Transfer-matrix scaling for anomalous dynamics of a vector spin-glass chain

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The transverse dynamics of a one-dimensional vector spin glass at zero temperature is treated by a new transfer-matrix scaling technique, and also by evaluating static critical properties related to dynamics by a crossover argument. A nontrivial dynamic exponent is found which implies that the system at zero temperature has dynamic critical behavior $w \propto k^{3/2}$, in contradiction to the hydrodynamic behavior normally assumed and to the results of previous work.

Most of the important advances resulting from spin-glass studies[1] over more than a decade have concerned the static properties of Ising spin glasses, usually in the Edwards-Anderson model.[2]

Less is known about the static properties of the vector spin glasses (Heisenberg and XY), and still less is known about the dynamics of either type (Ising or vector) of spin glass. This article is concerned with possibly the simplest spin-glass dynamics problem, the spin-wave dynamics of the (Edwards-Anderson) Heisenberg spin-glass chain, which turns out to have a nontrivial (critical) dynamics, different from the hydrodynamic behavior[3] normally assumed or from the forms arrived at in some previous (approximate) discussions.[4,5]

Spin glasses, i.e., magnetic systems involving competition between ferromagnetic and antiferromagnetic interactions, due to some quenched structural disorder, exhibit, for $d \geq d_l$ (where d, d_l are dimensionality and lower critical dimensionality), a continuous "freezing" transition at which the correlation length ξ diverges, to a phase with zero net magnetization. The divergence of ξ suggests the use of scaling techniques[6-14] and these provide us with our best knowledge about such things as the lower critical dimension, now thought to satisfy $2 < d_l < 3$ for the (short-range) Ising spin glass[8,9,15,16] and $3 < d_l$ for the Heisenberg spin glass.[12-14,17] The application of scaling techniques[18-20] to spin-glass dynamics has also now begun to yield firm results, one of which is given here.

In vector spin glasses it is normally assumed that the hydrodynamic theory of Halperin and Saslow[3] for elementary excitations from the ground state applies, in which spin waves occur, with linear dispersion relation between frequency w and wave vector k (for small k):

$$w = ck. \tag{1}$$

By virtue of the divergence of ξ there is however a possibility that the "velocity" c becomes "critical" near the transition (cf. critical slowing and mode softening at simple continuous thermal[21] or geometric[22] transitions) in which case (1) has to be replaced by the more general dynamic scaling[23] form

$$w = k^z f(k\xi), \tag{2}$$

where z is a dynamic exponent characterizing the dynamics near the spin-glass transition and k should now be understood as the inverse characteristic length of the spin fluctuations. It will be shown here that this generalized view, with z taking the nontrivial value $\frac{3}{2}$ (which implies critical behavior $w \propto k^{3/2}$ at $T=0$) is required for even the simple one-dimensional Heisenberg spin glass. This result thus contradicts the usual assumption that hydrodynamic behavior (1) occurs in the vector spin-glass chain at $T=0$. Previous explicit discussions of this system have normally started from mean-field/virtual-crystal views, which lead to (1), and perturbation extensions, possibly to all orders[4] which leave (1) unchanged, or from averaging a Green's function perturbation expansion which leads to a result different from (1) (Ref. 5); however, these approaches are limited to $k\xi < 1$, which does not apply at $T=0$, where ξ diverges.

The one-dimensional Heisenberg spin glass is studied in the usual Edwards-Anderson model, in which the spins occupy sites on a regular lattice, but each (nearest-neighbor) exchange interaction is taken to be an independent random variable with a symmetric distribution. One-dimensional models of spin glasses have a direct experimental application only for rather special substances, in which magnetic interactions are restricted to one lattice direction;[24,25] and, since one-dimensional models of spin glasses with nearest-neighbor interactions only, and in zero-magnetic field, are not frustrated, a study of their behavior provides information only on disorder effects; further, these disorder effects can be "gauged away"[26] by a transformation of spin variables when considering most static effects. The Mattis transformation does not, however, remove disorder effects in dynamics, which is nontrivial. So, for this reason and others given in the previous paragraph, we consider the proper treatment of these one-dimensional models an essential first step for the understanding of more general models describing the full behavior of spin glass in which frustration effects are also included.

A new, exact, scaling technique is here used, employing a transfer-matrix description. The transformation of a basic parameter (the frequency) under a lattice rescaling is derived and exploited; we shall find, for a dilatation in which the lattice constant a increases to $a' = ba$, the

linearized transformation of the frequency

$$w \rightarrow w' = \lambda w + O(w^2) ; \tag{3}$$

then, as in more usual scaling methods like decimation, the dynamic exponent is given in terms of the eigenvalue λ by[27]

$$z = \ln\lambda / \ln b . \tag{4}$$

Decimation has also been applied to this system and, though approximate, also results in $z > 1$, as will be reported elsewhere.[20] Starting from the usual Hamiltonian ($H = -\sum J_{nn+1} \mathbf{S}_n \cdot \mathbf{S}_{n+1}$) for the one-dimensional Heisenberg spin glass, with "plus-minus" distribution ($J_{nn+1} = \pm J$), the equations of motion governing the spin dynamics of transverse modes at zero temperature can be written in the form

$$(2 - \zeta_n w)\mu_n = \mu_{n-1} + \mu_{n+1} . \tag{5}$$

Here w is frequency divided by exchange constant J, and ζ_n equates to 1, -1 if the configuration of J's is such as to cause spin at site n to be up or down, respectively (at $T = 0$), and $\mu_n = \zeta_n S_n^+$ where S^+ is the usual combination $S^x + iS^y$ of transverse spin components. It can be seen that the Mattis transformation, from S_n^+ to μ_n, does not eliminate disorder effects from the dynamics; but it has formally transformed the original bond-disordered problem to one with diagonal site disorder. Further, we have ignored real quantum effects in taking the classical ground state. This is adequate for sufficient large spin.

Equation (5) can be written in the form

$$\begin{pmatrix} \mu_{n+1} \\ \mu_n \end{pmatrix} = \begin{pmatrix} 2 - \zeta_n w & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_n \\ \mu_{n-1} \end{pmatrix} = T_n(w) \begin{pmatrix} \mu_n \\ \mu_{n-1} \end{pmatrix}, \tag{6}$$

which involves the transfer matrix T_n. For a chain of $N+1$ spins with periodic boundary conditions, $\mu_n = \mu_{n+N}$ (periodic boundary conditions on the μ_n variables do not introduce frustration since they do not constrain the longitudinal spin components), the allowed frequences are determined by $\det(\Lambda_N - 1) = 0$ where

$$\Lambda_N(w) = \prod_{n=1}^{N} T_n(w) \tag{7}$$

is the transfer matrix across the whole chain; or equivalently (since $\det\Lambda = 1$) $\mathrm{Tr}\Lambda_N = 2$ determines the eigenfrequencies. Thus if we scale the system by a dilation factor b, the dynamics is preserved if we transform w to w' where (in a probabilistic sense to be clarified below)

$$\mathrm{Tr}\Lambda_N(w) = \mathrm{Tr}\Lambda_{N/b}(w') . \tag{8}$$

It is possible to use the algebra of the matrices

$$A \equiv \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, \quad B \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

(e.g., $A^l = lA - (l-1)I$, where I is the unit matrix, etc.) to obtain the coefficients (functions of the random variables ζ_n) of the expansion of $\mathrm{Tr}\Lambda_N(w)$ in powers of w:

$$\mathrm{Tr}\Lambda_N(w) = 2 - wN\sum_{l=0}^{N-1} \zeta_l + w^2 \sum_{l=0}^{N-1} \sum_{m=0}^{l-1} \zeta_l \zeta_m [N(l-m) - (l-m)^2] + \cdots . \tag{9}$$

We discuss first the scaling of w resulting from taking in (8) just the terms in (9) to first order in w. In the illustrative case of a ferromagnet, where all ζ_l equal $+1$, the matching (8) leads to $w' = b^2 w$ so, using (3) and (4) we get the result $z = 2$ corresponding to the usual quadratic dispersion of this trivial case.[28] For an antiferromagnet (where the ζ_l are alternately $+1$, -1, and it is appropriate to work with N and N/b both odd to scale antiferromagnet into antiferromagnet), (8) leads to $w' = bw$ and hence the usual[28] linear dispersion ($z = 1$). For the spin glass the occurrence of random variables ζ_n in (9) means that (8) should be understood as generating the scaling of its distribution: the distribution starts bimodal, $\zeta_n = \pm 1$ with equal probability, and after many scalings converges to a Gaussian fixed-point shape with fixed width if $w \rightarrow w' = \lambda w$ with $\lambda = b^{3/2}$. Equivalently, the first-order term in (9) can be considered in terms of the variable $\sum_{l=0}^{N-1} \zeta_l$, which for large N has a Gaussian distribution and characteristic (root mean square) size $\propto N^{1/2}$, again leading to $\lambda = b^{3/2}$. A further equivalent view is in terms of $\sum_{l=0}^{N-1} \zeta_l$ being like the end-to-end distance of a random walk of N steps. Including higher-order terms in (9) (in the required scaling regime of small w and large N) leads to the same eigenvalues λ. For ferro (F) and antiferromagnet (A), the forms resulting from (9) for $\mathrm{Tr}\Lambda_N(w)$ are, respectively, $f_F(N^2 w)$, $f_A(Nw)$, where the f's are different polynomials for the two cases: these forms merely provide an illustration of and check on the method; in the case of the spin glass, a form $f_S(N^{3/2}w)$ is found for any even moment of $\mathrm{Tr}\Lambda_N(w)$ (odd moments vanish). Similar statement apply to the quantity

$$\mathrm{Tr}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \Lambda_N(w)$$

which determines the dynamics for chains with open ends, confirming the insensitivity to boundary conditions. Hence for the spin glass we obtain the scaling (3) with $\lambda = b^{3/2}$ and hence, using (4), the critical exponent has the nontrivial value

$$z = 3/2 . \tag{10}$$

This result is exact (unlike some scaling methods for random problems even in one dimension) since under scaling no correlations are generated between the transfer matrices, originally considered independent. The result implies that $w \propto k^{3/2}$ in the Heisenberg spin-glass chain at $T = 0$, in contradiction to the results of hydrodynamic theories and of mean-field theory (linear dispersion) and CPA-like generalizations [which allow for disorder fluctuations but not the effects of self-similarity ($\xi \rightarrow \infty$)].

Hydrodynamic theory applies away from the transition at finite ξ. For, as the combination $k\xi$ changes from large

to small values, the dynamic scaling expression crosses over from the critical form $w \propto k^z$ to the hydrodynamic form (1), which needs the scaling function $f(x)$ in (2) to approach $x^{(1-z)}$ for small x. That implies that the velocity c in (1) behaves like

$$c \propto \xi^{(1-z)}, \tag{11}$$

and so, since $z > 1$, the velocity tends to zero (mode softening) at the transition where ξ diverges. The results of Halperin and Saslow[3] for the hydrodynamic regime ($k\xi < 1$) give

$$c \propto (\rho/\chi)^{1/2}, \tag{12}$$

where ρ is the spin stiffness and χ is an adiabatic isotropic spin-glass susceptibility. Since in the spin-glass chain in an equilibrium state at $T=0$ all the spins are up or down, χ in this case refers to a transverse susceptibility (i.e., generated by "torque free" fields which couple to the transverse components of the spins and depend on whether $\zeta_i = \pm 1$). Then critical behavior can be written

$$\chi \propto \xi^{\tau/\nu}, \quad \rho \propto \xi^{-t/\nu} \tag{13}$$

(using a common notation involving exponents τ, t, ν for temperature dependences of χ, ρ, ξ). Comparison with (11) gives the following relationship between dynamic and static exponents:

$$z = 1 + (t+\tau)/2\nu. \tag{14}$$

A scaling calculation of the static exponents hence gives an alternative way of obtaining z, and a check on the result (10). This calculation is simplified by the fact that the Mattis transformation $\mathbf{S}_n \rightarrow \mu_n = \zeta_n \mathbf{S}_n$ removes all disorder effects from ρ and allows gradient expansions, which gives

$$\rho \propto Ja^{(2-d)} \tag{15}$$

so the spin stiffness has the same scaling as the conductivity[29-32] of the uniform system (i.e., $t/\nu = 0$ for $d = 1$). The scaling of χ is less trivial because it still depends on the disorder because of the nature of the fields which generate it. The generating field h_n at site n occurs in the Hamiltonian through a term $h_n S_n^+ = h_n \mu_n \zeta_n$; h_n takes different values for up and down spins ($\zeta_n = \pm 1$) and is such that the net torque on the system vanishes (i.e., $\sum_n h_n \zeta_n$ is zero). To linear order in the h_n, the transverse spin component $\delta_n \equiv [\mu_n(h_n) - \mu_n(0)]/S$ generated at site n is given by

$$\sum_{(\mathrm{NN\ of}\ i)} (\delta_i - \delta_j) = c_0 h_i \zeta_i, \tag{16}$$

where $c_0 = g\mu_B/JS^2$ and NN is the nearest neighbor. The required transverse spin-glass susceptibility is

$$\chi = (g\mu_B S^2/a) \left(\frac{1}{N} \sum_n \delta_n \zeta_n \right) \Big/ \left(\frac{1}{N} \sum_n h_n \right). \tag{17}$$

For the chain, (16) is a second-order difference equation, which can be solved by writing it as first-order difference equation for $\delta_i - \delta_{i-1}$ and then iterating. Inserting the resulting solution,

$$\delta_n = -c_0 \sum_{k=1}^{n} \sum_{j=1}^{k-1} h_j \zeta_j,$$

into the numerator of (17) gives the susceptibility, which depends on the configuration of the ζ_j's. The N dependences in (17), which alone provide the scaling of χ since Ja is again a scaling invariant, are as follows. For the spin glass ($\zeta_n = \pm 1$), numerator and denominator of (17) are random variables. The numerator is a symmetrically distributed random variable with root-mean-square value proportional to N, while the denominator has mean value of order N^0. Hence under dilation by b, where $N \rightarrow N' = N/b$, the length scaling exponent of χ is $\tau/\nu = 1$. Inserting (with $t/\nu = 0$) into (14) gives again $z = \frac{3}{2}$, which provides an alternative derivation of the result (10) via static scaling alone.

A check on the scaling of χ is provided by the antiferromagnet. Here $\zeta_n = (-1)^n$ so, taking N and N' odd yields $\chi \propto N^0$; hence $\tau = 0$, which leads via (14) to the usual linear dispersion ($z = 1$).

It has been shown that the Heisenberg spin-glass chain at zero temperature does not have the linear dispersion ($z = 1$) of hydrodynamic theory, as usually assumed [or arrived at via for example perturbative expansions to all orders from mean-field (virtual-crystal) behavior]. The reason is that the zero-temperature spin-glass chain is at its critical ("freezing") temperature, and so it has intrinsic dynamic critical behavior (with dynamic exponent $z = \frac{3}{2}$). Similar critical dynamic behavior has to be expected in zero-temperature vector spin glasses in two and three dimensions: while these systems also have frustration effects, they again have $T_f = 0$ and therefore will exhibit critical dynamics ($w \propto k^z$, $z > 1$) rather than hydrodynamics at zero temperature. Our discussion of the crossover has assumed the validity of the dynamic scaling hypothesis (2). The assumption can in principle be checked by generalizing the treatment to finite (low) temperatures, where the length ξ over which μ_n's remain correlated (same sign) becomes finite. That leads to ξ competing with N in the scaling discussions above with the result that ξ appears as a characteristic controlling length, as expected in (2). Note that no activated dynamics is being considered, which might lead to a breakdown of dynamic scaling, as has been found near zero-temperature fixed points in random Ising systems.[22,33-35] The interpretation of the "dynamic" length $1/k$ appearing in (2), etc., has not so far been mentioned. This is merely the dynamic length to be associated with frequency w. For extended states this is usually a wavelength, but in the present case, where all states are expected to be localized,[36] it is the dependence of the frequency on inverse localization length which is characterized by the dynamic exponent. Further details of this work will be published elsewhere,[20] and extensions in progress as planned include numerical comparisons and treatments of simple higher-dimensional models.

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