Effective-medium theory of long-wavelength spin waves in magnetic superlattices

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We explore the theory of long-wavelength spin waves in superlattices which incorporate magnetically ordered films that interact either through dipolar fields generated by spin motions or exchange couplings at interfaces or those mediated by an intervening nonmagnetic film. We show that for long wavelengths one may replace the superlattice by an effective medium described by a permeability tensor composed of those of the constituent films. Our treatment applied to a semi-infinite lattice of ferromagnetic films reproduces results obtained earlier in a full theory by Camley, Rahman, and Mills. We also develop an effective medium theory of Y-Gd superlattices in the anti-phase-domain configuration, with transverse field applied. We obtain a rich spectrum of surface spin waves in this case. We also obtain Green's functions for the average medium description of the Y-Gd system and use these to discuss surface-mode contributions to the spectral densities explored by Brillouin scattering.

I. INTRODUCTION

Superlattice structures form an intriguing new class of materials, in that their macroscopic properties are subject to design or control by varying the thickness or composition of the constituent films. They also can possess a spectrum of elementary excitations that are influenced strongly by both geometry and composition; one also encounters modes unique to superlattices.[1]

An important new class of superlattices is fabricated with one or more films of magnetically ordered materials in each unit cell of the structure. The simplest example consists of films of a ferromagnetic metal such as Ni or Fe, with nonmagnetic "spacers" in between.[2,3] Recently rare-earth materials, including spiral spin materials, have been incorporated into superlattices.[4,5] Modest magnetic fields cannot only influence the frequency and dispersion relations of collective excitations in such materials, but can lead to major modifications in the ground-state spin configuration, illustrated by recent experimental[4] and theoretical studies.[6,7] Because the influence of small magnetic fields can be so substantial, magnetic superlattices will prove a rich field of study in the near future.

Theoretical studies of spin waves in magnetic superlattices proceed by considering the nature of the solutions for the appropriate wave field in each film, then linking these together with appropriate boundary conditions, and the assistance of Bloch's theorem. A semi-infinite superlattice can also possess an interesting spectrum of surface modes.[1]

Many magnetic superlattices are fabricated from very thin films, whose thickness is quite small compared to the wavelength of spin waves excited in various experiments. For instance, light scattering has proved a powerful means of studying the collective spin-wave modes of superlattices.[2,3] The wavelength of the modes excited are in the range of a few thousand Angstroms, while the films can be as thin as 10 Å. Under such conditions, a detailed theory of the spin waves, with attention to each film in the structure, should not be required, since the wave field amplitudes are constant to very good approximation across a given film. It should prove possible to describe the superlattice as a uniform, homogeneous medium characterized by a magnetic permeability tensor which is an appropriate average over the response functions of the various films within the superlattice unit cell. Such a description will prove particularly convenient for materials which incorporate rare-earth materials with complex spin structures, since a detailed theory of spin waves of the superlattice may prove complicated from the technical point of view.

The purpose of this paper is to address the issue just described. The approach we use may be regarded as the extension to gyrotropic media of the simple, but elegant effective-medium description of the response of dielectric superlattices given by Agranovich and Kravtsov.[8] These authors show how such structures may be described by a suitable anisotropic dielectric tensor; their treatment reproduces the results of more detailed theories,[9] in the long-wavelength limit.

We begin by considering a superlattice consisting of ferromagnetic films, separated by nonmagnetic films. We obtain an average magnetic permeability tensor which, when applied to the description of the bulk and surface excitations of the average medium, reproduces in the long-wavelength regime results obtained earlier in a detailed analysis by Camley, Rahman, and Mills.[10] This demonstrates that such an approach can indeed provide a description of the collective modes of such structures.

We then turn to new results for a structure realized experimentally in the Y-Gd superlattice system. Of interest to us is the anti-phase-domain configuration, where in the magnetic ground state in zero external field, the magnetization in the Gd films alternates in sign as one progresses down the structure.[4] Application of a magnetic field transverse to the easy axis leads to a canted spin configuration,[4] quite similar to that realized in the unaxial antiferromagnet in a transverse field. We have shown recently[11] that such a structure has a most intriguing spectrum of surface spin waves. The Y-Gd superlattices are a physical realization of a system where large canting angles are induced by a weak external magnetic field, a direct consequence of the fact that the interfilm exchange mediated by the Y is quite weak. In this paper, we develop an effective-medium description of the long-wavelength dynamical response of the antiphase structure in a transverse magnetic field, and explore its spectrum of surface spin waves.

After this work was completed, a paper by Raj and Tilley[12] appeared which also addresses an effective-medium description of magnetic superlattices. The spirit of their approach is very similar to ours; indeed, results we present in Sec. II are also found in their paper, and our method of derivation patterned after that in Ref. 8, is virtually the same as the one used in their paper. The application we make of the effective-medium permeability tensor of the ferromagnetic superlattice differ substantially from those in the work of Raj and Tilley, and our treatment of the antiphase ground state of the Y-Gd system is an extension of the basic scheme, in a sense described later.

In Sec. II, we derive the effective-medium description of a ferromagnetic superlattice (the results actually can be applied directly to a wider class of systems), and we recover results found earlier by Camley, Rahman, and Mills[10] through application to the description of surface spin waves in the semi-infinite superlattice. In our view, this provides us with an important check on the basic validity of the scheme. In Sec. III we derive the effective-medium description of the Y-Gd system, incorporating interfilm exchange transmitted through the Y films. Surface spin waves on these structures are explored in Sec. IV, and Sec. V derives the Green's functions which enter the theory of light scattering, and uses these to explore the nature of the spin fluctuations near the surface of the structure. Our aim is to motivate the experimental study of such surfaces by Brillouin spectroscopy.

II. EFFECTIVE MEDIUM THEORY OF THE FERROMAGNETIC SUPERLATTICE

In this section, we consider the effective medium theory of a structure such as that shown in Fig. 1(a). Each unit cell of the superlattice contains two films, one of thickness d_1 and one of thickness d_2. Each may be a ferromagnet, though the moment in film 1 and that in film 2 need not be equal. Indeed, the development here as given applies directly to the case where the magnetic moments of film 1 and film 2 are antiparallel, and also to the case where one or both is an antiferromagnet. The key assumption is that for each of the two films in the unit cell, the spin quantization axis is parallel or antiparallel to the z direction, an assumption not applicable to the antiphase ground state of the Y-Gd system, when a transverse magnetic field is applied. In the end, our explicit application is to the system discussed in Ref. 10, where film 1 is a ferromagnet, and film 2 is nonmagnetic.

We shall assume that within each film, the magnetic field $\mathbf{h}$ and the magnetic induction $\mathbf{b}$ is spatially uniform, although the value of each in film 1 may differ from that in film 2. This means our attention is confined only to the

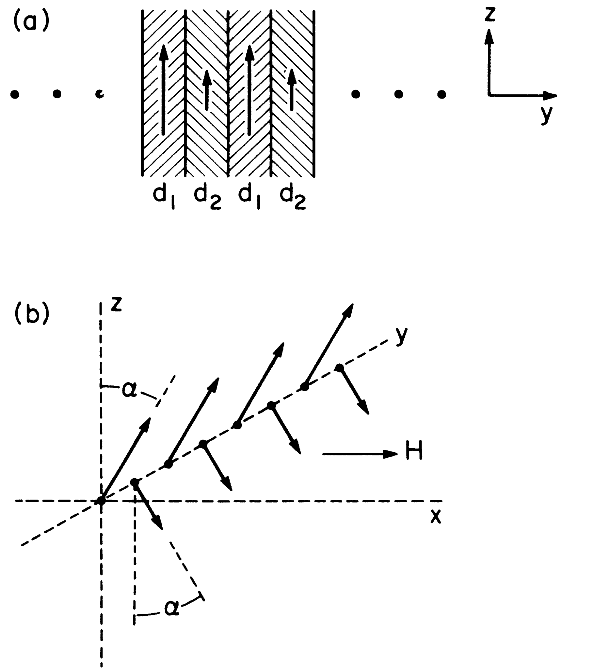

FIG. 1. (a) An illustration of the superlattice structure considered in Sec. II. There are two films in each unit cell, one of thickness d_1 and one of thickness d_2. (b) An illustration of the antiphase ground state realized in the Y-Gd system. This is the topic of Sec. III.

low-frequency spin-wave modes where the spin motion within a given film consists of a rigid precession of the net magnetization. Exchange modes, for which the time varying components of the magnetization possess one or more nodes within a given film, are not contained in the present description. Such modes have high frequency, for the thin films we expect effective-medium theory to prove useful, and will be well separated in frequency from the manifold of long-wavelength spin-wave modes considered here. We note that some years ago, for the isolated ferromagnetic film, the contribution of the exchange modes to the light-scattering spectrum was described theoretically,[13] in a framework that made explicit contact with experimental studies of rather thick ferromagnetic films.[14]

The present paper also ignores the influence of spin pinning at interfaces, not only here but throughout the discussion. In the thin-film limit, the influence of such pinning fields may be represented by introducing effective anisotropy fields whose strength varies inversely with film thickness.[3] The treatment here can readily be extended to incorporate such effects, but we do not address this question at present.

When a long-wavelength spin wave of frequency Ω is excited, the various films in the superlattice each will have a small amplitude magnetization $\mathbf{m}_i$ which oscillates with frequency Ω, and which lies in the xy plane, within the framework of the linearized theory of spin waves. Note from Fig. 1(a) that the coordinate axes are oriented so the xz plane is parallel to the interfaces. Let $\mathbf{h}^{(i)}$ and

$\mathbf{b}^{(i)}=\mathbf{h}^{(i)}+4\pi\mathbf{m}^{(i)}$ be the time-dependent magnetic field and magnetic induction in the ith film. These two fields are related by a magnetic permeability tensor of gyrotropic form. We write this as

$$b_x^{(i)}=\mu_a^{(i)}h_x^{(i)}-i\mu_b^{(i)}h_y^{(i)} , \quad (2.1a)$$

$$b_y^{(i)}=\mu_a^{(i)}h_y^{(i)}+i\mu_b^{(i)}h_x^{(i)} , \quad (2.1b)$$

and

$$b_z^{(i)}=h_z^{(i)} . \quad (2.1c)$$

If film i is a ferromagnetic material, we have, omitting explicit use of the subscript i,

$$\mu_a=1+\frac{4\pi\Omega_M\Omega_H}{\Omega_H^2-\Omega^2} \quad (2.2a)$$

and

$$\mu_b=\frac{4\pi\Omega\Omega_M}{\Omega_H^2-\Omega^2} , \quad (2.2b)$$

where $\Omega_M=\gamma M_s$ and $\Omega_H=\gamma H$. Here γ is the gyromagnetic ratio, M_s the static magnetization, and H an external magnetic field we assume applied parallel to the z direction. If film i is an antiferromagnet Eqs. (2.1) still apply, and explicit forms of μ_a and μ_b are given elsewhere.[15]

Now in our superlattice, $i=1$ or 2, and we wish to find the relationship between $\langle \mathbf{b}\rangle$ and $\langle \mathbf{h}\rangle$, the average value of the magnetic induction in the unit cell. The tensor response function which links these two is the effective-medium permeability tensor. If $f_1=d_1/(d_1+d_2)$, and $f_2=d_2/(d_1+d_2)$, then

$$\langle \mathbf{h}\rangle=f_1\mathbf{h}^{(1)}+f_2\mathbf{h}^{(2)} , \quad (2.3)$$

with a similar definition used for $\langle \mathbf{b}\rangle$.

Now conservation of tangential $\mathbf{h}$ at the interfaces requires $h_x^{(1)}=h_x^{(2)}=\langle h_x\rangle$ while conservation of normal $\mathbf{b}$ requires $b_y^{(1)}=b_y^{(2)}=\langle b_y\rangle$. Thus

$$\langle b_y\rangle=\mu_a^{(1)}h_y^{(1)}+i\mu_b^{(1)}\langle h_x\rangle$$
$$=\mu_a^{(2)}h_y^{(2)}+i\mu_b^{(2)}\langle h_x\rangle , \quad (2.4)$$

which when combined with

$$h_y^{(2)}=\frac{1}{f_2}(\langle h_y\rangle-f_1h_y^{(1)}) \quad (2.5)$$

leads to

$$h_y^{(1)}=\frac{\mu_a^{(2)}}{(f_2\mu_a^{(1)}+f_1\mu_a^{(2)})}\langle h_y\rangle+i\frac{(f_2\mu_b^{(2)}-f_2\mu_b^{(1)})}{(f_2\mu_a^{(1)}+f_1\mu_a^{(2)})}\langle h_x\rangle . \quad (2.6)$$

This may then be substituted into Eq. (2.4) to give

$$\langle b_y\rangle=\bar{\mu}_a^{(\perp)}\langle h_y\rangle+i\bar{\mu}_b\langle h_x\rangle , \quad (2.7)$$

where

$$\bar{\mu}_a^{(\perp)}=\frac{\mu_a^{(1)}\mu_a^{(2)}}{(f_2\mu_a^{(1)}+f_1\mu_a^{(2)})} \quad (2.8a)$$

and

$$\bar{\mu}_b=\frac{f_2\mu_a^{(1)}\mu_b^{(2)}+f_1\mu_a^{(2)}\mu_b^{(1)}}{(f_2\mu_a^{(1)}+f_1\mu_a^{(2)})} . \quad (2.8b)$$

Continuing on,

$$\langle b_x\rangle=f_1b_x^{(1)}+f_2b_x^{(2)}$$
$$=(f_1\mu_a^{(1)}+f_2\mu_a^{(2)})\langle h_x\rangle-i\mu_b^{(1)}f_1h_y^{(1)}-i\mu_b^{(2)}f_2h_y^{(2)} . \quad (2.9)$$

Then expressing $h_y^{(i)}$ and $h_y^{(2)}$ in terms of $\langle h_y\rangle$ and $\langle h_x\rangle$ as in Eq. (2.6), we find

$$\langle b_x\rangle=\bar{\mu}_a^{\parallel}\langle h_x\rangle-i\bar{\mu}_b\langle h_y\rangle , \quad (2.10)$$

where

$$\bar{\mu}_a^{\parallel}=f_1\mu_a^{(1)}+f_2\mu_a^{(2)}-f_1f_2\frac{(\mu_b^{(1)}-\mu_b^{(2)})^2}{(f_2\mu_a^{(1)}+f_1\mu_a^{(2)})} . \quad (2.11)$$

The above results, and in fact the above derivation, appear in the work of Raj and Tilley.[12] Our work and theirs both follow that of Agranovich and Kravtsov,[8] evidently. The application of the expressions in Eqs. (2.8a), (2.8b), and (2.11) we now make differs from those in Ref. 12, however.

We consider superlattices of the Ni-Mo type,[2] where film 1 is ferromagnetic Ni, film 2 is nonmagnetic Mo (thus $\mu_a^{(2)}=1$, $\mu_b^{(2)}=0$) and of course, all ferromagnetic films in the superlattice have their saturation magnetization M_s parallel. After a bit of algebra, one finds

$$\bar{\mu}_a^{\parallel}=1+\frac{4\pi f_1\Omega_M(\Omega_H+4\pi f_2\Omega_M)}{\Omega_H(\Omega_H+4\pi f_2\Omega_M)-\Omega^2} , \quad (2.12a)$$

$$\bar{\mu}_a^{\perp}=1+\frac{4\pi f_1\Omega_M\Omega_H}{\Omega_H(\Omega_H+4\pi f_2\Omega_M)-\Omega^2} , \quad (2.12b)$$

and

$$\bar{\mu}_b=\frac{4\pi\Omega_M f_1\Omega}{\Omega_H(\Omega_H+4\pi f_2\Omega_M)-\Omega^2} . \quad (2.12c)$$

We can rewrite the relationships just obtained in a notation that is perhaps more useful. We have found that

$$\langle b_x\rangle=\bar{\mu}_{xx}\langle h_x\rangle+\bar{\mu}_{xy}\langle h_y\rangle \quad (2.13a)$$

and also that

$$\langle b_y\rangle=\bar{\mu}_{yy}\langle h_y\rangle-\bar{\mu}_{xy}\langle h_x\rangle , \quad (2.13b)$$

where we make the identification $\bar{\mu}_{xx}=\bar{\mu}_a^{\parallel}$, $\bar{\mu}_{yy}=\bar{\mu}_a^{\perp}$, and $\bar{\mu}_{xy}=-i\bar{\mu}_b$.

In ferromagnetic superlattices, for which the frequency of long-wavelength spin waves lies in the microwave regime, retardation effects are small. Magnetostatic theory then describes the spin waves. In our average medium,

$$\nabla\times\langle \mathbf{h}\rangle=0 \quad (2.14a)$$

so we may write $\langle \mathbf{h} \rangle = -\nabla\Phi_M$, and we also have

$$\nabla\cdot\langle \mathbf{b} \rangle = 0 \tag{2.14b}$$

from which one finds easily

$$\mu_a^{\parallel}\frac{\partial^2\Phi_M}{\partial x^2}+\mu_a^{\perp}\frac{\partial^2\Phi_M}{\partial z^2}=0 \ . \tag{2.15}$$

In the infinitely extended superlattice, bulk spin waves are described by seeking solutions of Eq. (2.15) with $\Phi_M \sim \exp(i\mathbf{k}\cdot\mathbf{x})$. After a few lines of algebra, one finds the frequency $\Omega_B(\mathbf{k})$ of a bulk spin wave of wave vector $\mathbf{k}$ is given by

$$\Omega_B^2(\mathbf{k})=\Omega_H(\Omega_H+4\pi f_2\Omega_M)+4\pi f_1\Omega_H\Omega_M\left[\frac{k_x^2+k_y^2}{k^2}\right] + 16\pi^2 f_1 f_2\Omega_M^2\frac{k_x^2}{k^2} \ , \tag{2.16}$$

a result which agrees fully with Eq. (2.28) of Ref. 10.

Consider next a semi-infinite superlattice which resides in the half-space $y>0$, as described by our effective-medium theory. Surface spin waves are described by a magnetic potential of the form

$$\Phi_M(\mathbf{x})=\Phi_M^{>}\exp(ik_x x+ik_z z-\beta y) \tag{2.17a}$$

in the half-space $y>0$, and a magnetic potential

$$\Phi_M(\mathbf{x})=\Phi_M^{<}\exp(ik_x x+ik_z z+k_{\parallel} y) \tag{2.17b}$$

in the half-space $y<0$, where $\nabla\cdot\mathbf{h}=0$. We write $k_x=k_{\parallel}\cos\psi$ and $k_z=k_{\parallel}\sin\psi$.

Use of Eq. (2.15) gives one relation between β, $k_{\parallel}$ and the frequency of the wave which may be written as

$$\frac{\beta^2}{k_{\parallel}^2}=\frac{\sin^2(\psi)[\Omega_H(\Omega_H+4\pi f_2\Omega_M)-Q^2]+\cos^2(\psi)[\Omega_H(\Omega_H+4\pi\Omega_M)+16\pi f_1 f_2\Omega_M^2-\Omega^2]}{\Omega_H(\Omega_H+4\pi\Omega_M)-\Omega^2} \ . \tag{2.18}$$

As is standard in surface-wave theory, additional relations come from boundary conditions. Conservation of tangential $\langle \mathbf{h} \rangle$ is assured if $\Phi_M^{>}=\Phi_M^{<}$, and conservation of normal $\langle \mathbf{b} \rangle$ leads to a second relation between β and $k_{\parallel}$:

$$\frac{\beta}{k_{\parallel}}=\frac{4\pi\cos\psi f_1\Omega\Omega_M+\Omega_H(\Omega_H+4\pi f_2\Omega_M)-\Omega^2}{\Omega^2-\Omega_H(\Omega_H+4\pi\Omega_M)} \ . \tag{2.19}$$

The frequency of the wave is found by requiring Eq. (2.19) to be compatible with Eq. (2.18). After some algebra, we find the surface-wave frequency is given by

$$\Omega_s(\mathbf{k}_{\parallel})=\frac{1}{2}\left[\frac{\Omega_H}{\cos\psi}+(\Omega_H+4\pi\Omega_M)\cos\psi\right] \ , \tag{2.20}$$

a result also obtained in Ref. 10. It is remarkable that the result in Eq. (2.20) is identical to the expression which applies to a semi-infinite pure ferromagnet, which in our present notation is described by choosing $f_1=1$ and $f_2=0$.

Our derivation of Eq. (2.20) applies only in the limit where both $k_{\parallel}d_1 \ll 1$ and $k_{\parallel}d_2 \ll 1$, since it is only in this limit that effective-medium theory is applicable. The discussion in Ref. 10 shows, again remarkably in our view, that the frequency of the superlattice surface wave is given by Eq. (2.20) for all values of these ratios.

For the attenuation constant β, we have

$$\beta=k_{\parallel}\left[\frac{4\pi(2f_1-1)\Omega_M\cos^2\psi+\Omega_H\sin^2\psi}{(\Omega_H+4\pi\Omega_M)\cos^2\psi-\Omega_H}\right] \ . \tag{2.21}$$

The result displayed in Eq. (2.21) is more useful than the explicit formulas which appear in Ref. 10. This result can be obtained also by combining Eq. (2.17) and Eq. (2.42) of Ref. 10 with its Eq. (2.41).

For propagation perpendicular to the magnetization, where $\psi=0$, we have

$$\beta=k_{\parallel}(2f_1-1) \ , \tag{2.22}$$

a result quoted in Ref. 10. Equation (2.22) shows we have surface-spin-wave solution only when $f_1>\frac{1}{2}$, i.e., only when the ferromagnetic films are thicker than the nonmagnetic films, in the superlattice. Finally, $\beta\to\infty$ when $\psi\to\psi_c$, where $\cos(\psi_c)=\Omega_H/(\Omega_H+4\pi\Omega_M)$, as in the case of the simple semi-infinite ferromagnet.

This discussion demonstrates that through use of the effective-medium theory, we recover all of the results for the long-wavelength limiting behavior of the more general formulas derived by Camley, Rahman, and Mills, in their discussion of spin waves in the semi-infinite lattice of dipolar coupled ferromagnetic films. This demonstrates the correctness of the approach, and we now turn to development of an effective-medium description of a more complex ground-state spin geometry, for which a full theory would prove technically rather complex.

III. DYNAMIC RESPONSE OF THE ANTIPHASE GROUND STATE OF Y-Gd SUPERLATTICES

Quite recently, very high quality superlattices formed from alternating layers of Y and Gd have been synthesized and characterized.[16] In the bulk, Gd is ferromagnetic with very little anisotropy, while Y is paramagnetic. For a selected range of Y thicknesses, the magnetic ground state of the superlattice is one in which the magnetization alternates in sign, from the $+z$ to the $-z$ direction.[4] From the macroscopic point of view, the structure consists of sheets of ferromagnetically aligned spins, between which exchange couplings of antiferromagnetic character conspire to produce the antiparallel alignment of magnetizations in adjacent Gd films. This configuration, found only for selected thicknesses of the intervening Y film, has been referred to as the antiphase ground state. Application of an external magnetic field H transverse to the z direction produces the canted ground state illustrated in Fig. 1(b).

We have recently completed an analysis of surface spin waves in antiferromagnets that consist of such antiferromagnetically coupled spin sheets.[11] Examples would be the MnF_2 or FeF_2 structures with a (100) surface. We found a rich and interesting spectrum of surface modes when a transverse field is applied, to produce a ground state such as that illustrated in Fig. 1(b). Surface waves which propagate along the external field have a dispersion relation that is an even function of the wave vector $\mathbf{k}_\parallel$ parallel to the surface, while those which propagate normal to it display nonreciprocal propagation characteristics: The dispersion relation for traveling from right to left across the field differs from that for propagating from left to right.

We argued earlier[11] that superlattices such as the Y-Gd system in the antiphase ground state provide another physical realization of a system within which such a diverse spectrum of surface waves would be realized. The virtue of such a material is that very large canting angles can be produced with very modest magnetic fields. The present effective-medium approach described here allows us to make a precise connection between the dynamic response of the superlattice structure, and our earlier analysis of the uniaxial antiferromagnet in a transverse field.

To construct an effective-medium description of the Y-Gd structure described above, one notes there are four films in each unit cell, two Gd films and two Y films. The thickness of each Gd film is taken to be d_1 that of each Y film is d_2, and as before $f_1=d_1/(d_1+d_2)$ while $f_2=d_2/(d_1+d_2)$. Variables that refer to one of the two films in a given unit cell, that whose static magnetization has a positive z component, will be designated by appending the superscript a to various quantities, and those that refer to the film whose static magnetization has a negative z component will be designated by appending a superscript b. The magnetization in film a is then written

$$\mathbf{M}^{(a)}=M_s(\hat{z}\cos\alpha+\hat{x}\sin\alpha)+\mathbf{m}^{(a)} \tag{3.1}$$

and in film b we shall write

$$\mathbf{M}^{(b)}=M_s(-\hat{z}\cos\alpha+\hat{x}\sin\alpha)+\mathbf{m}^{(b)}\ , \tag{3.2}$$

where M_s is the static magnetization, and $\mathbf{m}^{(a)},\mathbf{m}^{(b)}$ are the time-dependent components associated with the spin wave in the structure. We assume $\mathbf{m}^{(a)}$ and $\mathbf{m}^{(b)}$ are small, and ultimately we linearize the equations of motion given below with respect to these variables.

The magnetizations in the a and b films interact via exchange mediated by the intervening Y, so a driving field applied to film a excites spins in film b, and conversely. Thus, if the spins in the a films are exposed to an oscillatory driving field $\mathbf{h}^{(a)}$, and those in the b films are exposed to $\mathbf{h}^{(b)}$, we shall have constitutive relations of the form

$$m_i^{(a)}=\sum_j \chi_{ij}^{(aa)}(\Omega)h_j^{(a)}+\sum_j \chi_{ij}^{(ab)}(\Omega)h_j^{(b)} \tag{3.3a}$$

and

$$m_i^{(b)}=\sum_j \chi_{ij}^{(ba)}(\Omega)h_j^{(a)}+\sum_j \chi_{ij}^{(bb)}(\Omega)h_j^{(b)}\ . \tag{3.3b}$$

In our effective-medium theory, we endow each of the four films in the unit cell with its own field, and then we find the relation between $\langle \mathbf{b}\rangle$ and $\langle \mathbf{h}\rangle$ by averaging over the four films in the unit cell.

Let $\mathbf{M}^{(a)}$ and $\mathbf{M}^{(b)}$ be written $\mathbf{M}^{(a)}=M_s\mathbf{a}$ and $\mathbf{M}^{(b)}=M_s\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors. Then the tensor response functions defined in Eq. (3.3a) and Eq. (3.3b) are extracted by analyzing the equations of motion

$$\dot{\mathbf{a}}=+\Omega_H(\mathbf{a}\times\hat{\mathbf{x}})+\Omega_e(\mathbf{b}\times\mathbf{a})+\gamma(\mathbf{a}\times\mathbf{h}^{(a)}) \tag{3.4a}$$

and

$$\dot{\mathbf{b}}=+\Omega_H(\mathbf{b}\times\hat{\mathbf{x}})-\Omega_e(\mathbf{b}\times\mathbf{a})+\gamma(\mathbf{b}\times\mathbf{h}^{(b)})\ . \tag{3.4b}$$

Here, if γ is the gyromagnetic ratio, $\Omega_H=\gamma H$ and Ω_e is a measure of the strength of the interfilm exchange.

The equilibrium condition is found by setting $\mathbf{h}^{(a)},\mathbf{h}^{(b)}$ to zero, then seeking time independent solutions of Eqs. (3.4), with $\mathbf{a}=\langle\mathbf{a}\rangle=\hat{\mathbf{z}}\cos\alpha+\hat{\mathbf{x}}\sin\alpha$, and $\mathbf{b}=-\hat{\mathbf{z}}\cos\alpha+\hat{\mathbf{x}}\sin\alpha$. One finds easily

$$\sin\alpha=\frac{\Omega_H}{2\Omega_e}\ . \tag{3.5}$$

We then linearize Eqs. (3.4) by writing $\mathbf{a}=\langle\mathbf{a}\rangle+\delta\mathbf{a}$, and $\mathbf{b}=\langle\mathbf{b}\rangle+\delta\mathbf{b}$, with $\delta\mathbf{a}$ small. When this task is completed, the relations in Eqs. (3.3) assume the form

$$m_x^{(a)}=\chi_1\cos\alpha h_\parallel^{(a)}-i\chi_2\cos\alpha h_y^{(a)} -\chi_1\cos\alpha h_\parallel^{(b)}+i\chi_3\cos\alpha h_y^{(b)}\ , \tag{3.6a}$$

$$m_x^{(b)}=\chi_1\cos\alpha h_\parallel^{(b)}+i\chi_2\cos\alpha h_y^{(b)} -\chi_1\cos\alpha h_\parallel^{(a)}-i\chi_3\cos\alpha h_y^{(a)}\ , \tag{3.6b}$$

$$m_z^{(a)}=-\chi_4 h_\parallel^{(a)}+i\chi_2\sin\alpha h_y^{(a)} +\chi_4 h_\parallel^{(b)}-i\chi_3\sin\alpha h_y^{(b)}\ , \tag{3.6c}$$

$$m_z^{(b)}=\chi_4 h_\parallel^{(b)}+i\chi_2\sin\alpha h_y^{(b)} -\chi_4 h_\parallel^{(a)}-i\chi_3\sin\alpha h_y^{(b)}\ , \tag{3.6d}$$

$$m_y^{(e)}=\chi_5 h_y^{(a)}+i\chi_2 h_\parallel^{(a)}+\chi_6 h_y^{(b)}+i\chi_3 h_\parallel^{(b)}\ , \tag{3.6e}$$

and

$$m_y^{(b)}=\chi_5 h_y^{(b)}-i\chi_2 h_\parallel^{(b)}+\chi_6 h_y^{(a)}-i\chi_3 h_\parallel^{(a)}\ , \tag{3.6f}$$

where we have defined

$$h_\parallel^{(a)}=\cos\alpha h_x^{(a)}-\sin\alpha h_z^{(a)} \tag{3.7a}$$

and

$$h_\parallel^{(b)}=\cos\alpha h_x^{(b)}+\sin\alpha h_z^{(b)}\ , \tag{3.7b}$$

with

$$\chi_1=\frac{\Omega_e\Omega_M}{\Omega_H^2-\Omega^2}\ , \tag{3.8a}$$

$$\chi_2=\frac{\Omega_M(\Omega_H^2-2\Omega^2)}{2\Omega(\Omega_H^2-\Omega^2)}\ , \tag{3.8b}$$

$$\chi_3=\frac{\Omega_M\Omega_H^2}{2\Omega(\Omega_H^2-\Omega^2)}\ , \tag{3.8c}$$

$$\chi_4=\frac{\Omega_M\Omega_H}{2(\Omega_H^2-\Omega^2)}\ , \tag{3.8d}$$

$$\chi_5=\frac{\Omega_M\Omega_H\sin\alpha}{2(\Omega_H^2-\Omega^2)}-\frac{\Omega_M\Omega_e\cos^2\alpha}{\Omega^2}\ , \tag{3.8e}$$

and

$$\chi_6=\frac{\Omega_M\Omega_H\sin\alpha}{2(\Omega_H^2-\Omega^2)}+\frac{\Omega_M\Omega_e\cos^2\alpha}{\Omega^2}\ . \tag{3.8f}$$

In these expressions, we define $\Omega_M=\gamma M_s$, with M_s the magnitude of the saturation magnetization in each film. While these expressions are rather complex in nature, in the end our effective-medium description of the structure will be in terms of a rather simple effective-medium permeability tensor.

As remarked above, the magnetic field and magnetic induction in the two ferromagnetic films within the unit cell are written $\mathbf{h}^{(a)}$, $\mathbf{h}^{(b)}$, $\mathbf{b}^{(a)}$, and $\mathbf{b}^{(b)}$, respectively. The superscripts c and d will be applied to the two nonmagnetic films, for which we assume $\mathbf{h}^{(c)}=\mathbf{b}^{(c)}$, and $\mathbf{h}^{(d)}=\mathbf{b}^{(d)}$. Thus, the average value $\langle b_\alpha\rangle$ of the αth Cartesian component of the magnetic induction is written

$$\langle b_\alpha\rangle=\tfrac{1}{2}f_1(b_\alpha^{(a)}+b_\alpha^{(b)})+\tfrac{1}{2}f_2(h_\alpha^{(c)}+h_a^{(d)})\ . \tag{3.9}$$

We have

$$h_x^{(a)}=\cdots=h_x^{(d)}=\langle h_x\rangle\ , \tag{3.10}$$

and similarly for the z components of $\mathbf{h}$, and the y component of the induction $\mathbf{b}$. Using these statements, along with Eqs. (3.8), we have

$$\langle b_x^{(a)}\rangle=\langle h_x\rangle-8\chi_1\cos\alpha\sin\alpha\langle h_z\rangle +4\pi i\cos\alpha(\chi_3h_y^{(a)}-\chi_2h_y^{(b)})\ , \tag{3.11a}$$

$$\langle b_x^{(b)}\rangle=\langle h_x\rangle+8\pi\chi_1\cos\alpha\sin\alpha\langle h_z\rangle +4\pi i\cos\alpha(\chi_2h_y^{(a)}-\chi_3h_y^{(b)})\ , \tag{3.11b}$$

from which one finds

$$\langle b_x\rangle=\langle h_x\rangle+4\pi i f_1\cos\alpha(\chi_2+\chi_3)(h_y^{(a)}-h_y^{(b)})\ . \tag{3.12}$$

Also, we can construct the relations

$$\langle b_z\rangle=(1+8\pi f_1\chi_4\sin\alpha)\langle h_z\rangle +2\pi i f_1(\chi_2-\chi_3)\sin\alpha(h_y^{(a)}+h_y^{(b)})\ , \tag{3.13}$$

$$\langle b_y\rangle=\langle b_y^{(a)}\rangle=(1+4\pi\chi_5)h_y^{(a)}+4\pi\chi_6h_y^{(b)} +4\pi i(\chi_2+\chi_3)\cos\alpha\langle h_x\rangle +4\pi i(\chi_3-\chi_2)\sin\alpha\langle h_z\rangle\ , \tag{3.14a}$$

$$\langle b_y\rangle=\langle b_y^{(b)}\rangle=(1+4\pi\chi_5)h_y^{(b)}-4\pi i(\chi_2+\chi_3)\cos\alpha\langle h_x\rangle +4\pi i\sin\alpha(\chi_3-\chi_2)\langle h_z\rangle\ . \tag{3.14b}$$

Upon equating the right-hand side of Eq. (3.14a) with Eq. (3.14b), we have

$$h_y^{(b)}-h_y^{(a)}=\frac{8\pi i(\chi_2+\chi_3)\cos\alpha}{1+4\pi(\chi_5-\chi_6)}\langle h_x\rangle\ . \tag{3.15}$$

Upon combining Eq. (3.15) with Eq. (3.12), we find

$$\langle b_x\rangle=\bar{\mu}_{xx}\langle h_x\rangle\ , \tag{3.16}$$

where

$$\bar{\mu}_{xx}=1-\frac{32\pi^2f_1(\chi_2+\chi_3)^2}{1+4\pi(\chi_5-\chi_6)}\ . \tag{3.17}$$

Note that Eq. (3.16) gives $\bar{\mu}_{xy}=\bar{\mu}_{xz}=0$.

Now we also have

$$\langle h_y\rangle=\tfrac{1}{2}f_1(h_y^{(a)}+h_y^{(b)})+\tfrac{1}{2}f_2(h_y^{(c)}+h_y^{(d)}) \equiv f_2\langle b_y\rangle+\tfrac{1}{2}f_1(h_y^{(a)}+h_y^{(b)})\ . \tag{3.18}$$

Hence, we have

$$h_y^{(a)}+h_y^{(b)}=\frac{2}{f_1}(\langle h_y\rangle-f_2\langle b_y\rangle)\ , \tag{3.19}$$

a relation which may be used in Eq. (3.13). In addition, Eq. (3.19) may be used with Eq. (3.15) to find explicit expressions for $h_y^{(a)}$ and $h_y^{(b)}$, in terms of the average fields. After some algebra, we find

$$\langle b_y\rangle=\bar{\mu}_{yy}\langle h_y\rangle+\bar{\mu}_{yz}\langle h_z\rangle\ , \tag{3.20a}$$

and

$$\langle b_z\rangle=\bar{\mu}_{zz}\langle h_z\rangle-\bar{\mu}_{yz}\langle h_y\rangle\ . \tag{3.20b}$$

One has

$$\bar{\mu}_{zz}=1+8\pi f_1\sin\alpha\left[\chi_4\frac{2\pi f_2(\chi_2-\chi_3)^2\sin\alpha}{1+4\pi f_2(\chi_5+\chi_6)}\right]\ , \tag{3.21a}$$

$$\bar{\mu}_{yy}=1+\frac{4\pi f_1(\chi_5+\chi_6)}{1+4\pi f_2(\chi_5+\chi_6)} \tag{3.21b}$$

and

$$\bar{\mu}_{yz}=1+\frac{4\pi f_1(\chi_5+\chi_6)}{1+4\pi f_2(\chi_5+\chi_6)}\ . \tag{3.21c}$$

When the explicit forms for the χ_i are used, we find

$$\bar{\mu}_{xx}=1+\frac{32\pi^2 f_1\Omega_M^2}{8\pi\Omega_M\Omega_e\cos^2\alpha-\Omega^2}\ , \quad (3.22a)$$

$$\bar{\mu}_{yy}=1+\frac{4\pi f_1\Omega_H\Omega_M\sin\alpha}{\Omega_H^2+4\pi\Omega_H\Omega_M f_2\sin\alpha-\Omega^2}\ , \quad (3.22b)$$

$$\bar{\mu}_{zz}=1+\frac{4\pi\Omega_M f_1(\Omega_H+4\pi f_2\Omega_M\sin\alpha)\sin\alpha}{\Omega_H^2+4\pi\Omega_H\Omega_M f_2\sin\alpha-\Omega^2}\ , \quad (3.22c)$$

and

$$\bar{\mu}_{yz}=+\frac{4\pi i f_1\Omega_M\Omega\sin\alpha}{\Omega_H^2+4\pi\Omega_H\Omega_M f_2\sin\alpha-\Omega^2}\ . \quad (3.22d)$$

The remaining elements of the permeability tensor vanish.

The permeability tensor displayed in Eqs. (3.22) is isomorphic in structure to that derived in our earlier treatment of the easy axis antiferromagnet in a transverse field, although here we have no anisotropy included in the treatment. The demagnetizing fields generated by the spin motions in the films, whose strength is measured by $4\pi\Omega_M$, stiffen the response of the system in a manner similar to, but not identical to, the anisotropy fields of our earlier analysis.

IV. BULK AND SURFACE SPIN WAVES IN THE CANTED ANTI-PHASE-DOMAIN GROUND STATE: THE NATURE OF SPIN FLUCTUATIONS NEAR THE FUTURE

As remarked at the end of Sec. III, the magnetic permeability tensor displayed in Eqs. (3.22) is isomorphic to that described in our earlier theory of the dynamic response of the uniaxial antiferromagnet in a transverse field. Following the notation of our earlier paper,[11] we write the diagonal components of the magnetic permeability tensor in the form

$$\bar{\mu}_{xx}=1+\frac{\omega_{xx}^2}{\Omega_\parallel^2-\Omega^2}\ , \quad (4.1a)$$

$$\bar{\mu}_{yy}=1+\frac{\omega_{yy}^2}{\Omega_\perp^2-\Omega^2}\ , \quad (4.1b)$$

and

$$\bar{\mu}_{zz}=1+\frac{\omega_{zz}^2}{\Omega_\perp^2-\Omega^2}\ . \quad (4.1c)$$

Much of the discussion of our earlier paper may be used quite directly in the present analysis, with these definitions.

In what follows, we discuss the long-wavelength spin waves of the superlattice structure, proceeding very much as we did in Sec. II. We begin with the infinitely extended superlattice, and examine those waves which generate a macroscopic magnetic field which, with retardation ignored may be described through use of a magnetic potential Φ_M introduced between Eqs. (2.14a) and (2.14b). It is a straightforward exercise to show that in the precent case, there are two bulk spin-wave bands with dispersion relation $\Omega_{B\pm}(\mathbf{k})$ given by

$$\Omega_{B\pm}(\mathbf{k})^2=1/2\left[\Omega_\parallel^2+\Omega_\perp^2+\omega_{xx}^2\frac{k_x^2}{k^2}+\omega_{yy}^2\frac{k_y^2}{k^2}+\omega_{zz}^2\frac{k_z^2}{k^2}\right] \pm\frac{1}{2}\left[(\Omega_\parallel^2-\Omega_\perp^2)^2+2(\Omega_\parallel^2-\Omega_\perp^2)\left[\omega_{xx}^2\frac{k_x^2}{k^2}-\omega_{yy}^2\frac{k_y^2}{k^2}-\omega_{zz}^2\frac{k_z^2}{k^2}\right]+\left[\omega_{xx}^2\frac{k_x^2}{k^2}+\omega_{yy}^2\frac{k_y^2}{k^2}+\omega_{zz}^2\frac{k_z^2}{k^2}\right]^2\right]^{1/2}, \quad (4.2)$$

a result identical in form to Eq. (3.3) of our earlier paper.[11] in Eq. (4.2), $k^2=k_x^2+k_y^2+k_z^2$ is the square of the total wave vector.

It is a straightforward matter also to generate the implicit dispersion relation for surface spin waves. The discussion proceeds exactly as in Ref. 11, and thus we omit the derivation. However, before we turn to the results, we review the special features of the present geometry, which is very interesting from the point of view of surface spin-wave propagation.

A striking aspect of surface spin waves is their nonreciprocal propagation characteristics. Let $\Omega_s(\mathbf{k}_\parallel)$ be the dispersion relation of a surface spin wave, where $\mathbf{k}_\parallel$ is its two-dimensional wave vector. Then if $\Omega_s(+\mathbf{k}_\parallel)\neq\Omega_s(-\mathbf{k}_\parallel)$, the propagation characteristics for the direction $+\mathbf{k}_\parallel$ differ from those for the direction $-\mathbf{k}_\parallel$. The term nonreciprocal describes this feature of the waves. The surface spin wave discussed in Sec. II is an example of such a wave. There the frequency $\Omega_s(\mathbf{k}_\parallel)$ is in fact an *odd* function of $\mathbf{k}_\parallel$, i.e., $\Omega_s(+\mathbf{k}_\parallel)=-\Omega_s(-\mathbf{k}_\parallel)$ [consider Eq. (2.20), and let $\psi\rightarrow\psi+\pi$]. This means that the wave crests only have one sense of propagation across the magnetization, from right to left. The combination $\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel-\Omega_x(\mathbf{k}_\parallel)t$ can never describe crests which propagate in the reverse sense. In the light scattering spectra, this feature of the surface spin-wave dispersion relation has the consequence that if the wave is seen on the Stokes side of the spectrum, it is missing from the anti-Stokes, and conversely.

This behavior is very different from that exhibited by the bulk spin waves in the same system. For a given choice of $\mathbf{k}$, we have two bulk-spin-wave frequencies from Eq. (2.16), $+\Omega_B(\mathbf{k})$ and $-\Omega_B(\mathbf{k})$. If the first choice describes wave crests moving from right to left, the second describes wave crests moving from left to right. The propagation characteristics of the two waves are identical. For example, the magnitude of the group velocity of the two waves is the same.

It is intriguing that nonreciprocal propagation characteristics are found for surface spin waves, but never bulk spin waves. Thus, the origin of the phenomena is not related to the fact that time-reversal symmetry is broken by the presence of the spontaneous magnetization, as one might first think. Some years ago, it was pointed out[17] that the breakdown of reflection symmetry at the surface, in combination with the pseudovector character of the magnetization in regard to reflections, can account for this fact, and the fact that nonreciprocal behavior in surface-wave propagation occurs only in the presence of a net spontaneous magnetization, or a magnetic field.

The argument just cited shows that in the present system, for propagation parallel to the external magnetic field and net magnetization (the x direction), the surface-wave propagation characteristics cannot display nonreciprocity, i.e., for this direction, $\Omega_s(+\mathbf{k}_\parallel)\equiv\Omega_s(-\mathbf{k}_\parallel)$. For all other directions, nonreciprocal propagation will be the rule. Thus, we have an interesting example of a magnetic system which presents both reciprocal and nonreciprocal propagation characteristics.

With these remarks in mind, we turn next to our results for the Y-Gd system, in the antiphase ground state with canting produced by an externally applied field. Here the magnetization in each film is taken to have the value 2.06 kG. The only other parameter is the interfilm exchange coupling Ω_e, which we choose to be 433 G, which reproduces the canting angles formed in Ref. 4. We have varied f_1, but kept the value of Ω_e fixed, so the calculations refer to a set of structures whose Y film thickness is fixed.

As f_1 is varied, say from 0.25 to 0.75, we do not find qualitative differences in the surface magnon spectrum, in contrast to the semi-infinite ferromagnetic superlattice, where the surface magnon exists only for $f_1>\frac{1}{2}$. Here the differences are quantitative, not qualitative, as f_1 is varied. There is, however, a strong dependence on the canting angle α. Thus, we present explicit results only for $f_1=0.75$.

In Fig. 2, for two values of the canting angle α, we show the frequency regimes in the k_x-k_z plane occupied by the bulk-spin-wave bands projected onto this plane as shaded areas. The upper of the two bulk-spin-wave bands extends to rather high frequency, and is not shown. The angle ϕ is defined by writing $k_x=k_\parallel\sin\phi$ and $k_z=k_\parallel\cos\phi$; the symmetry argument cited earlier[17] shows the surface spin-wave dispersion relation to be an even function of the angle ϕ. Thus, we need only show results for ϕ in the range $0\le\phi\le180°$.

For small canting angles, such as $\alpha=20°$ illustrated in Fig. 2(a), there is a surface magnon with frequency outside the bulk bands, but the nonreciprocal character of the propagation shows dramatically. For $0\le\phi\le90°$, we have the branch SM_2 below the bulk-spin-wave bands, while for $90°\le\phi\le180°$, we have the branch SM_1 between the two bulk bands. The calculations show that SM_1 joins the bulk-spin-wave manifold at $\phi\cong100°$.

As the canting angle increases, the points P where the bandwidth of the lower band vanishes move toward the boundaries $\phi=0°$ and $\phi=180°$, and at larger canting an-

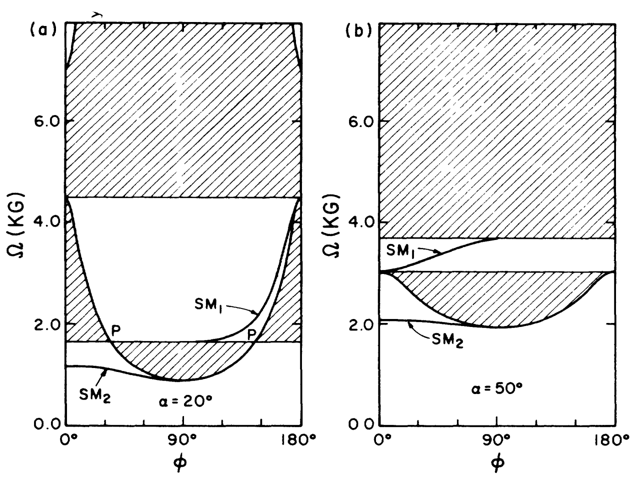

FIG. 2. For the case where the filling factor $f_1=0.75$, and for the Y-Gd superlattice in the field induced canted state, we show the frequency regimes occupied by the bulk spin waves (shaded areas), and the dispersion relation of the surface magnons as a function of propagation angle. We have $k_x=k_\parallel\sin\phi$ and $k_z=k_\parallel\cos\phi$, and the surface-wave frequencies are an even function of ϕ. We give results for two values of the field induced canting angle α: (a) $\alpha=20°$ and (b) $\alpha=50°$.

gles the mode structure illustrated in Fig. 2(b) is realized. We again have two surface magnon branches, but both lie in the regime $0\le\phi\le90°$, so we have a situation reminiscent of the semi-infinite ferromagnet, where surface-wave propagation is possible in one sense across the magnetization.

These calculations show that for the Y-Gd superlattice in the antiphase ground state, we have a rich spectrum of surface spin waves, whose form is influenced strongly by the canting angle α induced by the external magnetic field. As remarked earlier, it would thus be most interesting to explore such materials by the Brillouin scattering method. The reader will also note that the surface-wave spectrum is very different in nature than that found in our earlier study of the two sublattice, uniaxial antiferromagnet in a transverse field. While the magnetic permeability tensors for the two cases are very similar in structure, here we are in a very different parameter regime.

Dispersion curve plots such as those displayed in Fig. 2 outline the nature of the surface-wave propagation characteristics, for any choice of canting angle, but provide one with little feeling for whether they contribute importantly to the total mean square spin-fluctuation amplitude near the surface of the material, and thus will appear as prominent features in the light-scattering spectrum. Spin fluctuations near the surface are driven by both the thermally excited surface waves bound to the surface, and also by thermally excited bulk waves which propagate up to and reflect off the surface. The answer to this question requires one to assess the relative strength of these two contributions.

In earlier papers,[9,10,13] this issue is addressed within the framework of a Green's-function approach, structured as follows. One begins by assuming the system is driven by an external magnetic field $h_\beta(\mathbf{x},t)$, and constructs a dynamic susceptibility tensor $\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';t-t')$ which describes the response of the semi-infinite structure to the external field. The fluctuation-dissipation theorem is then used to relate the dynamic susceptibility tensor to the amplitude of the thermally induced spin fluctuations near the surface. Given this information, one may compute the Brillouin spectrum. The results of such analyses have provided a fully quantitative account of the shape and relative intensity of bulk and surface mode contributions to the Brillouin spectra of semi-infinite ferromagnets, ferromagnetic films and superlattices.[9,10,13]

A full theory of the Brillouin spectrum of the Y-Gd superlattice in the canted state is technically complex, because of the low symmetry of the ground state. Here we shall be content with a more schematic discussion which does provide one with an adequate feeling for the relative importance of the bulk and surface-wave contributions to the spin fluctuations near the surface.

Let $m_\alpha(\mathbf{x},t)$ be the αth component of the magnetization fluctuation near the surface, and form the correlation function $\langle m_\alpha(\mathbf{x}t)m_\beta(\mathbf{x}',t')\rangle_T$, which depends on only $(t-t')$, and the differences $\mathbf{x}_\parallel-\mathbf{x}_\parallel'$. Within our average medium approach, such a correlation function may be constructed through use of the fluctuation-dissipation theorem as just discussed, once the Green's functions associated with the average medium permeability tensor are known. A derivation of these Green's functions is given in the Appendix of the present paper.

For the cases explored earlier, the Brillouin intensities may be expressed in terms of certain spectral density functions $S_{\alpha\beta}(\mathbf{k}_\parallel,\Omega)$, given by

$$S_{\alpha\beta}(\mathbf{k}_\parallel\Omega)=\int_0^\infty dz\int_0^\infty dz'\int d^2x_\parallel\int_{-\infty}^{+\infty}dt\, e^{i\mathbf{k}_\parallel\cdot(\mathbf{x}_\parallel-\mathbf{x}_\parallel')}e^{-i\Omega(t-t')}e^{i(k_\perp^{(I)}z+k_\perp^{(s)}z')}\langle m_\alpha(\mathbf{x}_\parallel t)m_\beta(\mathbf{x}_\parallel' t')\rangle \ . \tag{4.3}$$

If $\mathbf{k}^{(I)}$ and $\mathbf{k}^{(s)}$ are the wave vectors of the incident and scattered light in the vacuum outside the sample, and $\Omega^{(I)}$ and $\Omega^{(s)}$ their frequencies, then $\mathbf{k}_\parallel=\mathbf{k}_\parallel^{(I)}-\mathbf{k}_\parallel^{(s)}$ and $\Omega=\Omega^{(I)}-\Omega^{(s)}$, while $k_\perp^{(I)}$ and $k_\perp^{(s)}$ are the complex wave vectors of the incident and scattered light inside the material. A proper discussion of light scattering from the Y-Gd structure in the antiphase ground state will require not only the correlation functions given in Eq. (4.3), but also a new set involving the staggered magnetization proportional to $\mathbf{a}-\mathbf{b}$, in the notation of Sec. III.

To obtain a feeling for the surface magnon contributions to the spin-fluctuation amplitude near the surface of the structure, we have calculated $S_{xx}(\mathbf{k}_\parallel,\Omega)$ and $S_{zz}(\mathbf{k}_\parallel,\Omega)$, with both $k_\perp^{(I)}$ and $k_\perp^{(s)}$ replaced by i/δ, with δ an estimate of the skin depth. The physical meaning of these two functions is the following. Consider the outermost portion of the structure, a slab with thickness δ. Then $S_{xx}(\mathbf{k}_\parallel\Omega)$, considered a function of Ω for fixed $\mathbf{k}_\parallel$, gives the frequency spectrum of the square of the amplitude of the thermal spin fluctuations within the optical skin depth with the wave vector $\mathbf{k}_\parallel$, in the direction parallel to the surface and to the external magnetic field. Similarly, $S_{zz}(\mathbf{k}_\parallel,\Omega)$ is the frequency spectrum of the fluctuations in the skin depth parallel to the surface, but perpendicular to the applied field. As remarked earlier, the frequency spectrum of both sets of fluctuations contains contributions from thermally excited surface magnons, and from bulk modes which propagate up to the surface, and reflect off it.

In Figs. 3–5, we show plots of the two spectral functions $S_{xx}(\mathbf{k}_\parallel,\Omega)$ and $S_{zz}(\mathbf{k}_\parallel,\Omega)$, for various propagation angles, and for the magnetic fields used in the dispersion curve plots given in Fig. 2. In the calculations, we have taken $k_\parallel^{(I)}=k_\perp^{(s)}=i/\delta$, with δ a measure of the skin depth, chosen here equal to $0.1k_\parallel$. With $\mathbf{k}_\parallel$ chosen equal to that employed in typical Brillouin scattering experiments, this corresponds to averaging the spin fluctuations over a skin depth of a few hundred Angstroms. In the figures, the structures labeled with the symbol B described structures in the spectral density produced by the bulk spin waves in the shaded region of Figs. 2(a) and 2(b), while the lines labeled SM_1 and SM_2 are lines associated with the various surface spin waves. Strictly speaking, the surface spin waves introduce Dirac-delta functions into the spectral densities, and these have been broadened artificially by endowing the frequency Ω which enters the Fourier transform of $\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';t-t')$ with a small imaginary part.

The spectral density plots show clearly that both surface wave branches make strong, observable contributions to the amplitude of thermal fluctuations within the skin depth of the superlattice. There should be no difficulty observing these modes in a light-scattering study, at least in principle. It would be of particular interest to examine the surface waves as a function of the propagation angle ϕ.

There is one point that remains to be discussed. This is the physical origin of the feature labeled $\Omega_\perp$ in Fig. 4.

Our discussion of the bulk normal modes of the spin system, here based entirely on macroscopic considerations, assume the spin motions generate a demagnetizing field which can be expressed as the gradient of the magnetic potential Φ_M introduced just after Eq. (2.14a). In general, for propagation in an anisotropic medium, for a wave vector removed from a high symmetry direction, all the normal modes will involve spin motions that have a component of dynamic magnetization parallel to the wave vector, and will thus generate a macroscopic demagnetizing field as a consequence.

However, if the wave vector is directed along a high-symmetry direction, then we may have modes that are purely transverse, and generate no macroscopic magnetic field as a consequence. All such modes do not emerge from a description such as that given earlier, which assumes that the macroscopic field is nonvanishing.

Consider the propagation of bulk magnons parallel to the y direction, for $k_x=k_z=0$ in Eq. (4.2). The frequencies given by Eq. (4.2) in this case are $\Omega_{B+}(\mathbf{k})=\Omega_\parallel$, a purely transverse mode with magnetization fluctuation parallel to the x axis, and $\Omega_{B-}=(\Omega_\perp^2+\omega_{yy}^2)^{1/2}$, a mode

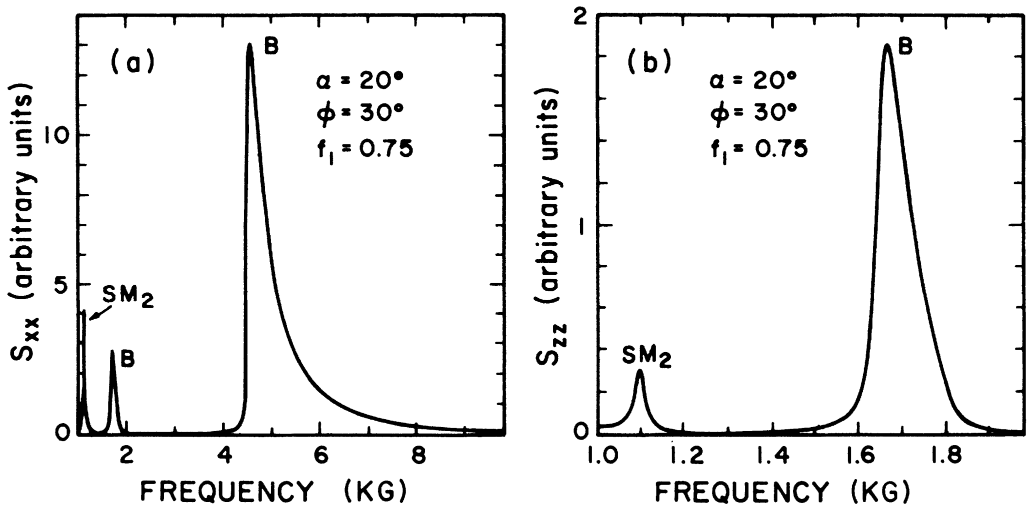

FIG. 3. For a magnetic field which induces a canting angle α of 20°, and the propagation direction $\phi=30°$, we show the spectral density functions (a) $S_{xx}(\mathbf{k}_{\parallel},\Omega)$ and (b) $S_{zz}(\mathbf{k}_{\parallel},\Omega)$, for $f_1=0.75$. The line SM_2 comes from the surface magnon branch displayed in Fig. 2(a), and the structures labeled B have their origin in bulk magnons present as thermal excitations within the skin depth.

polarized in the yz plane with a net magnetization fluctuation parallel to the y direction. The macroscopic field generated by the spin motion "stiffens" the response of the system; this is described by the contribution ω_{yy}^2. This mode is the analogue of the longitudinal optical phonon of an ionic dielectric, and the mode with frequency $\Omega_{\parallel}$ is equivalent to a transverse optical phonon, polarized parallel to the x axis.

There is also a mode analogous to a transverse-optical phonon polarized parallel to the z direction when the wave vector is directed along the y axis, and this has frequency $\Omega_{\perp}$. One may see this as follows, within the present framework. For spin fluctuations in the yz plane, we may invert the relation between the magnetic induction components $\langle b_\alpha \rangle$, and those of the magnetic field $\langle h_\alpha \rangle$, to find

$$\langle h_y \rangle = \frac{1}{(\bar{\mu}_{yy}\bar{\mu}_{zz}+\bar{\mu}_{yz}^2)}(\bar{\mu}_{zz}\langle b_y \rangle - \bar{\mu}_{yz}\langle b_z \rangle) \tag{4.4a}$$

and

$$\langle h_z \rangle = \frac{1}{(\bar{\mu}_{yy}\bar{\mu}_{zz}+\bar{\mu}_{yz}^2)}(\bar{\mu}_{yy}\langle b_z \rangle + \bar{\mu}_{yz}\langle b_y \rangle) \ . \tag{4.4b}$$

The modes under discussion have $\langle h_y \rangle = \langle h_z \rangle = 0$ since no macroscopic demagnetizing fields are present, but also $\langle b_z \rangle$ and $\langle b_y \rangle$ are nonzero, since there is a net dynamic magnetization associated with the spin motion. Thus, we require the frequency such that

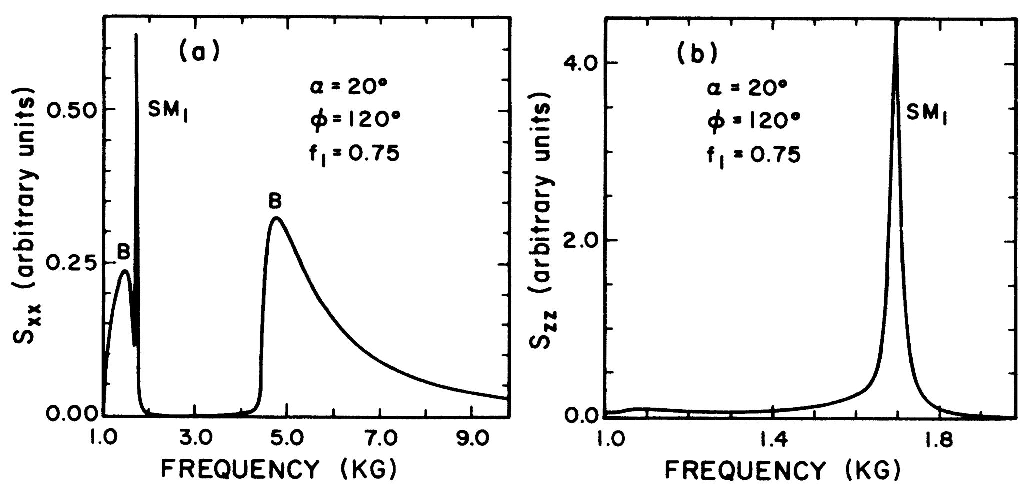

FIG. 4. The same as Fig. 3, but now the canting angle $\phi=120°$. The feature labeled SM_1 is produced by the surface spin-wave branch in Fig. 2(a).

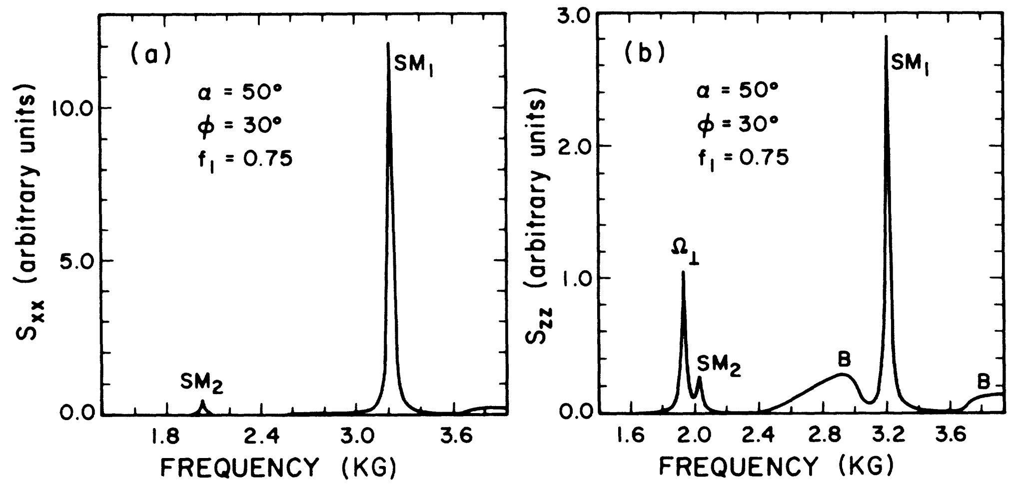

FIG. 5. The same as Fig. 3, but now the magnetic field has been increased so the canting angle α is 50°. The labeling is the same as that used in Figs. 3 and 4, save for the feature $\Omega_\perp$, whose physical origin is discussed in the text.

$$D=\frac{1}{(\bar{\mu}_{yy}\bar{\mu}_{zz}+\bar{\mu}_{yz}^2)^2}\begin{vmatrix}\bar{\mu}_{zz} & -\bar{\mu}_{yz}\\ \bar{\mu}_{yz} & \bar{\mu}_{yy}\end{vmatrix}=\frac{1}{(\bar{\mu}_{yy}\bar{\mu}_{zz}+\bar{\mu}_{yz}^2)}=0\ . \tag{4.5}$$

The condition in Eq. (4.5) is satisfied when $\Omega=\Omega_\perp$.

The Green's function G_{33} derived in the Appendix has a term proportional to $(\bar{\mu}_{yy}\bar{\mu}_{zz}+\bar{\mu}_{yz}^2)\delta(y-y')$. This is the term with origin in the modes just described, which are frequency independent of k_y, in the limit $k_y \gg k_\parallel$. Such modes are dispersionless, and contribute a delta function to the spectral density.[18]

There are also modes polarized parallel to the x direction (linearly polarized) which fail to generate a macroscopic magnetic field. Their frequency is determined by the condition

$$\langle h_x\rangle=\frac{1}{\bar{\mu}_{xx}}\langle b_x\rangle=0 \tag{4.6}$$

so we require

$$\frac{1}{\bar{\mu}_{xx}}=0\ , \tag{4.7}$$

satisfied when $\Omega=\Omega_\parallel$. We note that G_{xx} has a term of the form $\delta(y-y')/\bar{\mu}_{xx}$. This frequency coincides precisely with one of the bulk band edges in all of the examples given, and in the spectral densities we do not see this as a separate, distinct line, but rather a contribution from the bands of bulk spin waves discussed earlier.

The analysis presented here shows that the antiphase ground state of the Y-Gd superlattice, placed in a transverse field, possesses a rich spectrum of collective spin-wave modes, in the bulk and localized on the surface. In the limit all constituent films were viewed as very thin, compared to the wavelength of the modes of interest, the effective medium approach developed here provides a rather simple and appealing means of describing them.

ACKNOWLEDGMENTS

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APPENDIX: CONSTRUCTION OF THE DYNAMIC SUSCEPTIBILITY FOR THE ANTIPHASE GROUND STATE IN A TRANSVERSE FIELD: THE SEMI-INFINITE GEOMETRY

Suppose our system is subjected to a time and spatially varying magnetic field $h_\alpha^{(e)}(\mathbf{x})\exp(-i\Omega t)$ of external origin. Of interest here is the dynamic susceptibility tensor $\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';\Omega)$ that relates the amplitude $\langle m_\alpha(\mathbf{x})\rangle$ of the time-dependent component of magnetization induced at $\mathbf{x}$, to the amplitude of the external field, for the average medium description of the structure of interest:

$$\langle m_\alpha(\mathbf{x})\rangle=\sum_\beta\int d^3x'\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';\Omega)h_\beta^{(e)}(\mathbf{x}')\ . \tag{A1}$$

This function is the Fourier transform with respect to time of the function $\chi_{\alpha\beta}(\mathbf{x}\mathbf{x}';t-t')$ mentioned briefly in Sec. IV of the text, just before Eq. (4.3). As remarked there, given this function, through use of the fluctuation-dissipation theorem, we may find the frequency spectrum of thermal spin fluctuations in the medium, near the surface or elsewhere. The manner in which this is done is discussed in an early paper on the dynamic response of the semi-infinite ferromagnet.[9] Precisely the same formalism is used here, so our attention is confined only to the construction of $\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';\Omega)$ in the present paper.

We shall suppose $h_\alpha^{(e)}(\mathbf{x})$ varies in a wavelike manner in the two directions parallel to the surface, so $h_\alpha^{(e)}(\mathbf{x})=h_\alpha^{(e)}(y)\exp(i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel)$. Then we have, since $\chi_{\alpha\beta}(\mathbf{x},\mathbf{x}';\Omega)$ depends on only $\mathbf{x}_\parallel-\mathbf{x}'_\parallel$, $\langle m_\alpha(\mathbf{x})\rangle=m_\alpha(y)\exp(i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel)$. We then see easily that

$$m_\alpha(y)=\sum_\beta \int_0^\infty dy'\chi_{\alpha\beta}(y,y';\mathbf{k}_\parallel\Omega)h_\beta^{(e)}(y') , \quad \text{(A2)}$$

where

$$\chi_{\alpha\beta}(\mathbf{x}\mathbf{x}';\Omega)=\int \frac{d^2k_\parallel}{(2\pi)^3}e^{i\mathbf{k}_\parallel\cdot(\mathbf{x}_\parallel-\mathbf{x}'_\parallel)}\chi_{\alpha\beta}(yy';\mathbf{k}_\parallel\Omega) . \quad \text{(A3)}$$

In what follows, in the interest of compactness, we omit explicit reference to both $\mathbf{k}_\parallel$ and Ω, in the various quantities that depend on these variables.

The spin motions induced by the external field induce an internal magnetic field $\langle \mathbf{h}\rangle,=-\nabla\Phi_M$, very much as in our discussions of spin waves. Thus, the total field seen by a given spin is the sum $\langle h_\alpha\rangle+h_\alpha^{(e)}$ of the internally generated and the external field. We then may write for our system

$$m_x=\frac{1}{4\pi}(\bar{\mu}_{xx}-1)(\langle h_x\rangle+h_x^{(e)}) , \quad \text{(A4a)}$$

$$m_y=\frac{1}{4\pi}(\bar{\mu}_{yy}-1)(\langle h_y\rangle+h_y^{(e)})+\frac{1}{4\pi}\bar{\mu}_{yz}(\langle h_z\rangle+h_z^{(e)}) , \quad \text{(A4b)}$$

$$m_z=-\frac{1}{4\pi}\bar{\mu}_{yz}(\langle h_y\rangle+h_y^{(e)})+\frac{1}{4\pi}(\bar{\mu}_{zz}-1)(\langle h_z\rangle+h_z^{(e)}) , \quad \text{(A4c)}$$

while in addition $\nabla\cdot\langle \mathbf{b}\rangle=0$ is written, in the present case,

$$\left[k_\parallel^2-\frac{\partial^2}{\partial y^2}\right]\Phi_M+4\pi i k_x\langle m_x\rangle +4\pi i k_z\langle m_z\rangle+4\pi\frac{\partial}{\partial y}\langle m_y\rangle=0 . \quad \text{(A4d)}$$

If we let $\bar{\mu}_{xx}=1+4\pi\bar{\chi}_{xx}$, $\bar{\mu}_{yy}=1+4\pi\bar{\chi}_{yy}$, $\bar{\mu}_{zz}=1+4\pi\bar{\chi}_{zz}$, and $\bar{\mu}_{yz}=4\pi\bar{\chi}_{yz}$, then the first three of Eqs. (A4) can be arranged to read

$$\frac{1}{\bar{\chi}_{xx}}m_x+ik_x\Phi_M=h_x^{(e)} , \quad \text{(A5a)}$$

$$\frac{\bar{\chi}_{zz}}{(\bar{\chi}_{zz}\bar{\chi}_{yy}+\bar{\chi}_{yz}^2)}m_y-\frac{\bar{\chi}_{yz}}{(\bar{\chi}_{zz}\bar{\chi}_{yy}+\bar{\chi}_{yz}^2)}m_z+\frac{\partial\Phi_M}{\partial y}=h_y^{(e)} , \quad \text{(A5b)}$$

and

$$\frac{\bar{\chi}_{yy}}{(\bar{\chi}_{za}\bar{\chi}_{yy}+\bar{\chi}_{yz}^2)}m_z+\frac{\bar{\chi}_{yz}}{(\bar{\chi}_{zz}\bar{\chi}_{yy}+\bar{\chi}_{yz}^2)}m_y+ik_z\Phi_M=h_z^{(e)} . \quad \text{(A5c)}$$

Our task is to solve Eq. (A5), in combination with Eq. (A4d). We have a set of four differential equations, in the four variables m_x, m_y, m_z, and Φ_M. We introduce a four vector $\mathbf{u}=(m_x,m_y,m_z,\phi_M)$, with components $u_1=m_x$, $u_2=m_y$, $u_3=m_z$, and $u_4=\Phi_M$, and we let $\mathbf{F}=(h_x^{(e)},h_y^{(e)},h_z^{(e)},0)$. (A similar notation was used in Ref. 19, but the reader should be aware that the present notation is different in detail than that used earlier.) Then the above system of equations reads

$$\sum_{j=1}^{4} A_{ij}u_j=F_i , \quad \text{(A6)}$$

with $A_\parallel=(\bar{\chi}_{xx})^{-1}$, $A_{12}=A_{13}=0$, $A_{14}=ik_x$, etc. We may introduce a 4×4 Greens-function matrix $g_{ij}(y,y')$ which satisfies

$$\sum_j A_{ij}g_{jk}=\delta_{ik}\delta(y-y') , \quad \text{(A7)}$$

and the solution to Eq. (A6) is then

$$u(y)=\sum_j \int_0^\infty dy' g_{ij}(y,y')F_j(y') . \quad \text{(A8)}$$

The Green's functions just defined are the dynamic susceptibilities we seek. For example, $g_{11}(y,y')=\chi_{xx}(y,y';k_\parallel,\Omega)$ and $g_{33}(y,y')=\chi_{zz}(y,y';\mathbf{k}_\parallel\Omega)$.

We have concentrated on the equations within the medium. While we have interest only in the case where y' lies within the material, boundary conditions at the surface are required to ensure continuity of the tangential components of the magnetic field, and normal component of the magnetic induction generated by the spin motion. For y outside the material, $g_{ij}(y,y')$ vanishes for $i=1$, 2, or 3 because the magnetization vanishes there, while $g_{4j}(y,y')$ obeys $(k_\parallel^2-\partial^2/\partial y^2)g_4j(y,y')=0$ and is thus given by $g_{4j}(y,y')=C^{<}\exp(k_\parallel y)$, with the medium in the upper half space $y>0$. The boundary conditions at the surface read

$$g_{4j}\,|_{y=0-}=g_{4j}\,|_{y=0+} , \quad \text{(A9a)}$$

which ensures continuity of tangential components of magnetic field, and

$$\left.\frac{\partial g_{4k}}{\partial y}\right|_{y=0-}=\left.\frac{\partial g_{4j}}{\partial y}\right|_{y=0+}-4\pi g_{2j}\,|_{y=0+} , \quad \text{(A9b)}$$

which ensures continuity of the normal components of the magnetic induction.

For fixed j, and i ranging from 1 to 4, we have a set of four coupled differential equations. For example, for $j=1$, we may rearrange three of the four basic equations to obtain

$$g_{11}=-ik_x\bar{\chi}_{xx}g_{41}+\bar{\chi}_{xx}\delta(y-y') , \quad \text{(A10a)}$$

$$g_{21}=-\bar{\chi}_{yy}\frac{\partial g_{41}}{\partial y}-i\bar{\chi}_{yz}g_{41} , \quad \text{(A10b)}$$

and

$$g_{31}=\bar{X}_{yz}\frac{\partial g_{41}}{\partial y}-i\bar{X}_{zz}g_{41}\ , \tag{A10c}$$

which then lead to a single elementary equation for g_{41}:

$$\frac{\partial^2 g_{41}}{\partial y^2}-k_{\parallel}^2\kappa^2 g_{41}=\frac{4i\bar{X}_{xx}}{\bar{\mu}_{yy}}\delta(y-y') \tag{A11}$$

where

$$\kappa^2=\frac{4\pi}{\bar{\mu}_{yy}}[(\bar{X}_{xx}\sin^2\phi+\bar{X}_{zz}\cos^2\phi)+1]\ , \tag{A12}$$

where $k_x=k_{\parallel}\phi$, and $k_z=k_{\parallel}\cos\phi$.

It is now straightforward matter to find the various Green's functions. The spectral functions denoted by S_{xx} in the text were calculated by using the form

$$g_{11}(y,y')=\frac{(\bar{\mu}_{xx}-1)}{4\pi}\delta(y-y')+\frac{2\pi k_{\parallel}\bar{X}_{xx}^2\sin^2\phi}{\kappa\bar{\mu}_{yy}}\left[\left[\frac{1-\kappa\bar{\mu}_{yy}-4\pi\bar{i}X_{yz}\cos\phi}{1+\kappa\bar{\mu}_{yy}-4\pi i\bar{X}_{yz}\cos\phi}\right]e^{-k_{\parallel}\kappa(y+y')}-e^{-k_{\parallel}\kappa|y-y'|}\right]\ , \tag{A13}$$

while the spectral densities referred to as S_{zz} were calculated from (using the hierarchy with $j=3$)

$$g_{33}(y,y')=\left[\frac{4\pi\bar{X}_{yz}+\bar{X}_{zz}\bar{\mu}_{yy}}{\bar{\mu}_{yy}}\right]\delta(y-y')-2\pi k_{\parallel}\frac{(\kappa\bar{X}_{yz}+i\bar{X}_{zz}\cos\phi)^2(1-\kappa\bar{\mu}_{yy}-4\pi iX_{yz}\cos\phi)}{\kappa\bar{\mu}_{yy}(1+\kappa\bar{\mu}_{yy}-4\pi i\bar{X}_{yz}\cos\phi)}$$
$$-\frac{2\pi k_{\parallel}}{\kappa\bar{\mu}_{yy}}(\bar{X}_{zz}^2\cos^2\phi+\bar{X}_{yz}^2\kappa^2)e^{-\kappa|y-y'|}\ . \tag{A14}$$

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[18]This remark requires elaboration. The spectral density $S_{zz}(\mathbf{k}_{\parallel},\Omega)$ receives contributions from all surface modes of wave vector $\mathbf{k}_{\parallel}$ and appropriate polarization, and all bulk modes with waves vectors $\mathbf{k}=\mathbf{k}_{\parallel}+\hat{\mathbf{y}}k_y$, where $-\infty\le k_y \le +\infty$. The Dirac delta function has its origin in the contributions from those portions of the k_y axis where $|k_y|\gg|\mathbf{k}_{\parallel}|$, where we encounter the dispersionless bulk mode of frequency $\Omega_{\perp}$.

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