PHYSICAL REVIEW B VOLUME 38, NUMBER 10 1 OCTOBER 1988

Impossibility of spontaneous current in equilibrium

E. I. Blount
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 18 May 1988)

Eliashberg [JETP Lett. **38**, 220 (1983)] and Tavger [Phys. Lett. A **116**, 123 (1986)] have argued that in structures where electric current is not forbidden by symmetry it should be observed. I show that such current is forbidden by the nature of equilibrium.

Eliashberg[1] and Tavger[2] have stated that it is possible for electric current to flow without dissipation in thermal equilibrium if the crystal symmetry permits it and time-reversal symmetry does not hold. Eliashberg considered crystals without mirror symmetry in a magnetic field **B**, and predicted a current proportional to B. He calculated the appropriate tensor (incorrectly as will be shown later). Tavger gives no calculation and his argument is essentially that if the current is not forbidden by symmetry, including time reversal, it is compulsory.

The proposition sounds highly improbable and it is easy to see that it violates the second law of thermodynamics unless we postulate a peculiar interface with normal metals.

Starting with a finite simply connected piece of the material, we have to find a way to make the perpendicular current density vanish at the boundary. This task is trivial, requiring only that an electric field **E** be set up whose ordinary conduction current cancels the anomalous current. This is a simple problem with Neumann boundary conditions. Since the anomalous current density is constant inside the material, the result is that the total current density vanishes inside.

At this point we could argue on the one hand that there is no joule heating because there is no current. On the other hand, we could maintain that the canceling normal current should itself produce joule heating. The latter is clearly paradoxical as a steady state since it violates conservation of energy. The former does not so grossly violate thermodynamics, but if we now connect a normal resistance between the two ends of a cylinder of the material, some of the current will apparently go through it producing joule heat. In consequence, the integral of $\mathbf{j}\cdot\mathbf{E}$ in the anomalous material would be negative. This corresponds, in other words, to cooling a "reservoir" at the anomalous materials and heating one at the normal resistance. Thus, if the latter temperature is higher, we violate the second law. We cannot rule out the possibility of the temperatures being so related; thus, we have to rule out anomalous materials absolutely, or require that their interfaces with normal metals create absolute barriers against current flow. Then the total current is always zero, contrary to hypothesis.

The current under discussion is a bulk, not surface, effect, and in Eliashberg's case it is a local response to the field **B**.

I now want to show, directly, that there is no such current. In standard transport theory the current density is written as

$$\mathbf{j}=\sum_n \int d^3k\, f_{n\mathbf{k}}\mathbf{v}_{n\mathbf{k}} \; .$$

In doing the integration, we remember that $f_{n\mathbf{k}}$ is, in equilibrium, a function only of the energy $E_{n\mathbf{k}}$, and that $v_{n\mathbf{k}}$ is $\partial E_{n\mathbf{k}}/\partial \mathbf{k}$. Then the integral vanishes because of periodicity in **k** space. This remains true in a uniform magnetic field in the effective Hamiltonian formulation.[3]

This is a rather glib discussion, and you might think that a hole could be found in it if you really looked hard. This is not the case, however; the argument can be made as tight as necessary, but it seems excessive to overload this paper with it.

The conclusion is that spontaneous currents in equilibrium are not possible and that Eliashberg's argument must be incorrect. In the Appendix his error is discussed.

A consequence of this discussion is that if we take a crystal of a material and form it into a ring, there can be no bulk current around the ring in equilibrium. In Eliashberg's case, the current forbidden is also related to the local field **B**.

The preceding argument does not, therefore, apply to superconducting rings, whose current is at the surface and depends on the gross topology of sample. Furthermore, the persistent current of superconductors is not strictly an equilibrium effect—it flows only in metastable states.

It happens, however, that the fuller argument referred to earlier also forbids any net current around a ring in equilibrium in the limit of a very large ring. To reconcile this with superconductors, we have to consider how the material interacts with the magnetic field. The preceding discussion, and that of Eliashberg, is based on a field which is applied externally and not appreciably affected by the material. This is totally inappropriate in the case of superconductors, whose interaction with the field is fundamental. They cannot be discussed without including the self-energy of the field $B^2/8\pi$, which is responsible for flux exclusion and quantization.

The final conclusion is that the spontaneous current in equilibrium is forbidden fundamentally by the nature of equilibrium, not just by symmetry.

Note added in proof. P. B. Littlewood has called my attention to a paper by Volkov and Kopaev (Pis'ma Zh. Eksp. Teor. Fiz. 27, 10 (1978) [JETP Lett. 27, 7 (1978)]), in which it is claimed that a state of homogeneous current can occur in certain cases of electron-hole pairing. Specifically, they refer to the situation where the two band edges involved are at the same point, $\mathbf{k}_0$, in $\mathbf{k}$ space. At this point, the argument becomes a bit terse, but I think that what they actually show is that after pairing, the states at $\mathbf{k}_0$ become current carrying. This may well be true for some models, but it is irrelevant. The current for the state at the new band edge vanishes and the preceding argument still applies, and there will be no current.

I want to make clear the fact that this paper applies rigorously only in the limit of a large sample, and that Refs. 1 and 2 address only the same problem.

APPENDIX

Eliashberg's formula involves the integral R, where

$$R \equiv \sum_n \int d^3k\, f_{nk}(\nabla_k E)\cdot\nabla_k \times \mathbf{X}_{nn} \ .$$

Here, f is the Fermi function and $\mathbf{X}$ is a matrix arising in the representation of $\mathbf{x}$ in band theory.[4] $\nabla_k \times \mathbf{X}_{nn}$ will be called $\mathbf{\Omega}_n$. Defining F as the integral of f with respect to energy

$$R = \sum_n \int dS\cdot\mathbf{\Omega}_n F_n \ .$$

The integral is taken over the surface of the Brillouin zone, where it vanishes because the integrand is periodic.

Actually, we must be more careful and consider the effect of degeneracies between two bands. (Degeneracies between more than two bands arise only in the cubic system, where the current vanishes by symmetry.) Near such points the periodic parts of the wave functions depend only on the direction from $\mathbf{k}_0$ (the degeneracy) to $\mathbf{k}$, not on the distance. Therefore,[4] $|\mathbf{X}|$ goes like $1/|\mathbf{k}-\mathbf{k}_0|$, and Ω like $1/|\mathbf{k}-\mathbf{k}_0|^2$. The contribution from each spherical shell around $\mathbf{k}_0$ is finite and approaches a constant as $\mathbf{k}-\mathbf{k}_0\rightarrow\mathbf{0}$. The contribution of each band does not, in general, vanish, but is cancelled by that from the other band because F is a differentiable function of energy, while Ω has the same magnitude but opposite sign for the two bands at the point of contact. If the two bands are degenerate on a line, similar arguments apply, but the contribution of each band diverges logarithmically, while the sum vanishes. This is an unusual way of obtaining a vanishing result and it is unrelated to symmetry.

[1]G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 188 (1983) [JETP Lett. **38**, 220 (1983)].

[2]B. A. Tavger, Phys. Lett. A **116**, 123 (1986).

[3]W. Kohn, Phys. Rev. **115**, 1460 (1959); E. I. Blount, *ibid.* **126**, 1636 (1962); L. M. Roth, J. Phys. Chem. Solids, **23**, 433 (1962).

[4]E. I. Blount, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1962), Vol. 13, p. 306.