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Coherent phase slip in arrays of underdamped Josephson tunnel junctions

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In hysteretic *I-V* characteristics of two-dimensional Josephson junction arrays resistance steps are observed. These steps are explained by switching into a coherent phase-slip state of rows of junctions across the whole array.

Resistance in two-dimensional superconducting networks such as Josephson junction arrays or granular films is generally associated with the flow of vortices across the sample.[1-4] In a zero field these vortices are excited by dissociation of vortex-antivortex pairs or induced by an applied current. Moreover, in a finite system vortices may cross by thermal activation.[5] Experimental results on the resistance of junction arrays are usually interpreted using the analysis of Lobb, Abraham, and Tinkham.[4] In that analysis vortices move independently. The energy barrier for a vortex to move from one cell to the next is small. The probability to cross is derived by analogy to the probability for a phase slip in a single junction, for which Lobb *et al.* use the results of Ambegaokar and Halperin.[6] However, the latter results are valid only for junctions in the overdamped limit: i.e., the Mc Cumber parameter $\beta_c \ll 1$. For arrays of superconductor–normal-metal–superconductor junctions this limit is clearly applicable. For tunnel junctions, with a much higher resistance at the same value of the junction critical current, the β_c parameter may be of order 1 or well above, even for very small junctions.

We have studied arrays of Josephson tunnel junctions in a wide range of normal-state resistances. At low temperatures where the junctions are all in the underdamped limit $\beta_c > 1$, the *I-V* characteristics of the arrays show a stepwise increase of the resistance (as given in the inset of Figs. 1 and 2). Measuring at different voltage contacts made clear that the steps are localized. The large number of identical steps indicates that each step corresponds to a simultaneous switching of one single row of junctions across the width of the array into the dissipative state. The step size is largely independent of temperature and of small magnetic fields perpendicular to the array.

No theoretical calculation is available as yet of resistance due to vortex motion in arrays of underdamped junctions in the regime where the Josephson coupling energy E_J dominates the charging energy $E_c = e^2/2C$, with C the junction capacitance. Recently, theoretical descriptions of vortex motion in the opposite limit $E_c > E_J$ have been developed by Larkin, Ovchinnikov, and Schmid[7] and Korshunov.[8] For single junctions, the time dependence of the phase difference can be compared with the movement of a particle in a washboard potential, with the particle mass proportional to the junction capacitance. For low enough damping the particle, once started, continues moving; the single junction remains in the dissipative state. For vortices in a two-dimensional (2D) array a similar correspondence can be evoked where now the vortex is the particle and movement is in real space, the particle experiencing the periodic potential of the spatial lattice. Again the particle mass is proportional to the junction capacitance.[7] With underdamped junctions the vortex has a tendency to continue moving once started. Our steps combine both aspects: in a cross row all junctions are either in the zero-voltage state or they exhibit a phase-slip process

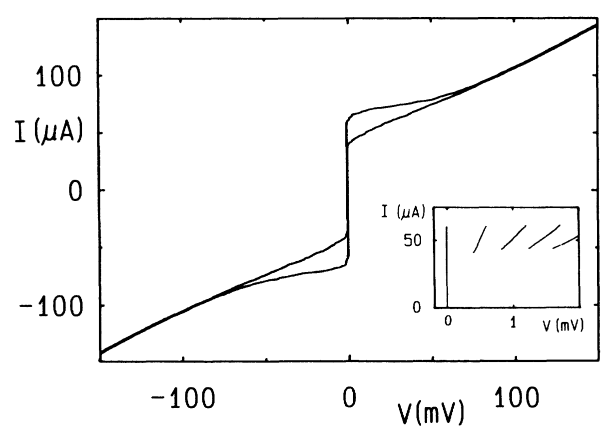

FIG. 1. The hysteretic *I-V* characteristic of sample 55. In the inset the first four resistance steps are given. These steps were measured in four different sweeps, each time the biased current was increased by a small amount.

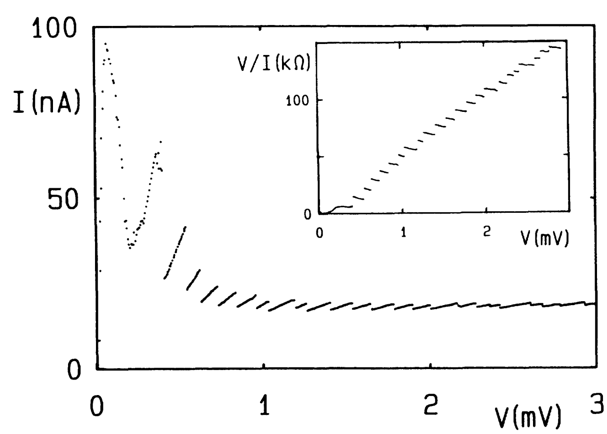

FIG. 2. The voltage biased *I-V* characteristic of sample 22 near the $V=0$ axis, showing the step behavior. In the inset the voltage of the *I-V* characteristic divided by the current is given as a function of this voltage.

that is continuous in time and coherent between the junctions along the row.

The Josephson junction arrays were fabricated using a shadow evaporation method. Between the two niobium electrodes a barrier was formed by evaporation of silicon and oxidation in a glow discharge. Different normal-state resistances were achieved by varying the oxidation time. The junctions measure 1 μm$\times$0.2 μm and one unit cell of the square array has an area of 50 μm^2. With $\varepsilon_r=10$, $d=50$ Å, the junction capacitance is estimated to be about 5 fF, corresponding to a temperature of 200 mK. The arrays have a width W of 128 junctions and are 384 or 512 junctions long. Over the length of the array two or three superconducting contacts are placed connecting to two islands of the array. The fabrication method is the same as for the arrays which are used in our group to study phase transitions.[5,9]

Measurements were performed using standard equipment. The noise level was reduced by the use of low-noise preamplifiers and the measuring circuit was optically coupled. I-V characteristics were registered with a digital plotter, connected to a computer. The low-resistance arrays were measured in a vacuum can in a standard He-4 cryostat and the high-resistance arrays, having a much smaller Josephson coupling energy, in a dilution refrigerator. For measurements in the He-4 cryostat the lowest accessible temperature was 1.35 K, for the dilution refrigerator 5 mK. Except for sample 3, the arrays were well shielded by μ-metal and superconducting lead. A magnetic field could be applied perpendicular to the array. In analogy with descriptions of phase transitions in arrays, the applied magnetic field f is defined as the flux through one unit cell, normalized to the flux quantum ($\phi_0=h/2e$).

The normal-state resistance per square R_n for six investigated arrays varies from 900 Ω to 300 kΩ. For our square lattice, R_n equals the junction normal-state resistance r_n. Figure 1 shows the I-V characteristic for sample 55 at 1.35 K, clearly indicating hysteresis. The array critical current is determined as the intersection of the upper part of the I-V characteristic with the $V=0$ axis. Division by W gives the measured junction critical current i_c. For low-resistance arrays ($R_n<2$ kΩ), the observed junction critical current-resistance product is typically 500 μV. For increasing resistance, i_c is found to be more reduced than expected from a junction critical current-resistance product of 500 μV. Previously, in single junctions with high normal-state resistances a reduction of the critical current by thermal fluctuations has been reported.[10] For two-dimensional systems the energy barrier for vortices to move from cell to cell is proportional to the junction critical current. For high r_n, thermal activation over these energy barriers is much easier, probably resulting in a preliminary switching to the dissipative state. We assume that the product of r_n and the intrinsic critical current i_{c0} for the junctions remains at 500 μV even for the high values of r_n:

$$i_{c0}=\frac{500\ \mu\text{V}}{r_n}\,. \tag{1}$$

For the low-resistance arrays at low temperatures, the measured i_c coincides with i_{c0}.

Table I contains the relevant parameters for the different samples. δ is the basic step resistance (ΔR_{step}) in terms of the normal-state resistance of a single cross row of the array:

$$\delta=\frac{\Delta R_{\text{step}}}{R_n/W}\,. \tag{2}$$

The junction return current (i_{ret}) is determined from the lower branch of the I-V characteristics in a similar way as the critical current from the upper branch. The Mc Cumber parameter $\beta_c(=2ei_{c0}r_n^2C/\hbar)$ has been calculated with a junction capacitance of 5 fF. Finally, the Josephson coupling energy $E_j=\hbar i_{c0}/(2e)$ is given.

In the inset of Fig. 1, the first four resistance steps are shown for zero magnetic field. At higher temperatures, with no hysteresis, the I-V characteristics are smooth curves. At reduced temperatures hysteresis sets in and the steps become visible. The four steps in the inset have been obtained in four different sweeps.

In Fig. 2, the steps are shown for a high resistance array. Because of the negative slope of the overall characteristic near $V=0$, voltage biasing is required. In the figure the I-V characteristic is given as observed with continuously decreasing bias voltage. If, from a point of the characteristic in Fig. 2 the bias voltage is increased rather than decreased, the same step is followed over a much wider current range up to about 80 nA. When this procedure is repeated for several steps, straight lines are found similar to those of the inset of Fig. 1. The corre-

TABLE I. Sample parameters. R_n is the array normal-state resistance per square, i_c the measured junction critical current, i_{ret} the measured return junction current, β_c the Mc Cumber parameter, and E_J the Josephson coupling energy. δ and i_{c0} are defined in formulas (1) and (2). For samples 55 and 54 i_c and i_{ret} were determined at 1.35 K, for the other arrays near 5 mK.

Sample no.	R_n (kΩ)	δ	i_c (nA)	i_{c0} (nA)	i_{ret} (nA)	β_c	E_J/k_B (K)
55	0.90	1.2	550	556	313	7	12.7
54	1.80	1.2	266	278	156	14	6.3
21	9.4	3.2	23	53	7.8	75	1.2
4	46	21	1.3	11	0.16	368	0.25
22	62	29	0.7	8	0.13	496	0.18
3	285	76	0.08	1.8	0.008	2280	0.04

sponding resistance V/I is given as a function of voltage in the inset. A significant difference between I-V characteristics of low- and high-resistance arrays is found near the $V=0$ axis. In zero field and at reduced temperatures the high-resistance arrays always show a negative resistance slope.

In the regime where steps occur, the resistance is equal to approximately an integer n times the step resistance ΔR_{step}. For arrays with R_n below 2 kΩ, ΔR_{step} is almost equal to the resistance of one single cross row R_n/W. The ratio δ increases strongly for higher values of R_n. The value of ΔR_{step} is roughly constant in a wide temperature range (25% decrease from 5 to 900 mK for sample 22). The negative resistance region near $V=0$ in high-resistance arrays disappears at a lower temperature (about 400 mK for sample 22). This peak is also very sensitive to applied flux, oscillating periodically in f with period 1. In contrast, ΔR_{step} is independent of f.

We determined simultaneously the voltage over the whole array as well as over parts by using the voltage contacts along the array. These parts contain of the order of 150 cross rows. When the array switches between two values of the resistance, clearly the additional resistance is localized in only one of the parts that are monitored. This observation excludes the possibility that the steps are associated with quantized modes of some kind of the whole array. A very large number of identical steps is observed, more than 120 for a length of 512 junctions. In smaller arrays, not included in the table, systematically the number of steps is equal to the number of cross rows. In larger arrays the resolution makes it difficult to distinguish steps for values of n above about 120.

In our two-dimensional system, resistance is necessarily due to phenomena that extend over the full width. As we observe that the steps are localized in parts of the array, that even for high values of n the step size remains unaltered and that the number of steps is very large, we can only conclude that each step is due to the switching of one particular cross row of junctions between $V=0$ and the dissipative state. Apparently the dynamic effects in one cross row do not influence the adjacent regions. From the number of observed steps in the large arrays we cannot completely exclude the possibility that each step is due to switching of up to four rows simultaneously. The results for smaller arrays, however, make this unlikely.

The experimental evidence indicates that each cross row behaves as if it is one single junction. In linear arrays of single Josephson junctions steplike behavior has been observed by many groups.[11] If the arrays are very inhomogeneous, conduction might occur mainly along one linear percolative path. However, our arrays very clearly behave two dimensionally. In all samples periodic variation as a function of flux per cell has been observed. The pronounced structure that we observe in magnetoresistance curves at fractional f values in low-resistance arrays[5,9] also indicates that our fabrication procedure is likely to produce very homogeneous samples. In these samples also a well defined two-dimensional transition is seen. So we conclude that this dissipative regime in our arrays is very different from the usual picture of diffusive vortex flow but is due to coherent phase slip in short regions, probably single rows across the array.

Single junctions that are underdamped ($\beta_c > 1$) can, over a certain current range below the critical current, be in either the $V=0$ state or in a dissipative state with V/I about constant and equal to the shunt resistance. For our low-resistance arrays the effective shunt resistance per junction is r_n. Comparison of the return current level of the hysteretic array characteristic yields good quantitative agreement with a single-junction Mc Cumber analysis. For the high-resistance arrays ($R_n > 2$ kΩ) the interpretation is less straightforward. It may be that the subgap resistance is higher than r_n, the ratio increasing with the normal-state resistance. It is likely that the quality of the tunnel junctions improves with longer oxidation times. In that case everything would be similar as in the low-resistance arrays, the increased δ solely due to the higher subgap resistance. On the other hand, it cannot be excluded that in the high-Ohmic arrays, where the ratio E_c/E_J is of order 1, a different type of vortex flow is encountered.

Hebard and Vandenberg[12] reported steps in I-V characteristics of high-resistance granular lead films. They interpreted their data with formation of a limited number of coherent clusters. We now believe that they observed switching of rows of junctions between grains, in the same line as our results.

In conclusion, in homogeneous two-dimensional arrays of underdamped Josephson tunnel junctions we have found a stepwise increase of the resistance. Each step is connected with switching of one row of junctions across the width of the array.

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