Magnetic-flux dependence of the resistive transition in a square Josephson-junction array

John P. Carini*
Department of Physics and The James Franck Institute, The University of Chicago, Chicago, Illinois 60637
(Received 28 October 1987)

We report measurements of the resistive behavior as a function of the perpendicular magnetic flux in a square proximity-coupled Josephson-junction array. The array was made by a novel ion-beam sputtering technique using the University of Chicago scanning ion microscope. The magnetic flux produces a uniform frustration of phase ordering between the superconducting islands in the array. We have measured both the Ohmic and non-Ohmic resistances as a function of temperature at several values of the frustration index $f=\phi/\phi_0$, the ratio of the applied flux ϕ to the superconducting flux quantum $\phi_0=hc/2e$. We compare the resistive transitions in the array with no frustration at $f=0$ and $f=1$, with weak frustration at $f=0.05$, with commensurate frustration at $f=\frac{1}{2}$, and with incommensurate frustration at $f=1-1/\tau$ and at $f=1/\tau$ [$\tau=(1+\sqrt{5})/2$, the golden ratio]. We find that the transitions at both $f=0$ and $f=1$ resemble the Kosterlitz-Thouless transition of a two-dimensional superfluid. The transition at $f=0.05$ is much broader and is not described by the Kosterlitz-Thouless theory. Instead, the resistance has a pinned flux-flow behavior at low temperatures. The transitions at $f=\frac{1}{2}$ and the irrational frustrations are similar to each other but different from the unfrustrated transitions, a result which differs from the expectations of arguments based on the ground-state properties of the array for those fluxes. Our data support the idea that some type of freezing occurs for incommensurate frustration; however, we did not observe the hysteretic effects one might expect to see when cooling and heating through a glass transition.

I. INTRODUCTION

In this paper we will report on the behavior of Josephson-junction arrays in the presence of a magnetic field. Such arrays provide an elegant realization of how frustration can be introduced into a system without any disorder in the underlying Hamiltonian. This frustration can be easily tuned by varying the strength of the field. We will compare the nature of the transitions that occur in the frustrated arrays with that which occurs in the unfrustrated cases when the field is zero. The arrays consist of a periodic lattice of superconducting islands coupled by the proximity effect through a normal metal region.[1] Applying a perpendicular magnetic flux to the array leads to a coupling energy between neighboring islands[2]

$$E_{ij}=-J_0\cos(\theta_i-\theta_j-A_{ij}) .$$

Here, $J_0=(\hbar/2e)i_c(T)$, $i_c(T)$ is the temperature dependent critical current in the junction between the islands, θ_j is the phase of the Cooper-pair wave function on the jth island, and $A_{ij}=(2e/\hbar c)\int_i^j \mathbf{A}\cdot d\mathbf{l}$ is a phase factor proportional to the line integral of the vector potential $\mathbf{A}$, between islands i and j. As a result, the arrays are physical realizations of the two-dimensional uniformly frustrated XY model.[3] The frustration index is $f=\phi/\phi_0$, which is the ratio of the flux ϕ per unit cell to the superconducting flux quantum $\phi_0=hc/2e$.

The frustration oscillates periodically as a function of f with a period equal to one. When f has an integral value the phase order in the array is unfrustrated. In this case, as in two-dimensional (2D) superconducting films,[4–6] it has been shown[7] that there should be Kosterlitz-Thouless vortex unbinding transition.[8] This has been verified by experiments.[9–11] When one applies a field to the array the nature of the ground-state changes, and it is no longer clear that a Kosterlitz-Thouless transition occurs. At rational values of the flux, ordered ground states of vortex superlattices exist.[12,13] Analyses of the ground state have led both to estimates for the zero-temperature properties, such as the critical current and ground-state energies, and to estimates for the transition temperatures. In particular, Teitel and Jayaprakesh have proposed[12] that, for the case $f=p/q$, where p and q are integers that do not share a common factor, the upper bound on the dimensionless transition temperature (kT/J_0) is proportional to $1/q$. The nature of the phase transition has not been determined. For example, in a square array at $f=\frac{1}{2}$, known as the fully frustrated XY model,[14] the transition could have critical fluctuations due to either a vortex-antivortex unbinding (leading to a Kosterlitz-Thouless transition), or to domain boundary motion (leading to an Ising-like transition).[3]

These ground-state arguments have led to the conclusion that no transition to a superconducting state occurs when f is irrational.[12,13] This conclusion has recently been challenged by Halsey[15] and Choi and Stroud,[16] who have suggested that there may be a transition into a vortex glass state: a disordered metastable state in which the mobility of the vortices is impeded by neighboring vortices. Halsey's Monte Carlo simulation found that the metastable states carry a nonzero critical supercurrent, in contrast to the ground-state prediction of $I_C(T=0,f\neq p/q)=0$. (We will use upper case I to denote the current through the entire array and lower case i to denote the current through a single junction.)

These ideas are related to the notion that the glass transition in supercooled liquids of close-packed spheres is due to the uniform frustration that results because tetrahedra cannot pack without defects in a Euclidean three-dimensional space.[17] Both this model of the structural glass and Halsey's "superconducting glass" rely on uniform frustration without disorder to give rise to the glassy properties of the system. We note, however, that the superconducting arrays are two-dimensional and may, as with 2D spin-glasses,[18] never truly freeze until zero temperature. Nevertheless the dynamics of the arrays may get progressively more slow as the temperature is lowered until the vortex pattern appears stationary on the time scale of any experiment. In this case the system, caught in a long-lived metastable state, appears to be a superconducting glass.

In this paper, we report measurements of the resistive transitions in a square proximity-coupled Josephson-junction array at several values of f. The array was made by a novel ion-beam technique using the University of Chicago scanning ion microscope. We will compare the unfrustrated transitions at $f=0$ and $f=1$, the weakly frustrated transition at $f=\frac{1}{20}$, the fully frustrated transition at $f=\frac{1}{2}$, and the incommensurately frustrated transitions at the irrational flux values $f=1-1/\tau$ and $f=1/\tau$ [$\tau=(\sqrt{5}+1)/2\approx1.618\ldots$, the golden ratio]. We find that the transitions at $f=0$ and $f=1$ are quite similar and resemble the Kosterlitz-Thouless transition previously seen at $f=0$ in arrays. In contrast, at $f=\frac{1}{20}$, the transition is substantially smeared out and no longer has the characteristics of a Kosterlitz-Thouless transition. No dramatic difference is seen between the resistive transitions at the commensurate flux $f=\frac{1}{2}$ and at the nearby incommensurate fluxes $f=1/\tau$ and $f=1-1/\tau$. These transitions show a behavior that is intermediate between the unfrustrated transitions and the transition at a small flux.

II. BACKGROUND THEORY AND EXPERIMENT

To compare the nature of the transitions occurring at the different fluxes, we have measured the Ohmic and non-Ohmic electrical resistance as a function of temperature at each flux. We will briefly discuss what information these quantities provide about the state of the vortices in the array.

The Ohmic resistance R is proportional to the density of unbound vortices in the array[4–6] which are free to move and also able, therefore to dissipate energy when a current is applied through the array[7]

$$R(T)/R_n=b^2 n_f(T)\mu(T)\ , \tag{1}$$

where R_n is the normal state resistance, b is the lattice constant, n_f is the density of free vortices, and $\mu(T)$ describes the temperature dependence of the vortex mobility caused by the thermal activation across a single junction in a square array:[7] $\mu(T)=[\mathcal{J}_0(J_0(T)/\ 10k_BT)]^{-2}$, where $\mathcal{J}_0$ is the hyperbolic Bessel function of order 0. Note that $n_f b^2$ is the number of vortices per cell in the array, which may be produced by thermal activation $n_{fT}b^2$ or by an applied magnetic flux $n_{fH}b^2\approx f$. The vanishing Ohmic resistance of the superconducting state occurs when the vortices bind in pairs with antivortices, as below the Kosterlitz-Thouless transition temperature in the unfrustrated XY model, or, possibly, organize in more complicated vortex lattices.

Higher measuring currents produce a non-Ohmic resistance, $R_{nl}=V_{nl}/I$. This dissipation results from current-assisted thermal activation of bound or pinned vortices. We will define a non-Ohmic power-law exponent $a(T)$ that is the low-voltage slope of the I-V curve on a log-log plot

$$a(T)=\{d\ln[V(I)]/d\ln(I)\}_{V\to0}\ . \tag{2}$$

This definition is used because, in a vortex unbinding transition, $a(T)$ is related to the renormalized interisland coupling strength $J^*(T)$. In the presence of a current density $I/(Nb)$ the potential energy for a vortex pair of separation s is[19]

$$U(s)=(\phi_0/c)\{[I/(Nb)]s-2\pi(I_c^*/N)\ln(s/b)\}\ , \tag{3}$$

where I_c^* is the critical current of the array renormalized by thermal effects such as vortex fluctuations and Nb is the width of the array. Essentially, a bound vortex pair in a current is a metastable state since the Lorentz force will exceed the vortex attraction for pair separations greater than $s_c/b=I_c^*/I$. The non-Ohmic resistance is proportional to the density of current-depaired vortex pairs n_{fI} which is determined by the equilibrium between the thermal activation rate over the energy barrier $E=2\pi J^*\ln(I_c/I)$ and the recombination rate with other free vortices, a binary process:[19]

$$R_{nI}/R_n\propto n_{fi}\approx\{\exp[(-2\pi J^*/k_BT)\ln(i_c/i)]\}^{1/2}$$
$$=(i/i_c)^{(\pi J^*/k_BT)}\ .$$

Therefore, in the limit of vanishing measuring current, the exponent gives direct information about the renormalized coupling between the islands at temperatures near to and below the transition [in contrast to the unrenormalized (or bare) coupling, J_0, which appears in the Hamitonian]: $a(T)-1=\pi J^*(T)/k_BT$ ($=\pi K$ is known as the renormalized reduced superfluid stiffness constant;[4] J^* is also known as the superfluid helicity modulus).

The theory[20] for the Kosterlitz-Thouless transition which includes renormalization of the bare coupling also leads to a universal relation[21] between the transition temperature and the renormalized superfluid density, represented by J^* in our experiment,[7] at the transition temperature: $k_BT_{\rm KT}/J^*(T_{\rm KT})=\pi/2$. Thus, $a-1$ will jump from 0 to the universal value of 2 at the temperature where the Ohmic resistance from free vortices vanishes in a Kosterlitz-Thouless transition.

Both the linear and nonlinear responses will contribute towards an understanding of the nature of the transitions at different fluxes. For the unfrustrated transition there are well-developed theories for the temperature dependence of the Ohmic resistance above and for the non-Ohmic resistance below the transition temperature.[4]

Lobb, Abraham, and Tinkham extended the theories for continuous films to include arrays.[7] Unfortunately no theory for the temperature dependence of the resistance exists for fractional values of f. Several authors have proposed that a transition occurs at $f=\frac{1}{2}$ with a nonuniversal jump in the value of a which may depend on the lattice structure.[3,22,23]

Experiments have only clarified the physics at $f=0$ where a Kosterlitz-Thouless transition has been bound both in proximity-coupled arrays by Resnick *et al.*[9] and by Abraham *et al.*[11] and also in insulator-junction arrays by van Wees, van der Zant, and Mooij,[24] In disagreement, however, Brown and Garland[25] found a jump to $a\approx 2$ instead of to $a=3$ at all fluxes in a triangular proximity-coupled array. They did not explain the absence of the "universal jump" in $a(T_c)$ from 1 to 3 in the unfrustrated transition. In one other study of an array with f different from an integer, van Wees and coworkers[24] have measured the Ohmic and non-Ohmic resistance in a square array at $f=0$ and $f=\frac{1}{2}$ finding a jump to $a=3.2$ at $f=0$ and to $a=5$ at $f=\frac{1}{2}$. No studies have yet reported the behavior of the transition at an incommensurate value of the flux. Leemann *et al.*[26] measured the sheet inductance (as distinct from resistance) in a square array, which showed a Kosterlitz-Thouless transition at $f=0$ and $f=1$. They also found an enhanced inductive response at $f=\frac{1}{2}$ and $f=\frac{1}{3}$ compared to other nonintegral fluxes, which they interpreted as evidence of the formation of an ordered state at those commensurate fractional fluxes.

III. EXPERIMENT

Our arrays were fabricated using a new focused ion-beam sputtering technique. The University of Chicago scanning ion microscope can focus a 40-keV Ga^+ beam to a spot less than 0.1 μm in diameter. Even though the beam has a small total current, on the order of 10^{-11} A, the focused beam has a substantial current density, about 0.1 A/cm^2. This current density is sufficient to sputter through metal films several hundreds of Angstroms thick with a line writing speed of 10 μm/s. The sputtered lines are typically 0.2-μm wide and their spacing can be controlled to 0.05 μm. The maximum size of our arrays was limited to less than 200 μm by the maximum undistorted field of view of the microscope.

The fabrication of an array is a simple two-step process. First, we evaporate onto a glass substrate a normal metal film (1000 Å of copper) immediately followed by a superconducting metal film (300 Å of $Pb_{0.50}Sn_{0.50}$). Second, to make a square array we cut two perpendicular sets of grooves through the top layer using the ion beam. We made our arrays with a lattice constant of 1 μm, which is substantially smaller than arrays that have been previously studied. The distance between the islands was about 0.2 μm, again a much smaller distance than previous proximity-coupled arrays. The array measurements reported here were made on a 128×128 square array.

Resistance measurements were made using superconducting leads evaporated with the original film used to make the array. We measured the I-V curves by passing a low-frequency square-wave current through the array and measuring the voltage in phase with the current across the array with a PAR 124a lock-in amplifier using the 100:1 transformer in the lock-in model 116 preamplifier. An external Helmholz coil produced the magnetic flux. One superconducting flux quantum per cell in the array corresponded to an applied field of approximately 20 Oe.

Since there are three independent experimental parameters, temperature (T), magnetic field (H), and current (I), we needed to determine an experimental protocol for changing them during an experiment. Hysteresis in the superconducting properties in the H-T and in the I-T planes has been suggested as a signature of the glassy superconducting state at irrational values of f. Brown and Garland have reported hysteresis in the behavior of the resistance in "zero-field cooled" arrays compared to "field-cooled" arrays.[25] In measurements below the transition temperature we took care to make measurements only after setting the magnetic field to the desired value, and then cooling the array through the transition temperature to 1.3 K. The I-V curves were measured at each temperature without changing the magnetic flux. We looked for hysteresis in the Ohmic resistance, nonlinear resistance, and low-temperature critical currents in the H-T plane as well as the I-T plane. We saw a measurable hysteretic effect only in the low-temperature properties at $f=0$. The increased resistance and decreased critical currents we observed after cooling the array at $f=0$, changing the flux and then returning to zero magnetic field, were probably due to a small amount of residual flux. The hysteretic behavior was noticeably solely near to $f=0$ because the transport properties are most sensitive to changes of flux near $f=0$, where changing f by $\frac{1}{2000}$ produces measurable effects.

The transition temperature of the superconducting islands and the sample leads is approximately 5 K. Below this temperature the resistance of the array drops to 0.2 Ω, which we define as the array's normal resistance R_n. We will use R to refer strictly to the Ohmic resistance measured in the lowest portion of the I-V curves. The resistive transition of the array occurs below 3.2 K.

In order to compare the behavior of the transitions for the different fluxes we will have to make several important corrections to take into account the fact that the coupling between islands in the array is a function both of the magnetic flux and of the temperature. These corrections are also necessary in order to compare our experiment to the theoretical predictions.

The geometry of our array introduces a different coupling between adjacent islands as the flux is changed from $f=0$ to $f=1$. The reason for this is that, since the superconducting islands expel the flux, almost all of the magnetic field is trapped in the region between the islands, i.e., in the junction itself. Since our array is square, with a relatively narrow island separation, the flux per junction is about half the flux per plaquette as is shown in Fig. 1. The flux in the junction reduces the critical current in the junction, and therefore the coupling between islands depends on the magnetic flux:[2] $J(f)=J(f=0)\sin(\pi\,|\,f\,|\,r)/(\pi\,|\,f\,|\,r)$, where the ratio of

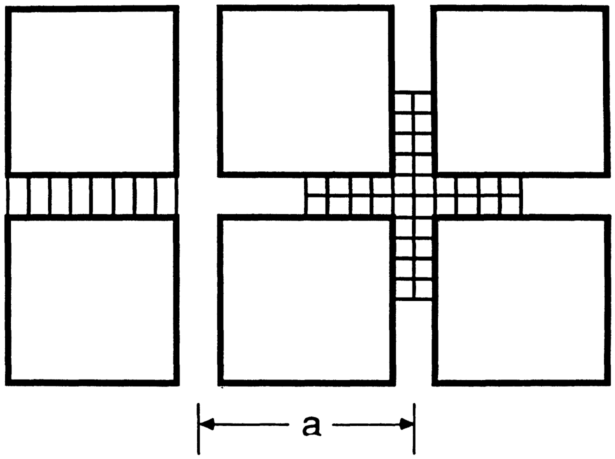

FIG. 1. Schematic drawing of a plaquette in a square array. The effective plaquette area A_p where the magnetic flux in the array is confined, is cross hatched; the junction area A_J where the proximity effect tunneling between neighboring islands occurs, is striped vertically. When the space between islands is small then $A_p \approx 2A_J$.

junction area to plaquette area r is approximately equal to $\frac{1}{2}$ in our array, as we argued above. As a result, the ratios between the coupling at the lowest integer values of f, $J(f=0){:}J(f=1){:}J(f=2)$, are 1:0.6366:0. The coupling actually is expected to vanish near $f=2$, and, indeed, we do not observe a minimum in the resistance there as we do at $f=0$ and $f=1$.

In addition to the flux dependence of the interisland coupling, we must also include the effects of the temperature dependence of the coupling between islands in order to compare the transitions at $f=0$ and $f=1$ and the other fluxes in more detail. Following Lobb and co-workers[7] we define an effective dimensionless temperature $t=k_BT/J(T,f)$, where $J(T,f)$ is the coupling between islands, which is proportional to the temperature-dependent single junction critical current: $J(T,f)=(\hbar/2e)i_c(T,f)$. We measured the critical current at $f=0$ for the array at low temperatures, where the effects of vortex fluctuations are small. We obtained the high-temperature single-junction critical current by fitting our data to the single proximity-coupled junction critical current formula:[27]

$$i_c(T)=i_c(0)(1-T/T_{cs})^2e^{-d/\xi(T)} ,$$

using the dirty limit for the normal metal coherence length,

$$\xi(T)=\{hD/[(2\pi)^2k_BT]\}^{1/2} ,$$

where D is the electron diffusion constant in the normal-metal region. The fitting parameters are $d=2500$ Å, $T_{cs}=5.0$ K, $i_c(0)=5.0$ mA, and $\xi(T_{cs})=200$ Å. The coherence length is considerably shorter than it would be for a pure copper film, probably due to alloying of the evaporated tin with the copper and also to the presence of implanted gallium in the junction region, both of which would increase the junction resistance. The flux dependence was put in using the factor described in the previous paragraph:

$$J(T,f)=J(T,f=0)\sin(\pi|f|/2)/(\pi|f|/2) .$$

Several other effects must be considered when analyzing the data which are not so simple to take into account as quantitatively as we have done for the flux and temperature dependence of the coupling. These effects will influence our interpretation of the data, however, and so we will briefly discuss the important additional corrections.

The theory of the Kosterlitz-Thouless transition predicts that the current-voltage exponent measured in the limit of zero measurement current jumps from 1 to 3 at the Kosterlitz-Thouless transition temperature in an infinite system. We measure the exponent at nonzero current and nonzero frequency in a finite sample, and so we expect that the sharp features of the transition will be broadened by several effects.

(i) Above the transition, where the low-current dissipation is primarily produced by the density of thermally activated free vortices, some of the dissipation in the array will also be due to unpaired vortices generated by the measurement current itself. Therefore the voltage-current exponent a will have a non-Ohmic contribution and will have a value somewhat greater than 1.

(ii) The finite size of the sample smears out the drop to zero density of thermally activated free vortices. As the transition temperature is approached, the activation energy to create a vortex in a finite sample will not diverge as it would in an infinite system.[7] The correlation length in the finite sample cannot exceed about $L/2$, where L is the size of the sample. The reduced resistance, which equals the density of free vortices, will therefore reach a value of about $(2b/L)^2$ instead of zero as it would in an infinite system at the transition temperature. The minimum reduced resistance in our 128×128 array, $R_{\min}/R_n$, is about $\frac{1}{4000}$.

(iii) There are additional sources of Ohmic dissipation due to performing the measurement at a nonzero frequency. Even in an infinite system below the transition temperature, where the density of free vortices is zero, there will still be an Ohmic dissipation due to the dynamics of the thermally generated vortex pairs.[19,28] The vortex pair density, unlike that of individual free vortices, does not drop to zero below the transition temperature. A current polarizes the pair orientation, which relaxes with a characteristic time $t_D=s^2/D$, which is determined by the vortex diffusion constant D and the pair separation s. In a finite sample there is an additional source of Ohmic dissipation due to thermally activated vortices generated near the edge of the array.[19] These vortices are not free since they are pinned to the edge of the sample by an attractive potential to the edge of the array, which is exactly half of the energy needed to create a vortex pair, Eq. (3), replacing the pair separation s by the distance from the vortex to the edge of the array. These vortices will relax in an applied current just as bulk vortex pairs do.

The magnitude of the dissipation due to vortex dynamic effects may be estimated from the vortex diffusion constant D. The vortex diffusion constant can be calculated from known quantities:

$$D = k_B T(2e/h)^2 b^2 R_n = \Omega b^2 \ ,$$

so that we can calculate a vortex hopping rate $\Omega \approx 1.7 \times 10^6$ s^{-1}. Measurements at frequency ω cannot distinguish vortex pair with separations $s/b \approx [(D/b^2)/\omega]^{1/2}$ ($\approx$40–130 lattice spacings at our measurement frequencies of 20–200 Hz). Since this distance is comparable to the size of the array, the additional dissipation in the array due to vortex-pair dynamic effects will be comparable to the dissipation due to the finite-size effects discussed above.

Evidently, the vanishing Ohmic dissipation caused by the rapid decrease in the density of thermally activated free vortices can only be followed in our experiment down to a value which is approximately 0.001 of the normal resistance. The resistance reaching this value indicates that the Kosterlitz-Thouless transition temperature for an infinite system has nearly been reached.

IV. RESULTS AND ANALYSIS

In Fig. 2 we plot the resistance as a function of temperature normalized by the normal resistance $R(T)/R_n$ for several different values of f. The resistance axis is on a logarithmic scale. As we have discussed above, a number of effects must be included in order to analyze this data. Nevertheless a few qualitative conclusions may be made by examining the raw data. The resistance drops rapidly as the temperature is lowered for all fluxes. For $f=0$ and $f=1$ this drop is steepest and become successively less steep at $f=0.05$, 0.50, 0.38, and 0.62. It is evident that the temperature dependence of the resistance changes as the flux is varied. This fact has been noted before,[24–26,29] and implies that for different values of f the nature of the transition is changing beyond a simple shifting of the transition temperature. The resistance oscillates as a function of flux, with deep minima at integral values of f and secondary minima at $f=\frac{1}{2}$, $\frac{1}{3}$, and $\frac{2}{3}$, as has been previously seen[30–33] (see the inset to Fig. 2, where the voltage is plotted as a function of f at constant current and temperature; note that this voltage is due in part to Ohmic and in part to non-Ohmic processes since as the flux is varied the transition temperature oscillates about the measurement temperature). A distinct thermally activated tail appears in the resistance at $f=0$ below $R/R_n = 10^{-3}$.

Figure 3 shows a representative sample of non-Ohmic I-V curves measured at several temperatures through the transition region for $f=0$. Similar I-V curves were measured for each value of f. The solid lines are plotted only to show slopes of 1 and 3 on the full logarithmic plot and are not meant to represent fits to the data. To characterize the non-Ohmic response and to compare between the different fluxes, we will use a single parameter from each I-V curve. For each flux, the exponent $a(T)$ was defined as the logarithmic derivative of the I-V curve evaluated at a voltage of 1 nV: $a = [(I/V)dV/dI]_{V=1\ \mathrm{nV}}$.

Clearly, this one parameter does not fully describe the entire I-V curve. What the slope $a(T)$ measures is the crossover in the resistive behavior, from a high-temperature Ohmic resistance dominated by the motion of free vortices ($a=1$), through the transition temperature, below which the dissipation is a nonlinear function of the current ($a>1$), caused by current-activated vortices such as is described by Eq. (2) in Sec. II. The choice of evaluating the slope of the I-V curve at $V=1$ nV represents a compromise between the sensitivity of the measurement and measuring the slope at a relatively low

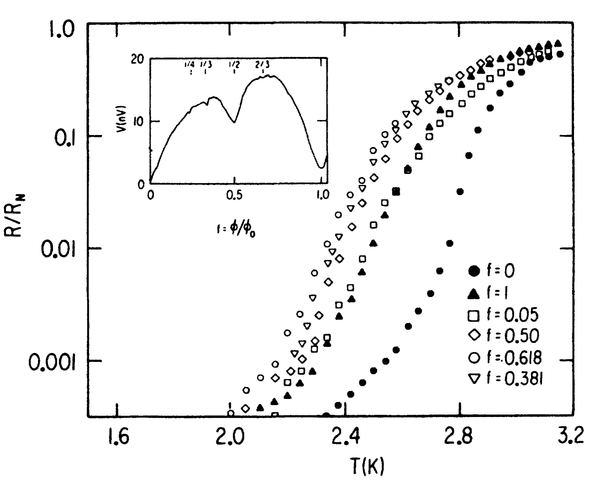

FIG. 2. The reduced resistance R/R_n as a function of temperature T plotted for several values of the flux per plaquette, $f=\phi/\phi_0$. The resistance is plotted on a $\log_{10}$ scale. Inset: The voltage V as a function of f, measured at $T=2.20$ K and $I=5$ μA. Several rational values of f are pointed out. Note that the voltage is not necessarily proportional to the Ohmic resistance as defined in the text.

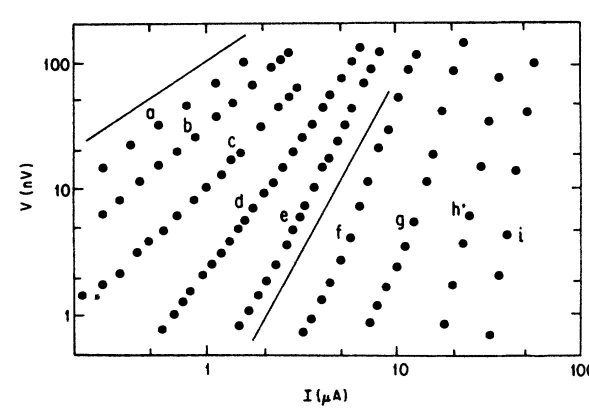

FIG. 3. A set of I-V curves measured at $f=0$ for the following temperatures (the effective temperature t follows in parentheses): (a) 2.979 K (4.51); (b) 2.893 K (3.41); (c) 2.804 K (2.56); 9d) 2.730 K (1.98) (e) 2.656 K (1.56); (f) 2.540 K (1.08); (g) 2.376 K (0.64); (h) 2.202 K (0.36); (i) 2.056 K (0.22). Similar I-V curves were measured for each value of the flux per cell f. The solid lines are plotted only to show slopes of 1 and 3 on the full logarithmic plot and are not meant to represent fits to the data.

amount of dissipation. As shown in the discussion of Eq. (2), the advantage of measuring the slope at lower currents in a Kosterlitz-Thouless transition is to obtain the renormalized intervortex interaction strength J^* at larger distances.[4]

In Fig. 4 we plot $a(T)$ for several values of f. From this graph it is evident that the temperature dependence of the non-Ohmic exponent changes as a function of the flux, as was also true for the Ohmic resistance, which again implies that the nature of the transition changes as the flux is changed. Our data disagree with the data of Brown and Garland for a triangular array which indicated a universal form for $a(T/T_c(f))$ at different values of flux.[25]

In examining the data more quantitatively we shall first compare the transitions at $f=0$ and $f=1$ to previous work and to the predictions for a Kosterlitz-Thouless transition in an unfrustrated array. We will then examine the data at $f=0.05$, and finally at the fluxes $f=0.5$, 0.381, and 0.618, where the frustration is greater.

A. Integer flux

In the transition at $f=0$ the value of a rises above 1 beginning at 2.80 K, when R/R_n of the array (measured at lower currents) has dropped to 0.01. The exponent then reaches a value of 3 at 2.60 K, when the reduced resistance has dropped to 0.001. At $f=1$ the results are similar, with a rising from 1 to 3 in a temperature interval of about 0.2 K where R/R_n drops from 0.1 to 0.001.

Our data for the rise in a from 1 to 3 when measured at a low voltage combined with a rapidly decreasing resistance is similar to earlier work on proximity coupled arrays. Abraham *et al.* measured a 0.2-K temperature width for the exponent (measured at 10 nV in a 1000×1000 square array) to rise from 1 to 3 at the temperature where their measured resistance vanished at $f=0$.[11] Brown and Garland measured a temperature width of 0.05 K at $f=0$ and $f=1$, but their data show a rising rapidly only from 1 to 2.[25] Unlike the Brown and Garland experiment, the resistance in our array has not quite dropped to zero when $a\approx3$. However, as we have argued in the last section, several effects prevent our measured reduced resistance from falling below about 0.001 at the transition temperature. We therefore conclude that the rapid rise in a from 1 to 3 at the temperature where $R/R_n=0.001$ is consistent with previous work at $f=0$ which found a Kosterlitz-Thouless transition.

In order to make more quantitative conclusions, we will next compare R/R_n and a at the different fluxes as a function of the inverse effective temperature $1/t$ which is essentially the reduced interisland coupling $J(T,f)/k_BT$. The reduced resistance and I-V exponent a, each as a function of inverse effective temperature, are shown in Figs. 5 and 6, respectively. Using the effective temperature as the experimental parameter shows that the transitions at $f=0$ and 1 are quite similar to each other as a function of the interisland coupling and that both transitions are clearly distinguished from those at the other fluxes. The resistance at all fluxes has a qualitatively similar behavior, with a rapid drop in resistance that has an activated effective temperature dependence down to $R/R_n\approx0.001$. Such thermally activated behavior has also been noted in insulator junction arrays.[24] The nonlinear exponent at $f=0$ and 1 depends linearly on $1/t$ at temperatures below which the exponent reaches a value near 3. The exponent at the noninteger fluxes rises more slowly as the effective temperature is decreased, increasing somewhat more rapidly at $f=\frac{1}{2}$.

The transitions at $f=0$ and 1 have two features distinguishing them from those at the other fluxes, the large effective activation energy describing the drop in resistance above the transition and the rapidly rising nonlinear resistance exponent a below the transition. The transition temperatures are also the highest at these two fluxes.

Some quantitative differences exist between the transitions at the integer values of f. Even after including the

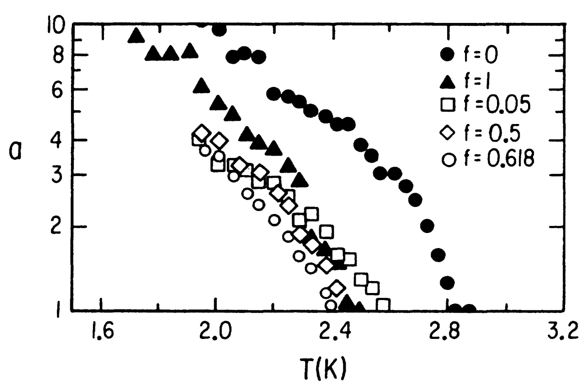

FIG. 4. The current-voltage exponent $a=[(I/V)dV/dI]_{V=1\ \mathrm{nV}}$, as a function of temperature T plotted for several values of the flux per plaquette $f=\phi/\phi_0$. The exponent is plotted on a $\log_{10}$ scale.

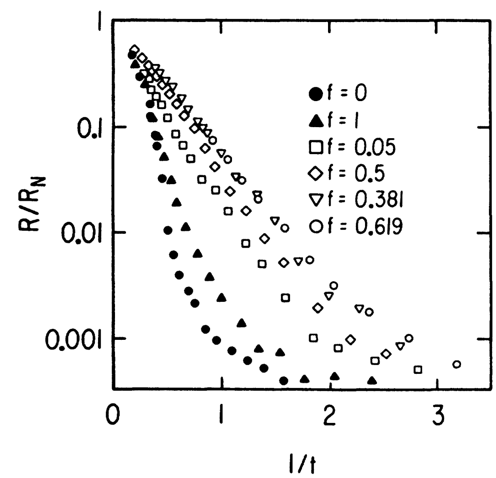

FIG. 5. The reduced resistance R/R_n as a function of inverse effective temperature $1/t$ (which is defined in the text), plotted for several values of the flux per plaquette $f=\phi/\phi_0$. The resistance is plotted on a $\log_{10}$ scale.

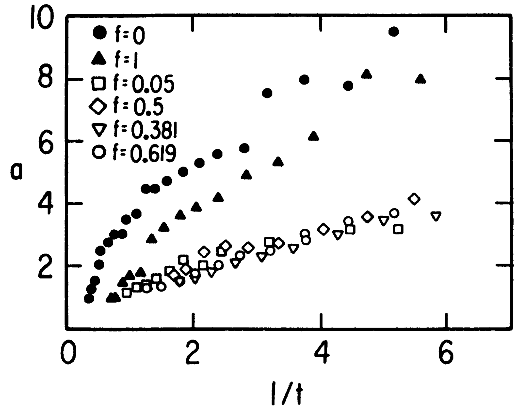

FIG. 6. The current-voltage exponent $a=[(I/V)dV/dI]_{V=1\ \mathrm{nV}}$ as a function of inverse effective temperature $1/t$ (defined in the text), plotted for several values of the flux per plaquette, $f=\phi/\phi_0$. The exponent is plotted on a linear scale.

temperature and flux dependences of the coupling, the resistive transition at $f=1$ occurs at a depressed effective temperature with respect to the $f=0$ transition. The theoretical estimate of Kosterlitz-Thouless for the effective transition temperature t_c is $\pi/2\approx 1.57\ldots$; however, this prediction takes into account all renormalizations of the interisland coupling. Our analysis, which uses estimates for the coupling extrapolated from lower temperatures, does not take into account all such renormalizing effects. This analysis results in an overestimate for the coupling near the transition and gives an experimentally estimated value of t_c that is lower than the theoretical value. Monte Carlo calculations of the specific heat for the XY model find a value for t_c of 0.95.[34] In our array we have estimated the transition as the point where $a\approx 3$ and $R/R_n\approx 0.001$, and we find the effective transition temperature is 1.06 ± 0.07 at $f=0$ which is consistent with the Monte Carlo results. For the case where $f=1$ we find that $t_c=0.64\pm 0.07$, which is a 40% depression in the t_c compared with the $f=0$ case. This is not in agreement with the simple hypothesis that only the strength of the single-junction coupling determines the state of the unfrustrated array. A large depression in t_c at $f=1$ compared with t_c at $f=0$ was also seen by Leemann *et al.*,[26] although they did not explicitly take into account the flux dependence of the single-junction critical current. Brown and Garland measured at a slight depression of t_c at $f=1$, but did not use the effective temperature in analyzing their data.[25]

The effective activation energies for the resistance data in the region above the transition are 12 and 9 at $f=0$ and $f=1$, respectively. An effective transition temperature of 0.92 at $f=0$ and a large activation energy of 14 was seen by van Wees and co-workers[24] who attributed the magnitude to the activation energy of a single vortex at low temperatures in their 128×384 array. The effective activation energies at the other fluxes are considerably lower than at the integer fluxes: 3.9 at $f=0.05$, 3.7 at $f=\frac{1}{2}$, and 3.1 at $f=1/\tau$ and $1-1/\tau$. Below $R/R_n=0.001$ there is a much slower decrease in resistance as a function of $1/t$ for all fluxes. In this range of temperature there is a small effective activation energy of 0.3 to 0.4.

The temperature dependence of a below the transition follows a $1/t$ dependence which approximately agrees with predictions[4] for the Kosterlitz-Thouless transition: $a(t)-1=2t_c/t$. The measured slopes of $a-1$ versus $1/t$ are 1.7 at $f=0$ and 1.3 at $f=1$, values that are within 25% of the predicted values of 2.12 and 1.28. At $f=0$ Van Wees and co-workers[24] measured a faster increase in a below the transition and Abraham *et al.*[11] measured a slower increase (as a function of $1/t$).

B. $f=0.05$

The application of a small amount of flux, $f=0.05$, significantly broadens the resistive transition compared to $f=0$. As seen in Fig. 5, the resistance drops much more slowly as a function of effective temperature than at both $f=0$ and at $f=1$. The behavior of the non-Ohmic resistance has also changed as compared to the integer fluxes: $a(1/t)$ rises much more slowly as temperature is decreased than at the integer fluxes. The I-V power-law exponent reaches a value near 2, when the reduced resistance has dropped to 0.001, and does not approach a value of 3 until much lower in temperature. The transition does not resemble a Kosterlitz-Thouless transition. Brown and Garland have also measured the Ohmic and non-Ohmic properties at a low flux in their triangular array, but they found no difference in the transition as compared with the integer fluxes if they plotted the exponent on a reduced temperature scale:[25] $a[T/T_c(f),f]$.

Notice, however, that the resistance still drops reasonably rapidly as a function of $1/t$ at $f=0.05$ even though the applied flux produces a fixed excess number of unpaired vortices. This is in contradiction to Eq. (1), which asserts that the resistance should flatten out at a value of $R/R_n\approx f$ at low temperatures. In this case the density of thermally activated free vortices is small since the density will be proportional to the applied flux. Our results finding a dropping resistance at nonzero values of f agree with the previous measurements of Kimhi, Leyvraz, and Ariosa[32] and Brown and Garland[25] at small values of the frustration, and calls into question the assumption that the mobility of the vortices is only weakly dependent on temperature.

The presence of pinning effects[35] has been previously noted in resistance measurements in a small magnetic field, both in continuous superconducting films,[36,37] where a nonzero current is required to depin the vortices from preferred sites usually associated with inhomogeneities in the film, and also in measurements of arrays.[32] Gray, Brorson, and Bancel have even called into question the interpretation of the temperature dependence of the resistance in the films according to predictions based on a renormalization treatment of the Kosterlitz-Thouless transition that do not take into account pinning effects.[36,37]

We certainly observe the effects of pinning at $f=0.05$ below the $f=0$ transition temperature, when the low current R/R_n falls below 0.05. Instead of analyzing the non-Ohmic I-V curves at $f=0.05$ by evaluating the logarithmic derivative as we have done above, an alternate analysis gives some insight into the non-Ohmic mechanism. We follow a procedure to treat the non-Ohmic resistance in granular aluminum films used by Gray and co-workers.[37] We subtract the $f=0$ non-Ohmic voltage measured at each value of current and temperature from the I-V curves measured for $f=0.05$ at the same current and temperature (see the left half of Fig. 7). This subtraction is an attempt to eliminate the effects of vortex-pair breaking effects on the $f=0.05$ non-Ohmic resistance. The resulting I-V curve has the classic "flux-flow" shape: a low-current Ohmic region and a high-current region with a constant value of the differential reduced resistance $(1/R_n)dV/dI=R_{\mathrm{ff}}/R_n \approx f$. The measured reduced flux-flow resistance has a temperature-independent value approximately equal to f $(R_{\mathrm{ff}}/R_n \approx 0.05-0.06)$. The intercept on the current axis obtained by extrapolating the constant differential resistance region of the I-V curves back to $V=0$ defines the depinning critical current $I_{c\mathrm{ff}}(T)$. We interpret the depinning current as the current required to move all of the vortices produced by the applied field.[35,37]

On the right-hand side of Fig. 7, we have plotted the temperature dependence of

$$I_{c\mathrm{ff}}(T,f=0.05)/I_c(T,f=0)\ ,$$

the ratio of the depinning current at $f=0.05$ to the critical current measured at $f=0$ at the same temperature (or extrapolated from measurements at lower temperatures as described above in the calculation of the effective temperature). The depinning current rises from zero at the $f=0$ Kosterlitz-Thouless transition temperature and reaches a value near to 0.5 at temperatures well below the transition temperature at $f=0.05$ that we have defined by our measurements of the Ohmic resistance.

Our interpretation of the Ohmic resistance that we measure in the low-current region must also be altered, at least in the temperature region below the $f=0$ transition temperature. Instead of representing the density of free vortices in the array, as we have assumed above, the reduced resistance is apparently proportional to the density of free vortices which is also thermally activated over a pinning potential barrier.

Finally, notice from Fig. 2 the large effect a small amount of flux has on the resistance above the $f=0$ transition temperature. In a first-order approximation the total reduced resistance would be the density of thermally activated vortices plus the density of vortices added by the external flux:

$$R(T,f=0.05)/R_n - R(T,f=0)/R_n \approx f\ .$$

The measured R/R_n at $f=0.05$ is increased significantly over R/R_n at $f=0$ at temperatures above $T_c(f=0)$. Qualitatively, the additional resistance is due to the breaking of vortex-antivortex pairs by the screening effects of the free vortices that are produced by the field. Pairs with a separation comparable to the free-vortex separation will be broken and will contribute to the Ohmic resistance. These effects have been treated self-consistently in continuous films by Doniach and Huberman[6] and also by Minnhagen.[38]

The enhancement of reduced resistance over the zero-field resistance at small values of f slightly above the zero-field transition temperature is also apparent in the data of Brown and Garland in triangular arrays,[25] where the effect is perhaps even more pronounced. The data of Kimhi and co-workers in square arrays[32] does not show such an enhancement at temperatures slightly above $T_c(f=0)$. Unlike the resistive behavior at the Kosterlitz-Thouless transition, the enhancement of the resistance by a small amount of flux is not expected to have universal properties. However, the existence of

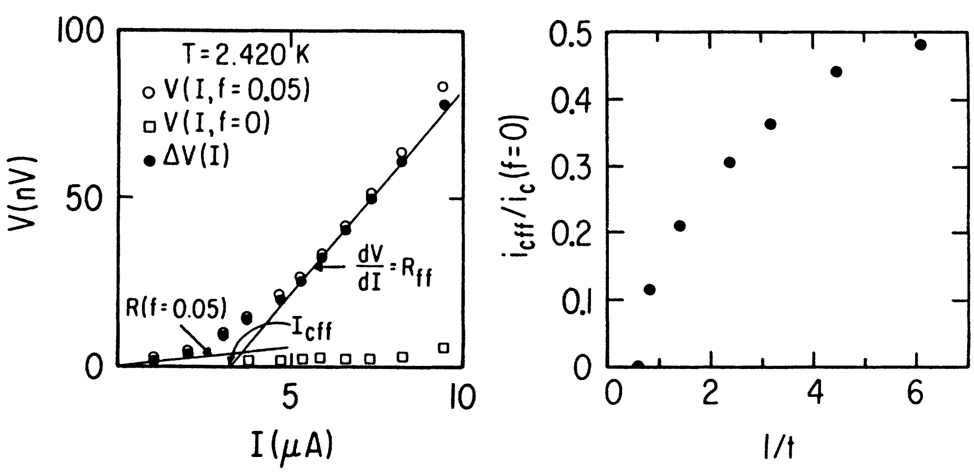

FIG. 7. Right: The method used to define $I_{c\mathrm{ff}}$. Shown are the current-voltage curves measured at $T=2.420$ K $V(I,f=0.05)$, $V(I,f=0)$, and $\Delta V(I)+V(I,f=0.05)-V(I,f=0)$. The difference curve $\Delta V(I)$ is then used to define the flux-flow resistance $R_{\mathrm{ff}}=[dV/dI]_{I\gg I_{c\mathrm{ff}}}$, and the flux-flow critical current for that flux and temperature $I_{c\mathrm{ff}}$ which is the current-axis intercept of the flux-flow regime of $\Delta V(I)$. Left: The ratio of the flux depinning current (measured at $f=0.05$) to the critical current at $f=0$, $I_{c\mathrm{ff}}(1/t,f=0.05)/I_c(1/t,f=0)$, plotted as a function of the inverse effective temperature $1/t$.

such large differences in the experimental literature probably indicates that the theoretical analysis applied is incomplete. Clearly, any theoretical interpretation of the experimentally observed effect of the field on the Ohmic resistance in the high-temperature region must first confront the contradictory results of these measurements.

Recently, measurements[39] of the magnetic-field dependence of the resistive transition in continuous two-dimensional superconducting films have also found more complicated behavior than expected theoretically, including evidence of vortex pinning with a field dependent pinning energy that dropped to zero at the $f=0$ Kosterlitz-Thouless transition temperature.

C. $f=\frac{1}{2}$, $f=1/\tau$, and $f=1-1/\tau$

The transitions at $f=\frac{1}{2}$, $1/\tau$, and $1-1/\tau$ are depressed the most in temperature and effective temperature compared with the integer fluxes and $f=0.05$, yet the transitions themselves represent an intermediate behavior between the integer and small flux transitions. This conclusion follows from comparing the rise in the non-Ohmic slope $a(1/t)$ in Fig. 6 with the decrease in the reduced resistance as a function of the inverse effective temperature in Fig. 5.

This behavior is most marked at $f=\frac{1}{2}$, where the resistance $R(1/t)/R_n$ is larger than at $f=0.05$, yet $a(1/t)$ increases at a similar or perhaps somewhat faster rate as the effective temperature is decreased. When $R(f=1/2)/R_n$ reaches a value of 0.001, at an inverse effective temperature of 2.16, the value of $a(f=\frac{1}{2})$ is 2.6 ± 0.2. For $f=0.05$ at the same reduced resistance value, $a(f=0.05)$ was 2.0 ± 0.2.

Our estimate of the transition temperature at $f=\frac{1}{2}$ in a square array differs from that determined by Van Wees and co-workers who used a different criterion. Van Wees and co-workers found an inverse effective transition temperature of about 4, which they determined by finding the highest temperature where a reached a constant value when measured at different voltage levels approaching zero voltage. They defined a critical value of the I-V exponent $a(t_c)\approx5.5$ by this procedure. The t_c measured by this method does not necessarily correlate with the behavior of the resistance, which in their experiment had reached a very low value of R/R_n, well below 10^{-5}, for their 128×384 island array. Our $f=\frac{1}{2}$ resistance data for values of R/R_n greater than 0.001 resembles what they measured over the same effective temperature range: Our resistance drops with an effective thermal activation energy of 3.7 (versus a value of 4.7 that they measured) below a higher-temperature regime where the effective activation energy is smaller.

Monte Carlo simulations provides the best theoretical estimate for the properties of the transition in the fully frustrated XY model on a square lattice. From the simulation of Teitel and Jayaprakash,[3] the transition occurs at an effective temperature near 0.45, about 50% reduction from an estimate based on ground-state energies for a vortex-antivortex unbinding transition at $f=\frac{1}{2}$. Due to the two degenerate ground states at $f=\frac{1}{2}$ a set of Ising-like excitations on the vortex lattice exists, and these excitations rather than vortex unbinding may be responsible for destroying the ordered state. The jump seen in the nonlinear exponent does not equal the universal value for a Kosterlitz-Thouless transition, corresponding instead to a value of $a(t_c)$ at the transition of about 3.5 to 4 (we have calculated the expected jump in a from the value of the renormalized coupling J^* measured in the simulations $a-1=\pi J^*/k_BT$; note that J^* is the same as the helicity modulus calculated in the Monte Carlo simulation). Other theories have also suggested that the transition occurs at a reduced effective temperature with a nonuniversal jump at the transition,[22] or even that a combined Kosterlitz-Thouless and Ising-like transition occurs at $f=\frac{1}{2}$ in a square array.[23]

Our measurements agree with the simulations in estimating a transition temperature of about 0.46, but we find a value of $a(t_c)\approx2.6$, much lower than 3.5. This result would seem to be inconsistent with the Kosterlitz-Thouless stability criterion for the superfluid, which predicts a minimum value of $a(t_c)\geq3$. On the other hand, the non-Ohmic dissipation at a noninteger value of f is not necessarily due to breaking of vortex-antivortex pairs by the current. The non-Ohmic mechanism may be more complicated since the vortex lattice itself can also be altered by a measuring current. Therefore a measurement of the nonlinear resistance may not be simply related to J^* as in the vortex-pair unbinding theory. Unfortunately, no theory for the non-Ohmic transport aside from the flux-flow model exists. The flux-flow model does not work well at the larger fluxes near $f=\frac{1}{2}$, as no reasonable region of constant dV/dI can be found in the I-V curves at these fluxes.

The temperature dependence of the nonlinear exponent at fluxes near $f=\frac{1}{2}$ has a behavior intermediate between the sharp rise integer values of f and the slow smooth behavior at $f=0.05$. At $f=\frac{1}{2}$, a rises from 1 to a value of 2.6 when $R/R_n=0.001$ in a temperature range which is about 0.25 K wide. At the incommensurate value $f=1/\tau$ (≈0.618) the initial rise in a is similar, from 1 to 2.4 upon a decrease in the temperature of 0.25 K. Compared with $f=0.05$, at $f=\frac{1}{2}$ and at $f=1/\tau$ the transition region is narrower and the value of a is greater when $R/R_n=0.001$. The temperature dependences of the two curves are the same within experimental error if the temperature axes are shifted by about 0.03 K. Thus, both the Ohmic and non-Ohmic data show that a transition temperature depression of 0.03–0.04 K is the major effect on the temperature dependence of the resistive transition on changing the flux from $\frac{1}{2}$ to $1/\tau$.

The effective temperature dependence of the resistance also shows little difference between the commensurate and the incommensurate transitions. The effective activation energy of the Ohmic resistance decreases from 3.7 at $f=\frac{1}{2}$ to 3.1 at $f=1/\tau$ and $f=1-1/\tau$. As a result, the effective temperature at which $R/R_n=0.001$ is depressed to $t_c=0.38$ at the incommensurate fluxes, down from $t_c(f=\frac{1}{2})=0.46$. Halsey's Monte Carlo simulation[15] found a glass transition at an effective temperature of approximately 0.25. The increase in $a(1/t)$ is substantially the same below the transition for the incommensurate fluxes and for $f=\frac{1}{2}$.

The lack of a significant difference between the resistive transitions at commensurate and incommensurate fluxes differs from the expectations of arguments based on the ground-state properties of the array in those fluxes.[12,13] On the other hand, the possibility that a freezing of vortices as the temperature is lowered in the incommensurate state could explain the similarity of the transitions at $f=1/\tau$ and $1-1/\tau$ with the transition at $f=\frac{1}{2}$. However, we did not observe the hysteretic effects one might expect to see when cooling and heating through a glass transition. The nature of the transitions with frustration remains a complex problem. A theory for non-Ohmic effects could provide some basis for trying to understand the transitions. At the same time, the effects of pinning, evident at $f=0.05$, must also be considered.

V. CONCLUSIONS

In conclusion, we have measured the resistive behavior as a function of temperature and frustration in a square Josephson-junction array. The transition for $f=0$ and 1, when the array is unfrustrated, are similar to each other and have characteristics similar to a Kosterlitz-Thouless vortex unbinding transition. This result agrees with several previous square array measurements[9,11,24] of the Ohmic and non-Ohmic behavior at $f=0$, and with measurements[26,30] of the Ohmic behavior for nonzero integer values of f. The measurements of Brown and Garland[25] in a triangular array do not agree with these results, although there is no theoretical reason to expect a different transition in the unfrustrated triangular array compared with the unfrustrated square array.

Applying a small amount of flux to the array, $f=0.05$, significantly broadens the transition compared to the integer fluxes, and the transition no longer resembles a vortex unbinding transition. In one other comparable measurement, Brown and Garland[25] do not see such a change in the transition between $f=0$ and 0.043. Our resistance measurements show that vortex pinning is probably occurring at low temperatures, which probably complicates the theoretical interpretation of any underlying transition.

We find that the transitions with a large frustration, either commensurate, as at $f=\frac{1}{2}$, or incommensurate, as at $f=1/\tau$, are similar to each other, but different from both the unfrustrated and weakly frustrated transitions if the Ohmic and non-Ohmic resistive response are considered together. The similarity of the transitions is not expected theoretically. The nature of the transition at $f=\frac{1}{2}$ is unclear from our experiment, and we do not find evidence of a sharp transition as described by van Wees and coworkers.[24] A better theoretical interpretation of the non-Ohmic behavior for these fluxes may help to improve our understanding of these transitions. The resistive behavior for the incommensurate values of f show that some type of vortex freezing is occuring, and thus our results do not preclude the possibility that a superconducting glass state forms for irrational values of the frustration.

ACKNOWLEDGMENTS

I am grateful to Thomas Halsey for introducing to me the problem of the Josephson-junction array in a magnetic flux for his help in understanding the physics involved in the experiment. The array fabrication with the ion microscope would not have been possible without the help of Riccardo Levi-Setti, Yuh-Lin Wang, Jan Chabala, and Geoffrey Crow. Dave Dennison was very helpful in thin film evaporation. Finally, I would like to thank Sid Nagel for his support throughout this work. This work was supported by a National Science Foundation—Materials Research Laboratory Grant at the University of Chicago, Grant No. 8519460.

*Present address: Department of Physics, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, California 90024-1547.

[1]D. H. Sanchez and J.-L. Berchier, J. Low Temp. Phys. **43**, 65 (1981).

[2]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[3]S. Teitel and C. Jayaprakash, Phys. Rev. B **27**, 598 (1983).

[4]B. I. Halperin and D. R. Nelson, J. Low Temp. Phys. **36**, 1165 (1979).

[5]M. R. Beasley, J. E. Mooij, and T. P. Orlando, Phys. Rev. Lett. **42**, 1165 (1979).

[6]S. Doniach and B. A. Huberman, Phys. Rev. Lett. **42**, 1169 (1979).

[7]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).

[8]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); J. M. Kosterlitz, *ibid.* **7**, 1046 (1974).

[9]J. Resnick, J. C. Garland, J. T. Boyd, S. Shoemaker, and R. S. Newrock, Phys. Rev. Lett. **47**, 1542 (1981).

[10]R. F. Voss and R. A. Webb, Phys. Rev. B **25**, 3446 (1982).

[11]D. W. Abraham, C. J. Lobb, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **26**, 5268 (1982).

[12]S. Teitel and C. Jayaprakash, Phys. Rev. Lett. **51**, 1999 (1983).

[13]T. C. Halsey, Phys. Rev. B **31**, 5728 (1985).

[14]J. Villain, J. Phys. C **10**, 1717 (1977).

[15]T. C. Halsey, Phys. Rev. Lett. **55**, 1018 (1985).

[16]M. Y. Choi and D. Stroud, Phys. Rev. B **32**, 7532 (1985); **35**, 7109 (1987).

[17]M. Kléman and J. F. Sadoc, J. Phys. (Paris) Lett. **40**, L569 (1979); S. Sachdev and D. R. Nelson, Phys. Rev. Lett. **53**, 1947 (1984).

[18]K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).

[19]V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. Lett. **40**, 783 (1978); B. A. Huberman, R. J. Myerson, and S. Doniach, *ibid.* **40**, 780 (1978); A. P. Young, Phys. Rev. B **19**, 1853 (1979).

[20]J. M. Kosterlitz, J. Phys. C **7**, 1046 (1974).

[21]D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[22]T. C. Halsey, J. Phys. C **18**, 2437 (1985); M. Y. Choi and S. Doniach, Phys. Rev. B **31**, 4516 (1985); M. Y. Choi and D.

Stroud, *ibid.* **32**, 5773 (1985); P. Minnhagen, *ibid.* **32**, 7548 (1985); E. Granato and J. M. Kosterlitz, *ibid.* **33**, 4767 (1986).
[23]B. Berge, H. T. Diep, A. Ghazali, and P. Lallemand, Phys. Rev. B **34**, 3177 (1986).
[24]B. J. van Wees, H. S. J. van der Zant, and J. E. Mooij, Phys. Rev. B **35**, 7291 (1987).
[25]R. K. Brown and J. C. Garland, Phys. Rev. B **33**, 7827 (1986).
[26]Ch. Leemann, P. Lerch, G.-A. Racine, and P. Martinoli, Phys. Rev. Lett. **56**, 1291 (1986).
[27]P. G. DeGennes, Rev. Mod. Phys. **36**, 225 (1964); G. Deutscher and P. G. DeGennes, *Superconductivity*, edited by R. D. Parks (Marcel-Dekker, New York, 1969), Vol. 2.
[28]S. R. Shenoy, J. Phys. C **18**, 5163 (1985).
[29]C. J. Lobb, Physica **126B**, 319 (1984).
[30]R. A. Webb, R. F. Voss, G. Grinstein, and P. M. Horn, Phys. Rev. Lett. **51**, 690 (1983).
[31]M. Tinkham, D. W. Abraham, and C. J. Lobb, Phys. Rev. B **28**, 6578 (1983).
[32]D. Kimhi, F. Leyvraz and D. Ariosa, Phys. Rev. B **29**, 1487 (1984).
[33]K. N. Springer and D. J. van Harlingen, Bull Am. Phys. Soc. **31**, 494 (1986).
[34]J. Tobochnik and G. V. Chester, Phys. Rev. B **20**, 3761 (1979).
[35]D. S. Fisher, Phys. Rev. B **22**, 1190 (1980).
[36]P. A. Bancel and K. E. Gray, Phys. Rev. Lett. **46**, 148 (1981).
[37]K. E. Gray, and J. Brorson, and P. A. Bancel, J. Low Temp. Phys. **59**, 529 (1985).
[38]P. Minnhagen, Phys. Rev. B **23**, 5745 (1981).
[39]J. C. Garland and H. J. Lee, Phys. Rev. B **36**, 3638 (1987).