Effect of uniaxial stress on the electron spin resonance in zinc-blende semiconductors

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We present a study of the effect of uniaxial stress on the electric-dipole spin resonance absorption in zinc-blende semiconductors. We show that previous work in this field omits an important contribution to the transition amplitude, leading to an underestimate of the strength C_2 of the stress-induced spin-orbit coupling in InSb. The necessary correction factor is between 3 and 4. We suggest experimental configurations that would permit an accurate measurement of this and other related material parameters. One of these exploits the possibility of an interference between stress-induced and stress-free spin-flip transition amplitudes. In this way, not only the magnitude of C_2, but also its sign, could be determined.

I. INTRODUCTION

Spin resonance absorption by conduction electrons in elemental semiconductors having the diamond structure is electric-dipole forbidden. However, in compound semiconductors crystallizing in the cubic zinc-blende structure these transitions are allowed. This results because, in the presence of a parity-violating spin-orbit interaction, the Landau levels in the external magnetic field $\mathbf{B}_0$ no longer have a well-defined parity. In the absence of the spin-orbit coupling the stationary states of an electron are the ordinary Landau levels characterized by a quantum number n ($n=0,1,2,\ldots$) and a wave vector k_ζ along the direction of $\mathbf{B}_0$. Each level has a high degeneracy which, classically, can be viewed as the degree of freedom associated with the center of the classical orbit in the plane normal to $\mathbf{B}_0$. Each of these levels is spin split due to the Zeeman interaction between the intrinsic magnetic moment of the electron and $\mathbf{B}_0$. The mixing of Landau levels of opposite parities and spin orientations causes the spin-flip transitions to be allowed in the electric-dipole approximation so that the intensity of the resonance is considerably stronger than would be expected assuming only a magnetic-dipole coupling. The resulting effect is called electric-dipole spin resonance (EDSR). It has been studied extensively both theoretically and experimentally. Work in this field, prior to 1975, has been reviewed by McCombe and Wagner.[1]

More recent magnetooptical studies in InSb by Dobrowolska, Chen, Furdyna, and Rodriguez[2] showed that the intensity of the EDSR displayed striking anisotropy. In particular, under certain circumstances, the intensity experienced large changes under reversal of either the applied magnetic field or of the direction of propagation of the incident radiation. The phenomenon was attributed[3,4] to an interference between electric-dipole and magnetic-dipole amplitudes. The mechanism for making the EDSR allowed in this case results from the parity-violating spin-orbit interaction introduced by Rashba and Sheka.[5,6] This interaction is cubic in the electron wave vector $\mathbf{k}$ and has the form

$$H_A=\delta_0[\sigma_x k_x(k_y^2-k_z^2)+\sigma_y k_y(k_z^2-k_x^2)+\sigma_z k_z(k_x^2-k_y^2)]\ , \tag{1.1}$$

where δ_0 is a parameter measuring the strength of the coupling, σ is the Pauli spin operator, and the subindices x,y,z refer to components along the cubic axes of the crystal.

Kuchar, Meisels, and Kriechbaum[7] and Kriechbaum, Meisels, Kuchar, and Fantner[8] performed far infrared magnetoabsorption experiments in InSb in the Faraday configuration while the sample was subjected to uniaxial stress. The presence of the stress breaks the cubic symmetry of the crystal giving rise to a spin-orbit coupling which is linear in the electron wave vector and in the components of the strain tensor. This leads to an observed enhancement of the intensity of the EDSR line. The authors of Refs. 7 and 8 interpreted their results using the expression

$$H'_\epsilon=\tfrac{2}{3}PC_2\left[\frac{1}{E_g}-\frac{1}{E_g+\Delta}\right]\times[\sigma_x(k_z\epsilon_{zx}-k_y\epsilon_{xy})+\text{c.p.}] \tag{1.2}$$

for the stress-induced spin-orbit coupling, where c.p. denotes cyclic permutations. Here ϵ_{ij} $(i,j=x,y,z)$ are the components of the strain tensor, E_g is the fundamental energy gap, Δ the separation between the $\mathbf{k}=0$ $p_{3/2}$ and $p_{1/2}$ states of the valence band, and P is proportional to the momentum matrix element between states at the top of the valence band and the lowest conduction band. The quantity C_2 is the deformation potential constant defined by Trebin, Rössler, and Ranvaud;[9] Kuchar *et al.*[7] and Kriechbaum *et al.*[8] found the value $|C_2|=1.6\pm0.5$ eV for InSb.

Jagannath and Aggarwal[10] studied the stress-induced generation at the EDSR frequency in InSb. The strength of the spin resonance is proportional to the square of the magnitude of the second-order nonlinear electric susceptibility $\chi^{(2)}$. These authors compared the magnitude of

$\chi^{(2)}$ in a geometrical arrangement in which the electric-dipole amplitude is enhanced under stress with one in which, in the absence of stress, only the magnetic-dipole interaction is effective.[11] In this way Jagannath and Aggarwal deduced the absolute value of the constant C_2. They found $|C_2|=1.0\pm0.3$ eV.

In this paper we discuss the effect of strain on the spin-orbit interaction and the consequent enhancement of the EDSR line. We show that the authors of Ref. 7 omitted a contribution to the electron-photon interaction leading to an underestimate of C_2 by a factor between 3 and 4. References 8 and 10 also omit this contribution. It is also interesting to note that Eq. (1.2) does not represent the most general form of the stress-induced spin-orbit coupling in zinc-blende semiconductors. An additional, independent, term, linear in $\mathbf{k}$ and $\underline{\epsilon}$ is present. However, the second contribution does not appear within the framework of the perturbation analysis of Ref. 9 and, hence, may be regarded as small. We analyze possible geometrical arrangements in which its presence may be significant.

We remark that only the magnitude of C_2 is determined on the basis of the experiments in Refs. 7, 8, and 10. In this connection we investigate the possibility of interference between electric-dipole amplitudes due to the stress-induced spin-orbit coupling and the cubic, stress-free, interaction H_A. We recall that, while the first interaction is linear in $\mathbf{k}$, the second is cubic. Thus, their contributions to the absorption amplitude are independent and linear in B_0, respectively. This allows, in principle, to alter the relative magnitudes of these amplitudes by judicious choices of B_0 and, hence, the appropriate frequency of the incident radiation and of the stress. The interference between the transition amplitudes caused by the stress-induced spin-orbit interaction H'_ϵ and by H_A can be used to determine the relative signs of C_2 and δ_0. Since, in InSb, δ_0 is positive, taking the polarity of the positive [111] direction along the line joining the anion to the cation[12] in the primitive cell, this effect can yield the sign of C_2. An interference with the magnetic-dipole amplitude, similar to that considered in Refs. 3 and 4, is also possible even though less likely to be measurable. It would have the advantage that it is revealed by a simple reversal of either $\mathbf{B}_0$ or of the direction of incidence of the infrared radiation.

The complete stress-induced spin-orbit interaction H_ϵ, linear in $\mathbf{k}$ and $\underline{\epsilon}$, is obtained forming invariants of the from $\mathbf{u}\cdot\boldsymbol{\sigma}$, where the components u_x,u_y,u_z of $\mathbf{u}$ transform according to the irreducible representation[13] Γ_4 of T_d. The components of $\underline{\epsilon}$ generate $\Gamma_1+\Gamma_3+\Gamma_5$. The quantity $\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz}$ belongs to Γ_1, $2\epsilon_{zz}-\epsilon_{xx}-\epsilon_{yy}$ and $3^{1/2}(\epsilon_{xx}-\epsilon_{yy})$ belong to Γ_3, and ϵ_{yz}, ϵ_{zx}, ϵ_{xy} generate Γ_5. The products of components of $\mathbf{k}$ and $\underline{\epsilon}$ belong to $\Gamma_3\times\Gamma_5=\Gamma_4+\Gamma_5$ or to $\Gamma_5\times\Gamma_5=\Gamma_1+\Gamma_3+\Gamma_4+\Gamma_5$. There are, therefore, two independent invariants of the form $\mathbf{u}\cdot\boldsymbol{\sigma}$, one from each set of products just mentioned, yielding the irreducible representation Γ_4. We obtain the appropriate invariants using the Clebsch-Gordan coefficients[14] for the group T_d. These combinations allow us to write

$$H_\epsilon=a[\sigma_x(\epsilon_{zx}k_z-\epsilon_{xy}k_y)+\sigma_y(\epsilon_{xy}k_x-\epsilon_{yz}k_z)+\sigma_z(\epsilon_{yz}k_y-\epsilon_{zx}k_x)]+b[\sigma_xk_x(\epsilon_{yy}-\epsilon_{zz})+\sigma_yk_y(\epsilon_{zz}-\epsilon_{xx})+\sigma_zk_z(\epsilon_{xx}-\epsilon_{yy})]\ . \tag{1.3}$$

This form replaces that in Eq. (1.2). We note that the identification of a with

$$a=\tfrac{2}{3}PC_2\left[\frac{1}{E_g}-\frac{1}{E_g+\Delta}\right] \tag{1.4}$$

is appropriate within the approximation in the model of Ref. 9.

In Sec. II we obtain the contribution of H_ϵ to the spin resonance amplitude under various physical and geometrical arrangements. Section III gives a discussion of the results and their comparison with previous experimental work.

II. INTENSITY OF THE SPIN RESONANCE UNDER STRESS

In this section we provide a complete derivation of the electron spin resonance intensity under arbitrary experimental configurations, i.e., for arbitrary directions of the applied magnetic field and of the uniaxial stress. The transition amplitude consists of three contributions: (i) the ordinary magnetic-dipole matrix element; (ii) the stress-independent electric-dipole amplitude originating from H_A; and (iii) the electric-dipole matrix element brought about by the stress-induced spin-orbit interaction H_ϵ.

In the presence of a static magnetic field $\mathbf{B}_0$ described by a vector potential $\mathbf{A}_0$, the effective mass Hamiltonian H is obtained replacing the wave vector of the electron by the operator $\mathbf{k}=(\mathbf{p}+e\,\mathbf{A}_0/c)/\hbar$. In addition there is also an explicit dependence on $\mathbf{B}_0$. We write $H=H(\mathbf{p}+e\,\mathbf{A}_0/c;\mathbf{B}_0)$. The components of $\mathbf{k}$ obey the commutation relation $\mathbf{k}\times\mathbf{k}=-i(e/\hbar c)\mathbf{B}_0$. In the presence of the alternating field of the incident radiation the time-dependent Hamiltonian is obtained substituting $\mathbf{A}_0$ and $\mathbf{B}_0$ by $\mathbf{A}_0+\mathbf{A}$ and $\mathbf{B}_0+\mathbf{B}$ in H, where $\mathbf{B}=\nabla\times\mathbf{A}$ is the magnetic induction of the incident light.

The electron-photon interaction is

$$H'=\frac{e}{2c}(\mathbf{A}\cdot\mathbf{v}+\mathbf{v}\cdot\mathbf{A})+\mathbf{B}\cdot\frac{\partial H}{\partial\mathbf{B}_0} \tag{2.1}$$

to first order in $\mathbf{A}$ (and $\mathbf{B}$). Here $\mathbf{v}$ is the velocity operator

$$\mathbf{v}=(i\hbar)^{-1}[\mathbf{r},H]=\hbar^{-1}\partial H/\partial\mathbf{k} \tag{2.2}$$

and $\partial H/\partial\mathbf{B}_0$ is the gradient of H with respect to $\mathbf{B}_0$ treating $\mathbf{A}_0$ and $\mathbf{B}_0$ as independent quantities.

Writing $\mathbf{A}$ in the form

$$\mathbf{A}=\mathrm{Re}(A\hat{\boldsymbol{\epsilon}}e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}) \tag{2.3}$$

we find

$$H'=\frac{e}{2c}(AVe^{-i\omega t}+A^*V^\dagger e^{i\omega t})\ , \tag{2.4}$$

where

$$V=\tfrac{1}{2}\{\hat{\epsilon}\cdot\mathbf{v},e^{i\mathbf{q}\cdot\mathbf{r}}\}+\frac{i\omega}{e}\sqrt{\epsilon}(\hat{\mathbf{n}}\times\hat{\epsilon})\cdot\frac{\partial H}{\partial \mathbf{B}_0}e^{i\mathbf{q}\cdot\mathbf{r}} . \tag{2.5}$$

Here, the curly brackets are used to indicate the anticommutator of the operators within and ω is the angular frequency of the incident radiation and $\mathbf{q}=\omega\sqrt{\epsilon}\hat{\mathbf{n}}/c$ its wave vector inside the sample. We denote by ϵ the dielectric constant of the material and by $\hat{\epsilon}$ a unit vector describing the polarization of the incident field. When $\hbar\omega$ matches the energy difference between an initial, occupied, state ψ_i and a higher, unoccupied, state ψ_f, resonant absorption takes place. The corresponding integrated intensity is

$$\alpha\Gamma=\frac{8\pi n_0 e^2}{\hbar\omega c n_r}\,|\langle\psi_f\,|\,V\,|\,\psi_i\rangle|^2 . \tag{2.6}$$

Here α is the absorption coefficient, Γ the full linewidth at half maximum, n_0 the density of electrons, and $n_r=\mathrm{Re}(\epsilon^{1/2})$ the index of refraction. Our task is, thus, to calculate the matrix element $\langle\psi_f\,|\,V\,|\,\psi_i\rangle$ for the spin-flip transition under uniaxial stress.

The effective mass Hamiltonian of the conduction electrons can be written as

$$H=H_0+H_A+H_c+H_\epsilon , \tag{2.7}$$

where

$$H_0=\frac{\hbar^2k^2}{2m^*}+\tfrac{1}{2}g_0\mu_B\mathbf{B}_0\cdot\boldsymbol{\sigma} , \tag{2.8}$$

and

$$H_A=\tfrac{1}{2}\delta_0(\sigma_x\{k_x,k_y^2-k_z^2\}+\text{c.p.}) . \tag{2.9}$$

The quantity H_A of Eq. (1.1) has been rewritten in a symmetrized way to ensure its Hermitian character. This is required because the components of $\mathbf{k}$ do not commute in the presence of the magnetic field. The parameters m^* and g_0 are the electron effective mass and the low-field g factor, respectively; $\mu_B=e\hbar/2mc$ is the Bohr magneton.

The term H_c in Eq. (2.7) results from the fourth-order terms in $\mathbf{k}$ in the effective mass expansion. It is given by

$$\begin{aligned}H_c=&\epsilon_0k^4+\alpha_0(\{k_y^2,k_z^2\}+\{k_z^2,k_x^2\}+\{k_x^2,k_y^2\})\\&+g'\mu_B\boldsymbol{\sigma}\cdot\mathbf{B}_0k^2+g''\mu_B\{\boldsymbol{\sigma}\cdot\mathbf{k},\mathbf{B}_0\cdot\mathbf{k}\}\\&+\gamma_0\mu_B(\sigma_xB_{0x}k_x^2+\sigma_yB_{0y}k_y^2+\sigma_zB_{0z}k_z^2) .\end{aligned} \tag{2.10}$$

Here ϵ_0, α_0, g', g'', and γ_0 are material parameters.[15] This term in the Hamiltonian can be disregarded in the calculation of the transition amplitude of the spin resonance except that it yields a magnetic-field-dependent contribution to the g factor. As such it makes an appearance in the absorption coefficient. The g factor is anisotropic and is defined by the tensor $\underline{g}$ in

$$\frac{\partial H}{\partial \mathbf{B}_0}=\tfrac{1}{2}\mu_B\underline{g}\cdot\boldsymbol{\sigma} . \tag{2.11}$$

We obtain

$$\begin{aligned}\underline{g}=&(g_0+2g'k^2)\underline{1}+2g''\{\mathbf{k},\mathbf{k}\}\\&+2\gamma_0(\hat{\mathbf{x}}\hat{\mathbf{x}}k_x^2+\hat{\mathbf{y}}\hat{\mathbf{y}}k_y^2+\hat{\mathbf{z}}\hat{\mathbf{z}}k_z^2) .\end{aligned} \tag{2.12}$$

The quantity H_ϵ was defined in Eq. (1.3).

In the following we will treat H_A and H_ϵ as perturbations in first order; the effect of H_c is negligible except for its contribution to the g factor.[16] We use a right-handed triad $\hat{\xi},\hat{\eta},\hat{\zeta}$ with $\hat{\zeta}$ parallel to $\mathbf{B}_0$ and the symmetric gauge

$$\mathbf{A}_0=\tfrac{1}{2}B_0(-\eta,\xi,0) . \tag{2.13}$$

Defining the operator

$$a=(iR_0/2^{1/2})(k_\xi-ik_\eta) , \tag{2.14}$$

where $R_0=(\hbar c/eB_0)^{1/2}$ is the Landau length, we find

$$H_0=\hbar\omega_c(a^\dagger a+\tfrac{1}{2})+\frac{p_\zeta^2}{2m^*}+\frac{g_0}{4}\left[\frac{m^*}{m}\right]\hbar\omega_c\sigma_\zeta . \tag{2.15}$$

The quantity $\omega_c=eB_0/m^*c$ is the cyclotron frequency and $[a,a^\dagger]=1$. The eigenstates of H_0 are the well-known Landau states whose eigenvalues are

$$\begin{aligned}E_n^{(s)}(k_\zeta)=&\hbar\omega_c(n+\tfrac{1}{2})+(\hbar^2k_\zeta^2/2m^*)\\&+(g_0m^*/2m)\hbar\omega_cs ,\end{aligned} \tag{2.16}$$

where $n=0,1,2,\ldots$ are the eigenvalues of $a^\dagger a$ and $s=\pm\frac{1}{2}$ those of $\frac{1}{2}\sigma_\zeta$. We consider only states with $k_\zeta=0$ and energies $E_n^{(s)}(k_\zeta=0)=E_n^{(s)}$. The corresponding eigenvectors will be denoted by[17] $|\,n,s\,\rangle$.

The eigenstates of H are written as

$$\psi_{ns}=e^{iS}|\,n,s\,\rangle , \tag{2.17}$$

where S is an Hermitian operator selected in such a manner that $e^{-iS}He^{iS}$ is diagonal in the unperturbed representation $|\,n,s\,\rangle$. We write

$$e^{-iS}He^{iS}|\,n,s\,\rangle=W(n,s)\,|\,n,s\,\rangle , \tag{2.18}$$

where the $W(n,s)$ are the perturbed energy eigenvalues. To find approximate solution for ψ_{ns} and $W(n,s)$ we use the expansion

$$\begin{aligned}e^{-iS}He^{iS}=&H_0+H_A+H_c+H_\epsilon+i[H_0,S]+i[H_A,S]\\&+i[H_c,S]+i[H_\epsilon,S]\\&-\tfrac{1}{2}[[H_0,S],S]+\cdots .\end{aligned} \tag{2.19}$$

To first order in H_A and H_ϵ, neglecting the off-diagonal part of H_c, we require

$$H_A+H_\epsilon+i[H_0,S]=0 . \tag{2.20}$$

Since H_A and H_ϵ have no diagonal matrix element in the representation $\{\,|\,n,s\,\rangle\}$ we can set[18]

$$S=S_A+S_\epsilon \tag{2.21}$$

with

$$\langle n',s' | S_A | n,s \rangle = i(E_{n'}^{(s')} - E_n^{(s)})^{-1} \times \langle n',s' | H_A | n,s \rangle \tag{2.22}$$

and

$$\langle n',s' | S_\epsilon | n,s \rangle = i(E_{n'}^{(s')} - E_n^{(s)})^{-1} \times \langle n',s' | H_\epsilon | n,s \rangle \tag{2.23}$$

when the states $|n,s\rangle$ and $|n',s'\rangle$ are nondegenerate and zero otherwise. The first-order perturbed states are $\psi_{ns} \approx |n,s\rangle + iS|n,s\rangle$. The diagonal part of H_c is taken into account using the actual g factor in $E_n^{(s)}$ instead of g_0.

The amplitude of the transition rate between states $\psi_i = e^{iS}|n,s\rangle$ and $\psi_f = e^{iS}|n',s'\rangle$ is

$$M_{fi} = \langle \psi_f | V | \psi_i \rangle \simeq \langle n',s' | V + i[V,S] | n,s \rangle \ . \tag{2.24}$$

For the spin-flip transition of interest in the present work $n=n'=0$ and $s=\frac{1}{2}$ while $s'=-\frac{1}{2}$. We have taken our initial state as that for which $s=\frac{1}{2}$ because in InSb g is negative.

The expression for V is approximated by

$$V = \hat{\boldsymbol{\epsilon}}\cdot\mathbf{v} + (i\omega\mu_B\sqrt{\epsilon}/2e)(\hat{\mathbf{n}}\times\hat{\boldsymbol{\epsilon}})\cdot\underline{g}\cdot\boldsymbol{\sigma} \ . \tag{2.25}$$

We have expanded the imaginary exponential $e^{i\mathbf{q}\cdot\mathbf{r}}$ in powers of $\mathbf{q}\cdot\mathbf{r}$ and neglected first- and higher-order corrections in the magnetic-dipole coupling. The electric-quadrupole term $i/2\{\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v},\mathbf{q}\cdot\mathbf{r}\}$ is not included because its lowest-order term, namely $i/2\{\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_0,\mathbf{q}\cdot\mathbf{r}\}$ yields zero matrix elements between states having opposite spin orientations. Terms involving $\{\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_A,\mathbf{q}\cdot\mathbf{r}\}$ and $\{\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_\epsilon,\mathbf{q}\cdot\mathbf{r}\}$ give negligible contributions to the spin-flip transitions. In addition, since we are only interested in transitions between states with the same Landau quantum number, we keep only the even-parity terms in $V+i[V,S]$. Thus, we replace this quantity by the even-parity operator

$$\tilde{V} = \hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_A + \hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_\epsilon + i[\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_0,S_A] + i[\hat{\boldsymbol{\epsilon}}\cdot\mathbf{v}_0,S_\epsilon] + (i\omega\mu_B\sqrt{\epsilon}/2e)(\hat{\mathbf{n}}\times\hat{\boldsymbol{\epsilon}})\cdot\underline{g}\cdot\boldsymbol{\sigma} \ . \tag{2.26}$$

We have used the symbols $\mathbf{v}_A$, $\mathbf{v}_\epsilon$, and $\mathbf{v}_0$ for the quantities $\hbar^{-1}\partial H_A/\partial\mathbf{k}$, $\hbar^{-1}\partial H_\epsilon/\partial\mathbf{k}$, and $\hbar^{-1}\partial H_0/\partial\mathbf{k} = (\hbar\mathbf{k})/m^*$, respectively.

In order to obtain $\langle 0,-\frac{1}{2} | \tilde{V} | 0,\frac{1}{2}\rangle$ we must express the operators with respect to the axes $\hat{\xi}$, $\hat{\eta}$, and $\hat{\zeta}$, where $\hat{\zeta}$ is parallel to $\mathbf{B}_0$. Let α,β,γ be the Euler angles[19] of the $(\hat{\xi},\hat{\eta},\hat{\zeta})$ triad with respect to the cubic axes.

For any vector $\mathbf{u}$,

$$u_i = \sum_\kappa u_\kappa R_{i\kappa} \ , \tag{2.27}$$

where $i=x,y,z$, $\kappa=+,-,\zeta$, and

$$u_\pm = u_\xi \pm i u_\eta \ . \tag{2.28}$$

The coefficients $R_{i\kappa}$ are given by

$$R_{x,+} = R_{x,-}^* = \tfrac{1}{2}e^{i\gamma}(\cos\alpha\cos\beta + i\sin\alpha) \ , \tag{2.29}$$

$$R_{x\zeta} = \cos\alpha\sin\beta \ , \tag{2.30}$$

$$R_{y,+} = R_{y,-}^* = \tfrac{1}{2}e^{i\gamma}(\sin\alpha\cos\beta - i\cos\alpha) \ , \tag{2.31}$$

$$R_{y\zeta} = \sin\alpha\sin\beta \ , \tag{2.32}$$

$$R_{z,+} = R_{z,-}^* = -\tfrac{1}{2}e^{i\gamma}\sin\beta \ , \tag{2.33}$$

and

$$R_{z\zeta} = \cos\beta \ . \tag{2.34}$$

The form of H_A is

$$H_A = \tfrac{1}{2}\delta_0 \sum_{\kappa\lambda\mu\nu} \sigma_\kappa \{k_\lambda, k_\mu k_\nu\} \sum_{kij} \epsilon_{kij} R_{i\kappa} R_{i\lambda} R_{j\mu} R_{j\nu} \ . \tag{2.35}$$

where ϵ_{kij} is the Levi-Cività antisymmetric tensor. After some transformations we obtain

$$H_A = \tfrac{1}{2}\delta_0(\sigma_\xi + i\sigma_\eta)\Omega + \tfrac{1}{2}\delta_0(\sigma_\xi - i\sigma_\eta)\Omega^\dagger + \delta_0\sigma_\zeta\Omega_\zeta \ , \tag{2.36}$$

where

$$\Omega = (k_+k_-^2 + k_-^2k_+ - 8k_-k_\zeta^2)F_0 + 2(3k_+k_-k_\zeta + 3k_-k_+k_\zeta - 4k_\zeta^3)F_1 + 2(k_+k_-k_+ - 4k_+k_\zeta^2)F_2 + 2k_+^2k_\zeta F_3 - 10k_-^2k_\zeta F_1^* - 2k_-^3F_2^* \ , \tag{2.37}$$

$$\Omega_\zeta = (k_+k_-k_+ - k_+^2k_- - k_-k_+^2 + 4k_+k_\zeta^2)F_1 + 4k_+^2k_\zeta F_2 - k_+^3F_3 + \text{H.c.} \tag{2.38}$$

with

$$F_0 = -\tfrac{3}{16}i\sin2\alpha\sin2\beta\sin\beta \ , \tag{2.39}$$

$$F_1 = \tfrac{1}{16}e^{i\gamma}[\cos2\alpha\sin2\beta + i\sin2\alpha\sin\beta(3\cos^2\beta - 1)] \ , \tag{2.40}$$

$$F_2 = \tfrac{1}{16}e^{2i\gamma}[2\cos2\alpha\cos2\beta + i\sin2\alpha\cos\beta(3\cos^2\beta - 1)] \ , \tag{2.41}$$

and

$$F_3 = \tfrac{3}{16}e^{3i\gamma}[\cos2\alpha\sin2\beta + i\sin2\alpha\sin\beta(1+\cos^2\beta)] \ . \tag{2.42}$$

The expression for H_ϵ is conveniently written in the form

$$H_\epsilon = \sum_{\mu\nu} D_{\mu\nu} k_\mu \sigma_\nu \ , \tag{2.43}$$

where

$$D_{\mu\nu}=a[\epsilon_{yz}(R_{y\mu}R_{z\nu}-R_{y\nu}R_{z\mu})+\text{c.p.}]+b[R_{x\mu}R_{x\nu}(\epsilon_{yy}-\epsilon_{zz})+\text{c.p.}] \quad (2.44)$$

We define

$$L_+=1+\frac{\hbar^2}{m^*R_0^2}\frac{1}{E_0^{(+)}-E_1^{(-)}}=\frac{|g|m^*/2m}{1+(|g|m^*/2m)}, \quad (2.45)$$

and

$$L_-=1+\frac{\hbar^2}{m^*R_0^2}\frac{1}{E_0^{(-)}-E_1^{(+)}}=-\frac{(|g|m^*/2m)}{1-(|g|m^*/2m)}, \quad (2.46)$$

where $E_n^{(\pm)}$ is short for $E_n^{(s)}$ with $s=\pm\frac{1}{2}$. The matrix element of the velocity operator associated with the stress-free interaction H_A is

$$\langle 0,-\tfrac{1}{2}|\hat{\epsilon}\cdot\mathbf{v}_A+i[\hat{\epsilon}\cdot\mathbf{v}_0,S_A]|0,\tfrac{1}{2}\rangle=\frac{4\delta_0}{\hbar R_0^2}[L_+F_0^*(\hat{\xi}+i\hat{\eta})+L_-F_2^*(\hat{\xi}-i\hat{\eta})+3F_1^*\hat{\zeta}]\cdot\hat{\epsilon}\,. \quad (2.47)$$

In a similar way we obtain

$$\langle 0,-\tfrac{1}{2}|\hat{\epsilon}\cdot\mathbf{v}_\epsilon+i[\hat{\epsilon}\cdot\mathbf{v}_0,S_\epsilon]|0,\tfrac{1}{2}\rangle=2\hbar^{-1}[D_{+-}L_+(\hat{\xi}+i\hat{\eta})+D_{--}L_-(\hat{\xi}-i\hat{\eta})+D_{\zeta-}\hat{\zeta}]\cdot\hat{\epsilon}\,. \quad (2.48)$$

The matrix element of the magnetic dipole interaction is

$$\langle 0,-\tfrac{1}{2}|(i\omega\mu_B\sqrt{\epsilon}/2e)(\hat{\mathbf{n}}\times\hat{\epsilon})\cdot\underline{g}\cdot\boldsymbol{\sigma}|0,\tfrac{1}{2}\rangle=\frac{i\omega\mu_B\sqrt{\epsilon}}{2e}(\hat{\mathbf{n}}\times\hat{\epsilon})\cdot\langle\underline{g}\rangle\cdot(\hat{\xi}+i\hat{\eta})\,, \quad (2.49)$$

where $\langle\underline{g}\rangle=\langle 0|\underline{g}|0\rangle$ is the expectation value of the g tensor in the lowest Landau level. This quantity is given by

$$\langle\underline{g}\rangle=(g_0+2g'R_0^{-2}+\gamma_0R_0^{-2})\underline{1}+2g''R_0^{-2}(\hat{\xi}\hat{\xi}+\hat{\eta}\hat{\eta})-\gamma_0R_0^{-2}(\zeta_x^2\hat{\mathbf{x}}\hat{\mathbf{x}}+\zeta_y^2\hat{\mathbf{y}}\hat{\mathbf{y}}+\zeta_z^2\hat{\mathbf{z}}\hat{\mathbf{z}})\,. \quad (2.50)$$

In the next section we discuss applications of these results to specific geometrical arrangements. We quote the explicit dependence of D_{+-}, D_{--}, and $D_{\zeta-}$ in terms of the Euler angles α,β,γ and the components of the strain tensor deduced from Eq. (2.44). They are

$$D_{+-}=\frac{ia}{2}(\epsilon_{yz}\zeta_x+\epsilon_{zx}\zeta_y+\epsilon_{xy}\zeta_z)-\frac{b}{4}[(\epsilon_{yy}-\epsilon_{zz})\zeta_x^2+(\epsilon_{zz}-\epsilon_{xx})\zeta_y^2+(\epsilon_{xx}-\epsilon_{yy})\zeta_z^2]\,, \quad (2.51)$$

where, of course, $\zeta_x\pm i\zeta_y=e^{\pm i\alpha}\sin\beta$ and $\zeta_z=\cos\beta$;

$$D_{--}=(b/4)e^{-2i\gamma}[(\epsilon_{yy}-\epsilon_{zz})(\cos\alpha\cos\beta-i\sin\alpha)^2+(\epsilon_{zz}-\epsilon_{xx})(\sin\alpha\cos\beta+i\cos\alpha)^2+(\epsilon_{xx}-\epsilon_{yy})\sin^2\beta]\,, \quad (2.52)$$

and

$$D_{\zeta-}=-(ia/2)e^{-i\gamma}[\epsilon_{yz}(\cos\alpha\cos\beta-i\sin\alpha)+\epsilon_{zx}(\sin\alpha\cos\beta+i\cos\alpha)-\epsilon_{xy}\sin\beta]+(b/2)e^{-i\gamma}\sin\beta[(\epsilon_{yy}-\epsilon_{zz})\cos\alpha(\cos\alpha\cos\beta-i\sin\alpha)+(\epsilon_{zz}-\epsilon_{xx})\sin\alpha(\sin\alpha\cos\beta+i\cos\alpha)-(\epsilon_{xx}-\epsilon_{yy})\cos\beta]\,. \quad (2.53)$$

III. APPLICATION TO InSb AND COMPARISON WITH EXPERIMENTS

The purpose of this section is to apply the general theory developed in Sec. II to InSb in specific geometrical configurations and to compare our results with the experimental work reported in Refs. 7, 8, and 10.

In the experiments by Kuchar *et al.*[7] and Kriechbaum *et al.*[8] the sample was subjected to uniaxial stress along [110]. The external magnetic field $\mathbf{B}_0$ and the direction of propagation of the incident radiation (Faraday geometry) were parallel to $[1\bar{1}2]$. The components of the strain tensor, in this case, are

$$\epsilon_{xx}=\epsilon_{yy}=\tfrac{1}{2}T(s_{11}+s_{12}),\quad \epsilon_{zz}=s_{12}T\,,$$

$$\epsilon_{xy}=\tfrac{1}{4}Ts_{44},\quad \epsilon_{yz}=\epsilon_{zx}=0\,,$$

where T is the force per unit area and s_{ij} are the elastic compliance coefficients. We take the Euler angles $\alpha=7\pi/4$, $\gamma=0$, and leave β arbitrary. The angle β is, thus, that between [001] and the direction of $\mathbf{B}_0$. We find

$$\langle 0,-\tfrac{1}{2}|\tilde{V}|0,\tfrac{1}{2}\rangle=\frac{i\cos\beta}{\hbar\sqrt{2}}L_-[bT(s_{11}-s_{12})+\tfrac{1}{2}\delta_0R_0^{-2}(3\cos^2\beta-1)]-(\omega\mu_B\sqrt{\epsilon}/4\sqrt{2}eR_0^2)\times\gamma_0\sin^2\beta(3\cos^2\beta-1) \quad (3.1)$$

for the cyclotron resonance active configuration [CRA: $\hat{\epsilon}_{\text{CRA}}=2^{-1/2}(\hat{\xi}+i\hat{\eta})$]. For the cyclotron resonance inactive geometry [CRI: $\hat{\epsilon}_{\text{CRI}}=2^{-1/2}(\hat{\xi}-i\hat{\eta})$] we obtain

$$\langle 0,-\tfrac{1}{2}\,|\,\tilde{V}\,|\,0,\tfrac{1}{2}\rangle=\frac{i\cos\beta}{2\sqrt{2}\hbar}L_+(aTs_{44}-6\delta_0R_0^{-2}\sin^2\beta)$$
$$-\frac{\omega\mu_B\sqrt{\epsilon}}{\sqrt{2}e}[g_0+2g'R_0^{-2}+2g''R_0^{-2}+\gamma_0R_0^{-2}(1-\tfrac{1}{4}\sin^2\beta-\tfrac{3}{4}\sin^2\beta\cos^2\beta)]\ . \quad (3.2)$$

In Refs. 7 and 8 the contributions due to the stress-free spin-orbit interaction (proportional to δ_0), the magnetic-dipole coupling [terms proportional to μ_B in Eqs. (3.1) and (3.2)] and the strain term proportional to b were not considered. Thus, for incident radiation in the CRI geometry, the transition amplitude would reduce to

$$\frac{i\cos\beta}{2\hbar\sqrt{2}}L_+aTs_{44}\ .$$

The quantity L_+ is given by Eq. (2.45). However, the authors of Refs. 7 and 8 calculated correctly only the second term, namely

$$(\hbar^2/m^*R_0^2)(E_0^{(+)}-E_1^{(-)})^{-1}$$

but omitted the first. The latter owes its origin to the contribution of H_ϵ to the velocity operator while the former is the matrix element of the zero-order velocity due to the first-order correction, proportional to $\underline{\epsilon}$, of the energy eigenstates.[20] We note that for InSb at $B_0=50$ kG

$$(\hbar^2/m^*R_0^2)(E_0^{(+)}-E_1^{(-)})^{-1}=-[1+(\,|\,g\,|\,m^*/2m\,)]^{-1}$$
$$\approx -0.76$$

while $L_+\approx 0.24$. Therefore these authors have underestimated C_2 by a factor of about 3.2.

Since b is expected to be small compared to a, this approach is satisfactory as long as the term in δ_0 is small compared to the stress-dependent term in Eq. (3.2). The magnetic-dipole amplitude in Eq. (3.1), being proportional to γ_0, is three orders of magnitude smaller than the contribution proportional to δ_0. It can, therefore, be neglected. The same conclusion is not permitted for the magnetic-dipole interaction in Eq. (3.2). In fact, it is this term which, in conjunction with the zero-stress spin-orbit coupling, is responsible for the interference effect discussed in Refs. 3, 4, and 12. We recall that this effect was used to determine the sign and magnitude of δ_0 from the magnetotransmission data.[3,12]

Using the numerical parameters for InSb in Refs. 8, 10, and 3, i.e., $P=9.4\times10^{-8}$ eV cm, $E_g=0.2352$ eV, $\Delta=0.803$ eV, $m^*=0.015$ m, $g=-41.6$ at 50 kG, $\delta_0=2.2\times10^{-22}$ eV cm^3 (Ref. 3), $s_{44}=3.18\times10^{-3}$ (kbar)$^{-1}$, and $|\,C_2\,|$ (corrected)= 5 eV, Eq. (3.2) yields

$$\langle 0,-\tfrac{1}{2}\,|\,\tilde{V}\,|\,0,\tfrac{1}{2}\rangle=(\pm 4.2i\cos\beta$$
$$-1.3i\sin^2\beta\cos+0.6)$$
$$\times 10^5\ \text{cm/s}\ , \quad (3.3)$$

for $T=1$ kbar and $B_0=50$ kG. The first two terms in this expression corresponds to the stress-induced and the stress-free spin-orbit couplings and the third to the magnetic-dipole amplitude. In the experiments of Refs. 7 and 8, $\mathbf{B}_0$ was directed along $[1\bar{1}2]$ so that $\cos\beta=(\frac{2}{3})^{1/2}$. The quantities in Eq. (3.3) are, respectively, $\pm 3.4i\times10^5$, $-0.36i\times10^5$, and 0.6×10^5 cm/s. The first is thus 10 times larger than the second and the third, being in quadrature with the first two, does not contribute more than 3% to the intensity of the spin-flip transition. Disregarding the term proportional to b, the contributions to the transition amplitude arising from the CRA polarization is

$$\langle 0,-\tfrac{1}{2}\,|\,\tilde{V}\,|\,0,\tfrac{1}{2}\rangle\approx -i0.33\times10^5\ \text{cm/s}$$

under the same conditions as above.[21] We note that this term is, like the similar contribution proportional to δ_0 in Eq. (3.3), one order of magnitude smaller than the stress-induced transition amplitude. Thus we conclude that, taking proper consideration of the correction (value of L_+) noted above, the value $|\,C_2\,|\approx 5$ eV is meaningful.

In the experiments of Jagannath and Aggarwal[10] the stress and the magnetic field $\mathbf{B}_0$ were directed along the [111] axis and the wave vectors of the incident and emitted beams were along $[1\bar{1}0]$. In these circumstances $\epsilon_{xx}=\epsilon_{yy}=\epsilon_{zz}=\frac{1}{3}(s_{11}+2s_{12})T$, $\epsilon_{yz}=\epsilon_{zx}=\epsilon_{xy}=(s_{44}T/6)$. In the ordinary Voigt configuration ($\hat{\epsilon}\|\hat{\zeta}$) we find

$$\langle 0,-\tfrac{1}{2}\,|\,\tilde{V}\,|\,0,\tfrac{1}{2}\rangle=(i\omega\mu_B\sqrt{\epsilon}/2e)\hat{\xi}\cdot\langle\underline{g}\rangle\cdot\hat{\xi}\ , \quad (3.4)$$

i.e., the spin-flip transition is due entirely to the magnetic-dipole radiation. When $\hat{\epsilon}\|\hat{\xi}\|[\bar{1}\bar{1}2]$ we find

$$\langle 0,-\tfrac{1}{2}\,|\,\tilde{V}\,|\,0,\tfrac{1}{2}\rangle=(i/\hbar\sqrt{3})L_+(\delta_0R_0^{-2}+\tfrac{1}{2}aTs_{44})\ . \quad (3.5)$$

We estimate that at $B_0=62$ kG and $T=2.24$ kbar, $\delta_0R_0^{-2}\approx 2\times10^{-10}$ eV cm, while $\frac{1}{2}\,|\,a\,|\,Ts_{44}\approx 3\times10^{-9}$ eV cm. Thus the term in δ_0, through interference with the stress term, contributes less than 15% to the transition probability in this experiment. We note that when $T=0$, the contribution of the stress-free spin-orbit coupling to the absorption coefficient is less than 1% of that when $T=2.24$ kbar. In Ref. 10 only the second term in L_+ was taken into account so that the value of C_2 must be corrected by the factor $(L_+-1)/L_+$. At $B_0=62$ kG and $T=2.24$ kbar, $g=-36$ which yields $L_+=0.21$. Therefore, the corrected value of $|\,C_2\,|$ is 3.8 eV.

As a final example we mention a geometrical arrangement in the extraordinary Voigt (EV) configuration ($\hat{\epsilon}$ and $\hat{\mathbf{n}}$ perpendicular to $\mathbf{B}_0$) which could be useful in the determination of both the magnitude and the sign of C_2. We take $\hat{\epsilon}=\hat{\xi}$, $\hat{\mathbf{n}}=\hat{\eta}$, $\mathbf{B}_0=B_0\hat{\zeta}$ parallel to [001] and the stress along [111]. The EV configuration is chosen because of the absence[22] of the magnetic-dipole interaction. The selection of the uniaxial stress along [111] eliminates the term proportional to b in H_ϵ. Taking $\alpha=\beta=0$ we find

$$\langle 0,-\tfrac{1}{2} \mid \tilde{V} \mid 0,\tfrac{1}{2} \rangle = (\delta_0/2\hbar R_0^2) L_- e^{-2i\gamma} + (ias_{44} T L_+/6\hbar) \; . \qquad (3.6)$$

An interference between the two amplitudes in Eq. (3.6) is strongest when $\gamma=\pi/4$ or $3\pi/4$. Those directions correspond to incident radiation propagating along $[\bar{1}10]$ and $[\bar{1}\,\bar{1}\,0]$, respectively. We note that L_+ and L_- have different signs so that constructive interference when $\hat{\mathbf{n}} \| [\bar{1}10]$ would imply that $C_2>0$. We now estimate the values of the quantities in Eq. (3.6) for $B_0=50$ kG and $T=1$ kbar taking $|C_2|=4.4$ eV, $g=-40$, $\delta_0=2.2\times10^{-22}$ eV cm^3, and $m^*=0.015$ m. We find $(\delta_0 L_-/2\hbar R_0^2)\approx-0.54\times10^5$ cm/s and $(|a|\,T L_+ s_{44}/6\hbar)\approx1.7\times10^5$ cm/s. Because of the comparable magnitudes of these quantities we expect the interference effect to be measurable.

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[16]This amounts to keeping the diagonal part of H_c in the representation generated by the eigenstates of H_0 and neglecting its nondiagonal matrix elements.

[17]These states are highly degenerate and an additional quantum number $n_b=b^\dagger b$ with $b=(iR_0/2^{1/2})[k_\xi+ik_\eta-iR_0^{-2}(\xi+i\eta)]$ would be required for their complete identification. However, since states with equal n and s but different values of n_b are, for our purposes, simple replicas of one another, we need not specify n_b.

[18]The degeneracy with respect to n_b presents no difficulty because $[H_A,n_b]=[H_\epsilon,n_b]=0$.

[19]The Euler angles are defined as follows. (i) α is the angle between $\hat{\mathbf{y}}$ and the line of nodes $\hat{\mathbf{z}}\times\hat{\zeta}$; (ii) β is the angle formed by $\hat{\mathbf{z}}$ and $\hat{\zeta}$; (iii) γ is the angle between the (ξ,ζ) and (ζ,z) planes. Clearly $0\le\alpha<2\pi$, $0\le\beta\le\pi$, and $0\le\gamma<2\pi$.

[20]Referring to Eq. (2.26) the omission comes from overlooking the term $\hat{\epsilon}\cdot\mathbf{v}_\epsilon$ and calculating only the contribution from $i[\hat{\epsilon}\cdot\mathbf{v}_0,S_\epsilon]$.

[21]The CRA configuration can be used to determine the parameter b in this type of experiment by setting $\mathbf{B}_0$ along $[1\bar{1}1]$. In that case the amplitude (3.1) reduces to $(i/\hbar)(L_-/\sqrt{6})bT(s_{11}-s_{12})$.

[22]Strictly speaking there is a small magnetic-dipole contribution proportional to γ_0. This is negligible in all circumstances.