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Simulation of dielectric failure by means of resistor-diode random lattices

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It is possible to obtain an experimental simulation of electrical failures in studying the onset voltage of the nonlinearity of a two-dimensional random lattice of resistors and diodes. If there are more diodes than resistors, the onset voltage V_b is identical to the breakdown voltage of an insulator made of resistors and insulating elements which can be broken. Near p_c, V_b goes to zero with an exponent equal to 1.1 ± 0.3 in agreement with computer simulation. If now there are more resistors than diodes, the onset voltage $V_{\rm NL}$ is similar to the "fusing" current of a conductor made of conductive parts that behave like fuses and insulating parts (fuse model). Near p_c, $V_{\rm NL}$ goes to zero with an exponent equal to 0.5 ± 0.2 and it is shown to be larger than the exponent of the fuse model equal to $\nu - 1 = 0.33$ in two dimensions.

The problems of electric failure are of great technological importance. Recently, these problems were the subject of intensive theoretical work.[1-9] In this paper, I want to show that it is possible to simulate them by means of a network of resistors and diodes as nonlinear components.

As is well known, two main approaches have been proposed for possible electric failures. In the dielectric model[4-6] the system to be studied is a metal-loaded insulator. The discrete version of this model is a lattice of resistors and insulator elements distributed at random at the lattice bonds. An insulator element cannot stand a voltage larger than $V_{\rm th}$, otherwise there is a breakdown, i.e., the insulator becomes conducting. The problem is to find the breakdown voltage V_b of the whole lattice as a function of p, the resistor concentration. In the fuse model,[1] the system is an insulator-loaded metal. The lattice version is a random mixture of insulator elements and resistors which can be fused (i.e., the resistor becomes an insulator) if a current larger than $i_{\rm th}$ flows through one of them. The problem is to find the whole current flowing through the system for which it is fused, for various p.

In the dielectric model, the insulator elements form a continuous path whereas it is the inverse in the fuse model. In two dimensions (2D), these two situations correspond to the two sides of the percolation threshold p_c, with different conditions. In the dielectric model $(p < p_c)$ the insulator elements can be broken and in the fuse model $(p > p_c)$ the resistors can be fused.

I. THE DIELECTRIC MODEL

In the dielectric model it is assumed that once an insulator element is broken it becomes a conductor. One can imagine two possibilities for this insulator-conductor transformation. In the first possibility, the conductor obtained from a broken down insulator is identical to a resistor. After the breakdown of the first element, a complete breakdown occurs through a cascade phenomenon. In the second possibility, the insulator becomes a superconductor with the voltage kept constant and equal to the threshold $V_{\rm th}$. Once one element is broken, it is necessary to increase the external voltage until a continuous path of broken elements is formed. The solution of this problem, $V_b(p)$, is identical to that of the "shortest path"[10] or the "minimum insulator gap,"[7] i.e., the minimum number of bonds to be added to get a continuous path. Near p_c, the breakdown voltage of these two possibilities goes to zero as $(p_c - p)^\nu$ where ν is the correlation length ξ exponent, but with different amplitudes.

In my experiment, I simulated the second possibility in taking as insulator elements light emitting diodes. At low voltage $(V < V_{\rm th})$, the resistance of one diode is much larger $\sim 10^7 \Omega$ than that of one resistor $(300\ \Omega)$ such that the voltage distribution is almost identical to that given by elements with an infinite resistance. At the threshold $(V_{\rm th} = 1.15$ V) there is a sudden and very large increase of the current, from 10^{-7} to 10^{-3} A, when the voltage is changes from $V_{\rm th}$ to $V_{\rm th}+0.1$ V. Thus up to 10^{-3} A, a diode represents very well the behavior of the insulator elements in the second possibility.

A square lattice of 20×20 was built and resistors and diodes were randomly distributed in the lattice bonds. The experiment consists in measuring the curve $I(V)$ of the whole lattice for $p < p_c$ and in determining the voltage V_b for which there is a sudden increase in the current. At the same time, since the diodes emit light, it is possible to observe the breakdown path. In Figs. 1 and 2, V_b is presented as a function of p, in a regular plot (Fig. 1) and in a log-log plot (Fig. 2).

Near $p = 0$, it is possible to show that the mean breakdown voltage is given by $V_b = V_b(0)(1-p)$. In the case of the most probable state, the mean number of resistors in a lattice of size L in one column (i.e., in the current direction) is $n/2L$ if n is the absolute number of resistors in the lattice. (There are also $n/2$ resistors distributed in the L rows, but they do not influence the breakdown voltage in this dilute limit.) The breakdown voltage in $V_b = V_{\rm th}(L - n/2L) = V_{\rm th}L(1 - n/2L^2)$. Recalling that $V_{\rm th}L = V_b(0)$ and $p = n/2L^2$, one gets $V_b = V_b(0)(1-p)$.

However, I do not get this result whereas it is found by Manna and Chakrabarty.[7] The difference between my measurements and their results is that in their computer simulation they took the mean value after 1000

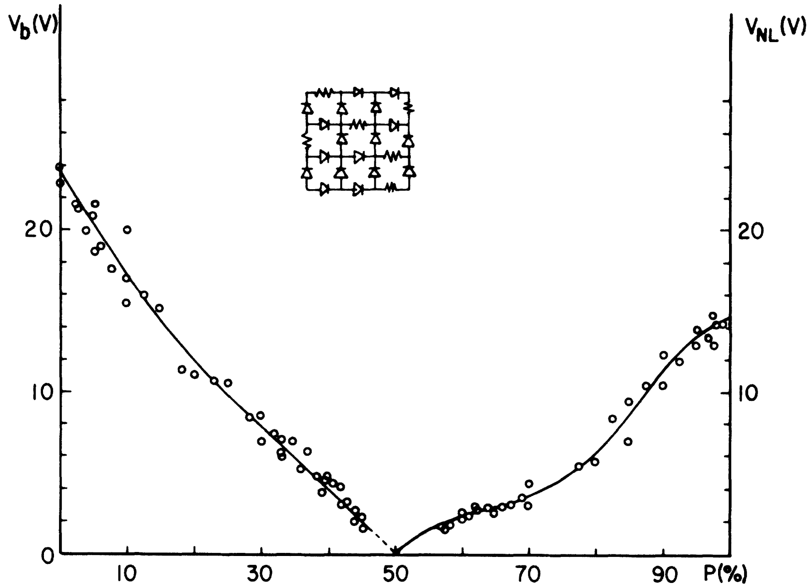

FIG. 1. Variations of the threshold voltages giving the onset of the nonlinearity, V_b and V_{NL} vs p, in the random resistor-diode lattice.

configurations. The measurements have been repeated only 5 times. This means that each configuration was far from the mean one. This result emphasizes the observation made by Duxbury, Leath, and Beale[5] that the breakdown is controlled by the "most extreme configurations."

Near p_c, the exponent of V_b is found to be equal to 1.1 ± 0.3 and it is near the exact value of $\nu = \frac{4}{3}$ (in 2D). Values near $\frac{4}{3}$ were obtained by Manna and Chakrabarty[6] and by Beale and Duxbury[9] by means of computer simulation on a sample of size $L = 100$. Thus, one can explain the lower value found here of 1.1 by the smaller size of the sample. This is supported by the results of Manna and Chakrabarty who found an exponent of 1 ± 0.05 in a 25×25 sample. It is important to note that the same exponent is found, although in the measurements directed elements were used. It is because the breakdown path is not very tortuous (see Fig. 3.) (at least not too near to p_c, as in such a small sample) but branched, such that paths with backward direction are not very likely.

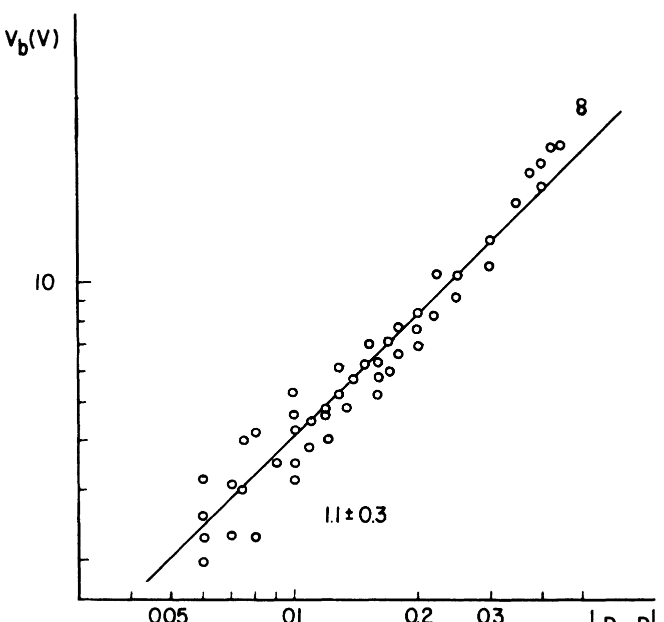

FIG. 2. The threshold voltage V_b vs $(p_c - p)$ $(p \leq p_c)$.

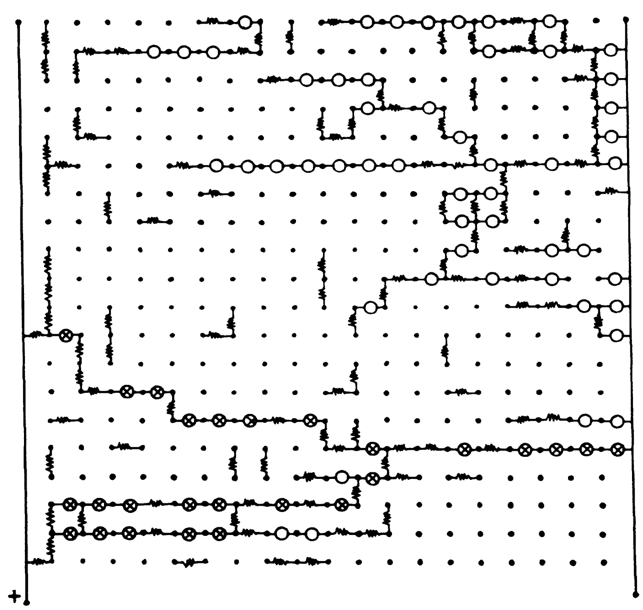

FIG. 3. Breakdown path and broken insulator elements for $p = 0.15$. The nonbroken elements are not shown. The circles are the broken elements given by the computer simulation and the crosses form the breakdown path experimentally determined from the light emitting diodes.

Computer simulations of our system were made in calculating the voltage distribution of a 20×20 square lattice with two types of resistors. Those which simulate the resistors of the experiment have a low resistance and those which simulate the diodes a much larger resistance (ratio of the resistance 10^6). One looks for the first insulator with a voltage of 1 V and this voltage is kept constant in the subsequent steps. The voltage is now increased until a second insulator with a voltage equal to 1 V appears and this voltage is kept constant and so on. The calculation is stopped when a continuous path of broken elements (i.e., with the voltage across them kept constant) is formed between the two electrodes. In Fig. 3, the results of the computer simulation and those of the measurements are given. One remarks that all the crosses (results of the measurements) coincide with the circles (results of the simulation). But there are many other broken elements given by the simulation that it is not possible to observe visually. This is because the current flowing in these diodes is too low for the diode to emit light. However, this observation explains why for a single diode the curve $I(V)$ is perfectly linear for $V<V_{\rm th}$ but not for the whole lattice for $V_{\rm ext}<V_b$. The computer simulation gives, in the case of Fig. 3 ($p=0.15$), $V_b=13$ V since there are 13 diodes in the shortest path and this is in agreement with the measurement of $V_b=15.3\ {\rm V}\simeq1.15\times13$.

II. THE FUSE MODEL

Now, consider the situation for $p>p_c$ where the system is conducting and we shall see that the results are very similar to those of the fuse model. Here also the measurement consists of measuring $I(V)$ of the whole lattice. The beginning of this curve is linear up to $V_{\rm NL}$. Above $V_{\rm NL}$ a departure from linearity is observed. In Figs. 1 and 4, $V_{\rm NL}$ as a function of p is given in a regular plot and in log-log plot. The behavior of $V_{\rm NL}$ is very different from that of V_b, in particular, $V_{\rm NL}$ goes to zero with an exponent smaller than 1. The estimate of this exponent is 0.5 ± 0.2. For $p\to1$, $V_{\rm NL}/L$ is equal to 0.75 V or $0.65V_{\rm th}$.

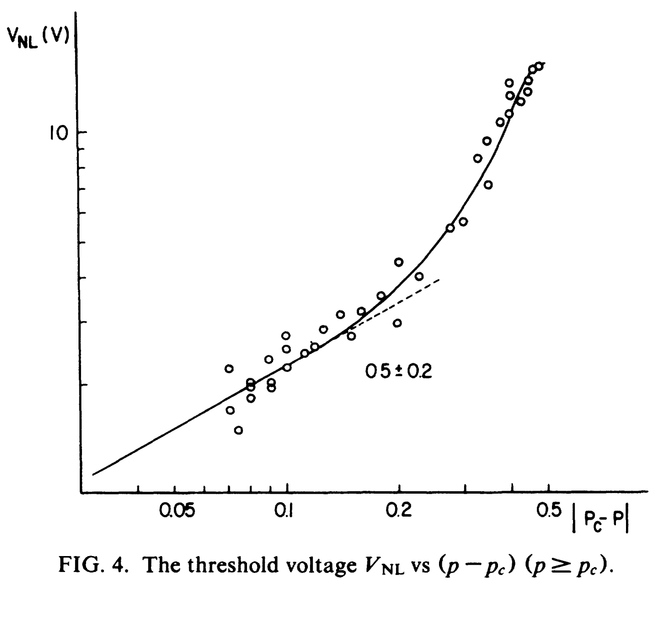

FIG. 4. The threshold voltage $V_{\rm NL}$ vs $(p-p_c)$ $(p\geq p_c)$.

One can easily explain this last value of $V_{\rm NL}/L$ if one considers the resistor lattice less one bond AB in the direction of the current and asking for the external voltage for which V_{AB} is equal to $V_{\rm th}$. At this voltage, the nonlinearity begins. Since for $V<V_{\rm th}$ the resistance of a diode is much larger than that of a resistor, it is safe to admit in this calculation that a diode resistance is infinite. The voltage and the current distribution can be calculated, as proposed by Duxbury *et al.*,[5] in considering our problem as the superposition of two problems. The first consists of the lattice without any defects (i.e., diodes) and the current in the bond AB is i and the voltage $V_{AB}=Ri=V_{\rm ext}/L$. The second problem is that of the lattice where the bond AB is replaced by a voltage source giving a current $-i$ in AB. In this second problem the voltage across AB is $Ri/2$, since the equivalent resistance of the lattice between AB is $R/2$. Thus, in the real problem the voltage across AB is $Ri+Ri/2$ or $V_{AB}=3Ri/2=3V_{\rm ext}/2L$. If $V_{AB}=V_{\rm th}$, $V_{\rm ext}/L=2V_{\rm th}/3$ in excellent agreement with the measurement.

To explain the value of the exponent, it is possible to use the node-blob-link (NBL) model of the backbone. The important point is that the voltage distribution in the sample is controlled by the current distribution in the backbone. The largest voltage is found where the current is the largest, in the singly connected bonds (SCB's). We assume that the first diode reaching the voltage $V_{\rm th}$ is that very near a SCB. The voltage across a link of length ξ is $V_\xi\sim\xi V_{\rm ext}$ ($V_{\rm ext}$ is the voltage applied to the whole lattice). If we call V_1 the voltage across a SCB, one has $V_\xi=L_1V_1$, where L_1 is the mean number of SCB in a link. Recall that $L_1\sim(p-p_c)^{-1}$. Thus, one has

$$V_1\sim\frac{\xi}{L_1}V_{\rm ext}\,. \tag{1}$$

Because a diode cannot be directly in parallel on a SCB one has $V(\text{diode})>V_1$. When $V(\text{diode})=V_{\rm th}$, one can write

$$V_{\rm th}>\frac{\xi}{L_1}V_{\rm ext}\,, \tag{2}$$

or

$$V_{\rm ext}=V_{\rm NL}<\frac{L_1}{\xi}V_{\rm th}\sim(p-p_c)^{-1+\nu}\,. \tag{3}$$

Thus the exponent of $V_{\rm NL}$ is larger than $\nu-1=0.33$ in agreement with the measurements.

The results $V_{\rm NL}(p\to1)\to\frac{2}{3}V_{\rm th}$ and the exponent slightly larger than 0.33 are reminiscent to those concerning the threshold current in the fuse model.[5,8] If $(p\to1)$, $I_f\to\pi/4i_{\rm th}$ (I_f is the current necessary to fuse the whole lattice in the fuse model) and the exponent is exactly $\nu-1=0.33$ in 2D. The difference stands in that the diode which is broken is not directly in parallel with the bond with the largest current but separated by horizontal resistors. Thus, $I_f/i_{\rm th}>V_{\rm NL}/V_{\rm th}$ at $p\to1$ and near p_c, $V_{\rm NL}<(p-p_c)^{\nu-1}$. Nevertheless, the qualitative behavior of the resistor diode system is the same as that of the fuse model.

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