able to capture, to store, and to release charge carriers.

Accordingly, in the semiconductor devices, such sources of noise are the SRH defect centers which, unlike what happens in the case of UED, are able to exchange carriers both with conduction and with valence bands.

Therefore, the extension of the preceding model from UED to BED, which we will make, requires referring some bases of the SRH theory[19,20] in order to compute,

firstly, the steady-state current through a single center and the relevant shot noise associated to its carrier transition and, then, its fluctuations, relaxation time, and Langevin equation.

Let us consider defects with one or more energy levels E_j for which the separation and the number of the levels are not affected by the electrons occupying them. In this case, according to the SRH theory corpuscularly applied to each defect center,[26] currents i_N and i_P entering it from the conduction and valence bands, respectively, at time t become

$$i_N = i_n + \eta_n, \quad i_n = i_n^+ - i_n^- , \tag{2.1}$$

$$i_P = i_p + \eta_p, \quad i_p = i_p^+ - i_p^- , \tag{2.2}$$

being

$$i_n^+ = q \sum_j c_{nj} N_j n_j f_j, \quad i_n^- = q \sum_j c_{nj} N_j n (1 - f_j) , \tag{2.3}$$

$$i_p^+ = q \sum_j c_{pj} N_j p f_j, \quad i_p^- = q \sum_j c_{pj} N_j p_j (1 - f_j) , \tag{2.4}$$

where q is the electron charge, N_j and f_j are the state number and the occupation factor, respectively, of the jth level, c_{nj} (c_{pj}) is the electron (hole) capture probability, n (p) is the electron (hole) concentration, and n_j (p_j) is the same concentration when the quasi-Fermi-level coincides with E_j.

Such concentrations and the intrinsic one n_i, for a nondegenerate semiconductor, are given by

$$n = N_C \exp\left[\frac{F_n - E_C}{kT}\right], \quad p = N_V \exp\left[\frac{E_V - F_p}{kT}\right] , \tag{2.5}$$

$$n_j = N_C \exp\left[\frac{E_j - E_C}{kT}\right], \quad p_j = N_V \exp\left[\frac{E_V - E_j}{kT}\right] , \tag{2.6}$$

$$n_i^2 = n_j p_j = N_C N_V \exp(-E_G/kT) , \tag{2.7}$$

in which N_C (N_V) and E_C (E_V) are the effective state density and the edge energy, respectively, of the conduction (valence) band, k is the Boltzmann constant, T is the absolute temperature, $E_G = E_C - E_V$ is the forbidden energy gap, F_n, F_p, and F_T are the quasi-Fermi-energy-levels for the free electrons and holes and for the electron trapped by the defect being considered, respectively, i.e., f_j is given by

$$f_j = \left[1 + \exp\left[\frac{E_j - F_T}{kT}\right]\right]^{-1} . \tag{2.8}$$

In (2.1) and (2.2), η_n (η_p) is the stochastic component of the current, with null time average, due to the random passages of the electrons (holes) between the island and the conduction (valence) band. Moreover, the kinetic equations (2.3) and (2.4) hold true on the assumption that the relationships between the capture and emission coefficients deduced at the thermal equilibrium are verified outside it too.[26]

Let us now indicate with $\bar{y}$ ($\mathbf{Y}_0$) and Δy ($\Delta \mathbf{Y}$) the time-average value and the fluctuation of the quantity y (vector $\mathbf{Y}$), respectively, and with $\delta y(j\omega)$ the phasor of its component at the frequency $f = \omega/2\pi$.

First, let us compute the time average value of F_T, i_N, and i_P. For this purpose it is sufficient to observe that, from the defect charge conservation, it follows that $\bar{i}_N = -\bar{i}_P$ and

$$\bar{i}_n = -\bar{i}_p , \tag{2.9}$$

which, by means of (2.1)–(2.8), allows us to determine $\bar{F}_T$. (It may be obtained by means of a trial and error procedure by making $(1 - \bar{f}_j) \simeq \exp[(\bar{E}_j - \bar{F}_T)/kT]$ and $\bar{f}_j \simeq \exp[(\bar{F}_T - \bar{E}_j)/kT]$ for $\bar{E}_j < \bar{F}_T$ and $\bar{E}_j > \bar{F}_T$, respectively, according to (2.8).)

Then, by indicating the density of the states at the point $\mathbf{r}$ with $N_S(\mathbf{r})$ and their distribution with respect to $\bar{F}_T$, $\bar{E}_j$, $\bar{c}_{nj}$, and $\bar{c}_{pj}$ by $D_S'(\mathbf{r}, \bar{F}_T, \bar{E}_j, \bar{c}_{nj}, \bar{c}_{pj})$, from (2.1), (2.3), and (2.9) we obtain the electron-hole recombination rate per unit volume $U(\mathbf{r})$ in the form

$$U = -\frac{\sum_\nu \bar{i}_{n\nu}}{q\,\Delta\Omega} = N_S \int\int\int\int D_S' \bar{c}_{nj} [\bar{n}(1 - \bar{f}_j) - \bar{n}_j \bar{f}_j] \times d\bar{F}_T d\bar{E}_j d\bar{c}_{nj} d\bar{c}_{pj} , \tag{2.10}$$

where $\bar{i}_{n\nu}$ is the average current across the νth defect contained in the volume element $\Delta\Omega$ around $\mathbf{r}$ being considered and the sum is extended to all its defects. Of course, $-qU$ represents the time-average value of the generation-recombination current per unit volume.

B. Single-energy-level case

When the N energy levels of the defect being considered occupy an interval smaller than kT or in the case of $N = 1$, i.e., of a single-level defect, by omitting the index j and by setting $n_1 \equiv n_j$ and $p_1 \equiv p_j$, from (2.1)–(2.9) we obtain

$$\bar{f} = \tau_I (\bar{c}_n \bar{n} + \bar{c}_p \bar{p}_1) , \tag{2.11}$$

where[6,8]

$$\tau_I = [c_n(\bar{n} + \bar{n}_1) + \bar{c}_p(\bar{p} + \bar{p}_1)]^{-1} . \tag{2.12}$$

In particular, according to (2.8), (2.11), and (2.12), $\bar{F}_T$ assumes a value $\phi_T(\bar{E}, \bar{c}_n, \bar{c}_p)$ depending on $\bar{E}$, $\bar{c}_n$, and $\bar{c}_p$ alone and, hence, we also have $D_S' = D_S(\mathbf{r}, \bar{E}, \bar{c}_n, \bar{c}_p)\delta(\bar{F}_T - \phi_T)$. Therefore, from (2.3), (2.5)–(2.7), and (2.9)–(2.12) we obtain

$$\bar{i}_n = -\bar{i}_p = -qN\bar{c}_n\bar{c}_p\tau_I(\bar{n}\,\bar{p} - n_i^2) \tag{2.13}$$

$$U = (\bar{n}\,\bar{p} - n_i^2) N_S \int\int\int \bar{c}_n \bar{c}_p \tau_I D_S \, d\bar{E} \, d\bar{c}_n \, d\bar{c}_p , \tag{2.14}$$

where, of course, for $N = 1$, N_S also represents the defect density.

Since Eqs. (2.10) and (2.14) take the parameter dispersion of the defect centers into account, they are an extension of the results of the SRH model.

C. Defect shot noise

The stochastic components η_n and η_p of the currents feeding the defect, which are constituted by the independent random passages of carriers between the defect and

the bands, produce shot noises associated with each capture and emission process. Therefore, according to the Schottky theorem and to (2.1)–(2.4) and (2.11), the power spectral densities $\sigma_{\eta_n}=2q(\bar{i}_n^+ +\bar{i}_n^-)$ and $\sigma_{\eta_p}=2q(\bar{i}_p^+ +\bar{i}_p^-)$ of the random sources η_n and η_p, respectively, and their sum $\sigma_\eta=\sigma_{\eta_n}+\sigma_{\eta_n}$, in the case of energy levels contained in an interval smaller than kT, according to van Vliet's result[8,10] become

$$\sigma_{\eta_n}=2q^2N\tau_I\bar{c}_n[\bar{c}_p(\bar{n}\,\bar{p}+n_i^2)+2\bar{c}_n\bar{n}\,\bar{n}_1] , \tag{2.15}$$

$$\sigma_{\eta_p}=2q^2N\tau_I\bar{c}_p[\bar{c}_n(\bar{n}\,\bar{p}+n_i^2)+2\bar{c}_p\bar{p}\,\bar{p}_1] , \tag{2.16}$$

$$\sigma_\eta=4q^2N\bar{f}(1-\bar{f})/\tau_I . \tag{2.17}$$

Such relationships will be used later on to compute the noise spectra of the defect and those at the device terminals.

III. FLUCTUATION, RELAXATION TIME, AND INTERACTION OF THE DEFECTS

A. Fluctuation

The stochastic currents η_n and η_p entering the defect being considered and the action of the other defects on it produce fluctuations in its charge

$$Q=-q\sum_j N_jf_j , \tag{3.1}$$

and in its other quantities. The purpose of this section is to compute such fluctuations and the relationship between them in order to determine the Langevin equation of ΔQ, its relaxation time, and the GR current modulation.

Let $\Delta v_I(\mathbf{R})$ and $\Delta v(\mathbf{R},\mathbf{r})$ be the variation of the potential inside and outside the defect being considered, respectively, generated by the fluctuation ΔQ of its charge, $\mathbf{R}$ being the barycenter of the defect. At the same time let $\Delta v_E(\mathbf{R})$ be the potential fluctuation produced inside the defect and in its neighborhood by all the others. Then it is

$$\Delta E_j=-q(\Delta v_I+\Delta v_E),\quad \Delta E_C=\Delta E_V=-q(\Delta v+\Delta v_E) . \tag{3.2}$$

Moreover, let ΔF_T, ΔF_n, and ΔF_p, be the fluctuation of the quasi-Fermi-levels, with

$$\Delta F_T=-q(\Delta u_I+\Delta v_E) , \tag{3.3}$$

where $-q\,\Delta u_I$ represents the F_T change due to the variation of the electron number of the defect.

In order to determine the eight variables Δv_I, Δv, Δu_I, ΔF_n, ΔF_p, Δi_n, Δi_p, and ΔQ, and, in particular, the Langevin equation which connects ΔQ to η_n, η_p, and Δv_E alone, as many equations are to be found.

From (2.8) and (3.1)–(3.3) we obtain the first equation

$$\Delta Q=C_I(\Delta u_I-\Delta v_I) , \tag{3.4}$$

where the internal capacitance C_I is given by

$$C_I=q^2(kT)^{-1}\sum_j N_j\bar{f}_j(1-\bar{f}_j) . \tag{3.5}$$

On the assumption that

$$\Delta F_n\simeq 0,\quad \Delta F_p\simeq 0 , \tag{3.6}$$

and $\bar{n}(\mathbf{r})$ and $\bar{p}(\mathbf{r})$ are about constant, at least in a region around the island which has sizes of the order of the Debye length λ given by

$$\lambda=\left[\frac{\varepsilon kT}{q^2(\bar{n}+\bar{p})}\right]^{1/2} , \tag{3.7}$$

another relationship may be obtained from the Poisson and Gauss theorem in the form[23]

$$\Delta Q=C_E\,\Delta v_I , \tag{3.8}$$

where the external capacitance C_E, for a spherical defect with radius r_I and surface $A_I=4\pi r_I^2$, is given by

$$C_E=\varepsilon A_I(r_I^{-1}+\lambda^{-1}) , \tag{3.9}$$

ε being the permittivity.

In order to simplify the determination of the Langevin equation of the defect, and for this purpose alone, to (3.6) we add the other assumption

$$\Delta v\simeq 0 . \tag{3.10}$$

According to the previous defect model,[23] the physical meaning of (3.6) and (3.10) is that one disregards the resistance of the medium surrounding the defect in relation to that of the defect itself which, normally, is much greater.

It should also be observed that the assumptions (3.6) and (3.10), i.e., $n=\bar{n}$ and $p=\bar{p}$ only in the continuity equation of the defect [and in its shot noises, see (2.15)–(2.17)], are equivalent to neglect, with regard to its dominant relaxation time, the second much smaller one.[4,8,10]

By taking into account that the capture coefficients c_{nj} and c_{pj}, after (3.10), depend on Δv_I alone, from (2.1)–(2.8), (3.2), (3.3), (3.6), and (3.10) we obtain

$$\Delta i_n=-G_n(\Delta u_I-\Delta v_I)-(G_n^+ +G_{c_n})\Delta v_I-G_n^-\,\Delta v_E , \tag{3.11}$$

$$\Delta i_p=-G_p(\Delta u_I-\Delta v_I)-(G_p^- +G_{c_p})\Delta v_I-G_p^+\,\Delta v_E , \tag{3.12}$$

where

$$G_d=(q^2/kT)\sum_j\bar{c}_{dj}N_j(\bar{d}_j+\bar{d})\bar{f}_j(1-\bar{f}_j) , \tag{3.13}$$

$$G_d^s=q\bar{i}_d^s/kT , \tag{3.14}$$

$$G_{c_d}=q\sum_j N_j\frac{\partial\bar{c}_{dj}}{\partial\bar{v}_I}[\bar{d}(1-\bar{f}_j)-\bar{d}_j\bar{f}_j] , \tag{3.15}$$

d being equal to n or p and s being the sign $+$ or $-$ [in the square brackets of (3.15) d and d_j must be interchanged for $d=p$].

Finally, to the seven equations (3.4), (3.6), (3.8), and

(3.10)–(3.12), the charge-conservation equation

$$\frac{\partial \Delta Q}{\partial t} = \Delta i_n + \Delta i_p + \eta_n + \eta_p \tag{3.16}$$

must be added.

In this way the eight equations we are looking for have been obtained.

B. Relaxation time and the Langevin equation

The problem is now to determine the dominant relaxation time and the Langevin equation for the defect charge fluctuations.

From (3.4), (3.6), (3.8), and (3.10)–(3.12) we obtain

$$\Delta i_n = -\Delta Q/\tau_n - G_n^- \Delta v_E \ , \tag{3.17}$$

$$\Delta i_p = -\Delta Q/\tau_p - G_p^+ \Delta v_E \ , \tag{3.18}$$

where the partial relaxation times τ_n and τ_p are given by

$$1/\tau_n = G_n/C_I + (G_n^+ + G_{c_n})/C_E \ , \tag{3.19}$$

$$1/\tau_p = G_p/C_I + (G_p^- + G_{c_p})/C_E \ . \tag{3.20}$$

Then, from (3.16)–(3.18), we get the Langevin equation in the form

$$\frac{\partial \Delta Q}{\partial t} = -\frac{\Delta Q}{\tau} - G \Delta v_E + \eta_n + \eta_p \ , \tag{3.21}$$

where total relaxation time τ and conductance G are given by

$$1/\tau = 1/\tau_n + 1/\tau_p \ , \tag{3.22}$$

$$G = G_n^- + G_p^+ = q(\bar{i}_n^- + \bar{i}_p^+)/kT \ . \tag{3.23}$$

The preceding results hold true for any distribution and number N of the defect energy levels.

For $N=1$, we have $(\partial \bar{c}_n/\partial \bar{v}_I) = (\partial \bar{c}_p/\partial \bar{v}_I) = 0$ as a consequence of the definition of c_n (c_p) which is computed for the state filled by a hole (electron), so that from (2.12), (3.5), (3.13), (3.15), (3.19), (3.20), (3.22), and (3.23) we get

$$1/\tau = 1/\tau_I + (G_n^+ + G_p^-)/C_E \ , \tag{3.24}$$

that is, the transient relaxation time τ is different from the value τ_I used in the previous noise models[3,4,6–10] in which the term $(G_n^+ + G_p^-)/C_E$, i.e., the electric effects of the defect center, is neglected.

However, indeed, as shown in Appendix A, such effects may be disregarded in most cases, even if not always, so that from (3.19), (3.20), and (3.24) we finally have

$$\tau_n = [\bar{c}_n(\bar{n}_1 + \bar{n})]^{-1}, \quad \tau_p = [\bar{c}_p(\bar{p}_1 + \bar{p})]^{-1} \ , \tag{3.25}$$

$$\tau = \tau_I \ . \tag{3.26}$$

C. Modulation of the GR current and interaction between defects

Let us now compute the variations Δi_n and Δi_p of the currents feeding the defect being considered, allocated at $\mathbf{R}$, due to all the others through the fluctuations $\Delta v_E(\mathbf{R})$ of the potential that they produce at $\mathbf{R}$.

For this purpose, from (3.17)–(3.23), where we must put $\eta_n = \eta_p = 0$ because we now are considering the effects of Δv_E alone, we obtain

$$\delta i_n = A_n \frac{1 + j\omega\tau_n'}{1 + j\omega\tau} \delta v_E \ , \tag{3.27}$$

where, here $j^2 = -1$, and

$$A_n = \tau \left[\frac{G_p^+}{\tau_n} - \frac{G_n^-}{\tau_p} \right], \quad \frac{1}{\tau_n'} = \frac{1}{\tau_p} - \frac{G_p^+}{\tau_n G_n^-} \ . \tag{3.28}$$

For energy levels grouped in an energy interval smaller than kT, as shown in Appendix B, we have $\tau_n' \simeq \tau_I = \tau$ and $A_n = (\partial \bar{i}_n/\partial \bar{v}_i) = -(\partial \bar{i}_p/\partial \bar{v}_i)$; $\bar{E}_i = -q\bar{v}_i$ represents the average intrinsic potential so that we have $\bar{E}_C = E_G/2 - q\bar{v}_i$, $\bar{E}_V = -E_G/2 - q\bar{v}_i$, and $\bar{E}_j = \bar{E}_j' - q\bar{v}_i$, E_j' being the energy of the jth state evaluated from the intrinsic level itself.

Therefore (3.27), and the corresponding relationship relevant to δi_p, yields

$$\Delta i_n = -\Delta i_p = \frac{\partial \bar{i}_n}{\partial \bar{v}_i} \Delta v_E \ , \tag{3.29}$$

that is, the GR current crossing the defect in question is modulated by the charge fluctuations of all the others which generate Δv_E.

Vice versa the defect, through the potential fluctuation $\Delta v(\mathbf{R}, \mathbf{r}, t)$ which it creates, according to (2.13), (2.14), and (3.29), produces the modulation

$$\Delta U = \frac{\partial U}{\partial \bar{v}_i} \Delta v = \frac{q \Delta v}{kT} (\bar{n}\,\bar{p} - n_i^2) N_S \times \int\int\int \bar{c}_n \bar{c}_p \tau_I^2 (\bar{c}_p \bar{p} - \bar{c}_n \bar{n}) \times D_S \, d\bar{E} \, d\bar{c}_p \, d\bar{c}_n \ , \tag{3.30}$$

of the electron-hole recombination rate.

Here, such a GR current modulation, achieved by means of a corpuscular approach, is proposed for the first time and, as is shown in the following part, it has to be taken into account in order to correctly evaluate the output noise of the device.

Let us now consider in more general terms the problem of the interaction between the defects which may arise in the semiconductors (and in the insulators), especially in the space-charge regions where the Debye length may become greater than the distance between the defects so that they are not screened from each other by the free carriers.

For this purpose let $\delta Q_\mu / C_{\nu\mu}$ represent the potential generated by the μth defect (located at $\mathbf{R}_\mu$) at the point $\mathbf{R}_\nu$ where it is another νth defect [in its turn the coefficient $1/C_{\nu\mu}(\mathbf{R}_\nu, \mathbf{R}_\mu, j\omega)$ should be computed by means of the transport and Poisson equations]. If, moreover, we set $(\tau_\nu^{-1} + j\omega) \equiv G_\nu / C_{\nu\nu}$, the Langevin equation (3.21) leads to the coupled equation system

$$\sum_{\mu=1}^{N_I} \frac{G_\nu}{C_{\nu\mu}} \delta Q_\mu = \delta \eta_{n\nu} + \delta \eta_{p\nu} \ , \tag{3.31}$$

where N_I is the total number of the defects of the device, $\nu=1,2,\ldots,N_I$, and τ_ν and G_ν are the values of τ and G, respectively, for the νth defect.

Unfortunately, we cannot solve such an interaction problem because of its great mathematical complexity due to the very high number N_I of equations of the system (3.31), so that we are obliged to assume

$$\Delta v_E=0 \ . \tag{3.32}$$

Indeed, in most cases, the term proportional to Δv_E of (3.17), (3.18), and (3.21) may be negligible with respect to the terms depending on ΔQ and the others owing to the screening effect of the free carriers, to the distance of the other defects generating Δv_E, and to their uncorrelated contribution to Δv_E itself.

Of the interaction phenomenon, on the other hand, we retain the effects of each defect on the GR current of the others, because in this case such effects, even though they are small, are correlated and they may yield a high cumulative contribution, through a large number of defects, to the variation of the electron-hole recombination rate given by the space integration (3.30).

To conclude this section, we point out that, from (2.17), (3.21), and (3.32), it follows that the spectrum of the ΔQ fluctuations is the usual Lorentzian one, and that the variance $\langle(\Delta Q)^2\rangle_t=q^2\bar{f}(1-\bar{f})\tau/\tau_I$ agrees with the value given for it by the quantum-mechanical statistics when the electric effects of the island are negligible and (3.26) holds true.

Equations (3.21), (3.22), (3.25), (3.26), (3.30), and (3.32) will be used in the following section to compute the noise at the terminals of the device.

IV. PROBLEMS AND COEFFICIENTS OF COUPLING

A. Coupling coefficients and terminal noise

The question which we now have to deal with is the coupling problem, i.e., we must find the mechanism and the coefficients that couple the fluctuations of the defect to those of the terminal quantities, currents, or voltages.

For this purpose let us consider a volume Ω of the device whose surface is divided into i areas A_h $(h=1,2,\ldots,i)$, where A_h may or may not be situated on the external surface of the device itself. Let $\mathbf{J}_n$, $\mathbf{J}_p$, and $\mathbf{J}_e=[\partial(\varepsilon\xi)\partial t]$ be the current density of electrons, holes, and of displacement, respectively, $\xi=-\nabla v$ being the electric field, and let

$$I_{dh}=\int_{A_h} d\,\mathbf{A}\cdot\mathbf{J}_d \tag{4.1}$$

be the current of the d type which crosses the surface A_h outwards, with $d=n$, p, and e.

Now we can define the coupling coefficient between currents,

$$\Gamma_{Bdh}\equiv\delta I_{dh}/\delta I_B \ , \tag{4.2}$$

where δI_{dh} is the phasor, at a given frequency f, of the variation of I_{dh} induced by a small ac current of phasor δI_B injected into the defect in question at $\mathbf{R}$ from the conduction $(B=C)$ or valence $(B=V)$ band.

After such a definition, by setting $\Phi_{Bh}=(\Gamma_{Bnh}+\Gamma_{Bph}+\Gamma_{Beh})$, the total current at the terminal h induced by $\delta\eta_n$ and $\delta\eta_p$ becomes $\delta I_h=(\Phi_{Ch}\delta\eta_n+\Phi_{Vh}\delta\eta_p)$ so that the noise spectrum S_{I_h} of the current I_h produced by the defects, since they and the stochastic sources η_n and η_p are independent, according to (2.15) and (2.16) may be written in the form

$$S_{I_h}=\int\int\int\int(|\Phi_{Ch}|^2\sigma_{\eta_n}+|\Phi_{Vh}|^2\sigma_{\eta_p})N^{-1}N_S D_S \times d\bar{E}\,d\bar{c}_p\,d\bar{c}_n\,d^3x \ , \tag{4.3}$$

where the integral is extended to the volume of the whole devices.

Therefore, the problem of computing the noise spectrum of the terminal currents is shifted to that of evaluating the coupling coefficients Γ_{Bdh}. The following sections are devoted to this task.

B. Coupling equations and their reduction to one dimension

The coupling equations which bind ΔI_B to ΔI_{dh} and hence allow us to compute coupling coefficients Γ_{Bdh} according to (4.2) are the defect equations (3.16)–(3.18), (3.21), and (3.32), where for the computation of Γ_{Bdh} it now has to be

$$\eta_n=\Delta I_C, \quad \eta_p=\Delta I_V \ , \tag{4.4}$$

together with the Poisson, continuity, and transport equations for the variations of the quantities outside the defect in question at $\mathbf{R}$, into which currents ΔI_C and ΔI_V are injected, causing such variations.

The Poisson equation is

$$\nabla\cdot(\varepsilon\,\Delta\xi)=q(\Delta p-\Delta n)+\Delta Q\delta(\mathbf{r}-\mathbf{R}) \ , \tag{4.5}$$

while the continuity equations for electrons and holes become, respectively,

$$q\frac{\partial\,\Delta n}{\partial t}=\nabla\cdot\Delta\mathbf{J}_n-q\,\Delta U+(\Delta i_n+\Delta I_C)\delta(\mathbf{r}-\mathbf{R}) \ , \tag{4.6}$$

$$q\frac{\partial\,\Delta p}{\partial t}=-\nabla\cdot\Delta\mathbf{J}_p-q\,\Delta U-(\Delta i_p+\Delta I_V)\delta(\mathbf{r}-\mathbf{R}) \ . \tag{4.7}$$

It has to be observed that through ΔU, which, according to (3.30), comprehends also the defect being considered, one eliminates from (4.6) and (4.7) the approximation (3.10) of $\Delta v=0$ which acts on Δi_n and Δi_p and which we have used in the defect continuity equation (3.21).

Apart from the particular transport mechanism of the charge carriers, i.e., drift, diffusion, thermal, and tunnel emission across an energy barrier, the linearization of the relevant laws allows us to write the transport equations for the variations induced by ΔI_B in the most general form

$$\Delta\mathbf{J}_d=\mathbf{a}_{dv}\,\Delta v+\mathbf{a}_{dd}\,\Delta d+b_{de}\,\Delta\xi+b_{dd}\nabla(\Delta d) \ , \tag{4.8}$$

where, of course, the linearization coefficients $\mathbf{a}_d$ and b_d depend on the transport mechanism, and $d=n$ and p.

Indeed, by expressing ΔU by means of Δn and Δp through (2.12) and (2.14), the eight linear and indepen-

dent equations (3.16)–(3.18) [for which (3.32) and (4.4) hold good] and (4.5)–(4.8), together with the boundary conditions of the system, allow us to compute as many variations $\Delta\xi=-\nabla(\Delta v)$, ΔQ, Δn, Δp, Δi_n, Δi_p, $\Delta \mathbf{J}_n$, and $\Delta \mathbf{J}_p$ produced by ΔI_B and hence, through (4.1) and (4.2), to determine the coupling coefficients Γ_{Bdh}.

The difficulty of such a coupling problem may be noticeably reduced, according to the following general method, by reducing from three to one the dimensions of the space in which Eqs. (4.5)–(4.8) are to be dealt with and solved.

Such a method is based on the linearity of the system for the small variations and its symmetries, and on the fact that, in the coupling problem, the variation $\Delta y=\Delta y_\nu(\mathbf{R}_\nu,\mathbf{r})$ of a quantity $y(\mathbf{r})$ produced by $\Delta I_B=\Delta I_{B\nu}$ injected in the νth defect in question at $\mathbf{R}=\mathbf{R}_\nu$ does not need to be known as a function of $\mathbf{r}$, but rather, according to (4.1) and the following relationships, it is sufficient to get its integral

$$\Delta Y=\Delta Y_\nu=\int_\Omega W\,\Delta y_\nu\,d^m x\ , \tag{4.9}$$

where $W(\mathbf{r})$ is any function of $\mathbf{r}$ and $m=2$ or 3.

Indeed, owing to the linearity of the system for the small variations, variation $\Delta y'$ of y produced by M defects stimulated by the respective current $\Delta I_{B\nu}$ is given by $\Delta y'=\sum_{\nu=1}^{M}\Delta y_\nu(\mathbf{R}_\nu,\mathbf{r})$, so that, after the variable substitution $\mathbf{r}=\mathbf{R}_\nu+\mathbf{r}_\nu$, we have

$$\Delta Y'\equiv\int_\Omega W(\mathbf{r})\Delta y'\,d^m x =\sum_{\nu=1}^{M}\int_\Omega W(\mathbf{R}_\nu,\mathbf{r}_\nu)\Delta y_\nu(\mathbf{R}_\nu,\mathbf{r}_\nu)d^m x_\nu\ . \tag{4.10}$$

If then currents $\Delta I_{B\nu}=\Delta I_B$ are equal and if, owing to the symmetries of the system, loci L of points $\mathbf{R}_\nu$ exist in which the integrands $W\,\Delta y_\nu$ or their integrals of (4.10) are equal, from (4.9) and (4.10) we have

$$\Delta Y=\int_\Omega W\,\Delta y\,d^m x=M^{-1}\int_\Omega W\,\Delta y'd^m x\ . \tag{4.11}$$

In the systems which, apart from the effects of the defect, are one dimensional, i.e., their macroscopic properties depend on x alone, the loci L are all the planes perpendicular to the x axis. If, in an artificial way, we uniformly distribute, with density μ, $M=\mu S$ equal defects on a surface S of a plane $x=X$, X being the abscissa of the particular defect whose effects we are computing, and if the system size along the x axis is much smaller than the linear ones of S, the variation $\Delta y'(X,x)$ due to such a uniform distribution depends on x alone and it has to be computed by means of Eqs. (4.5)–(4.8) which, accordingly, after the substitution of $\delta(\mathbf{r}-\mathbf{R})$ with $\mu\delta(x-X)$ and of Δy with $\Delta y'$, are now to be considered as one dimensional in x. Finally, (4.11) becomes

$$\Delta Y(X)=\mu^{-1}\int W(x)\Delta y'(X,x)dx\ , \tag{4.12}$$

$$\Delta Y(X)=\mu^{-1}W(x_a)\Delta y'(X,x_a)\ , \tag{4.13}$$

for $m=3$ and $m=2$, respectively, x_a being the abscissa of the plane, perpendicular to the x axis, to which the integration surface belongs. (ΔY, of course, is independent of μ since, owing to the linearity of the system, we have $\Delta y'\propto\mu$.)

In particular, according to (4.1), the current ΔI_{dh} necessary to compute Γ_{Bdh} by means of (4.2) may be obtained in the simple form (4.13).

The general procedure described above greatly simplifies the coupling problem and the computation of the noise produced by defects and it may also be extended to devices which have different symmetries and structures from the planar one discussed above.

C. Relationships between coupling coefficients from continuity and Poisson equations

The proportionalities $\delta y\propto\delta I_B$ and $\delta Q\propto\delta I_B$ due to the linearity of the system for the small variations allow us to define the coupling coefficient

$$\alpha_y\equiv\delta Y/\delta Q=\int_\Omega W\,\delta y\,d^m x/\delta Q\ , \tag{4.14}$$

between the quantity y and the island charge variation δQ; such a coefficient, in turn, may be computed by means of the preceding method.

If, by making $W=q$ and $y=n$, p, and U, we now define the coefficients α_n, α_p, and α_U, respectively, and

$$\alpha_{eh}=\int_{A_h} d\,\mathbf{A}\cdot(\varepsilon\delta\xi)/\delta Q\ , \tag{4.15}$$

we can express the coupling coefficient Γ_{Bdh} through them by means of continuity and Poisson equations (3.16) and (4.5)–(4.7) without using the transport equation (4.8).

Indeed, first of all, from (3.16) and (4.4)–(4.7) we see, as must be, that the current is solenoidal, i.e., we get the usual relationship $\nabla\cdot(\Delta\mathbf{J}_n+\Delta\mathbf{J}_p+\Delta\mathbf{J}_e)=0$ from whose integration over the volume Ω and from the divergence theorem we obtain the expected Kirchoff law

$$\sum_{h=1}^{i}(\Delta I_{nh}+\Delta I_{ph}+\Delta I_{eh})=0\ . \tag{4.16}$$

Then, from (4.2), according to which we have

$$\delta I_{dh}=\Gamma_{Cdh}\delta I_C+\Gamma_{Vdh}\delta I_V\ , \tag{4.17}$$

from the arbitrariness and independence of δI_C and ΔI_V and from (4.16) we get the first two relationships

$$\sum_{h=1}^{i}(\Gamma_{Bnh}+\Gamma_{Bph}+\Gamma_{Beh})=0 \tag{4.18}$$

between the coupling coefficients.

If we now integrate (4.6) over Ω and we take the previous definition of α_n and α_U through (4.14) into account, from (3.17) and (3.32) we obtain

$$\sum_{h=1}^{i}\delta I_{nh}=(j\omega\alpha_n+\alpha_U+\tau_n^{-1})\delta Q-\delta I_C\ , \tag{4.19}$$

so that then, from (3.21), (3.32), (4.4), (4.17), and (4.19) and, again, from the arbitrariness and independence of δI_C and δI_V, we get two other relationships

$$\sum_{h=1}^{i}\Gamma_{Cnh}=\frac{\alpha_U-\tau_p^{-1}-j\omega(1-\alpha_n)}{\tau^{-1}+j\omega}\ , \tag{4.20}$$

$$\sum_{h=1}^{i} \Gamma_{Vnh} = \frac{\alpha_U + \tau_n^{-1} + j\omega\alpha_n}{\tau^{-1} + j\omega} \ . \tag{4.21}$$

Moreover, from (3.21), (3.32), (4.1), (4.4), (4.15), and (4.17) we also have

$$\Gamma_{Ceh} = \Gamma_{Veh} = \frac{j\omega\alpha_{eh}}{\tau^{-1} + j\omega} \ , \tag{4.22}$$

while, from the integration of (4.5) over Ω and from (4.15), we obtain

$$\alpha_n - \alpha_p = \theta \ , \tag{4.23}$$

where

$$\theta = 1 - \sum_{h=1}^{i} \alpha_{eh} \ . \tag{4.24}$$

If to the sum $\sum_{h=1}^{i}(\Gamma_{Cnh} + \Gamma_{Ceh})$ $[\sum_{h=1}^{i}(\Gamma_{Vnh} + \Gamma_{Veh})]$ of the coefficients given by (4.20) and (4.22) [(4.21) and (4.22)] we now added the sum $\sum_{h=1}^{i} \Gamma_{Cph}$ $(\sum_{h=1}^{i} \Gamma_{Vph})$ similar to (4.20) [(4.21)] obtained from (4.7) with the previous procedure, we should again obtain (4.18) as a result of (4.23) and (4.24). This result, which makes the equations deduced from (4.7) useless since in actual fact they are not independent of (4.18) and (4.20)–(4.24), was to be expected because (4.7), together with (3.16), (4.5), and (4.6), has already been utilized to obtain (4.18) itself.

Equations (4.18) and (4.20)–(4.24) which express the current coupling coefficients Γ_{Bdh} through the coupling coefficients α_y with the defect charge variations are not subjected to any adiabatic approximation or to other assumption, i.e., they are totally general, they hold good both in quasineutral and space-charge regions and for all boundary conditions, quite independently of the fact the coupling equations (4.5)–(4.8) are or are not reducible to one dimension.

Moreover, the use of the charge coupling coefficients α_y and hence of (4.20)–(4.24) becomes particularly important in the case of the assumption (3.6) which has not been used until now for the coupling problem, and which allows us to compute all the coefficients α_n, α_p, α_U, and α_{eh} by means of Δv and the relative Poisson equation (4.5) alone. In fact on this assumption, from (2.5), (3.2), (3.30), and (3.32), (4.14) and (4.15) become, respectively,

$$\begin{aligned} \alpha_y &= \frac{1}{\Delta Q} \int_\Omega W \frac{\partial y}{\partial \bar{v}_i} \Delta v \, d^m x \ , \\ \alpha_{eh} &= -\frac{1}{\Delta Q} \int_{A_h} d\mathbf{A} \cdot [\varepsilon \nabla(\Delta v)] \ , \end{aligned} \tag{4.25}$$

where, in turn, Δv is given by (4.5) which is now reduced to the form

$$\nabla^2(\Delta v) = -\lambda^{-2} \Delta v - \varepsilon^{-1} \Delta Q \, \delta(\mathbf{r} - \mathbf{R}) \ , \tag{4.26}$$

in which the Debye length λ is given by (3.7). Then, in particular, (4.25) and (4.26) may be reduced to one dimension according to the procedure described above in Sec. IV B.

That is, when the assumption (3.6) holds true, the charge coupling coefficients α_y and, through (4.18) and (4.20)–(4.24), at least a part of the current ones Γ_{Bdh} may be computed without using the current equations (4.8).

In its turn assumption (3.6) means that Δn and Δp follow Δv instantaneously. That happens up to frequencies for which the diffusion delay and the transit time of the carriers in the quasineutral and space-charge regions, respectively, may be disregarded.[24] Otherwise, as a matter of fact, the current equations (4.8) has to be used also in order to determine α_y and α_{eh}.

Equations (4.25) and (4.26), of course, hold good both in quasineutral and space-charge regions. However, in the former case, if $\bar{n}$ and $\bar{p}$ are about constant in a region around the defect which is of a few λ in size, from the integration of (4.26), or directly of (4.5), over the volume Ω and from (2.5), (3.26), (3.30), (4.15), (4.24), and (4.25) we directly obtain

$$\int_\Omega \Delta v \, d^3x / \Delta Q = \theta k T / q^2 (\bar{n} + \bar{p}) \ , \tag{4.27}$$

$$\alpha_n = \theta \bar{n} / (\bar{n} + \bar{p}), \quad \alpha_p = -\theta \bar{p} / (\bar{n} + \bar{p}) \ , \tag{4.28}$$

$$\alpha_U = (\bar{n}\,\bar{p} - n_i^2) / n_i^2 \tau_U \ , \tag{4.29}$$

$$\frac{1}{\tau_U} = \frac{n_i^2 N_S}{\bar{n} + \bar{p}} \int\int\int \bar{c}_n \bar{c}_p \tau^2 (\bar{c}_p \bar{p} - \bar{c}_n \bar{n}) D_S \, d\bar{E} \, d\bar{c}_p \, d\bar{c}_n \ , \tag{4.30}$$

whereas, in the space-charge regions of planar devices, coefficients α_y, according to (4.12), (4.13), and (4.25), are given by

$$\alpha_y = \frac{1}{\mu \Delta Q} \int W \frac{\partial y}{\partial \bar{v}_i} \Delta v' \, dx, \quad \alpha_{eh} = -\frac{\varepsilon}{\mu \Delta Q} \frac{\partial \Delta v'}{\partial x} \bigg|_{x = x_a} \ , \tag{4.31}$$

where, from (4.26), $\Delta v'$ is given by the one-dimensional equation

$$\frac{\partial^2 \Delta v'}{\partial x^2} = -\lambda^{-2} \Delta v' - \varepsilon^{-1} \mu \Delta Q \, \delta(x - X) \ , \tag{4.32}$$

and by its proper boundary conditions.

D. Remarks

In order to determine the current-noise spectra S_{I_h} at the i terminals of the device by means of (4.3), $6i$ current coupling coefficients Γ_{Bdh} are to be computed, as shown, from the continuity equations of the defect charge and of the charge carriers, from their current equations and from Poisson's equation, or from relationships, such as (4.18), deduced from them.

If we introduce the $(i+2)$ charge coupling coefficients α_n (or α_p), α_U, and α_{eh}, as many coefficients Γ_{Bdh} may be determined by means of the $(i+2)$ independent equations (4.20)–(4.22). In any case, the total number parameters Γ_{Bdh}, α_{eh}, and α_y to be directly determined remain $6i$, or more exactly $(5i-2)$ if one considers (4.18) and that, owing to (4.22), $\Gamma_{Ceh} = \Gamma_{Veh}$.

However, on assumption (3.6) verified below proper frequencies, the use of the parameters α_y and α_{eh} greatly simplifies the coupling problem because they may be

computed directly by means of the Poisson equation (4.26), or (4.32), alone [without using the transport equation (4.8)] and then by utilizing relationships (4.25) or (4.31), or even, for the quasineutral regions, the explicit expressions (4.28)–(4.30) of α_n, α_p, and α_U.

In any case, apart from the ways of computing α_y and α_{eh}, relationships (4.18) and (4.20)–(4.24), as well as the preceding procedures, are quite general, i.e., they may be used for any semiconductor device, terminal number, bias and temperature condition, and for any distribution and density of impurities and carriers.

In particular, they allow us to compute the coefficients Γ_{Beh} relevant to displacement current I_{eh} across surface A_h. This is especially useful when A_h intersects a space-charge or an insulator region. If, rather, surface A_h of the volume Ω being considered intersects a region with free carriers which screen the electric field $\Delta\xi$, and in any case if the defects are a few λ away from A_h, $\Delta\xi$ (and especially δI_{eh} at low frequency) gives negligible contributions in comparison with those of the conduction current, i.e., we can put $\Delta\xi=0$ on A_h so that from (4.15), (4.22), and (4.24) we get

$$\Gamma_{Beh}=0, \quad \theta=1 . \tag{4.33}$$

Moreover, in some cases we may even find that the terminal current I_{dh} is due to a single type of carrier, so that from (4.1) and (4.2) we also get

$$\Gamma_{Bnh}=0 \quad \text{or} \quad \Gamma_{Bph}=0 . \tag{4.34}$$

In case of (4.33) and (4.34), the number of coefficients Γ_{Bdh} may be reduced to $2i$ and hence, in particular, for two terminal devices the four equations (4.18), (4.20), and (4.21) become sufficient to compute them completely through the charge coupling coefficients α_n, or α_p, and α_U which, in turn, in the case of (3.6), are given by (4.25), (4.26), and (4.28)–(4.30) or, if the devices are planar, by (4.28)–(4.32).[24]

The general model proposed, apart from the thermal noise of dissipative parts, should be able to account for all the main noises of the semiconductor devices, i.e., GR, burst, flicker, and shot noises.

However, we cannot demonstrate this here because the paper would become too long. For the junction devices, this will be shown elsewhere.[24]

V. CONCLUSIONS

A corpuscular-collective model of noise of the bipolar semiconductor devices has been proposed and several new, general results have been obtained.

According to preceding models, apart from thermal noise, the origin of the other main types of noise has been identified as being in the single SRH defect centers which, by interacting between themselves and with the conduction- and valence-band carriers, modulate their concentrations and currents and the GR current itself.

The SRH theory, applied in a corpuscular way to the single defect, has allowed us to compute its average quantities and shot noises, the generalized expressions of the GR current and of its modulation, the defect dynamic relaxation time, and the Langevin equation by achieving general formulations which take into account the parameter dispersion of the defects themselves.

The coupling problem between the defect fluctuations and the terminal currents has been dealt with by taking into account the collective effects of the defect fluctuations themselves through coupling coefficients between the output currents, and the currents and charges injected into each defect from the conduction and valence bands.

General relationships for such current and charge coupling coefficients, which in their turn make it possible to evaluate the output noise spectra, have then been obtained, without any adiabatic approximation, from the continuity, current, and Poisson equations, and by means of a new method which reduces the fluctuation problems from three- to one-dimensional space.

The model and its results hold true for any type, shape, size, and terminal number of the semiconductor device, for any bias condition, distribution of the energy levels, and capture coefficients of the defects, for any density of the impurities, defects, and carriers, and for any conduction mechanism and shape of electric field, both in space-charge and neutral regions. In this last case, expressions in closed form of the charge coupling coefficients have also been obtained.

According to the parameter dispersion and to the structure and working conditions of the device, the model should be able to account for the shot, burst, flicker, and GR noise of any bipolar electron device.

In order to keep this paper a reasonable length, we will discuss elsewhere[24] the applications and direct verifications of the proposed model which appears to be able to provide basic equations for the analysis of the noise phenomena in semiconductor devices.

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APPENDIX A

For energy levels contained in an interval smaller than kT, from (2.3), (2.4), (2.11), (2.12), (3.9), (3.14), (3.23), and (3.24) it follows that

$$(G_n^+ + G_p^-)/C_E = a\tau_I(2\bar{c}_p\bar{c}_n n_i^2 + \bar{c}_n\bar{n}_1\bar{n} + \bar{c}_p\bar{p}_1\bar{p}) , \tag{A1}$$

$$a = q^2N/kTC_E = q^2N/4\pi\varepsilon r_I kT = 46N/r_I . \tag{A2}$$

where, according to (3.9), the third term holds true for $r_I \ll \lambda$ and the fourth is true for silicon at room temperature and r_I expressed in Å.

According to (2.12) and (A1), we have

$$\tau_I \ll C_E/(G_n^+ + G_p^-) , \tag{A3}$$

for both $\bar{n} \gg \bar{p}, \bar{p}_1, a\bar{n}_1$ and $\bar{p} \gg \bar{n}, \bar{n}_1, a\bar{p}_1$. In the space-charge regions, where $n = p = 0$, (A3) is obtained for both

$\bar{c}_n\bar{n}_1 \gg 2a\bar{c}_p\bar{p}_1$ and $\bar{c}_p\bar{p}_1 \gg 2a\bar{c}_n\bar{n}_1$. Therefore, except in special cases, (3.25) and (3.26) hold true.

APPENDIX B

When energy levels E_j are contained in an interval smaller than kT, and G_{cp} and G_{cn} may be disregarded as far as the other term in (3.19) and (3.20) is concerned, from (2.3), (2.4), (2.11)–(2.13), (3.14), (3.19), (3.20), (3.28), and (A2) we have the relationships

$$A_n = -\frac{q}{kT}\bar{i}_n\tau[(\bar{c}_n\bar{n}-\bar{c}_p\bar{p})+a\tau_I(\bar{c}_n^2\bar{n}\,\bar{n}_j-\bar{c}_p^2\bar{p}\,\bar{p}_j)]\ , \tag{B1}$$

$$\frac{1}{\tau_n'} = \frac{\bar{i}_n}{\bar{i}_n^-}[(\bar{c}_n\bar{n}-\bar{c}_p\bar{p})+a\tau_I(\bar{c}_n^2\bar{n}\,\bar{n}_j-\bar{c}_p^2\bar{p}\,\bar{p}_j)]\ , \tag{B2}$$

which, when the conditions leading to (A3) and (3.24)–(3.26) exist, i.e., $a \to 0$, because of (2.12) and (2.13), become

$$A_n = -\frac{q}{kT}\bar{i}_n\tau_I(\bar{c}_n\bar{n}-\bar{c}_p\bar{p}) = \frac{\partial\bar{i}_n}{\partial\bar{v}_i} = -\frac{\partial\bar{i}_p}{\partial\bar{v}_i}\ , \tag{B3}$$

$$\frac{1}{\tau_n'} = \frac{\bar{i}_n}{\bar{i}_n^-}(\bar{c}_n\bar{n}-\bar{c}_p\bar{p}) \simeq \frac{1}{\tau_I} = \frac{1}{\tau}\ , \tag{B4}$$

so that (3.27) is reduced to (3.29).

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