Universality in two- and three-dimensional growth and deposition models

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We simulate the three-dimensional single-step model of deposition and evaporation as well as a three-dimensional ballistic-deposition model with surface diffusion. We find that the dynamic structure factor $S(\mathbf{k},t)$ obeys dynamic scaling: $S(\mathbf{k},t)\sim k^{-2+\eta}g(k^z t)$, where in all three-dimensional cases studied $z-\eta/2\approx 2$, consistent with a recently proposed scaling law. For the single-step model $z=2$ if the process is time-reversal invariant, $z\approx 1.63$ if it is not. The ballistic-deposition model seems to have the dynamic exponent $z=2$ in both two and three dimensions. These results are discussed in the context of the Burgers equation which has been proposed as a model for interface dynamics.

I. INTRODUCTION

A number of nonequilibrium growth and deposition models have been proposed and extensively studied in recent years, partly because they are crude models for phenomena such as tumor growth, vapor deposition and sedimentation, and partly because they have intrinsically interesting properties. Among the models that we have in mind are the Eden model,[1] the ballistic-deposition model,[2] with or without surface diffusion[3] or restructuring,[4] and the so-called single-step model.[5,6] These models have the common feature that they lead to a compact structure (the fractal dimension is equal to the Euclidean dimension) with a rough surface. In a strip geometry this roughness can be quantitatively described in terms of the width $\xi(L,t)$ of the surface of the cluster where L is the linear dimension of the substrate on which the cluster is grown and t is the time of growth. Empirically, one finds from computer simulations[3-11] that this function obeys a scaling form[12]

$$\xi(L,t)\approx L^{\alpha} f(t/L^{z}) ,$$

where $f(t)\rightarrow$const. as $x\rightarrow\infty$ and $f(x)\rightarrow x^{\alpha/z}$ as $x\rightarrow 0$. Thus the steady-state ($t=\infty$) width diverges as L^{α} as the substrate size is increased and the approach to the steady state is characterized by the dynamic exponent z. In two dimensions (one-dimensional substrate) the aforementioned models all have exponents consistent with the rational numbers $\alpha=\frac{1}{2}$ and either $z=2$ or $z=\frac{3}{2}$. In three dimensions, extensive numerical work[5,9,10] has yielded results consistent with either $\alpha\approx\frac{1}{3}$ together with $z\approx\frac{5}{3}$ or $\alpha\approx 0$ (logarithmic divergence of the steady-state width) together with $z\approx 2$.

The scaling form of the surface width is reminiscent of the behavior of thermodynamic functions near critical points, and one may ask whether the notion of universality, so powerful in classifying critical behavior, is a useful concept in these nonequilibrium processes. Specifically, one would like to determine which properties of a growth algorithm determine the values of the exponents α and z and how one can calculate these exponents from a renormalization-group analysis.

An important step in this direction was taken by Kardar, Parisi, and Zhang[13] who argued that Eden-model dynamics (and presumably that of other simple growth models) could be described in terms of the Burgers equation.[14] Their renormalization-group analysis of this equation yields two fixed points in two dimensions, a free-field fixed point with exponents $\alpha=\frac{1}{2}$ and $z=2$ and a strong-coupling fixed point with $\alpha=\frac{1}{2}$ and $z=\frac{3}{2}$. Numerical work on the Eden model,[7-9,11] the single-step model,[5,6] and ballistic-deposition models[3,4] are consistent with these predictions in the sense that only $z\approx 2$ and $z\approx\frac{3}{2}$ have been measured. In the two-dimensional single-step model, where the parameter responsible for the crossover from the free field to a strong-coupling fixed point could be explicitly identified, this crossover was clearly observed.[6] The remaining difficulty in two dimensions, in our mind, is the result of Family[3] on a ballistic-deposition model with surface diffusion. He, and later Meakin and Jullien,[4] find the free-field exponents for this model which it seems, at first glance, should be governed by the strong-coupling fixed point.

In three dimensions the situation is less clear. The renormalization-group equations of Kardar *et al.*[13] do not yield a strong-coupling fixed point; $d=3$ is the critical dimension of the Burgers equation and the perturbation theory of Ref. 13 fails. Kardar *et al.* conjectured that the two-dimensional strong-coupling exponent $z=\frac{3}{2}$ might be superuniversal (independent of dimensionality). Support for this conjecture came from numerical work on a directed polymer model[15] which yielded $z\approx 1.61$, in reasonable proximity to $z=\frac{3}{2}$ but closer to the subsequently discovered value $z\approx\frac{5}{3}$ for the three-dimensional versions of the single-step and Eden models. Although there are at present no renormalization-group predictions in three dimensions, a scaling solution of the Burgers equation can be shown[5,16] to imply the equation $z+\alpha=2$, consistent with numerical results.

In this article, we present the results of further simulations on two- and three-dimensional growth models. We investigate a generalized version of the single-step model,

already discussed in two dimensions in Ref. 6, which allows the evaporation as well as the deposition of particles. As in two dimensions, we find that a moving interface behaves differently from a stationary one, and that in the case of time-reversal invariance of the microscopic growth algorithm (stationary interface), the free-field exponents are recovered. We also investigate a simple ballistic-deposition model with restricted surface diffusion both with a square-lattice and hexagonal-lattice substrate. In both cases only the free-field exponents are found, and we find no evidence that substrate anisotropy is a relevant variable. Finally, we return to the two-dimensional version of the latter model[3] in order to examine the possibility of a slow crossover to the strong-coupling fixed point which may have escaped detection in previous simulations. We were unable to observe such a crossover and thus can shed no further light on the question of whether or not the Burgers equation generally applies to these processes.

The outline of this article is as follows. In Sec. II we describe the growth models that we have investigated in more detail, discuss the simulation methods and the quantities of interest, and briefly review the theoretical results of Kardar *et al.*[13] Section III contains the results of our simulations, and we conclude in Sec. IV with a short discussion.

II. MODELS AND THEORETICAL FRAMEWORK

As already mentioned, we have carried out simulations for a number of three-dimensional growth models which we now describe in more detail.

A. Single-step model

In the three-dimensional single-step model one deposits particles at the sites of a two-dimensional substrate which we take to be an $L \times L$ square lattice. The particles are of "height" 2 and one imposes the constraint that neighboring columns can only differ in height by ± 1, i.e.,

$$h(\mathbf{r}+\mathbf{a},t)-h(\mathbf{r},t)=\pm 1 \ ,$$

where $h(\mathbf{r},t)$ is the height of column $\mathbf{r}$ at time t of the process and where $\mathbf{a}$ is a basis vector of the substrate lattice. The initial state at $t=0$, in conformity with this constraint, is given by

$$h(x,y,0)=\tfrac{1}{2}[1+(-1)^{x+y}], \ 1 \le x, \ \ y \le L \ ,$$

where L is an even integer in order to allow periodic boundary conditions. Thus deposition can occur only at local minima of the interface. The process of deposition is stochastic, and each eligible site is selected with the same probability. This model has already been investigated by Meakin *et al.*[15] who found $\alpha=0.363\pm0.005$ and $z \approx 1.64$.

It is possible to generalize the single-step model[6] by also allowing the evaporation of particles subject to the same constraint that height differences between neighboring columns be restricted to ± 1. Evaporation can thus only occur at local maxima of the surface. If one assigns a probability p_+ to deposition and $p_-=1-p_+$ to evaporation, the velocity of the interface is proportional to p_+-p_-. In two dimensions, where this generalized single-step model is equivalent to a one-dimensional kinetic Ising model, the case of $p_+=p_-=\frac{1}{2}$ is exactly solvable[6] and one finds $z=2$, $\alpha=\frac{1}{2}$. For any $p_+ \neq p_-$, the results of Ref. 6 are consistent with $z=\frac{3}{2}$, $\alpha=\frac{1}{2}$. Thus the growth velocity is a relevant variable in two dimensions. As discussed in Ref. 6, the stationary or equilibrium process, $p_+=p_-$, is microscopically time-reversal invariant; the case of a moving interface is not. Plischke *et al.*[6] thus conjectured that the breaking of time-reversal invariance is responsible for the change in "universality class" that occurs when the surface is allowed to move.

In three dimensions we have not been able to solve the kinetic equations for the time-reversal invariant case $p_+=p_-$. For all growth velocities, the single-step model is equivalent to a kinetic six-vertex model[5] or, equivalently, to a kinetic Ising model with spins on the nearest-neighbor bonds of the square lattice substrate, but the resulting equations are complicated. Nevertheless, in a continuum approximation, one can show that the parameter λ which is the strength of the nonlinear term in the Burgers equation (see the following) is zero in the time-reversal invariant case. Thus we expect to find different values for the exponents in the two cases which we have simulated in three dimensions, namely $p_+=1$ and $p_+=p_-=\frac{1}{2}$. These simulations are discussed in Sec. III.

B. Ballistic deposition with surface diffusion

We have also examined a simple deposition model with some surface diffusion in two and three dimensions. In three dimensions particles are randomly deposited in columns above a square or hexagonal lattice of size $L \times L$. If the surface is locally flat the particle remains at the point of deposition. If one or more of the nearest-neighbor columns are at a lower height than the point of deposition, the particle moves to the nearest-neighbor column of lowest height. If several neighboring columns are at the same lower height, the direction of diffusion is randomly selected. One could allow this process to continue until the particle reached a local minimum, but this seems not to be important for the scaling properties. We note that in the absence of surface diffusion there are no correlations between columns and the width of the surface simply diverges as $t^{1/2}$, independent of L. Diffusion introduces correlations between the columns, and a nontrivial scaling form of the surface width is found.[3] Our motivation for studying this model is from the results of Family[3] and Meakin and Jullien[4] which seem to indicate that in two dimensions surface diffusion or rearrangement changes the universality class from the strong-coupling class ($\lambda \neq 0$ in the Burgers equation) to the free-field class. Since the hexagonal substrate is less anisotropic than the square substrate, we also obtain some indication as to the role of substrate anisotropy in these processes.

Finally, we have reexamined the two-dimensional version of the above-mentioned model for rather large substrate lengths and in more detail than in the original work of Family.[3] Since the surface height $h(x,t)$

$(1 \leq x \leq L)$ is a single-valued function of the variable x, one can define an equivalent spin-∞ kinetic Ising model by

$$\sigma(x,t)=h(x+1,t)-h(x,t) .$$

The corresponding master equation, analyzed as in Ref. 6 for the single-step model, seems to indicate that the coupling constant λ in the Burgers equation is nonzero. Thus, even if this coupling constant is small, we would expect to see some indication of crossover to the strong-coupling fixed point.

C. Simulations and quantities of interest

The models described above share the common feature that the height of the interface, $h(\mathbf{r},t)$, is a single-valued function of the substrate lattice position vector $\mathbf{r}$. Thus, the interfacial width $\xi(L,t)$ is simply given by

$$\xi^2(L,t)=\left\langle \frac{1}{L^d}\sum_{\mathbf{r}}[h(\mathbf{r},t)-\bar{h}]^2 \right\rangle , \tag{1}$$

where d is the dimensionality of the substrate and h the mean height at time t, and where the angular brackets indicate averaging over different realizations of the process. As mentioned in the Introduction, the quantity $\xi(L,t)$ is known to obey the scaling form

$$\xi(L,t)=L^{\alpha}f(t/L^z) , \tag{2}$$

where the time t is the number of events (deposition or evaporation) divided by L^d. Most authors[3–5,7,9–12] have determined the exponents α and z directly from expression (1). It is well known, however, that the divergence of the width in the limit $L\to\infty$, $t\to\infty$ is due, as in statistical models of roughening such as the solid-on-solid model,[17] to the divergence of long-wavelength fluctuations. We have therefore, as in Refs. 6 and 8, examined these fluctuations directly. We calculate the structure factor

$$S(\mathbf{k},t)=\langle \hat{h}(\mathbf{k},t)\hat{h}(-\mathbf{k},t)\rangle , \tag{3}$$

where

$$\hat{h}(\mathbf{k},t)=\frac{1}{L^{d/2}}\sum_{r}[h(\mathbf{r},t)-\bar{h}]e^{i\mathbf{k}\cdot\mathbf{r}} .$$

We expect[6,8] that the structure factor will, in the long-wavelength limit, be of the scaling form

$$S(\mathbf{k},t)=k^{-2+\eta}g(k^z t) , \tag{4}$$

where $g(x)\to$const. as $x\to\infty$. The width is given in terms of the structure factor by

$$\xi^2(L,t)=\frac{1}{L^d}\sum_{\mathbf{k}}S(\mathbf{k},t) \approx \left[\frac{1}{2\pi}\right]^d K_d\int_{\pi/L}^{\pi} dk\, k^{d-3+\eta}g(k^z t) , \tag{5}$$

where K_d is the surface area of a d-dimensional unit sphere. From (5) and (2) we find $\alpha=1-d/2-\eta/2$. The use of (1) to calculate the exponents α and z has the disadvantage that all modes including the noncritical short-wavelength modes contribute to the sum in (5). Thus we expect a stronger finite-size dependence than in the structure factor itself. Concentrating on the long-wavelength part of the structure factor should also have some of the effect of the noise reduction procedure of Wolf and Kertesz[11] since noise due to the stochastic nature of the growth processes is essentially a short-wavelength phenomenon.

To determine the dynamic exponent z we examine the relaxation function

$$\Psi(\mathbf{k},t)=1-\frac{S(\mathbf{k},t)}{S(\mathbf{k},\infty)}=1-g(k^z t) , \tag{6}$$

where we have used the scaling ansatz in the last expression. We attempt to find the value of z which provides the best collapse of the data for small k when $\Psi(\mathbf{k},t)$ is plotted as function of the scaled variable $k^z t$. In some cases we have also fitted the function $\Psi(\mathbf{k},t)$ to an exponential of the form

$$\Psi(\mathbf{k},t)=\exp(-ak^z t)$$

and found estimates of z in agreement with those obtained by simple inspection of the type of plot presented in Sec. III and with comparable uncertainties.

D. The Burgers equation

Before the work of Kardar *et al.*,[13] Edwards and Wilkinson[18] proposed a Langevin equation for the description of surface fluctuations in a settled granular material. This equation, which we refer to as the free-field equation, is, in the comoving frame of reference, of the form

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu\nabla^2 h(\mathbf{r},t)+\zeta(\mathbf{r},t) , \tag{7}$$

where the diffusion constant ν can be thought of as a surface tension and where $\zeta(\mathbf{r},t)$ represents Gaussian white noise. Equation (7) is easily solved in any dimension and one obtains $\eta=0$ and $z=2$. Kardar *et al.*[13] argued that, in the case of Eden-model growth, Eq. (7) should be generalized to the form

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu\nabla^2 h(\mathbf{r},t)+\frac{\lambda}{2}[\nabla h(\mathbf{r},t)]^2+\zeta(\mathbf{r},t) , \tag{8}$$

where the nonlinear term, the potential importance of which was recognized by Edwards and Wilkinson,[18] compensates for the lateral growth which can occur in the Eden process. In the single-step model, where lateral growth is not possible, it can be shown[6] that the nonlinear term is nevertheless present and that the coefficient λ is proportional to the mean velocity of the interface. Similar arguments applied to the ballistic-deposition model with surface diffusion indicate that λ is nonzero in this case as well.

The dynamic renormalization-group analysis of Ref. 13 demonstrates that for $d=1$ (d denotes the substrate dimension, $d=2$ for three-dimensional deposits) λ is a relevant coupling constant. The linearized recursion relations at the nontrivial fixed point predict $\eta=0$ and $z=\frac{3}{2}$ consistent with simulations of the Eden[7,8,11] and single-step[5,6] models but not with the results of Family,[3]

who finds $\eta=0$ and $z=2$ for the ballistic-deposition model with surface diffusion.

For $d=2$, no strong-coupling fixed point is found, but the recursion relations indicate that λ is relevant. Thus one should expect to find nontrivial exponents in three dimensions for the Eden model and for the single-step model with a moving surface and, indeed, $\eta\approx -0.73$, $z\approx 1.64$ have been measured.[5,11] As already mentioned, one can obtain a scaling law, valid in two and three dimensions, relating η and z by simply assuming,[5] in the absence of noise, that

$$h(\mathbf{r},t)\sim t^{\nu}\phi(r/t^{1/z}) .$$

For $\lambda\neq 0$, this ansatz leads to the equation

$$z-\eta/2=1+d/2 , \tag{9}$$

consistent with simulations. This scaling law can also be obtained[16] in the mode-coupling approximation. For $d>2$ the renormalization-group analysis indicates that the free-field fixed point is stable.

The Burgers equation provides an attractive framework in which to discuss the dynamic properties of growth and deposition models. In two dimensions it possesses the two fixed points with the appropriate exponents seen in simulations. The more general scaling law (9) is also consistent with numerical results in two and three dimensions. However, it is not yet entirely clear as to which features of a growth algorithm are responsible for a nonzero λ in the Burgers equation, or more generally, whether a single nonlinear equation captures the essential physics. The single-step and ballistic-deposition models are described, in spin language, by an infinite hierarchy of kinetic equations for spin-spin correlation functions. The Burgers equation is obtained if one decouples the second-order correlation functions and expresses them approximately[6] in terms of spatial derivatives of $\langle\sigma(\mathbf{r})\rangle$. There is no guarantee that this approximation is generally valid. In an attempt to elucidate these points we have carried out the simulations reported in Sec. III.

III. RESULTS OF SIMULATIONS

In this section we report the results of computer simulations for the models described in Sec. II.

A. Three-dimensional single-step model

We have simulated the three-dimensional single-step model for square-lattice substrates up to size 100×100. The two limiting cases discussed above, namely the case of a stationary surface (equal amount of deposition and evaporation, $p_+=p_-=\frac{1}{2}$) and the case of deposition only ($p_+=1$, $p_-=0$) were considered. Figure 1 is a log-log plot of $S(\mathbf{k},\infty)$ as function of k for these two models for substrate sizes 40×40, 60×60, and 100×100. The curves are the functions $S_1=1.92k^{-2}$ for the time-reversal invariant model (stationary interface) and $S_2=2.30k^{-2.75}$ for the case of maximum growth rate. The data points are in excellent agreement with these curves, even for k as large as $\pi/3$, indicating that $\eta\approx 0$ for the time-reversal invariant single-step model and $\eta\approx -0.75$ when the interface moves. These results indicate that the three-dimensional single-step model has the same feature as the two-dimensional one,[6] namely that the difference between the deposition rate and evaporation rate is a relevant field. In two dimensions this manifests itself only in the dynamic exponent z; in three dimensions it also affects the divergence of the steady-state width. Translating these results to the scaling form for the width (2) we find $\alpha\approx 0$ (logarithmic divergence) for the stationary interface, $\alpha\approx 0.375$ for the moving interface. This last result is consistent with the work of Meakin *et al.*[5] who find $\alpha\approx 0.364$ for the moving interface.

In Figs. 2 and 3 we plot the relaxation function (6) for these two models as function of $k^z t$ with $z=2$ in the stationary case and $z=1.625$ in the full growth case for the 12 smallest values of k for 100×100 substrates. Our choice of $z=1.625$ in the full-growth case is motivated by the scaling law (9) and the choice $\eta=-0.75$. Any value of z in the range $1.60<z<1.65$ provides a comparable collapse of the data.

Thus, for this model, we find the same general behavior in three as in two dimensions. If one assumes, on the basis of the mapping of the Burgers equation on the directed polymer problem[13,15] and on the basis of Eden-

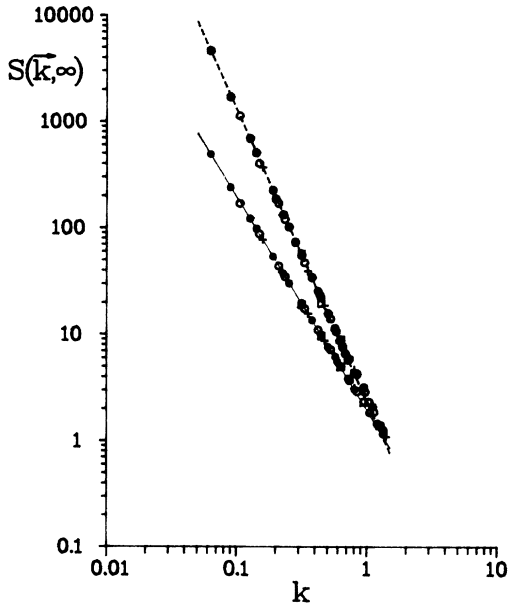

FIG. 1. Log-log plot of the steady-state structure factor $S(\mathbf{k},\infty)$ as function of k for the single-step model for square-lattice substrates of size 40×40 (+), 60×60 (○), and 100×100 (●). Both the case of deposition only and equal deposition and evaporation rates are shown and fitted approximately by the curves $S_1=2.30k^{-2.75}$ (dashed curve) and $S_2=1.92k^{-2}$ (full curve), respectively. For the 100×100 substrate 250 samples were grown to a maximum time of 1000; for the smaller substrates 500 samples were generated to a maximum time of 600 and 400, respectively.

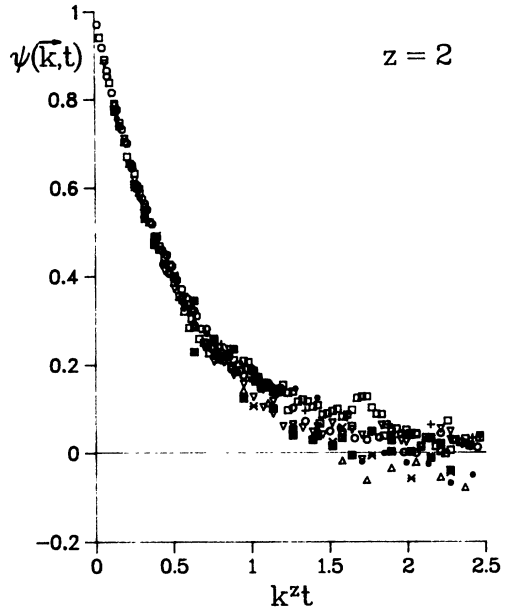

FIG. 2. Plot of the relaxation function $\Psi(\mathbf{k},t)$ [Eq. (6)] for the single-step model ($p_+ = p_- = 0.5$) for $L = 100$ as function of $k^2 t$ for the 12 smallest values of k. Data are based on 500 samples. Similar curves are obtained for smaller substrates.

model results,[9,10] that the strong-coupling fixed point of the Burgers equation is characterized by the exponents $\eta \approx -0.375$ and $z \approx 1.625$, then the single-step model is well represented by the Burgers equation with the difference between deposition and evaporation rates playing the role of the parameter λ.

B. Ballistic deposition with surface diffusion

We have simulated the ballistic-deposition model described above for square-lattice and hexagonal-lattice substrates. We have also generalized the algorithm to allow evaporation with appropriate modifications to make the resulting process time-reversal invariant. In the latter case, the interface, as in the single-step model, is stationary on the average. Since the results for the case of maximum growth and stationary interface are identical, we report only on the case of deposition.

Figure 4 contains a plot of $k^2 S(\mathbf{k}, \infty)$ as function of k^2 for both substrates and with $L \leq 50$ for the hexagonal substrate, $L \leq 100$ for the square-lattice substrate. The data points are consistent with a finite limit, as $k \to 0$, of this function and we conclude that $\eta \approx 0$. Other methods of data analysis yield the same results. Assuming that there is no significant finite-size effect we conclude that $|\eta| \leq 0.025$ with the choice $\eta = 0$ consistent with the predictions of the free-field equation (7). Analysis of the width, calculated directly from (1), also yielded results consistent with $\alpha = 0$ or equivalently $\eta = 0$.

Figure 5 shows the relaxation function (6) plotted versus $k^2 t$ for the square-lattice substrate (100×100, 12 smallest values of k). Once again a very reasonable collapse of the data is obtained indicating that $z \approx 2$, consistent with free-field behavior.

These results agree with those of Family[3] in two dimensions, but are nevertheless puzzling. As already discussed, we believe that this model should fall into the same category as the Eden and single-step models. If the Burgers equation applies the parameter λ might be small

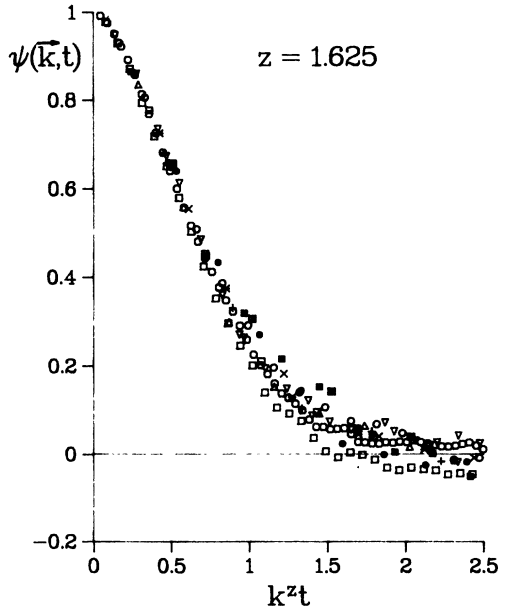

FIG. 3. The relaxation function (6) for the single-step model with deposition only for $L = 100$ plotted, for the 12 smallest values of k, as function of $k^z t$ with $z = 1.625$. Data are based on 500 samples.

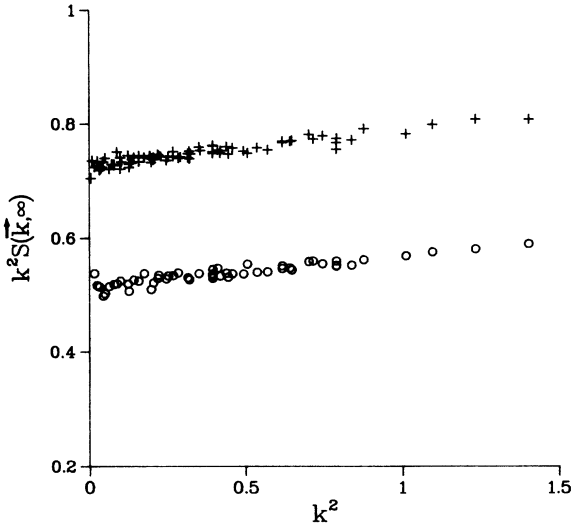

FIG. 4. Plot of $k^2 S(\mathbf{k}, \infty)$ as function of k^2 for the three-dimensional ballistic-deposition model with surface diffusion. The + are for square-lattice substrates of size $L = 30$ (2000 samples, maximum time 100), $L = 40$ (1000 samples, $t_{max} = 200$), $L = 50$ (2000 samples, $t_{max} = 250$), and $L = 100$ (640 samples, $t_{max} = 1000$); ○ are for hexagonal-lattice substrates of size $L = 30$, 40, and 50 (1000 samples in each case, $t_{max} = 100$, 200, and 250, respectively).

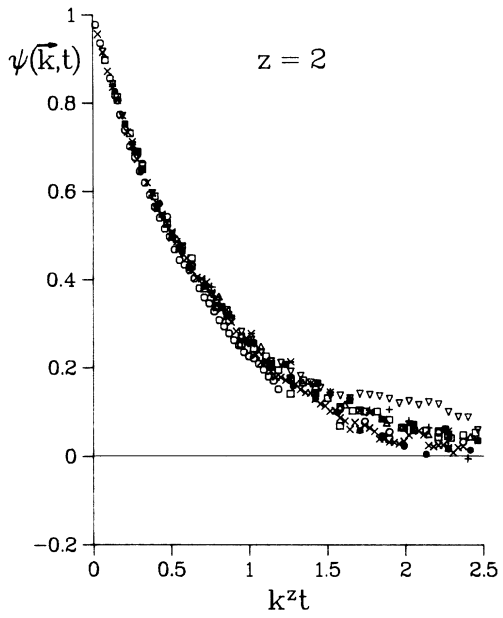

FIG. 5. The relaxation function for the three-dimensional ballistic-deposition model on a square-lattice substrate of dimension $L=100$ (12 smallest values of k, 1100 samples) plotted as function of $k^z t$ with $z=2$.

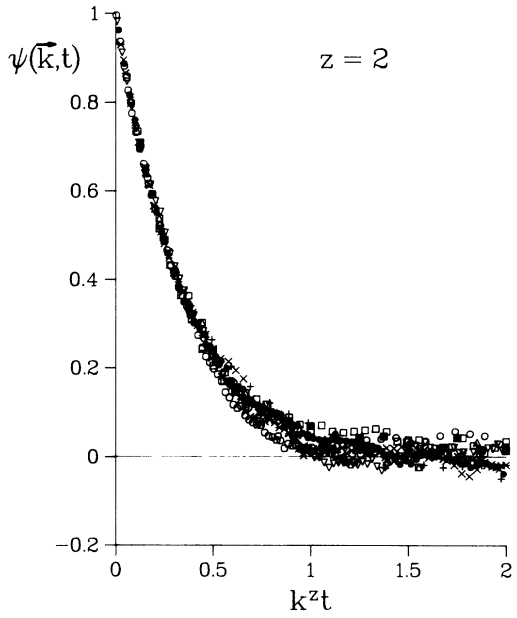

FIG. 6. The relaxation function $\Psi(k,t)$ plotted as function of $k^z t$ with $z=2$ for the *two*-dimensional ballistic-deposition model. In this case the particle is allowed to diffuse to a local minimum. The substrate length is 640, and the 12 smallest values of k are used. The best collapse of the data (by visual inspection) for all values of $L \leq 640$ is obtained with $1.95 \leq z \leq 2.0$. The data are obtained from 400 samples grown to a time of 32 000 (in order to obtain the steady-state structure factor), and 1800 samples grown to a time of 12 800.

but should be nonzero. Thus one would expect some indication of the strong-coupling behavior at least if the substrate is large enough. Since the analytic situation is clearer in two than in three dimensions we have searched for evidence of crossover as function of substrate length in the two-dimensional case. We have simulated the model with deposition only for substrates of length up to 320 in two dimensions and up to 640 for the case where the particle is allowed to diffuse to a local minimum (rather than only for a single step). We have seen no indication of crossover. In Fig. 6 we have plotted the relaxation function for $L=640$ for the 12 smallest k vectors and with $z=2$. It is clear that a very good collapse of the data results, indicating either that this value of L is still much too small or that surface diffusion somehow forces the model into the free-field universality class. In the two-dimensional single-step model we found,[6] for $p_+=0.75$, $p_-=0.25$, that the effective exponent z_{eff} which provided the best collapse of the data changed noticeably for very much smaller values of L and had reached the strong-coupling value (to within numerical uncertainty) for $L=384$.

IV. CONCLUSION

The results of our simulations present further evidence that two universality classes exist for simple growth and deposition models, a free-field class, characterized by $\eta=0$ and $z=2$, and a strong-coupling class with exponents $\eta \approx -0.75$, $z \approx 1.64$ ($d=2$), $\eta=0$, $z=\frac{3}{2}$ ($d=1$). The symmetries and conservation laws which determine the universality class to which a given process belongs are, however, still not understood. In our previous work[6] we conjectured that symmetry under time reversal might be important in determining the universality class. As far as the single-step model is concerned, breaking of time-reversal symmetry is accompanied by a change of universality class in two and, as we have shown in this article, three dimensions. Conversely, the ballistic-deposition model, which is clearly not time-reversal invariant, seems to belong to the free-field universality class. Thus we now believe that time-reversal invariance, while sufficient to guarantee free-field exponents, is not a necessary condition. This subject remains an interesting topic for further study by analytic and numerical methods.

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