Localization and electron-electron interaction effects in thin Bi wires and films

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(Received 13 October 1987)

We report the results of a comprehensive study of localization and electron-electron interaction effects in thin Bi wires and films. Measurements of the resistance as a function of both temperature and magnetic field over a wide range have allowed us to separate the contributions of localization and interactions to the resistance, and to determine the electron inelastic-scattering time. The overall behavior is in general accord with the theory. The contribution of interactions is characterized by a screening parameter, F, whose value is consistent with theoretical expectations. The inelastic scattering appears to be due to electron-electron scattering, and the absolute magnitude of the scattering rate agrees reasonably well with the theory. However, several aspects of our results are not understood. First, relatively thick ($\gtrsim 600$ Å) films exhibit an anomalously large resistance change as a function of temperature, which cannot be simply ascribed to a transition to three-dimensional behavior. Second, contrary to theoretical predictions, the dimensionality of the electron-electron scattering does not appear to always be the same as that of the electron-electron interaction effects. There is a range of wire diameter and of film thickness in which the scattering is three dimensional, while the interaction effects are one or two dimensional, respectively. This suggests that the important length scales for these processes are still not completely understood.

I. INTRODUCTION

The electrical properties of disordered systems in reduced dimensionalities have been the subject of much interest in recent years. A fairly complete, quantitative picture of the behavior of these systems has emerged from experimental and theoretical work in this area.[1,2] Precisely because the current understanding is so good, it is possible to consider the behavior of specific systems in great detail, and thereby test this understanding very thoroughly. In this paper we report a comprehensive study of localization and electron-electron interaction effects in Bi wires and films. We have used measurements of the resistance as a function of temperature and magnetic field to quantitatively separate the contributions of localization and electron-electron interaction effects to the resistance, and to study the electron inelastic scattering time. We have studied samples with a wide range of sizes, i.e., film thickness or wire diameter, and have observed a correspondingly wide variety of behavior. The majority of our results are in good agreement with previous experimental and theoretical work, but several of our findings are unexpected, and suggest that all is not understood in these systems.

This paper is organized as follows. Section II reviews the theory relevant to our experiments, while Sec. III describes the sample fabrication and measurement techniques. In Sec. IV we present our results for Bi films. We find that the behavior of the films depends on their thickness. The behavior of films less than about 500 Å thick without a magnetic field present is in good agreement with that expected for two-dimensional localization and electron-electron interaction effects. The overall temperature dependence and magnitude of the zero-field resistance increase at low temperatures of these films is also in good agreement with previous experimental results for Bi films. However, relatively thick ($\gtrsim 500$ Å) films exhibit a zero field resistance increase which is much larger than expected. Moreover, it does not appear possible to attribute this behavior to crossover from two to three-dimensional behavior. The behavior of the thick films is not at present understood, but is qualitatively similar to results reported recently for single crystal Au films.[3] Section IV also contains results for the magnetoresistance of Bi films, which can be used to derive various electron scattering times. The electron inelastic scattering appears to be due to electron-electron scattering in the presence of disorder. However, the temperature dependence of the inelastic scattering time suggests that this scattering process is three dimensional, while we would have expected it to be two dimensional. Section V contains results for thin Bi wires. The behavior at relatively high temperatures ($\gtrsim 1$ K) agrees well with previous results for wires made from other materials, and with the theory. The behavior at lower temperatures is complicated by several effects. For wires with lengths less than about 50 μm, universal conductance fluctuations[4] become important,[5] and also make it difficult to measure either the temperature dependence of the resistance, or the magnetoresistance. In addition, electron heating effects make it difficult to obtain results below about 0.2 K. Nevertheless, it is possible to determine the inelastic scattering time above about 0.3 K over about a decade of temperature. As in the films, this scattering appears to be due to electron-electron scattering. However, while this scattering process seems to be one dimensional in the thinnest wires, it appears to be *three* dimensional in the wires with the largest diameters. This is a surprising result since all of these wires behave one dimensionally as far as both localization and interaction effects are concerned. In Sec. VI we present our conclusions. A preliminary account of some of the results described here has been given elsewhere.[6]

II. THEORY

A. Localization

In one and two dimensions, the presence of disorder results in what is now known as weak localization of the electronic wave functions.[1,2] This effect makes a contribution to the conductance of the system which is a function of magnetic field and of various electron scattering times. The functional form of this contribution depends on the dimensionality of the system.

1. Two dimensions

In two dimensions it is convenient to consider the resistance per square, $R_\square$ (i.e., the sheet resistance), which is just the resistivity divided by the thickness t, $R_\square=\rho/t$. Similarly, the conductance per square, $G_\square$, is given by $G_\square=\sigma t$. Detailed calculations have shown that with a magnetic field, H, perpendicular to the plane of the film[7-9,1,2]

$$\Delta G_\square(T,H)=-\frac{e^2}{2\pi^2\hbar}\left[\psi\left[\frac{1}{2}+\frac{H_1}{H}\right]-\psi\left[\frac{1}{2}+\frac{H_2}{H}\right]+\frac{1}{2}\psi\left[\frac{1}{2}+\frac{H_3}{H}\right]-\frac{1}{2}\psi\left[\frac{1}{2}+\frac{H_2}{H}\right]\right] . \tag{1}$$

Here ψ is the digamma function, and the "fields" H_1, H_2, and H_3 are defined by[10]

$$\begin{aligned} H_1&=H_e+H_{\text{s.o.}}+H_s \ , \\ H_2&=H_i+\tfrac{4}{3}H_{\text{s.o.}}+\tfrac{2}{3}H_s \ , \\ H_3&=H_i+2H_s \ , \end{aligned} \tag{2}$$

where

$$H_n=\frac{\hbar}{4eL_n^2} . \tag{3}$$

In (3) "n" takes on the various meanings: e represents elastic scattering, i represents inelastic scattering, s.o. represents spin-orbit scattering, and s represents spin-spin scattering. The diffusion lengths L_n are related to the corresponding scattering times by $L_n=(D\tau_n)^{1/2}$, where D is the electron diffusion constant.

In the systems we consider the elastic scattering rate is generally much larger than the other scattering rates, and as a result the elastic field, H_e, is much larger than any of the other fields in the problem (including the applied magnetic field, H). If, in addition, $H_s, H_{\text{s.o.}} \ll H_i$, then in the limit $H\to 0$ (1) reduces to the now familiar form[11]

$$\Delta G_\square(T)=-\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{H_e}{H_i}\right]=-\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{\tau_i}{\tau_e}\right] , \tag{4}$$

or in terms of resistance

$$\frac{\Delta R_\square(T)}{R_\square}=-R_\square\frac{e^2p}{2\pi^2\hbar}\ln\left[\frac{T}{T_0}\right] , \tag{5}$$

where in (5) we have also used the fact the inelastic scattering time will in general vary as $\tau_i\sim T^{-p}$. Hence, in two dimensions the resistance varies logarithmically with temperature in zero field. If, however, as is the case with Bi, the spin-orbit scattering is very strong, then $H_{\text{s.o.}}\gg H_i$. In this case one can show from (1) that the contribution from localization is not only reduced in magnitude by half, but is *opposite* in sign. This result,

$$\frac{\Delta R_\square(T)}{R_\square}=R_\square\frac{e^2p}{4\pi^2\hbar}\ln\left[\frac{T}{T_0}\right] , \tag{6}$$

means that in the case of strong spin-orbit scattering the contribution from localization *decreases* as the temperature is reduced, an effect known as antilocalization.[1,2]

When measurements are made in a magnetic field, it is usually at a fixed temperature. In this case, using (1) we have

$$\Delta G_\square(H)=-\frac{e^2}{2\pi^2\hbar}\left[\frac{1}{2}\psi\left[\frac{1}{2}+\frac{H_3}{H}\right]-\frac{3}{2}\psi\left[\frac{1}{2}+\frac{H_2}{H}\right]-\frac{1}{2}\ln\left[\frac{H_3}{H}\right]+\frac{3}{2}\ln\left[\frac{H_2}{H}\right]\right] . \tag{7}$$

Here again, we have assumed that H_e is much larger than any of the other fields. Examination of (7) shows that there are two types of scattering that can reduce the effect of localization, i.e., destroy phase coherence; namely, inelastic scattering and spin-spin scattering. It is convenient to define the phase breaking time, τ_ϕ, by [see (2)]

$$\frac{1}{\tau_\phi}=\frac{1}{\tau_i}+\frac{2}{\tau_s} , \tag{8}$$

and also the phase breaking length

$$L_\phi=(D\tau_\phi)^{1/2} . \tag{9}$$

L_ϕ is simply the average distance which an electron diffuses between phase breaking (i.e., inelastic or spin-spin scattering) events. Also, since L_ϕ is the length scale over which phase coherence is maintained, it is this length which determines the effective dimensionality of a system as far as localization is concerned. In order to be two dimensional, one must have $L_\phi>t$. We should note

that it is possible to distinguish between the inelastic and spin-spin scattering by virtue of the fact that the latter should generally be temperature independent.

Returning to (7), it can be seen that the effects of localization are destroyed by the application of the magnetic field. This magnetoresistance provides a very convenient experimental means[2] of determining L_ϕ. It is common to define a magnetic length, given by

$$L_H = \left[\frac{\hbar}{4eH}\right]^{1/2} . \tag{10}$$

In analogy with (9), L_H must be greater than the film thickness t if the system is to behave two dimensionally. If $L_H < t$, the system is effectively three dimensional, and (1) is no longer applicable.

All of the above discussion applies to the case of a field applied perpendicular to the plane of the system. There is also a magnetoresistance for parallel fields, but we will not be concerned quantitatively with that case here, other than to note that it is generally much smaller than the perpendicular magnetoresistance.

2. One dimension

The behavior in one dimension is qualitatively similar to that found in two dimensions, although the detailed functional forms are different. The theory predicts[12–14]

$$\frac{\Delta R(T,H)}{R_0} = \frac{e^2\rho_e}{2\pi\hbar A}\left[\frac{3}{2}\left[L_\phi^{-2}+\frac{4}{3}L_{\text{s.o.}}^{-2}+L_H^{-2}\right]^{-1/2} - \frac{1}{2}\left[L_\phi^{-2}+L_H^{-2}\right]^{-1/2}\right], \tag{11}$$

where A is the cross sectional area of the wire, ρ_e is the impurity (elastic) resistivity, and[12]

$$L_H = \left[\frac{3\hbar^2}{e^2AH^2}\right]^{1/2} . \tag{12}$$

Note also that the field is applied perpendicular to the direction of the current. We also note that in order for (11) to be applicable, both L_H and L_ϕ must be greater than the transverse dimensions of the wire.

In the limit $H \to 0$ and strong spin-orbit scattering (11) reduces to

$$\frac{\Delta R}{R_0} = -\frac{e^2\rho_e}{4\pi\hbar A}L_\phi . \tag{13}$$

Here again, the spin-orbit scattering changes the sign of the contribution of localization to the resistance, and produces antilocalization.

3. Three dimensions

While our work has not involved any three-dimensional systems as such, we will need to compare our observations with the behavior predicted for three dimensions. Briefly, in three dimensions localization makes a contribution to the zero-field conductivity of the form[1]

$$\Delta\sigma = \frac{e^2}{\pi^3\hbar}L_\phi^{-1} \sim T^{p/2}. \tag{14}$$

Hence, in contrast to the behavior in one or two dimensions, this contribution does not diverge as $T \to 0$. Equation (14) applies for the case of very weak spin-orbit scattering; for the opposite limit, we would expect a contribution of opposite sign, with a magnitude reduced by a factor of 2, in analogy with (6) and (13).

B. Electron-electron interactions

The theory of electron-electron interactions has been considered by a number of workers.[15,1] It has been shown that in two dimensions interactions make a contribution to the resistance

$$\frac{\Delta R_\square(T)}{R_\square} = -\frac{e^2}{2\pi^2\hbar}(1-\tfrac{3}{4}F)R_\square \ln\left[\frac{T}{T_o}\right], \tag{15}$$

where F is a screening factor[1] whose value lies between zero and (approximately) unity. It can be seen that (15) is very similar in form to (6), although since Bi has strong spin-orbit scattering, the contribution from localization is negative (antilocalization) and hence is opposite in sign relative to (14). In one dimension the contribution of interactions is given by[15]

$$\frac{\Delta R}{R_0} = \frac{\rho_e e^2}{2^{3/2}\pi\hbar A}(4-\tfrac{3}{2}F)\left[\frac{D\hbar}{k_B T}\right]^{1/2}, \tag{16}$$

where k_B is the Boltzmann's constant, and F is again a screening factor (note that the screening factor depends only very weakly on dimensionality,[1] so that it should be approximately the same in different dimensions). The quantity $L_T \equiv (D\hbar/k_B T)^{1/2}$ is sometimes referred to as the "cutoff" or thermal length. This length scale determines the system dimensionality for interaction effects in precisely the same way that L_ϕ determines the effective dimensionality for localization. The effect of interactions in three dimensions is given by[1]

$$\Delta R = \frac{1.3\rho_e e^2}{4\pi^2\sqrt{2}\hbar}(\tfrac{4}{3}-\tfrac{3}{2}F)\left[\frac{k_B T}{\hbar D}\right]^{1/2} . \tag{17}$$

While the effects of localization are quite sensitive to a magnetic field, this is not the case for interactions. The theory does predict a positive, isotropic magnetoresistance.[16,1] However, since this arises from the splitting of the electronic spin-up and spin-down energy bands, it is very small compared to the low field magnetoresistance due to localization.

We have discussed the contributions from localization and interactions separately, but in a real system, both effects will of course be present. To lowest order, the two effects are believed to be simply additive,[1] so that, for ex-

ample, in two dimensions the behavior should be described by just the sum of (6) and (15).

C. Inelastic scattering

It is clear from the above discussion that inelastic scattering plays a key role in localization. Indeed, all of the temperature dependence of this effect is due to the temperature dependence of the inelastic scattering time, τ_i. In general, τ_i can be written as

$$\tau_i = CT^{-p} , \tag{18}$$

where C and p are constants. Many experiments[2,17] have shown that electron-electron scattering is the dominant inelastic mechanism at temperatures below about 5 K, while at higher temperatures electron-phonon scattering has sometimes been observed as the dominant process.

It is well known that for inelastic electron-electron scattering in a perfect, periodic potential the scattering rate is proportional to T^2 (i.e., $p=2$), regardless of the dimensionality.[18] The effect of disorder on this scattering process was first considered by Schmid.[19] He studied the problem in three dimensions and found that the electron-electron scattering rate is enhanced when the electronic motion is diffusive. Schmid showed that for this case $p=\frac{3}{2}$ with the constant C being a function of the amount of disorder.

The problem of electron-electron scattering in one and two dimensions in the presence of disorder has been studied more recently. In two dimensions it is found that[20]

$$\frac{\hbar}{\tau_i} = \frac{e^2 R_\square}{2\pi\hbar} k_B T \ln\left[\frac{T k_B \tau_i}{\hbar}\right] . \tag{19}$$

For one dimension, e.g., a thin wire,

$$\tau_i = \left[\frac{\nu\sigma A^2 \hbar^4}{2e^2 T^2 k_B^2}\right]^{1/3} , \tag{20}$$

where A is again the cross sectional area of the wire, and ν is the electronic density of states which can be estimated through the conductivity $\sigma = e^2 D \nu$.

While inelastic electron-electron scattering and electron-electron interaction effects are often considered as separate phenomena, it is clear on physical grounds that they must be very closely related. An important result of this relationship is that these two processes are governed by the same length scale $(D\hbar/k_B T)^{1/2}$. Thus, we would expect that in a given system these two processes should have the *same* effective dimensionalities. We will return to this point in Secs. IV and V.

III. SAMPLE FABRICATION AND MEASUREMENT

In order to make thin films which are continuous and uniform, it is important that the grain size of the films be as small as possible. We therefore investigated several different methods of depositing Bi films, and studied the grain sizes obtained. The grain size was determined from transmission electron microscopy of films which were lifted off from their substrates. We found that thermally evaporated films had larger grain sizes than sputtered films. In both cases, cooling the substrates to 77 K during the deposition significantly reduced the grain size. The smallest grain sizes, 100–200 Å, were obtained with films prepared by dc sputtering onto cooled substrates, and the vast majority of the measurements reported below were obtained with samples derived from Bi films which were produced in this way. In all cases, 99.999% pure Bi was used as the starting material.[21] The low temperature (4 K) resistivity of these films was ≈ 2400 $\mu\Omega$ cm, and was independent of the thickness for film thicknesses greater than about 100 Å, suggesting that the films were continuous and uniform. Hall effect measurements[22] were also made, and the result was ~ 0.025 cm^3/C for a 200 Å thick film at 4.2 K, which is, to within a factor of 3 or better, the same as that found by previous workers.[23–25] If the Hall constant is interpreted in terms of simple free electron theory assuming only a single type of carrier with only one band, a carrier density of 6×10^{20} cm^{-3} is found, although the actual carrier density is likely to be somewhat lower. Previous workers[23] have assumed a density of $\sim 6\times 10^{17}$ cm^{-3} (for holes) and $\sim 2\times 10^{17}$ cm^{-3} (for electrons), but these values are based on the assumption that the densities are the same as in pure, bulk Bi. This assumption is certainly not justified,[24] but unfortunately it is very difficult to directly determine these parameters in highly disordered films such as the ones studied here. In any case, all of the properties of our films are very similar to those studied by previous workers.[23–26] In the following section, we also report a few measurements on films which were produced by thermal evaporation onto substrates held at 77 K. These films had the same low temperature resistivity as the sputtered films, but with an average grain size about twice as large.[22] Unless noted specifically otherwise, the samples discussed in this paper were derived from sputtered films prepared as described above.

Films were patterned for measurement by scribing with a tungsten needle attached to a micromanipulator. Sample dimensions were typically $\gtrsim 10$ μm wide with a uniformity of order 1 μm, with lengths ranging from tens of μm to several mm. Small diameter wires were made from the same Bi films, using substrate step techniques which have been described in detail elsewhere.[27] Substrate step methods generally produce wires with right triangular cross sections, but it is also possible to make wires which are closer to "L" shaped, with widths several times their height.[28] Most of our wire samples had triangular cross sections, but some (which will be noted specifically) had shapes which were more nearly "L" shaped. For these latter samples the effective width was generally no larger than 3–5 times the effective thickness.

The Bi samples were relatively easy to produce, but were found to degrade with time due to oxidation and/or coalescing of the grains into larger islands. The rate of degradation could be slowed down somewhat by overcoating with a layer of photoresist. However, it appeared that grain growth occurred at a significant rate at room temperature so that even though the photoresist layer essentially stopped the oxidation, the samples still degraded, from having a fairly uniform thickness, to a state

consisting of (eventually) discontinuous islands. While we found that this degradation could be arrested by storing the samples at 77 K, this was not always convenient. As a result, samples were seldom measured in more than two separate runs (which involved cycling the temperature from room temperature to low temperature and back), although the behavior of samples which were maintained at temperatures at or below 77 K between measurements was very reproducible.[22]

Two different cryostats were used for the measurements. Initially a standard ^{4}He cryostat[29] was employed for measurements in the range 1–10 K. Later, a dilution refrigerator system was used to obtain results over a much wider range. While the refrigerator could attain temperatures below 50 mK, electron heating limited most of our measurements to $\gtrsim 0.1$ K. The refrigerator also contained an 8 T magnet, allowing magnetoresistance measurements as well. The measurements in both cryostats utilized standard low frequency ac bridge techniques. Details of the refrigerator system, and the computer system used to control its functions, are described in detail elsewhere.[22]

IV. Bi FILMS

A. Zero-field results

In our initial studies of Bi films we concentrated on the behavior in zero field. According to the theory we expect the behavior to be described by a sum of (6) and (15). Hence, the resistance should vary logarithmically with temperature, with a positive contribution from interactions and a negative one from antilocalization. Some typical results for the resistance as a function of temperature are shown in Fig. 1. It is seen that the logarithmic behavior begins below about 3 K. The behavior at higher temperatures is consistent with that seen by previous workers,[24,26] who have attributed it to a small residual variation of carrier concentration with temperature. However, this variation appears to be negligible below about 3 K, and does not seem to interfere with the study of localization and interactions in that range. We have obtained results like those in Fig. 1 for a large number of films, both sputtered and evaporated, with various thicknesses. In Fig. 2 we plot the fractional resistance change per decade of temperature, as a function of $R_\square$, or equivalently, film thickness, t. For films with t less than about 500 Å, the fractional rise is proportional to $R_\square$, as predicted by the theory (6) and (15). Note also that an extrapolation of the linear variation seen for large $R_\square$ to $R_\square = 0$ yields a vanishing fractional rise in this limit. This implies that "bulk" contributions to the resistance increase, such as the Kondo effect, are negligible. The linear variation of the resistance rise with $R_\square$ can be used to estimate the quantity $\beta \equiv -p/2 + 1 - 3F/4$ which characterizes the strength of localization and interactions, (6) and (15), and we find $\beta \sim 0.45$. This value is very much in line with what we would expect; namely F small, and close to zero ($F \sim 0.1$–0.3 has been found by previous workers[23,26]), and $p \sim 1$, corresponding to two-dimensional electron-electron scattering. Note, however, that our magnetoresistance results (discussed below) indicate a somewhat different value of p (~ 1.5). In any case, the zero-field behavior of the thinnest films considered in Fig. 2 is quite reasonable.

The behavior for small $R_\square$ (thick films) is, however, not in accord with the theory. We see from Fig. 2 that the fractional increase in resistance for films thicker than about 500 Å is *much* larger than expected from the results for large $R_\square$, or from the theory. We can nevertheless estimate the parameter β for the thick films, and in Fig. 3 we plot β as a function of $R_\square$ for all of the samples.

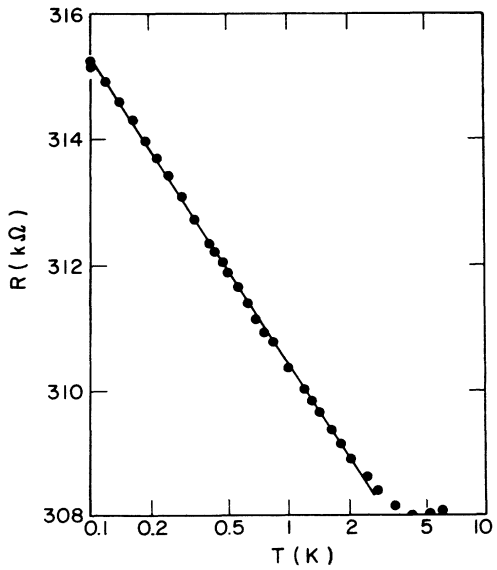


FIG. 1. Resistance as a function of temperature for a 270 Å thick sputtered Bi film.

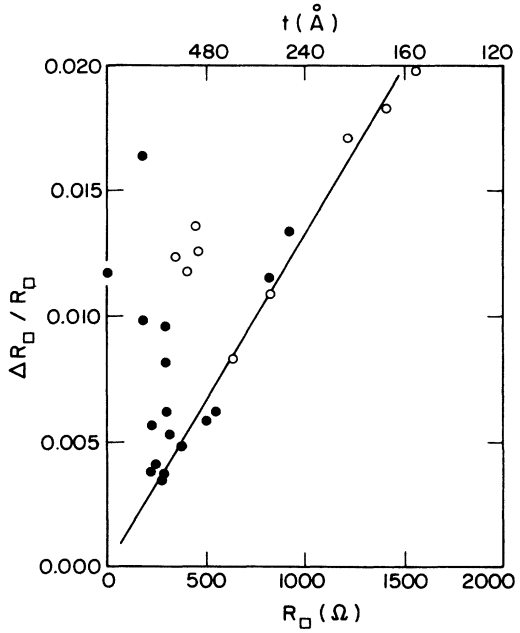


FIG. 2. Fractional resistance rise per decade of temperature as a function of $R_\square$ for sputtered (closed symbols) and evaporated (open symbols) films.

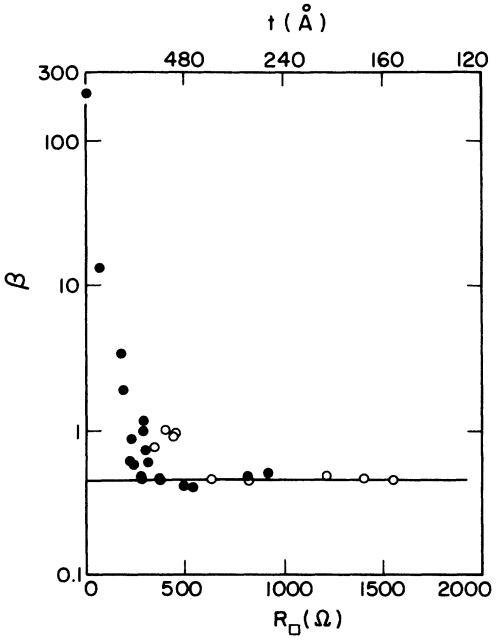


FIG. 3. β as a function of $R_\square$ for sputtered (closed symbols) and evaporated (open symbols) films.

For large $R_\square$, i.e., small film thickness, β is seen to be a constant, with a value of ~ 0.45, as noted above. However, the fractional rise seen for small $R_\square$ is again seen to be anomalously large. In fact, β increases very rapidly as $R_\square \to 0$. It is not possible to account for the large values of β found for the thick films simply in terms of magnetic impurities, etc., since we would expect such bulk (i.e., dimensionality-independent) impurity effects to be present in the thinner films as well, and Fig. 2 shows that this is certainly not the case. In addition, such a bulk effect would give rise to a fractional resistance change, $\Delta R/R$, which approaches a constant as $R_\square \to 0$, while we see from Fig. 2 that this quantity increases substantially in this limit. One might be tempted to attribute this behavior to a crossover to three-dimensional behavior. Such a dimensional crossover would be expected when the film thickness becomes comparable to the phase breaking length. However, the behavior seen in the thick films is not consistent with three-dimensional behavior. First, the variation of R with temperature for the thick films remains approximately logarithmic, while the theory (see Sec. II) predicts a power law form, T^δ, with $\delta \geq 0.75$. Second, the magnitude of the resistance rise seen in the thick films is much larger than predicted by the three-dimensional theory. For three-dimensional localization (14) the theory predicts $\Delta R/R \sim 1 \times 10^{-3}$ in Fig. 2 (here we use the value $L_\phi \sim 1000$ Å at 1 K as estimated from magnetoresistance measurements presented below). Three-dimensional interaction theory (17) predicts $\Delta R/R \sim 5 \times 10^{-5}$. Both of these predictions are significantly lower than the value $\Delta R/R \gtrsim 0.01$ seen in the thickest films, so our results do not seem to be consistent with three-dimensional behavior.

Behavior which is somewhat similar to that seen in our thick films has been observed by Chaudhari *et al.*[3] in thin single crystal Au films. They found that β became anomalously large when the film was made relatively thick, and attributed this to the effect of specular scattering at the very perfect surfaces of their single crystal films. A theory which predicts just such behavior has also been developed,[30] although other explanations, such as the Kondo effect, have been also proposed.[31,32] In any event, while the behavior we have observed for small $R_\square$ is similar to that seen in Au, our samples are quite different. They are, as discussed in Sec. III, polycrystalline, and we would not expect the surface scattering in our films to be specular. Hence, even though the behavior we observe is qualitatively similar to that seen in single crystal Au films, it is not at all clear that the origin is the same.

B. Magnetoresistance

We have measured the magnetoresistance of our Bi films over a wide range of field and temperature. Some typical results for $R_\square$ as a function of T and B are given in Figs. 4 and 5, which show results for perpendicular and parallel fields, respectively. For this sample the two-dimensional regime is limited to fields less than 1 kOe, and from Fig. 5 we see that the parallel magnetoresistance is essentially zero in this region. This is in good agreement with the theory, and confirms the two-dimensional nature of these films. From Fig. 4 we again see that the variation in zero field is logarithmic with temperature. In addition, the magnetoresistance is positive and hence from (6) and (7) we can conclude that, as expected, the spin-orbit scattering must be strong, yielding antilocalization. A logarithmic variation with temperature is also seen in relatively large fields ($\gtrsim 1$ kOe). Since a field of this magnitude largely quenches the effects of localization, the resistance variation seen in this case is due predominantly to interactions. The fact that the high field behavior is logarithmic implies from (15) that the interaction effects are two dimensional. This is

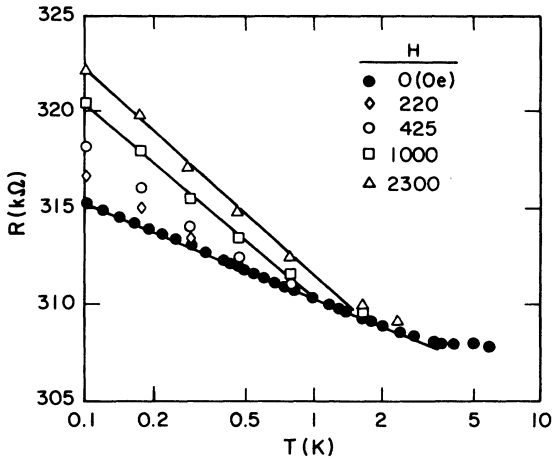


FIG. 4. Resistance as a function of temperature for several values of the magnetic field, for a 270 Å thick film. The field is applied *perpendicular* to the plane of the film. The lines are guides to the eye, which illustrate a logarithmic temperature dependence.

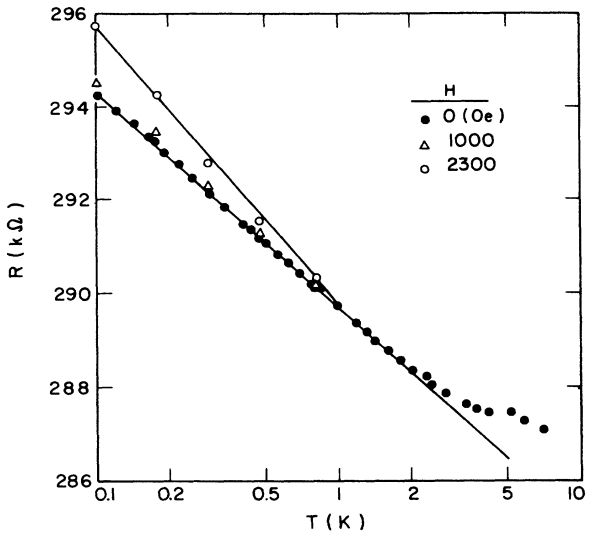


FIG. 5. Resistance as a function of temperature for several values of the magnetic field, for a 240 Å film. The field is applied *parallel* to the plane of the film.

not surprising, since using our best estimates[33] for quantities such as the diffusion constant, etc. (see Sec. III), we find[22] $L_T \approx 1000$ Å at 1 K, which is much larger than the film thickness.

We have used (7) to analyze the magnetoresistance in perpendicular fields. A typical least squares fit is shown in Fig. 6; here we plot the conductance, since the theory (7) is more naturally expressed in terms of this quantity. The solid line in Fig. 6 is a least squares fit to the theory (7); in order to ensure that L_H is less than the film thickness [see the discussion of (10)], the fit was restricted to fields $\lesssim 1$ kOe. From least squares fits like those shown in Fig. 6, we found that the spin-orbit scattering rate is sufficiently large that the magnetoresistance is essentially independent of its precise value, and we are only able to set an upper limit on the spin-orbit length, which we estimate (conservatively[22]) to be ~ 300 Å. As a result, the fits of the theory to the experimental magnetoresistance essentially involved only two parameters at each temperature, the zero-field conductance, and the phase breaking length, L_ϕ. Typical results for L_ϕ for a film are shown in Fig. 7, where it is seen that this quantity varies as a power of temperature. As discussed in Sec. II, $L_\phi = (D\tau_\phi)^{1/2}$ and the phase breaking time varies as $\tau_\phi \propto T^{-p}$. Hence, we expect that $L_\phi \propto T^{-p/2}$. A fit of the data in Fig. 7 to this form yields[34] $p = 1.45 \pm 0.10$ over nearly two decades of temperature. This fit was restricted to the data shown in Fig. 7, even though data at higher temperatures were obtained, so as to ensure that the values of L_ϕ were always greater than the film thickness. This result for p is surprising, since it is very close to the value expected for *three*-dimensional electron-electron scattering ($p = \frac{3}{2}$), in contrast to the value $p = 1$ predicted for two dimensions. As discussed above, the behavior of the resistance in large fields seems to show conclusively that the interaction effects are two-dimensional. Since the length scale L_T should determine the dimensionality of *both* the inelastic scattering and the interactions, these results appear to be at odds with the theory.

Other workers[26,35] have reported results for somewhat thinner (< 150 Å) Bi films, for which it is reported that $p \approx 1$, as expected for two dimensional electron-electron scattering. Thus, for films which are sufficiently thin, the behavior does become "fully" two dimensional. However, this does not explain how it is possible for the inelastic scattering and the interactions to have different effective dimensionalities in the *same* sample, as we have observed.

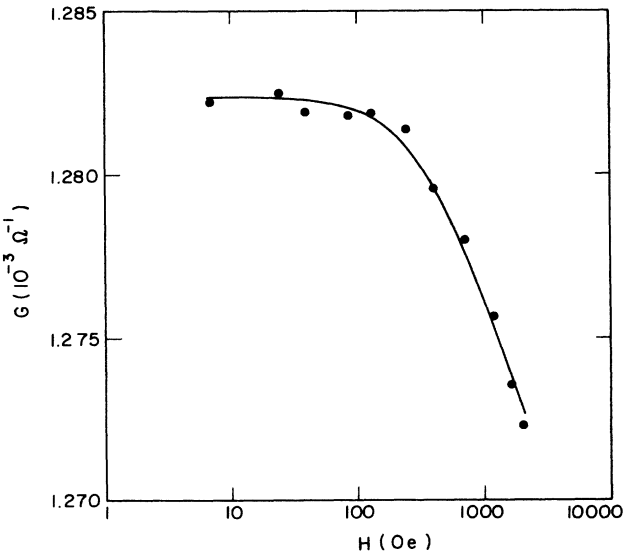


FIG. 6. Conductance per square as a function of magnetic field for a 314 Å thick film. The line is a least squares fit to the theory (7).

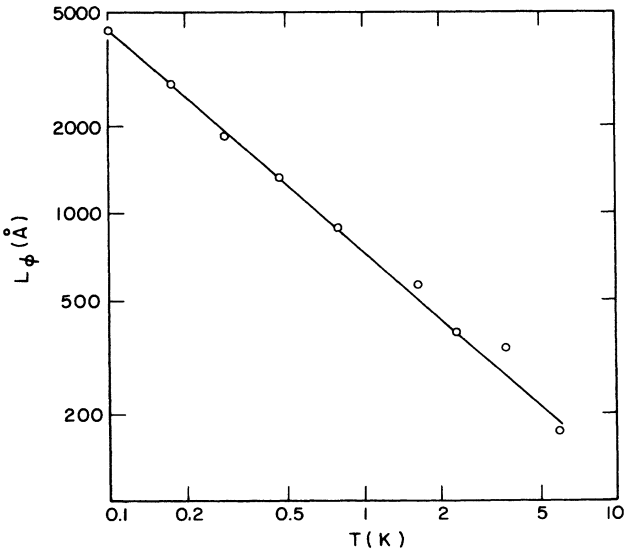


FIG. 7. Phase breaking length, L_ϕ, as a function of temperature, for the film considered in Fig. 4.

V. Bi WIRES

Some typical results for the resistance of a thin Bi wire as a function of temperature, at relatively high temperatures, are shown in Fig. 8. Note that the temperature scale in Fig. 8 is logarithmic, so it is clear that the variation of R with temperature is faster than logarithmic; i.e., $R \approx T^{-\gamma}$, with $\gamma > 0$, in qualitative agreement with the theory (15). The precise form of this variation, and the value of γ, will be discussed below. In Fig. 9 we show the fractional resistance increase as a function of the cross sectional area of the wire. Figure 9(a) shows results for high temperatures (1.4–3.2 K), while Fig. 9(b) shows results for a wider temperature range (0.3–3.2 K). It can be seen that at high temperatures [Fig. 9(a)] where the most data are available, the fractional resistance rise varies as A^{-1}, in good agreement with the theory for either localization or interactions, (13) and (15). The results at lower temperatures are also consistent with this functional form, although there are certainly not enough data available to really test the theory in this case.

In order to separate the effects of localization and interactions we have performed magnetoresistance measurements. Figure 10 shows results for R as a function of T and H for three different wires. The resistance of the smallest wire [365 Å, Fig. 10(a)] increases monotonically as the temperature is decreased, although dR/dT is seen to decrease at the lowest temperatures. This "flattening" out of R versus T is due in part to simple electron (Joule) heating from both the measuring current and external noise. This is also evident from the magnitude of the magnetoresistance, which can be seen to approach a constant at the lowest temperatures. Hence, data below about 0.3 K must be treated with caution, as the electron temperature is probably somewhat higher than the lattice (i.e., refrigerator) temperature. Data for successively larger wires are shown in Figs. 10(b) (465 Å) and 10(c) (525 Å). We see that as the wire is made larger, the variation of R changes qualitatively. For the larger wires R actually *decreases* at the lowest temperatures. Note that this decrease is *not* due to electron heating. Simple heating would cause the resistance to approach a constant value at low temperatures, not decrease. In addition, it can be seen from Fig. 10(c), for example, that the magnitude of the magnetoresistance continues to increase at temperatures well below the temperature at which R exhibits a maximum in zero field. This indicates [from (11)] that the phase breaking time is still increasing, which in turn implies that the electron temperature is

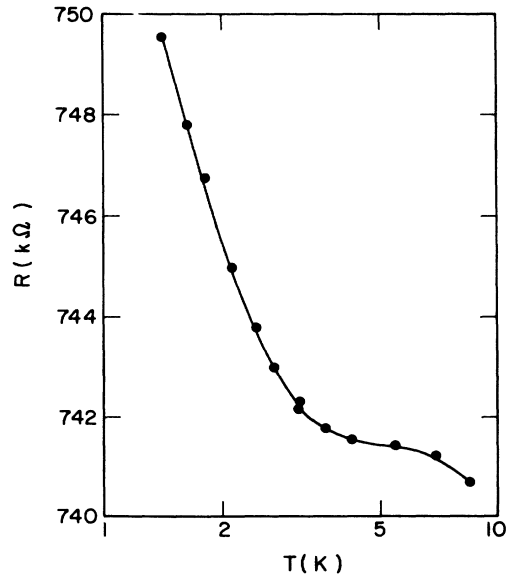


FIG. 8. Resistance as a function of temperature for a 315 Å wire. The line is a guide to the eye.

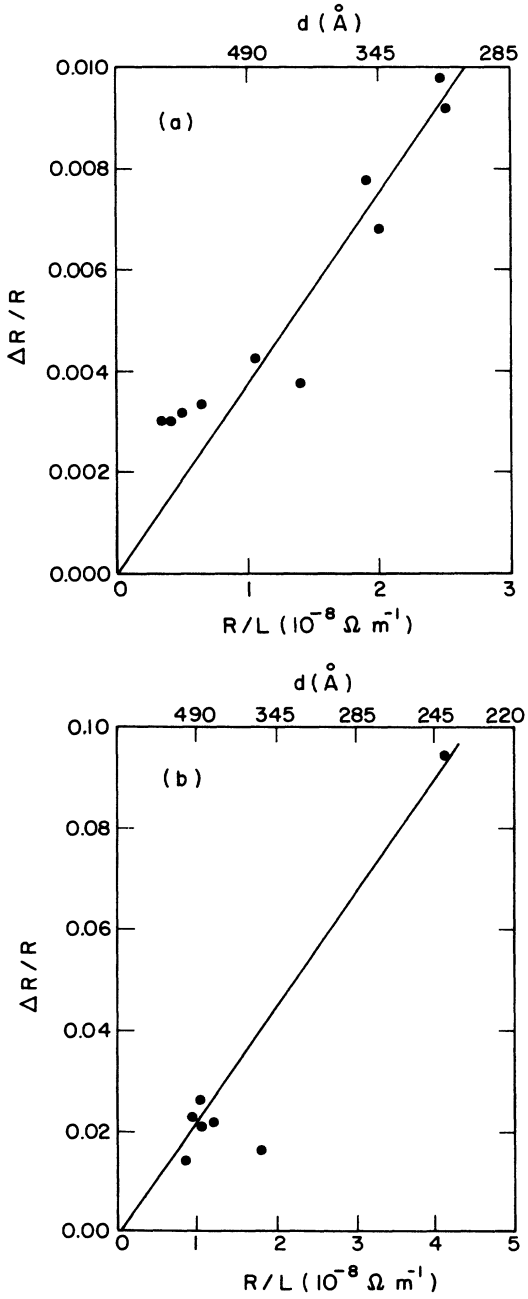
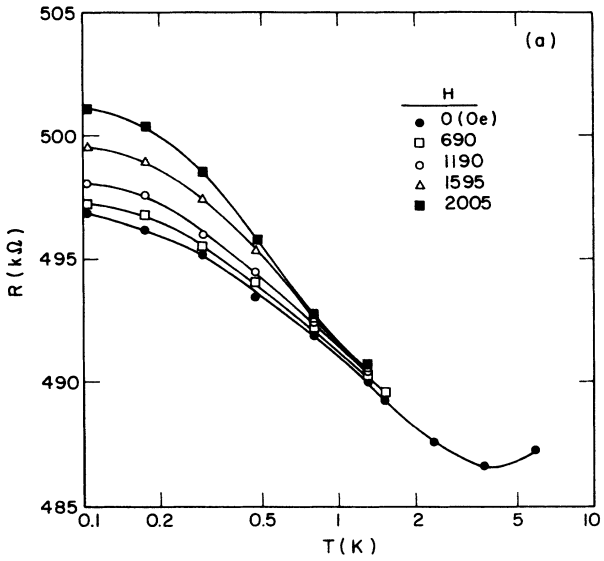
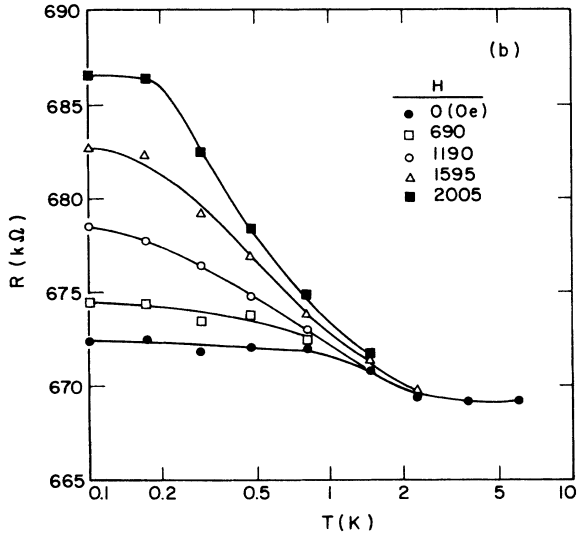


FIG. 9. Fractional resistance change as a function of wire diameter for (a) 1.4 to 3.2 K; and (b) 0.3 to 3.2 K. The lines are guides to the eye drawn proportional to A^{-1}.





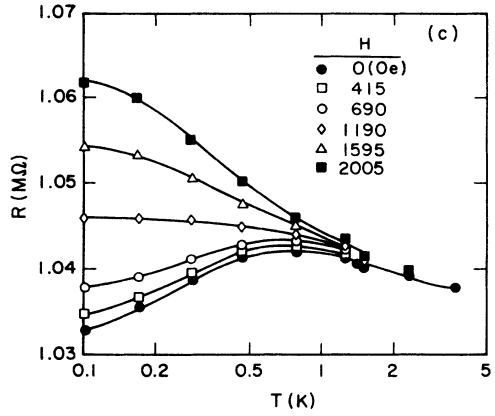


FIG. 10. Resistance as a function of temperature at several values of magnetic field for (a) a 365 Å wire, (b) a 465 Å wire, and (c) a 525 Å wire. The lines are guides to the eye.

still decreasing as the lattice temperature is reduced below 0.2 K.

Fits of the magnetoresistance data to the theory (11) indicated that, as was the case for the films, the spin-orbit length is very short, and only an upper limit on this quantity (which was similar to that found for the films) could be estimated. These fits also yielded the phase breaking length, L_ϕ, as a function of temperature. Results for the wires considered in Figs. 10(a) and 10(c) are shown in Fig. 11. For the smaller wire (the filled circles in Fig. 11) problems with Joule heating below about 0.3 K restrict the usable temperature range to a little less than one decade.[34] Nevertheless, if we fit the data for $T > 0.3$ K in Fig. 11 to a power law, (18), we can obtain a rough estimate for p. The results of such a fit are shown as the solid lines in Fig. 11. Note also that we have not included data above about 1.5 K (which are not shown in Fig. 11) in this analysis because in this range the values of L_ϕ derived from the magnetoresistance fits were comparable to or less than the diameter of the wire. Thus, the one-dimensional theory for the magnetoresistance which was used in these fits should not be applicable here.[36] From the fit for the 365 Å wire shown in Fig. 11 we find $p \approx 0.8$, which is consistent to the value $p = \frac{2}{3}$ expected for one dimensional electron-electron scattering. The absolute magnitude of the inelastic scattering length at 1 K [i.e., the constant C in (18)] derived from the fit is 620 Å. If we hold p fixed at the value $\frac{2}{3}$ expected theoretically, we find a similar value, 700 Å. These both agree fairly well with the theory (20), which predicts 900 Å for this sample. Note that this comparison involves no adjustable parameters.[22] We therefore conclude that the inelastic scattering in this case is due to one-dimensional electron-electron scattering.

For the 525 Å wire considered in Fig. 10(c) the behavior of L_ϕ is somewhat different. A least squares fit to the data for that sample (the open circles in Fig. 11) yields $p = 1.4 \pm 0.1$. This wire is larger than the one considered in Fig. 10(a), and this makes electron heating somewhat

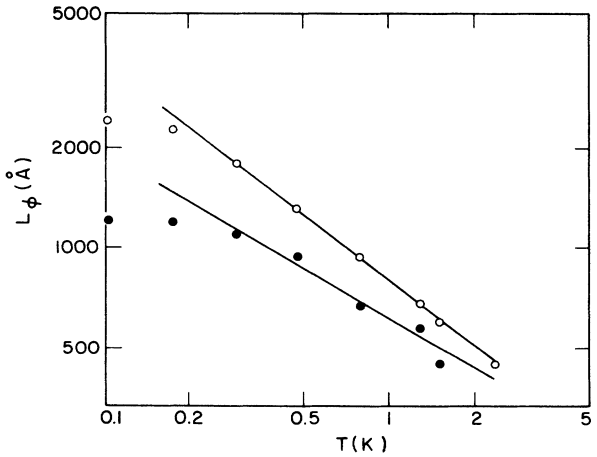


FIG. 11. Phase breaking length as a function of temperature for a 365 Å wire (filled circles), and a 525 Å wire (open circles). These are the same samples as considered in Fig. 10. The solid lines are least squares fits to the theory (18) as described in the text.

less of a problem at the lowest temperatures as can be seen from Fig. 11. Hence, this least squares fit, and the value of p for this sample are more precise than for the smaller wire. The value found for p is quite close to that expected for *three*-dimensional electron-electron scattering, in contrast to the one-dimensional value found for the smaller wire. At first sight this result seems plausible, since if the wire diameter is larger than L_T, we would expect it to behave three dimensionally as far as electron-electron scattering is concerned. However, our best estimate[20] of L_T is ≈ 1000 Å at 1 K, so we would not have expected to observe three-dimensional behavior for a sample with this diameter (525 Å). Nevertheless, it is difficult to reliably estimate the quantities which enter L_T, so it is conceivable that our estimate of L_T is in error. In any case, as we saw in our analysis of our two-dimensional data, there is another way to obtain a limit on the value of L_T. As noted in Sec. II C, L_T is the fundamental length scale for both electron-electron scattering and electron-electron interaction effects. Since L_ϕ is now known from the magnetoresistance, we can use the theory (13) to subtract the contribution of localization to the zero-field resistance, and obtain the contribution of interactions. The results for both the 365 Å wire and the 525 Å wire are shown in Fig. 12. The solid lines are least squares fits to the function $C_1 + C_2 T^{-\gamma}$, which yielded $\gamma = 0.48 \pm 0.05$ for the small sample and $\gamma = 0.45 \pm 0.05$ for the large sample. These values of γ are thus both in very good agreement with that expected for *one*-dimensional electron-electron interactions ($\gamma = \frac{1}{2}$).

The results for the 525 Å wire thus indicate that the inelastic scattering is three dimensional, while at the same time the effects of interactions are one dimensional. This appears to be at odds with the theory, which predicts that these phenomena are controlled by the *same* length scale, L_T. As noted above, our best independent estimate of L_T is 1000 Å at 1 K so one-dimensional behavior is consistent with the estimated material parameters for these samples. The three-dimensional behavior found for the inelastic scattering is thus hard to understand. It should be noted that some of the largest wires were made using large substrate steps, and this meant that they had "L" shaped cross sections[28] as compared with the triangular cross sections of the smallest wires. It is conceivable that this change in shape may have played a role in the change of dimensionality we have observed. However, the transverse dimensions of the samples were always less than our best estimate of L_T, so we still would have expected one-dimensional behavior. In addition, the behavior varied smoothly and systematically as the wire diameter was reduced. Finally, the fact remains that for a given sample the dimensionalities of the interactions and the electron-electron scattering were *not* always the same.

VI. SUMMARY AND CONCLUSIONS

We have studied the effects of localization and electron-electron interactions in thin Bi wires and films at low temperatures. The overall behavior we have observed is in good agreement with the theory, and with previous results for other materials. Our results are also in close agreement with those of other studies of similar Bi films, in cases where comparisons can be made. The two-dimensional behavior seen in films less than about 500 Å thick is well described by a combination of interactions with $F \sim 0.1$ and localization. However, the variation of the resistance with temperature seen in the thick films is much larger than that found in the thin films. Moreover, the resistance change becomes *larger* as the film thickness is increased. This behavior is seen in films which, based on our measured values of L_ϕ, would be expected to behave three-dimensionally as far as location is concerned. However, the magnitude and temperature dependence of the observed effect cannot be explained even qualitatively in terms three-dimensional localization or interactions.

A second unexplained result concerns the dimensionalities of the electron inelastic scattering and the electron-

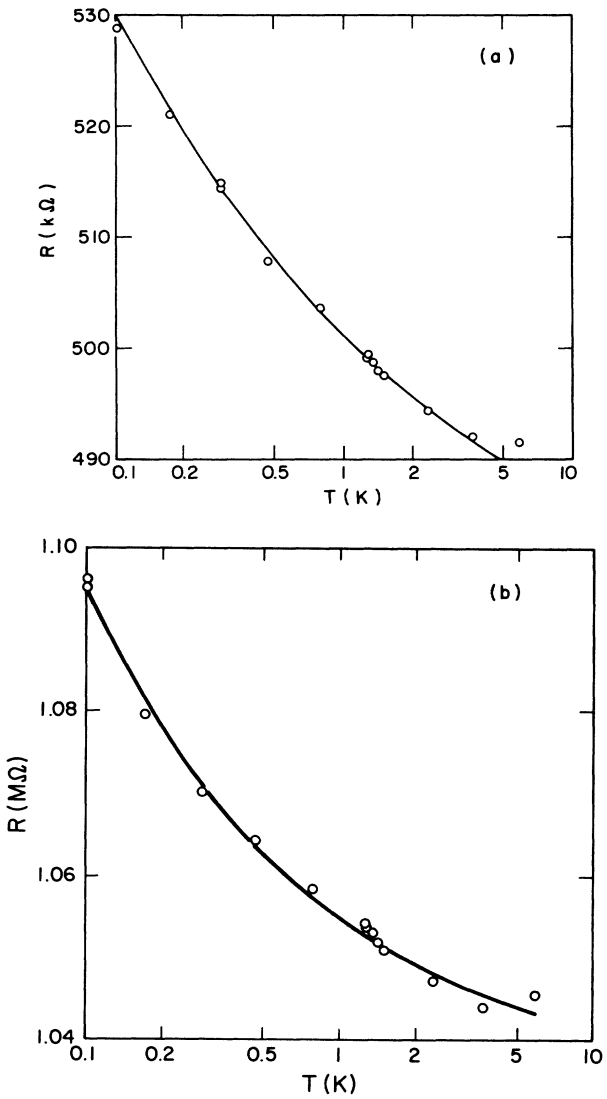


FIG. 12. Contribution of electron-electron interactions to the resistance of (a) the 365 Å wire, and (b) the 525 Å wire, considered in Figs. 10 and 11. The solid lines are least squares fits as described in the text.

electron interaction effects. From the temperature dependence of L_ϕ, its variation with sample dimensions (i.e., wire diameter, or in going from wires to films, etc.), and the results of many previous experiments in this area, we believe that the inelastic scattering in our samples is due to electron-electron scattering in the presence of impurities. This allows us to use the temperature dependence of τ_ϕ to directly determine the effective dimensionality of the scattering process. In this way we find that in some cases the dimensionality of the scattering process is *not* the same as the dimensionality of the contribution of electron-electron interactions to the resistance. This is surprising, since according to the theory these two processes should be controlled by the same length scale, L_T. The proper explanation of these two puzzles is not clear at this time. It is possible that this behavior is peculiar to Bi because of its low carrier concentration. However, such a dependence on carrier concentration is not predicted by the theory.

We should also note that, as discussed earlier, parameters such as D and the elastic mean free path, L_e, are difficult to estimate for our Bi films. For *bulk* Bi with the same resistivity as we have for our films, L_e would be[23] ~ 900 Å. This is comparable to the values of L_ϕ we find from our analysis of the magnetoresistance. If this value of L_e is correct, then our assumption that the electron motion is diffusive, which underlies the entire theoretical picture we have used to interpret our results, would not be appropriate. It has been shown theoretically that the behavior of the magnetoresistance is altered when L_e becomes larger than the film thickness or wire diameter.[37] The predictions of that theory[37] in the low field limit (appropriate for our experiments) are quantitatively very similar to the theory for small L_e which we have used in our analysis, suggesting that an analysis with that theory would yield similar results for L_ϕ, etc., and hence not resolve our discrepancy. Moreover, there are very good reasons to believe that L_e is *not* extremely long in our films. First, our Hall measurements show that, as found by previous experiments and calculations,[24,25] the carrier density in films this thin is much larger than in the bulk. The carrier density in Bi films is typically a factor of 10 or more larger than in the bulk, resulting in a corresponding reduction in the estimate for L_e. This then gives $L_e \lesssim 50$ Å, in accord with our previous estimates. Second, it is very difficult to see how polycrystalline films with grain sizes of ~ 150 Å and rough surfaces could have elastic mean free paths longer than ~ 100 Å. Third, the behavior of our thinnest films and wires (as well as the films studied by previous workers[23,26]) are very well described by the predictions of the theories of localization and interactions; theories which assume diffusive motion. In particular, we find that there is no dimensionality problem in the thinnest films and wires. Any breakdown of, or deviations from, the theoretical predictions because of a large value of L_e should be largest in the thinnest wires and films, but this is definitely *not* the case here. This is indirect but strong evidence that L_e must be smaller than L_ϕ and the film thicknesses and wire diameters for the samples studied. Finally, our "mixed" dimensionality problem can be inferred quite simply and unambiguously from the temperature dependence of the resistance of our wires in different magnetic fields. While our detailed analysis of the magnetoresistance has allowed us to draw quantitative conclusions, it seems likely that the conclusion that the dimensionalities of the inelastic scattering and interactions are different is in a sense "model" independent, and would not be altered if a different theory of the magnetoresistance were in fact appropriate.

Thus, we have no convincing explanation for the puzzling behavior we have observed. One- and two-dimensional electron-electron scattering has, of course, been observed previously in other materials, but most of these experiments have been performed with samples which were clearly in one dimensional regime or the other. That is, they involved films which were much thinner than L_T, etc. for wires, in contrast to our samples in which L_T was only a factor of typically 3–10 (depending on temperature, etc.) larger than the film thickness or wire diameter. It will be very interesting to see if the effects we have observed are present in other materials.

ACKNOWLEDGMENTS

We thank P. A. Lee, P. Muzikar, and J. Rammer for a number of useful comments and suggestions, and J. J. Lin and T. L. Meisenheimer for helpful conversations as well as permission to quote some of their unpublished results. We also thank M. D. Feuer and D. E. Prober for providing the mask from which the wires were made. This work was supported by the National Science Foundation through Grants No. DMR-84-03995 and No. DMR-86-14862.

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[34]Since the phase breaking length, and hence also the phase breaking time, for the films varies as a power of T over the entire temperature range, this implies that the spin-spin scattering is negligible in the films. Since the spin-spin scattering is a bulk property, we will assume that this scattering is negligible in the wires as well.

[35]Studies of very thin Bi films ($\lesssim100$ Å) in our lab have shown that they seem to exhibit two-dimensional electron-electron scattering; i.e., $p\approx1$ (T. L. Meisenheimer, unpublished).

[36]Also visible, but not pronounced, are small, nonmonotonic variations of the resistance at temperatures below about 0.8 K. These are due to universal conductance fluctuations (Ref. 4). These fluctuations become larger as the sample is made shorter, and are discussed elsewhere (Ref. 5). The samples considered in this paper were sufficiently long ($\gtrsim100$ μm) that these fluctuations could be neglected in the analysis discussed here.

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