Critical exponents of the gauge glass

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(Received 21 March 1988)

The spin-glass phase suggested for granular superconductors in an externally applied magnetic field is discussed in detail. The spin-glass order parameter is an $n\times n$, $n\equiv 0$ Hermitian matrix. As a consequence, we show, by explicit calculation of the critical exponents to order $\epsilon=6-d$, that the "gauge" glass is not in the same universality class as any of the vector spin-glass models.

It has been suggested[1,2] that superconductivity in a granular material or otherwise suitably disordered superconducting-nonsuperconducting composite may exhibit the analog of a spin-glass in the presence of an externally applied magnetic field. In this paper we analyze in detail the nature of the spin glass in one of the models widely used to discuss granular superconductivity; the same model, it has been speculated widely,[3] might describe the glassy characteristics observed in granular high-T_c superconductors. We shall explicitly demonstrate that the glass phase that we call the gauge glass is not in the same universality class as any of the vector spin-glass models.

The model is a pseudospin model that has been used in different manifestations to describe regular Josephson junction arrays as well as granular superconductivity. Each superconducting grain acquires a gap, as the temperature is lowered below the single-grain transition temperature T_g. In the absence of intergrain coupling the amplitude of the gap is fixed but its phase is not. The gap, therefore, behaves as a two-component XY spin. The weak coupling between grains due to proximity or Josephson effects acts as a ferromagnetic interaction between spins, with the result that at temperature T_c, lower than T_g, there is a phase-coherent transition. On switching on a magnetic field, frustration is introduced into the system leading to the possibility of spin-glass order. If the grains are large enough such that charging effects can be neglected this system can be described by the Hamiltonian

$$H=-\sum_{(i,j)} J_{ij}\cos(\phi_i-\phi_j-A_{ij})\,. \tag{1}$$

Here ϕ_i is the phase of the order parameter of the ith grain, $A_{ij}=2\pi/\Phi_0\int_i^j \mathbf{A}\cdot d\mathbf{l}$, $\mathbf{B}=\nabla\times\mathbf{A}$, $\Phi_0=h/2e$ is the elementary quantum of flux, and J_{ij} is the intergrain coupling. Assuming a random distribution of grains and that the magnetic field B was constant throughout the sample, Shih, Ebner, and Stroud[1] found in their Monte Carlo experiments evidence of glassy behavior such as hysteresis and time dependence of supposedly equilibrium quantities. Later John and Lubensky[2] studied a random-bond version of Eq. (1) near the percolation threshold. Analytic progress was possible after taking the continuum limit of the resulting replicated Hamiltonian: they were then able to demonstrate within mean-field theory the occurrence of a phase transition from a state of macroscopic phase coherence to spin-glass order above a critical field.

The transition from normal to spin-glass phase can be described by the replicated Hamiltonian density[2]

$$H=\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}\left[\frac{1}{8}[Q_{\alpha\beta}(r-\nabla^2)Q^*_{\alpha\beta}+\text{c.c.}]+\frac{1}{3!}u\,\text{Tr}Q^3\right]. \tag{2}$$

Here it is important to recognize that the $n\times n$ order-parameter field, $n=0$, is a Hermitian matrix in replica space $Q_{\alpha\beta}=Q^*_{\beta\alpha}$ and $Q_{\alpha\alpha}=0$. Assuming replica symmetry, $Q_{\alpha\beta}\alpha\sum_i\langle\exp i\phi_i\rangle\langle\exp-i\phi_i\rangle/N$. We shall see as a direct consequence of this description in terms of an Hermitian order-parameter field that the gauge glass is not in the same universality class as the vector spin glasses. Decomposing the Q fields into real and imaginary parts, $Q_{\alpha\beta}=X_{\alpha\beta}+iY_{\alpha\beta}$, the Hamiltonian becomes

$$H=\tfrac{1}{4}\sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}}[X_{\alpha\beta}(r-\nabla^2)X_{\alpha\beta}+Y_{\alpha\beta}(r-\nabla^2)Y_{\alpha\beta}] +\frac{1}{3!}\text{Tr}(uX^3-3vXY^2) \tag{3}$$

with $v=u$; because Q is Hermitian $X_{\alpha\beta}=X_{\beta\alpha}$ but $Y_{\alpha\beta}=-Y_{\beta\alpha}$. If follows immediately from this symmetry that the propagators of the X and Y fields are given by

$$\langle X_{\alpha\beta}X_{\gamma\delta}\rangle=\frac{1}{k^2+r}X_{\alpha\beta\gamma\delta} \tag{4}$$

and

$$\langle Y_{\alpha\beta}Y_{\gamma\delta}\rangle=\frac{1}{k^2+r}Y_{\alpha\beta\gamma\delta}\,, \tag{5}$$

where[4]

$$X_{\alpha\beta\gamma\delta}=\tfrac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}-2T_{\alpha\beta\gamma\delta}) \tag{6}$$

$$T_{\alpha\beta\gamma\delta}=\begin{cases}1, & \alpha=\beta=\gamma=\delta,\\ 0 & \text{otherwise}\,,\end{cases} \tag{7}$$

and

$$Y_{\alpha\beta\gamma\delta}=\tfrac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})\,. \tag{8}$$

Notice in particular that $\langle Y_{\alpha\beta}Y_{\alpha\beta}\rangle=-\langle Y_{\alpha\beta}Y_{\beta\alpha}\rangle=(k^2+r)^{-1}$. Recursion relations can now be obtained as an expansion in $\epsilon=6-d$ about six dimensions[5] by integrating out fluctuations with wave vectors between Λ/b and Λ $(b>1)$, rescaling all lengths by b^{-1} and fields by $b^{(d+2-\eta)/2}$. Allowing the two cubic couplings to differ initially we find the recursion relations

$$r'=b^{2-\eta}[1+2(n-2)(u^2+v^2)K_d\ln b]\,,$$

$$u'=b^{(\epsilon-3\eta)/2}\{u+[(n-2)u^3+(n-4)v^3]K_d\ln b\}\,, \tag{9}$$

and

$$v'=b^{(\epsilon-3\eta)/2}\{v+[(n-2)v^3+(n-4)v^2u]K_d\ln b\}\,.$$

Here $\eta=K_d(u^2+v^2)(n-2)/3$ and $K_d=\Omega_d/(2\pi)^d$. These recursion relations should be compared with those for the Ising spin glass.[5] There are three fixed points; the Gaussian fixed point $u^*=v^*=0$, and unstable Ising-spin-glass fixed point $K_d u^{*2}=-\epsilon/(n-2)$, $v^*=0$, and a stable gauge glass fixed point

$$u^*=v^*=[-\epsilon/2(n-4)K_d]^{1/2}\,, \tag{10}$$

from which we find in the limit $n\rightarrow 0$, $\eta=-\epsilon/6$ and $\nu=1/2+5\epsilon/24$. This is to be compared with the corresponding results for m-component vector spin glasses of $\eta=-m\epsilon/3(2m-1)$ and $\nu=\frac{1}{2}+5m\epsilon/12(2m-1)$. Hence the gauge glass is not in the same universality class as any vector spin glass with a finite value m.

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