Magnetoconductance in the variable-range-hopping regime due to a quantum-interference mechanism

O. Faran and Z. Ovadyahu
Racah Institute of Physics, The Hebrew University of Jerusalem, Givat Ram, 91 904 Jerusalem, Israel
(Received 31 December 1987)

Results of systematic magnetoconductance measurements on highly disordered In_2O_{3-x} films are described. Measurements were performed as a function of magnetic field, electric field, temperature, system dimensionality, and amount of static disorder. It is shown that in the hopping regime, the low-field magnetoconductance is always positive, and anisotropic in sufficiently thin films. The latter feature is suggestive of a nonlocal (orbital) mechanism. We demonstrate that the spatial range of phase coherence, involved in the phenomenon, scales with the hopping length. This length may be controlled by either the temperature or the electric field. It is further shown that several aspects of the experimental results support the basic ideas of a newly proposed quantum-interference mechanism. An intuitive physical description of the reason for the positive magnetoconductance is discussed based on the percolation model for the hopping transport.

I. INTRODUCTION

In this paper we describe results of extensive magnetoresistance (MR) measurements performed on In_2O_{3-x} films as a function of temperature T, magnetic field H, electric field F, amount of static disorder, and dimensionality. In particular, we focus on the MR observed in strongly localized systems, i.e., samples that exhibit exponential temperature dependence of the conductivity. Some peculiar features of the MR observed in the strong localization (SL) regime have been reported previously[1,2] and will be further elucidated in the present paper.

The main motivation for this work may be best seen from the following basic findings, pertaining to a two-dimensional (2D) system. Figure 1 depicts results of the fractional change of resistance $\Delta R/R$ due to a constant magnetic field H, as a function of disorder (taken as being proportional to the sheet resistance, $R_\square$). Figure 2 shows the MR anisotropy observed on some of these films. To facilitate discussion, we use the following notations throughout this paper: $\delta=\Delta R/R$, $\beta=\delta_\perp/\delta_\parallel$, where $\delta_\perp$ and $\delta_\parallel$ are the measured δ at the perpendicular and parallel field orientation (relative to the samples plane), respectively. First, we note that the MR is *always negative.* That holds true for most of the range of temperatures, fields, and disorder described below (the only exception is the case of very high electric fields where a small, positive MR component is detected for fields <2 kOe). It is seen that δ tends to increase with $R_\square$ or decrease with it, depending on whether $R_\square$ is much smaller or much larger than ~ 10 kΩ. This presumably marks the transition from weak-localization (WL) to SL behavior. There is a striking difference between the temperature dependence of the resistance $R(T)$, in the two regimes, i.e., logarithmic correction[3] in WL and exponential dependence in SL. By contrast, the MR observed in the SL regime is not dramatically different than that seen in WL samples. In particular, the MR may be anisotropic (Fig. 2) which is indicative of a quantum-interference (QI), orbital mechanism. For WL samples, the relevant QI mecha-

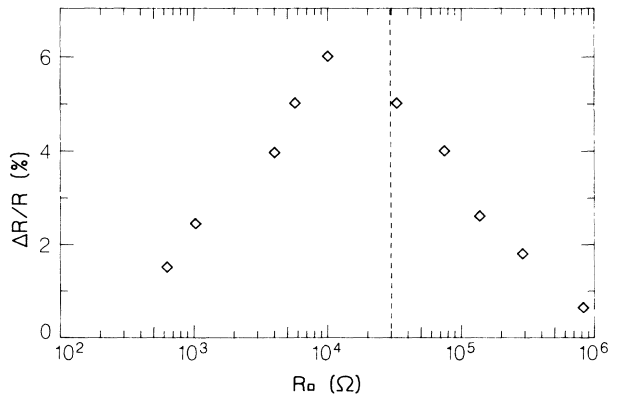

FIG. 1. MR as function of $R_\square$, measured at 4.2 K in a perpendicular field of 5.6 kOe. Data points are for a single batch of a 100-Å film. (The sheet resistance has been changed by heat treatment.)

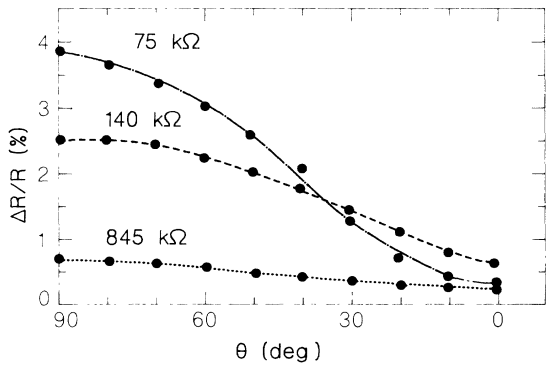

FIG. 2. MR for three SL films (with indicated sheet-resistances), measured at 4.2 K as a function of the angle between the field direction and the sample plane ($d=100$ Å).

nism is that due to backscattering (BS) suppression[3] which, for small fields, gives: $\delta=-f(L_\varphi/L_H)$. Here L_φ is the spatial range of the quasiparticle phase coherence, L_H is the magnetic length $\sim(ch/eH)^{1/2}$, and f is a monotonically increasing function. In the WL regime L_φ is the inelastic diffusion length, $L_{\rm in}$. When $R_\square$ increases, the localization length, ξ, becomes smaller and eventually, the condition: $\xi=L_{\rm in}$ will be met (for noninteracting, 2D systems, this should occur once $R_\square \gtrsim 30$ kΩ). In the SL regime, it might then be expected that it is ξ rather than $L_{\rm in}$ that determines the cutoff length for QI effects. This assumption may be consistent with the decline of δ for $R_\square > 30$ kΩ (Fig. 1) but it is in conflict with the MR anisotropy data of these samples (Fig. 2): Detailed analysis shows that β may be larger than unity even in samples where $\xi < d$ (d is the film thickness). One is then led to the conclusion that the appropriate L_φ in the SL regime can be larger than ξ. Our main aim in the present research was to find out what is this length and what parameters determine its size.

In the next section, we give details of sample preparation and describe the measurement techniques employed in this research. Sec. III contains the experimental results, discussed in the light of current theoretical ideas, and our main conclusions, which are as follows.

(1) The phase-coherence length associated with the negative MR in the hopping regime is, essentially, the hopping length r. This length may be controlled by either temperature or electric field.

(2) The negative MR is not necessarily related to the increase of the localization length with field. We believe that a more appropriate mechanism (at least, in the limit of strong disorder) is the one suggested by Nguyen *et al.*[4] and Sivan *et al.*[5] and we show that some important features of these models are borne out by our experiments.

II. EXPERIMENT

The samples used in this research were films of polycrystalline In_2O_{3-x} prepared by vacuum-depositing pure (99.997%) In_2O_3 onto glass substrates held at ~150 °C. Deposition rate and film thickness were monitored, *in situ*, by a quartz-crystal device calibrated against an optical interferometer. Stainless-steel masks were used to obtain the desired sample geometry usually, in the form of a strip, 10 mm long and 6 mm wide with two sets of evaporated voltage contacts (for resistance and Hall-effect measurements). A four-point dc technique was employed in all measurements, using the high-impedance, Keithley's current source (K220) and electrometer (617). The as-deposited samples had bulk resistivity ρ, of the order of 10^{-3} Ω cm at room temperature. To obtain SL behavior at the liquid-He range of temperatures, ρ had to be increased by, typically, one order of magnitude. This was achieved by heat treatment (quench cooling from ~250 °C, being the crucial stage, followed by annealing at 40–60 °C for fine tuning of ρ) Fuller descriptions of sample preparation, heat treatment, and structure determination and characterization are given elsewhere.[6] For purposes of the present study, we note that, for films thicker than 40 Å, full physical continuity was observed (by electron microscopy). Such samples were composed of randomly oriented polycrystals, with the basic (bcc) structure of the stoichiometric material (observed by electron and x-ray diffractometry).

Hall-effect measurements yielded an electron concentration N of the order of 4×10^{19}–8×10^{19} cm^{-3}. The value of the Hall coefficient, measured at room temperature, was virtually constant even for samples that underwent massive heat treatment.[7] This carrier concentration yields a density of states $N(0)$, of the order of $\sim10^{32}$ erg cm^{-3} (the effective mass of In_2O_{3-x} is[8] $\sim0.3m_0$). $N(0)$ is assumed to be constant, independent of disorder.

Most of the experiments described in the next section involve measuring the change of the resistance due to the application of a magnetic field. The fractional change of the resistance, in almost all cases, is of the order of 0.1–1 %. For samples in the SL regime [where $R(T)$ is an exponential function], one has to stabilize the temperature to better than 0.1% to get meaningful results. For this reason, most of our MR measurements were performed in the 1.25–4.2 K temperature range, with the sample immersed in liquid He. Vapor pressure or a calibrated Ge resistor were used for thermometry and temperature regulation. The cryostat was mounted in the air gap of a split-coil electromagnet with provision for varying, continuously, the angle between the field direction and the sample plane. The magnetic field direction was always perpendicular to the direction of current flow.

III. RESULTS AND DISCUSSION

In this section we present and discuss the main experimental results using two different approaches. First, we list basic findings in support of the conjecture that QI are responsible for the observed MR and, in particular, we demonstrate that the coherence length associated with the effect is the hopping length. This is done on a purely empirical basis and does not involve more than a few physically plausible assumptions. We then proceed to compare the finer details of the results with existing theoretical models. The single most important tool that we use in this section is the anisotropy of the MR. The "anisotropy parameter" β (cf. Sec. I), is found to be dependent on film thickness, temperature, electric and magnetic fields strength, and disorder. Qualitatively, all these parameters affect β in the same way: The larger they are, the smaller is β. However, the film thickness d has by far the biggest quantitative influence on β. It seems, therefore, reasonable to start by considering β-versus-d data, taken under such conditions where the other parameters (which will be considered in turn), have little influence on the results.

Pertinent data are illustrated in Figs. 2, 3, and 4 for d equal to 100, 250, and 500 Å, respectively. (Note the reduced field for the thicker films, which was needed in order to get a field independent β.) β was measured for eleven 100-Å, two 250-Å, two 350-Å, three 500-Å, and four 1200-Å-thick samples. The low-field values of β, measured at the liquid He range of temperatures, were

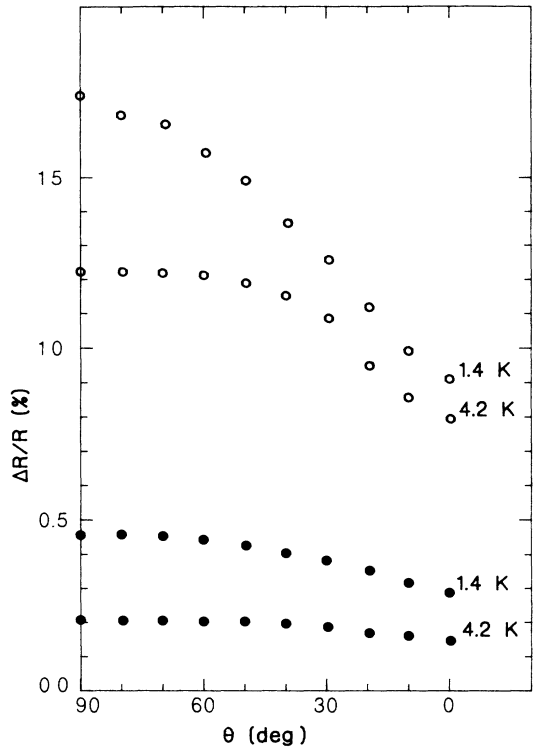

FIG. 3. MR of two 250-Å-thick films as function of the angle between a 2.4-kOe field direction ans samples plane: solid circles, sample with $\xi=35$ Å; open circles, sample with $\xi=85$ Å. (Note that anisotropy is stronger at the lower temperature.)

4–20, 1.5–3, 1.1–1.4, 1–1.1, and 1 respectively. The fact that β is larger than unity for the thinner films is interesting: Either the films structure is inherently anisotropic, or else the MR mechanism is a nonlocal one. The first possibility is not unrealistic; vacuum-deposited films often show preferred orientation that may be quite pronounced at the initial stage of growth (thin deposits). That, however, cannot be the cause for the anisotropic MR in this case. In the first place, no preferred orientation was detected in our films (cf. Sec. II above). Secondly, the crystallites of which the samples are composed

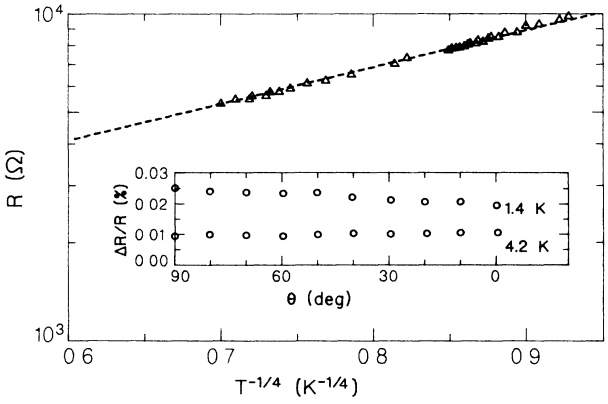

FIG. 4. Resistance as a function of temperature for a 500-Å sample. The insert shows the MR as a function of the angle between a 510-Oe field direction and the sample plane.

have a cubic symmetry.[6] Thus, even if there were a preferred orientation, the conductivity and (a local) MR ought to have been isotropic.[9] Our findings seem, therefore, to imply that the MR is due to an orbital mechanism with an associated coherence-length L_φ that for the above specified conditions is of the order of 200 to 500 Å. In other words, it is claimed that $\beta>1$ implies $L_\varphi>d$. The β-versus-d study is, then, a physically plausible way to estimate L_φ, and most of our conclusions are based on this assumption.

If one accepts the interpretation of the MR as being due to QI, then the following question naturally arises: What determines L_φ in this (SL) regime? There is a correlation between the *effective dimensionality* of the hopping process and β which, in our view, suggests that L_φ is, essentially, the hopping length r. Consider the $R(T)$ data for several thin films (Fig. 5) and that for thicker ones (Figs. 6 and 7). The former set of data is consistent with the Mott's law[10] for variable-range hopping (VRH) in the 2D system:

$$R(T)\sim\exp[(T_*/T)^{1/3}]\ , \tag{1a}$$

$$k_B T_*\sim 1/N(0)\xi^2 d\ . \tag{1b}$$

Similarly, the data of Figs. 6 and 7 illustrate the respective law in 3D:

$$R(T)\sim\exp[(T_0/T)^{1/4}]\ , \tag{2a}$$

$$k_B T_0\sim 1/N(0)\xi^3\ . \tag{2b}$$

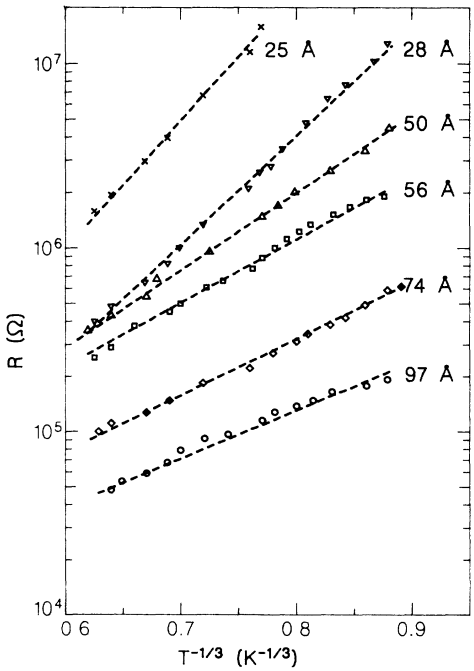

FIG. 5. Resistance as a function of temperature for several 100-Å-thick samples. The curves are labeled by the respective ξ values derived through Eq. (1) using $N(0)=10^{32}$ erg^{-1} cm^{-3} (cf. Sec. II) and T_* was taken from the logarithmic slope of the $R(T)$ data.

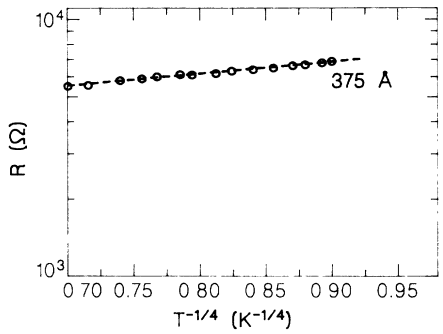

FIG. 6. Same as in Fig. 5, but for a 1200-Å sample and using Eq. (2) for the determination of ξ.

We have satisfied ourselves that samples with $R(T)$ that conform to a "$T^{-1/4}$ law" cannot be made to fit a "$T^{-1/2}$ law" (which would have indicated significant contribution due to Coulomb correlation effects[11]). A conclusive distinction between a $T^{-1/4}$ law and a $T^{-1/3}$ law cannot be made just on the basis of data over the rather limited range shown. It must be observed that a "forced" fit of a 2D sample to a 3D law (or vice versa) would result in an unreasonably small (or unreasonably large, respectively) value for ξ [which is determined from the logarithmic slopes of $R(T)$ through Eqs. (1b) or (2b)]. This, in turn, will result in an inconsistent interpretation of the data. In other words, it is the *physics* involved, by which one expects comparable localization length for comparable bulk disorder (when the latter is strong enough), that makes our dimensional analysis plausible.

$R(T)$ can be also written as:

$$R(T)\sim \exp(r/\xi)\ , \tag{3a}$$

with

$$r(T)\sim(T_*/T)^{1/3}\xi \quad (\text{for } 2D)\ , \tag{3b}$$

and

$$r(T)\sim(T_0/T)^{1/4}\xi \quad (\text{for } 3D)\ , \tag{3c}$$

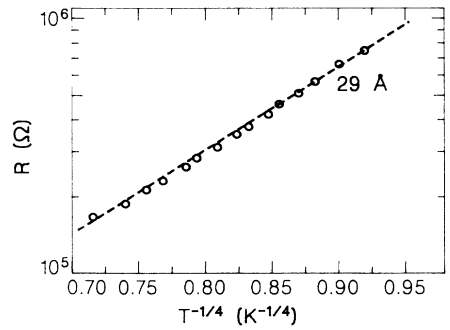

FIG. 7. Same as in Fig. 6, with different ξ.

where $r(T)$ is the temperature-dependent hopping length. As is well known,[10,12] $r(T)$ determines the dimensionality of the VRH, namely, 2D or 3D hopping occurs for $r>d$ or $r<d$, respectively. It is noteworthy that $\beta>1$ and $\beta=1$ for films that show 2D and 3D VRH, respectively. (This is found to be generally true). An interesting case is illustrated in Fig. 4 for a 500-Å sample: At 4.11 K β is unity, but at 1.35 K, some anisotropy is already evident. This seems to correlate with a gradual deviation from a 3D VRH, towards a faster temperature dependence that might be due to 3D→2D dimensional crossover. The quantitative consistency of the data with the conjecture that $L_\varphi\sim r$ can be checked by calculating r [through Eq. (3)] and comparing it with the measured value of β. Such a comparison can be made with the parameters listed in Table I for typical samples. Figure 8 illustrates the dependence of β on the dimensionless parameter r/d (including several measurements at 1.34 K not listed in Table I). Evidently, β scales with r/d. We recall that in WL, the effective dimensionality of the quantum corrections[3] to the Boltzmann conductivity, *as well as* the MR anisotropy,[13] are determined by the size of L_φ relative to d. In this respect there is then a close analogy between WL and SL, with $r(T)$ playing in SL the same role that L_{in} plays in WL.

We can further demonstrate the pertinence of the hopping length to the QI associated with the MR by studying δ and β as a function of the electric field F. Both, the

TABLE I. Relevant parameters for several films for which β was measured at 4.2 K [Brookhaven National Laboratory (BNL)] or 4.11 K [Hebrew University (HU)]. Not included are parameters for 350-Å samples, for which no definite variable-range-hopping (VRH) dimensionality could be assigned from the $R(T)$ data at the liquid-He temperature range. Films with $d\lesssim 250$ Å exhibited 2D VRH and their T_* and r (at 4 K) were calculated through Eqs. (1a), (1b), and (3b), respectively. Films with $d\gtrsim 500$ Å showed 3D VRH at the 1.3–4.2 K temperature range and their respective parameters were calculated through Eqs. 2(a), 2(b), and 3(c).

d (Å)	$T_{*,0}$ (K)	ξ (Å)	r (Å)	β
100	73	170	450	20
100	260	90	360	14
100	385	74	320	12
100	500	65	320	8
100	670	56	305	10
100	780	52	300	4.5
100	840	50	295	5.5
100	1300	40	270	5
100	2050	32	250	4
100	2680	28	240	3
100	3360	25	232	4
250	150	85	280	1.7
250	1180	35	230	1.4
500	37	130	225	1
500	3000	30	155	1
500	3800	28	154	1
1200	3020	30	160	1
1200	3200	29	152	1
1200	3900	27	150	1
1200	6500	23	145	1

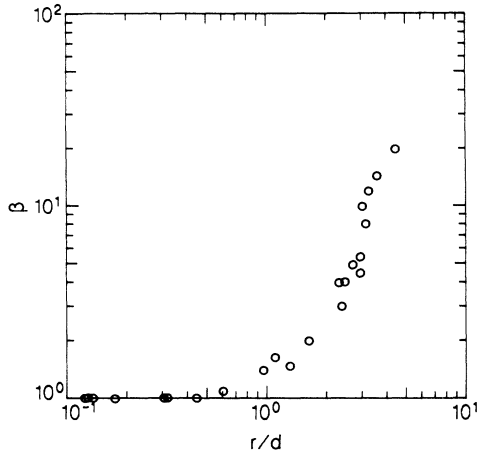

FIG. 8. The anisotropy parameter β as a function of the ratio of the calculated hopping length r to the film thickness d.

conductivity and the MR are sensitive to the value of the electric field used in the measurements. Up to this point, we described results obtained in the low-field limit (typically, $F<0.5$ V/cm) where the effect of F is marginal; namely, where δ (*and* R) is independent of F within our experimental error. However, for sufficiently intense fields, the film resistance becomes temperature independent. In this so called activationless-hopping regime, the current-voltage characteristics of a 2D system are given[14] by

$$\ln I \sim -(F_*/F)^{1/3} , \tag{4}$$

where $F_* \sim k_B T_*/e\xi$.

The characteristic field F_c, above which relation (4) is valid, is given by $eF_c\xi \sim k_B T$. In this regime, r has a power-law field dependence: $r \sim F^{-1/3}$, but is independent of temperature. Figure 9 depicts the current-voltage characteristics and the MR of a typical 2D sample. (The same systematic behavior was noticed on five different films). It is observed that, at high fields, the data are consistent with Eq. (4) and, more importantly, for $F > F_c$, both the resistance and the MR are *temperature independent.* This is not a trivial result (caused by, e.g., heating): Let $\delta_1(F_0, H_0, R)$ be the fractional change of the resistance due to the application of a magnetic field H_0 while $F_0 > F_c$. Let $\delta_2(T', H_0, R)$ be the respective value taken at T' (and for small F values), such that $R(T') = R(F_0)$. It is generally found that $\delta_1 > \delta_2$. The fact that δ is temperature independent cannot then be a result of simple heating. We interpret this result as yet another manifestation that the hopping length, whether limited by field or temperature, is the natural cutoff length for the QI associated with the MR. The decrease of L_φ with F can be more directly inferred from β as a function of F measurements. A characteristic case is illustrated in Fig. 10, which depicts δ for parallel and perpendicular magnetic field orientations. Note that above a threshold field, δ goes down with F. Both $\delta_\perp$ and $\delta_\parallel$ are affected but the former is apparently more sensitive, i.e., β is reduced with field, consistent with our basic conjecture.

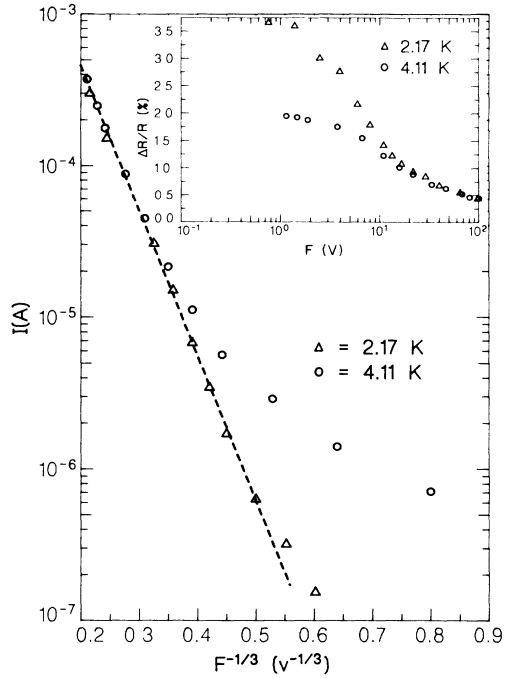

FIG. 9. Current-field characteristics of a typical 2D film ($d = 100$ Å, $\xi = 23$ Å) measured at two temperatures. (Distance between voltage probes is 4 mm.) The insert depicts the respective MR vs field of this sample measured at a perpendicular field of 6.5 kOe.

The similarity between the behavior of the MR in WL and the present (SL) case, has been noted before.[2] In reaching the conclusions above, an implicit use was made of this similarity by assuming that $\delta \sim -f(L_\varphi/L_H)$. However, no particular mechanism for the QI, nor a specific form for f, have been assumed so far. In the remainder of this paper, we attempt to answer these questions, by reference to theoretical models.

A negative MR has been found in a numerical study of a 2D, SL system by Lee and Fisher.[15] These authors solved an Anderson Hamiltonian on a 32×32 lattice with random site energies for various degrees of disorder. The negative MR found in the limit of strong disorder was at-

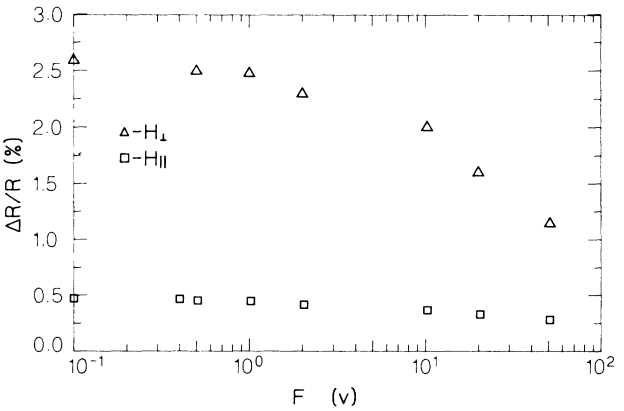

FIG. 10. MR at perpendicular and parallel field orientations ($H = 6.5$ kOe) for a 100-Å film ($\xi = 50$ Å) taken at 4.11 K.

tributed to the "delocalizing" effect of the field, i.e., to the increase of ξ with H. Recently, Nguyen *et al.*[4] and Sivan *et al.*[5] used a simplified model of VRH transport to show that a negative MR may result even when QI effects associated with backscattering (BS), are suppressed. These authors maintained that the MR reflects field modulation of QI by partial waves, which enter in a two-sites transfer integral. This, in turn, affects the values of the Miller-Abrahams[16] conductors involved in the VRH. In effect, this theoretical concept is closely akin to the QI responsible for conductance fluctuations[17] and the h/e oscillation term observable in metallic small structures and rings.[18] In WL, the corrections to the conductivity due to QI of the directed paths (which have no definite phase relationship) vanish upon averaging in the thermodynamic limit. Thus, in macroscopic samples QI due to BS dominates and one gets the well known negative MR. The distinguishing feature of SL, according to Nguyen *et al.*, is that a nonlinear average should be taken. These authors claim that it is the logarithmic averaging, rather than the alleged ξ increase, that gave rise to a negative MR in Ref. 15. They were able to show that their model, where BS was deliberately eliminated, can still reproduce the results of Lee and Fisher.

To facilitate comparison between these models and experiments, several assumptions will have to be made, the first of which has been mentioned above; namely, that for small fields δ is given by

$$\delta \sim f(L_\varphi / L_H) \tag{5a}$$

or

$$\delta \sim f'(\phi/\phi_0) \tag{5b}$$

where ϕ is the flux threading an elementary coherent-loop over which the QI operates and $\phi_0 = hc/e$. This merely reflects a basic Aharonov-Bohm behavior in a general form. The second assumption we make is that f' is a quadratic function, i.e., $\delta \sim \phi^2 \sim H^2$. Such a form is quite common in weak-field MR phenomena, it is apparently the form found in numerical simulations[4] and it is also the form predicted by the analytical result of the model by Sivan *et al.*[5] discussed later. It should be observed that, strictly speaking, the H^2 dependence of the MR is *not* borne out by our experiments. In fact, a persistent feature[1,2] of the low-field MR in SL In_2O_{3-x} films is that δ is not describable by a power law. (cf. Figs. 11 and 12.) Nevertheless, we are going to ignore this complication for the time being and proceed with the analysis on the assumption that the MR is quadratic with field. There is no real justification for this step, but, we note that the deviation from a quadratic dependence is severe only for the weakest fields. So, by restricting the analysis to fields of the order of a few kOe, no significant error is introduced.

A unique assumption of the models of Nguyen *et al.*[4] and Sivan *et al.*[5] is that the area of a coherent loop is that of an ellipse with length $\sim r$ and width $\sim (ra)^{1/2}$ (a being the microscopic length). This means that $\phi \sim r^{3/2}$ a form which is peculiar to these models and, therefore, suggests a critical test of them. Making use of the above assumptions, one gets

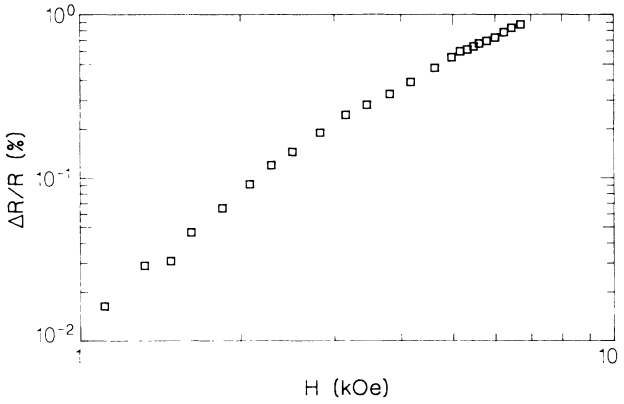

FIG. 11. MR as a function of magnetic field strength taken at 4.11 K for a typical 2D film ($d = 100$ Å, $\xi = 25$ Å).

$$\delta \sim r^3 H^2 , \tag{6a}$$

which, with the help of (3b) and (3c), gives for the temperature dependence of the MR

$$\delta \sim T^{-1} \quad (2D) , \tag{6b}$$

$$\delta \sim T^{-3/4} \quad (3D) . \tag{6c}$$

The temperature dependence implied by Eq. (6) has, in a way, a universal character: The exponent of the $\delta(T)$ relation depends *only* on dimensionality (for Mott's VRH systems). In particular, it is independent of the disorder strength. These specific predictions are compared with experimental results in Figs. 13 (2D system, cf. Fig. 5) and 14 (3D system, cf. Figs. 6 and 7). The exponents of the $\delta(T)$ plots for 2D and 3D samples are 0.93 ± 0.03 and 0.76 ± 0.03, respectively, in fair agreement with (6a) and

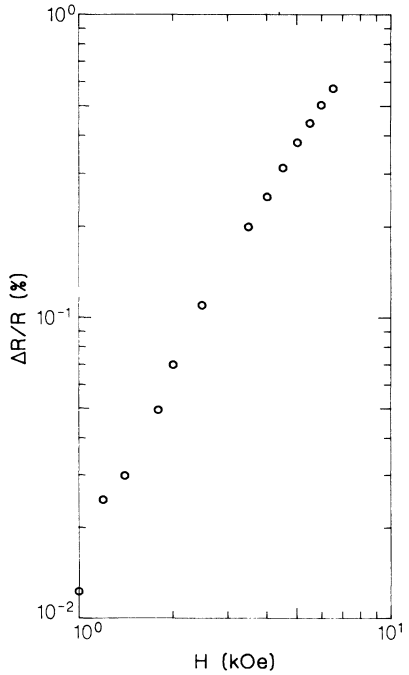

FIG. 12. Same as Fig. 11 for a 3D film ($d = 1200$ Å, $\xi = 29$ Å).

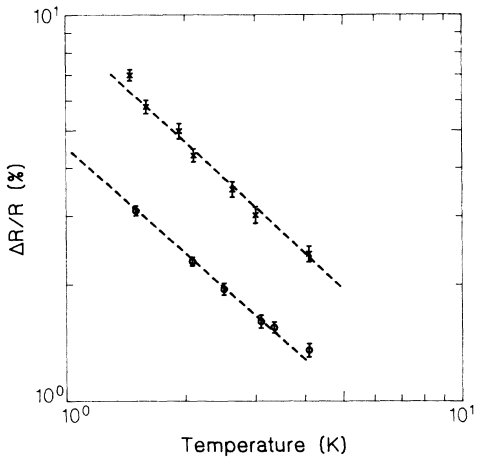

FIG. 13. MR taken at a perpendicular field of 6.5 kOe as a function of temperature for two 2D films (cf. Fig. 5). Lower curve: $d=100$ Å, $\xi=28$ Å. Upper curve: $d=100$ Å, $\xi=56$ Å.

(6b) above. Despite the limited temperature range shown, it is clear that 2D samples have a faster $\delta(T)$ dependence than 3D ones. [For the 100-Å samples, δ was also measured at 77 K and $\delta(77)/\delta(4.11)$ was found to be within $\pm 10\%$ of 77/4.11 suggesting that $\delta \sim T^{-1}$ may be even better obeyed than conveyed by Fig. 13.] It should also be observed that the temperature dependence of δ bears no relation to ξ. The directed-path approach seems, then, to be consistent with our data in two important aspects: the spatial cutoff for QI is r (which was shown to be true, independent of any model) and in terms of the scaling of the "effective area" with temperature. It seems also conceivable to predict the qualitative dependence of δ on the disorder. Neglecting a possible disorder dependence of the prefactor in Eq. (6a), $\delta(H_0,T')$ should first decrease and then saturate with increased disorder (in fact, numerical simulations[4] suggest that the prefactor saturates at high disorder). The decline of δ with $R_\square$ (for SL samples) is illustrated in Fig. 1. In addition, we have measured the MR for 25 similar films with

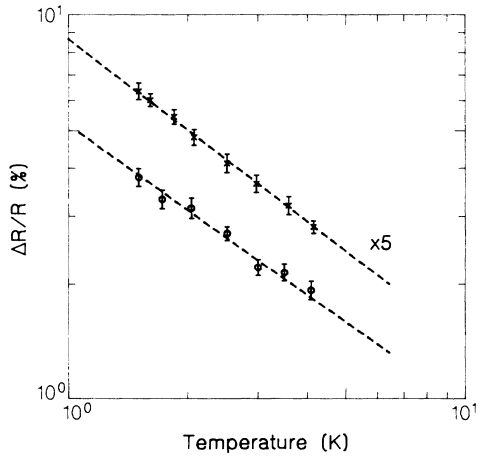

FIG. 14. Same as Fig. 13 but for two 3D films (cf. Figs. 6 and 7). Lower curve: $d=1200$ Å, $\xi=375$ Å. Upper curve: $d=1200$ Å, $\xi=29$ Å.

$R_\square=500\ k\Omega-3.5\ G\Omega$ (with $\xi=28-11$ Å) at 4.11 K and in a field of 6.5 kOe. Under these conditions, $\delta_\perp$ spanned the narrow range of 0.9–1.3 %. It may be argued that these observations, however, are also consistent with any flux-sensitive mechanism (the directed-path model being a particular example) and one wonders whether, e.g., a BS mechanism is not as likely an explanation for our observations. In other words, is it not possible that BS is significant on scales much larger than ξ (BS is certainly relevant on scales smaller than the localization length), in which case the negative MR has a "natural" explanation? The argument in favor of the directed-path interference being dominant, relies on the exponential decay of probability amplitudes at distances larger than ξ. That holds true for both types of QI but the relative contribution of BS might be expected to be exponentially small. To put it in physical terms, BS increases the probability for localization. In SL, where this probability is already substantial due to *short-scale* QI, large-scale contribution is an exponentially small correction to a large effect. The directed-path interference, on the other hand, is also a small correction but to an *equally* small entity (the transfer integral). In the limit $L_\varphi \gg \xi$, this argument seems indisputable. The real issue, then, is to understand, on physical grounds, the reason for the nonzero effect of a magnetic field operating on a macroscopic ensemble of resistors whose values are *randomly* modulated by QI. In other words, what is the physical reason for the nonlinear averaging advocated by Nguyen *et al.*[4] A possible answer to this question follows naturally from the picture of Sivan *et al.*[5] which is based on the percolation model for hopping transport. We wish to offer the following simple interpretation[19] of the main idea involved. The percolation model for VRH conductivity (Ambegaokar, Halperin, and Langer[20] and Pollak[21]) was invoked as a means of coping with the inherently inhomogeneous nature of charge transport in this regime. Due to the exponential dependence of the Miller-Abrahams conductors, g, on the intersite energy and space separations (being random variables), the former are distributed over an enormous range of values. The percolation model asserts that the macroscopic conductance of such a system is, essentially, dictated by the value of the smallest g that percolates in the infinite system. If $P(g)$ is the probability distribution for the Miller-Abrahams conductors, then, the medium conductance will be of the order of g_c given by:

$$p=\int_{\infty}^{g_c} P(g)dg\ , \tag{7}$$

where p_c is the critical value for percolation. Let us consider now the effect of a magnetic field on $P(g)$. The QI part of an individual conductor will be affected (through the Aharonov-Bohm effect) in a random way: Some g's will increase while others will decrease. It seems obvious that, in general, the g distribution in the presence of a weak field ($L_H \gg r$) $P_H(g)$, will be *wider* than $P(g)$. Thus, unless $\partial P/\partial g \mid_{g_c}=0$, it follows from Eq. (7) that $g_c(H)-g_c(0)\neq 0$, i.e., a macroscopic MR should be observed. In particular, a negative MR should result if

$\partial P/\partial g \mid_{g_c} < 0$ which is not unlikely in heavily doped semiconductors.

MR in the hopping regime is quite common. Mell[22] reports on MR measurements for the amorphous semiconductors Si, Ge, GaSb, and InP that show both positive and negative MR components. For amorphous Ge, different groups reported[23] opposite Mr signs. Apparently, the MR in these systems is dependent critically on preparation method and post-preparation treatment. Zabrodskii *et al.*[24] reported negative MR in heavily doped and strongly compensated crystalline Ge. The results of Zabrodskii *et al.* resemble ours in many respects: Their samples show Mott's VRH, activationless conductivity is observed for high electric fields, and, in particular, the MR for $H < 10$ kOe is similar to ours in terms of size, temperature and electric field dependence. Zabrodskii *et al.*, as well as the other researchers mentioned above, made their observations on bulk samples (or very thick films). Understandably, no anisotropy has been reported in any of these studies. Thus, it is not possible to tell whether the MR was due to orbital effects of the type discussed above. It is noteworthy, then, that anisotropic MR *has* been found in the VRH regime of[25] Si inversion layers (a 2D system) for weak magnetic fields. Recently, Webb *et al.*[26] measured the MR of a quasi-1D inversion layer that presumably enabled them to focus on the MR of a single critical conductor at a time. Interestingly, it was found that the MR was positive for some critical links while negative for others. Since for the *same* macroscopic system the MR was negative it seems quite possible that the heuristic model described above may be relevant to the QI effects observed by these authors. Clearly, a systematic work on restricted-geometry samples may be a powerful tool to test these ideas.

To summarize, we have presented data for the temperature and field dependencies of the MR in the VRH regime of In_2O_{3-x} films. The anisotropy observed for the thinner films suggested that the MR is due to a QI orbital effect. The correlation between the dimensionality of the VRH process and the MR anisotropy led us to the conjecture that the relevant phase-coherent length involved in this phenomenon is dictated by the hopping length. This conjecture was shown to be consistent with many of our experimental observations and its validity has been demonstrated without a recourse to any particular theoretical model.

A detailed analysis of the data seems to support the basic ideas of the models for MR recently proposed by Nguyen *et al.*[4] and Sivan *et al.*[5] The latter model is a major step forward in terms of supplying a physical reason for the nonlinear averaging involved. It also appears to give a better account of the functional dependence of the MR on the magnetic field.

We have not considered in this paper the possible effects due to electron-electron interactions. Coulomb correlations effects are usually of little concern in the magnetotransport properties of In_2O_{3-x} samples in the range of temperatures and fields investigated,[27] which may account for the good qualitative agreement with theoretical models that ignore these effects.

We have offered an intuitive, possible physical reason for a macroscopic MR arising from a "conductance-fluctuation" type of QI which is the essential ingredient of the directed-path approach. In this picture, a QI effect that is usually negligible in the diffusive regime may assume a dominant role in the case of extreme disorder. The main reason for that may be traced to the (inherently) inhomogeneous nature of the conductivity in the hopping regime. The latter, in turn, leads to a macroscopic conductance, which does not give equal weight to the basic elements of which it is composed.

An important corollary of these considerations is that a MR should appear in all disordered systems that exhibit Mott's VRH. Unfortunately, only limited relevant data exist in the literature. Thus it is not, as yet, possible to tell how general is this phenomenon in real systems. Clearly, measurements on restricted-dimensionality systems are needed in the first place to establish the existence of orbital QI effects. It should also be observed that the effects described above may be overshadowed by other MR mechanisms, especially for very intense magnetic fields: Once $L_H < \xi$, BS could become significant, and for $L_H < a$, the shrinkage of the wave-functions[28] will probably be the dominant factor in the MR (which will be then positive and isotropic). Also, spin-dependent contributions are more likely to be important in high fields. Nonetheless, there seem to be no *a priori* reasons to believe that such QI effects are less general in SL systems than those associated with BS are in WL ones.

ACKNOWLEDGMENTS

The authors gratefully acknowledge fruitful discussions with J. Chalker, U. Sivan and Y. Imry. This research was partially supported by a grant administered by the Israeli Academy for Sciences and Humanities.

[1] Z. Ovadyahu and Y. Imry, J. Phys. C **16**, L471 (1983).
[2] Z. Ovadyahu, Phys. Rev. B **33**, 6552 (1986).
[3] G. Bergmann, Phys. Rep. **107**, 1 (1984).
[4] V. I. Nguyen, B. Z. Spivak, and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **89**, 1770 (1985). [Sov. Phys.-JETP **62**, 1021 (1985)].
[5] U. Sivan, O. Entin-Wohlman, and Y. Imry, Phys. Rev. Lett.
[6] Z. Ovadyahu, J. Phys. C **19**, 5187 (1986).
[7] E. Tousson and Z. Ovadyahu, Solid State Commun. **60**, 407 (1986).
[8] H. K. Muller, Phys. Status Solidi **27**, 723 (1968).
[9] W. A. Harrison, *Solid State Theory* (McGraw-Hill, New York, 1970), pp. 13–15.
[10] N. F. Mott and G. A. Davis, *Electronic Processes in Non-Crystalline Materials*, 2nd ed. (Clarendon, Oxford, 1979).
[11] A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).
[12] S. Pollitt, Commun. Phys. **1**, 207 (1976).
[13] Z. Ovadyahu, Y. Gefen, and Y. Imry, Phys. Rev. B **32**, 781 (1985).
[14] B. I. Shklovskii, Fiz. Tekh. Poluprovodn. **6**, 2335 (1972) [Sov. Phys.—Semicond. **6**, 1964 (1973)]. O. Faran and Z. Ovadyahu, Solid State Commun. (to be published).

[15]P. A. Lee and D. S. Fisher, Phys. Rev. Lett. **47**, 882 (1981).
[16]A. Miller and E. Abrahams, Phys. Rev. **120**, 745 (1960).
[17]A. D. Stone, Phys. Rev. Lett. **54**, 2692 (1985), and references cited therein.
[18]Y. Gefen, Y. Imry, and M. Ya Azbel, Phys. Rev. Lett. **52**, 129 (1984); R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, *ibid.* **54**, 2696 (1985); V. Chanrasekhar, M. J. Rooks, S. Wind, and D. E. Prober *ibid.* **55**, 1610 (1985).
[19]J. Chalker (private communication).
[20]V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).
[21]M. Pollak, J. Non-Cryst. Solids **11**, 1 (1972).
[22]H. Mell, Thesis, University of Marburg (unpublished).
[23]A. H. Clark, M. M. Cohen, M. Campi, and H. P. D. Lanyon, J. Non-Cryst. Solids **16**, 117 (1974); A. Yoshida, S. Nakashimi, and M. Ieda, *ibid.* **35**, 421 (1980).
[24]A. G. Zabrodskii, A. I. Ionov, and I. S. Shlimak, Fiz. Tekh. Poluprovodn **8**, 503 (1974) [Sov. Phys.—Semicond. **8**, 322 (1974)].
[25]A. Hartstein, A. B. Fowler, and K. C. Woo, Physica B + C **117&118B**, 655 (1983).
[26]R. A. Webb, J. J. Wainer, and A. B. Fowler (unpublished).
[27]E. Tousson, V. Volterra, E. P. Rubenstein, R. Rosenbaum, and Z. Ovadyahu, Philos. Mag. B **55**, (1987).
[28]B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors,* Vol. 45 of *Springer Series in Solid-State Sciences* (Springer, Berlin, 1984), pp 210–216.