Uniaxial superconducting particle in intermediate magnetic fields

V. G. Kogan
Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011
(Received 2 May 1988)

The problem of an anisotropic uniaxial superconducting particle of a general ellipsoidal form in the magnetic field H_0 such that $H_{c1} \ll H_0 \ll H_{c2}$, is considered. It is shown—in agreement with experimental data—that particles of high-T_c materials at $T < T_c$ must orient with their c axis normal to the applied field. The torque acting on a particle is evaluated and shown to be independent of the ellipsoid shape. The torque is associated with the magnetization component perpendicular to the field direction.

All known high-T_c superconductors are strongly anisotropic. Although most of them are orthorhombic, the anisotropy between the $\hat{\mathbf{c}}$ crystal direction (long side of the primitive cell) and either $\hat{\mathbf{a}}$ or $\hat{\mathbf{b}}$ (in the Cu-O plane) is much larger than a relatively small "in-plane" anisotropy. Therefore, to describe major anisotropy effects, one can consider these materials as being uniaxial. It was observed in studies of the behavior of fine powders in a magnetic field, that in the superconducting state ($T < T_c$) the particles orient so that their $\hat{\mathbf{c}}$ axis is normal to the external field $\mathbf{H}_0$.[1,2] [In the normal phase ($T > T_c$) the $\hat{\mathbf{c}}$ axes align parallel to $\mathbf{H}_0$.[3] This effect is not considered in the present communication.]

To approach theoretically the orientation effect, one should be able to compare the proper thermodynamic potential of a particle in an external field $\mathbf{H}_0$ for different particle orientations with respect to $\mathbf{H}_0$. The free energy of a uniaxial superconductor has been discussed in Ref. 4 for the field domain near the upper critical field H_{c2}; this, however, is not the region in which the observations[1,2] have been made. There exists another field domain, where the free energy can be evaluated in materials of interest: $H_{c1} \ll H \ll H_{c2}$ (H_{c1} is the lower critical field). This region is quite broad in the high-T_c superconductors because their penetration depth λ ($\sim 10^3$ Å) is much longer than the coherence length ξ (~ 10 Å). In fact, this is the field domain in which the experiments[1,2] have been done. The magnetization in the intermediate fields is small in comparison to the applied field (see, e.g., Ref. 5). In particular, the demagnetization effects in this field region are weak; that simplifies the situation considerably.

To obtain the free energy density F in this domain one follows the procedure outlined in Ref. 5 for isotropic superconductors. Using the London equations (which suffice in the field region of interest at any temperature) one can evaluate F of the flux line lattice. The anisotropic version of the approach is discussed in Refs. 6 and 7. Although the anisotropic London equations are difficult to solve in real space, one can utilize the periodicity of the flux line lattice and use the Fourier transform to express F in terms of a sum over the reciprocal lattice. Actual derivation in the field domain of interest for the uniaxial case is given in Ref. 8:

$$8\pi F = B^2 + 2H^*(m_1 B_X^2 + m_3 B_Z^2)^{1/2}, \qquad 8\pi\lambda^2 H^* = \phi_0 \ln(\eta H_{c2}/B). \tag{1}$$

Here B is the magnetic induction, λ is the average penetration depth (see the definition in Ref. 6), and η is a number of the order unity depending upon the particular flux line lattice structure. The term $\ln(\eta H_{c2}/B)$ comes from the cutoff at the reciprocal-lattice vectors on the order ξ^{-1}, a common way to treat divergent London contributions at vortex axes. The direction $\hat{\mathbf{Z}}$ coincides with the $\hat{\mathbf{c}}$ axis of the single-crystal particle, as is shown in Fig. 1; axis $\hat{\mathbf{X}}$ is chosen as the intersection of the $\hat{\mathbf{Z}}$-$\mathbf{B}$ plane with the basal plane normal to $\hat{\mathbf{Z}}$. The "effective masses" m_1 and m_3 are the components of the mass tensor m_{ik} along the principal crystal directions $\hat{\mathbf{X}}$ and $\hat{\mathbf{Z}}$, respectively (e.g., for $Y_1Ba_2Cu_3O_7$ the ratio m_3/m_1, estimated from the ratio of H_{c2}'s in two principal crystal directions, is in the range 25–90). The field H^* is of the order H_{c1} and, therefore, small with respect to the external field H_0 (as well as to the internal fields B and H). Then the correction to $B^2/8\pi$ in the free energy is small; one can, therefore, replace B under the log sign on Eq. (1) with H_0, and develop a theory linear in the parameter $H^*/H_0 \ll 1$.

To this end one first obtains the thermodynamic field $\mathbf{H} = 4\pi\partial F/\partial \mathbf{B}$ inside the particle:

$$H_X = B_X + H^* m_1 m(\theta)^{-1/2}\sin\theta, \qquad H_Z = B_Z + H^* m_3 m(\theta)^{-1/2}\cos\theta, \tag{2}$$

where

$$m(\theta) = m_1 \sin^2\theta + m_3 \cos^2\theta, \tag{3}$$

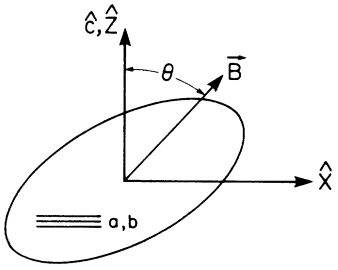

FIG. 1. The axis $\hat{\mathbf{Z}}$ coincides with the principal crystal direction $\hat{\mathbf{c}}$; $\hat{\mathbf{X}}$ is in a-b crystal plane. The angle θ is between the vortex axes (or induction $\hat{\mathbf{B}}$) and $\hat{\mathbf{Z}}$. The ellipsoidal particle (shown schematically as an ellipse) can be oriented arbitrarily with respect to the crystal frame.

and θ is the angle between $\hat{\mathbf{Z}}$ and $\mathbf{B}$. In particular, Eqs. (2) show that the magnetization $\mathbf{M}=(\mathbf{B}-\mathbf{H})/4\pi$ is of the order H^*, i.e., M is much smaller than both B and H.

To relate the fields $\mathbf{H}$ and $\mathbf{B}$ to the applied field $\mathbf{H}_0$, one should make an assumption concerning the particle shape. Recall that for an arbitrary shape the internal fields are not uniform. The only shape, for which the inside fields are homogeneous, is that of a general ellipsoid.[9] In this case

$$H_{0\alpha}=H_\alpha+4\pi N_{\alpha\beta}M_\beta , \tag{4}$$

where $N_{\alpha\beta}$ is the dimagnetization tensor (note that the principal axes of $N_{\alpha\beta}$ do not necessarily coincide with those of the crystal).

The thermodynamic potential $\tilde{F}$ of a finite body with respect to the applied field $\mathbf{H}_0$ as an independent variable ($\delta\tilde{F}=-\mathbf{M}\cdot\delta\mathbf{H}_0$) for an arbitrary relation $\mathbf{H}=\mathbf{H}(\mathbf{B})$ reads[9]

$$\tilde{F}=F-\mathbf{H}\cdot\mathbf{B}/8\pi-\mathbf{M}\cdot\mathbf{H}_0/2 . \tag{5}$$

In a given applied field $\mathbf{H}_0$, the minimum of $\tilde{F}$ corresponds to the thermodynamic equilibrium of the ellipsoidal body [for an arbitrary shape, the integral of (5) over the body volume has the minimum property].

Substituting (1) in (5) one notes that the quantity

$$[B^2-\mathbf{H}\cdot\mathbf{B}-(\mathbf{B}-\mathbf{H})\cdot\mathbf{H}_0]/8\pi=(\mathbf{B}-\mathbf{H}_0)\cdot(\mathbf{B}-\mathbf{H})/8\pi$$

is proportional to H^{*2} and, therefore, it should be neglected. Indeed, $|\mathbf{B}-\mathbf{H}|\propto H^*$ as is seen from Eqs. (2); to see that the factor $\mathbf{B}-\mathbf{H}_0$ has the same property, rewrite Eq. (4) in the form $B_\alpha-H_{0\alpha}=4\pi(\delta_{\alpha\beta}-N_{\alpha\beta})M_\beta$; recall that all N's are smaller than unity and $M\propto H^*$. Thus, the only contribution to $\tilde{F}$ linear in H^* comes from the second term in Eq. (1), in which, for the same reason, one can replace B with H_0:

$$\tilde{F}=H_0H^*\sqrt{m(\theta)}/4\pi \tag{6}$$

with $m(\theta)$ given in Eq. (3).

The angle θ in this expression should be taken in the zero order because $\tilde{F}$ is already proportional to the small field H^*; Eqs. (2) and (4) show that, in the lowest order, the direction of $\mathbf{B}$ (i.e., θ) coincides with that of $\mathbf{H}_0$. Thus, expression (6) (multiplied by the particle volume V) represents, in the first order, the particle "potential energy" in the field $\mathbf{H}_0$, inclined at an angle θ to the $\hat{\mathbf{c}}$ axis.

The torque $T_\theta=-V\partial\tilde{F}/\partial\theta$, experienced by the particle, is

$$T_\theta=V\frac{H_0H^*}{4\pi}\frac{(m_3-m_1)\sin\theta\cos\theta}{(m_1\sin^2\theta+m_3\cos^2\theta)^{1/2}} . \tag{7}$$

Thus, the torque vanishes if $\mathbf{H}_0$ is oriented along one of the principal crystal directions $\theta=0$ or $\pi/2$. The second derivative $\tilde{F}''(\theta)$ is easily evaluated. One obtains $\tilde{F}''(0)<0$, while $\tilde{F}''(\pi/2)>0$, i.e., the equilibrium state corresponds to the $\hat{\mathbf{c}}$ axis normal to the applied field.

It is worth noting that, in the approximation considered, which is quite good in the domain $H_{c1}\ll H_0\ll H_{c2}$, both the energy (6) and the torque (7) are shape independent. This explains why the orientation effects in the experiments of Refs. 1 and 2 were clearly seen despite the fact that there was no control over particle shape in the powders examined. The physical reason for the torque existence is that the magnetization has a component perpendicular to the applied field, an intrinsic feature of anisotropic superconductors.[4,8] To the best of the author's knowledge, this component has never been directly measured. The torque experiment, therefore, provides a possibility of this measurement. Also, such an experiment will provide an independent way to extract the anisotropy ratio m_3/m_1, the quantity estimated usually from the difficult measurements of H_{c2} and H_{c1}.

Only the orientation effects due to intrinsic anisotropy have been considered in this communication. The actual crystalline grains of orthorhombic high-T_c compounds are usually heavily twinned. The twinning is believed to be responsible for inhomogeneity of the superconducting phase even in zero field[2] due to the enhancement of superconductivity close to the twin boundaries.[10-13] (Another point of view is given in Ref. 14 where the twin boundaries are considered as pair breakers; that again results in a nonuniform superconducting order parameter.) This inhomogeneity may result in yet another orientation effect, in which, in addition to the tendency of $\hat{\mathbf{c}}$ axes to be perpendicular to $\mathbf{H}_0$, the particle orients with its twinning planes parallel to the applied field.[2]

This work was supported by the Office of Basic Energy Sciences of the Department of Energy and by the Electrical Power Research Institute.

[1] J. M. Tranquada, A. I. Goldman, A. R. Moodenbaugh, G. Shirane, S. K. Sinha, D. Vakin, and A. J. Jacobson, Phys. Rev. B **37**, 519 (1988).

[2] S. A. Solin, N. Garsia, S. Vieira, and M. Hortal, Phys. Rev. Lett. **60**, 744 (1988).

[3] D. E. Farrell, B. S. Chandrasekhar, M. R. DeGuire, M. M. Fang, V. G. Kogan, J. R. Clem, and D. K. Finnemore, Phys. Rev. B **36**, 4025 (1987).

[4] V. G. Kogan and J. R. Clem, Phys. Rev. B **24**, 2497 (1981).

[5] P. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), p. 69.

[6] V. G. Kogan, Phys. Rev. B **24**, 1572 (1981).

[7] A. V. Balatskii, L. I. Burlachkov, and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **90**, 1478 (1986) [Sov. Phys. JETP **63**, 866 (1986)].

[8] L. J. Campbell, M. Doria, and V. G. Kogan, Phys. Rev. B **38**, 2439 (1988).

[9] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1983).

[10] I. N. Khlustikov and A. I. Buzdin, Adv. Phys. **36**, 271 (1987).

[11] M. M. Fang, V. G. Kogan, D. K. Finnemore, J. R. Clem, L. S. Chumbley, and D. E. Farrell, Phys. Rev. B **37**, 2334 (1988).

[12] A. I. Buzdin, M. Kulić, and S. V. Polonskii, in *Proceedings of the International Conference on High-T_c Superconductors and Materials and Mechanisms of Superconductivity, Interlaken, Switzerland, 1988,* edited by J. Müller and J. L. Olsen [Physica C **153-155** (1988)]

[13] A. A. Abrikosov and A. I. Buzdin, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 204 (1988) [JETP Lett. (to be published)].

[14] G. Deutscher and K. A. Muller, Phys. Rev. Lett. **59**, 1745 (1987).