PHYSICAL REVIEW B VOLUME 38, NUMBER 7 1 SEPTEMBER 1988

Dechanneling and stopping power of relativistic channeled particles

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(Received 13 July 1987; revised manuscript received 4 February 1988)

A theory of multiple and inelastic scattering of relativistic channeled particles is developed starting with relativistic quantum mechanics. We obtain general formulas for the local diffusion function and local stopping power which enable us to calculate any type of inelastic scattering including relativistic effects. By making use of these formulas, the diffusion function and stopping power due to incoherent bremsstrahlung can be calculated. A simple formula for the local stopping power due to incoherent bremsstrahlung is also presented.

I. INTRODUCTION

In recent years there has been growing interest in relativistic channeling phenomena.[1] Especially radiation by channeled electrons and positrons,[2] bent-crystal channeling,[3] and crystal-assisted quantum electrodynamics[4] have been intensively investigated. In all these phenomena, the effect of dechanneling[5] becomes very important when we try to make quantitative comparisons between theoretical models and experimental results.

A few authors[6-8] have calculated the dechanneled fraction of GeV electrons and positrons using the diffusion approximation. In these calculations, a set of phenomenological diffusion functions is introduced, but this is not enough from the theoretical point of view. Since the most important quantity of dechanneling is the mean-square transverse momentum fluctuation of channeled particles expressed as diffusion functions, calculations of the dechanneled fraction are mainly affected by diffusion functions. Hence, when we go beyond a simple estimation of dechanneling effects, we should use reliable (nonphenomenological) expressions of the diffusion functions. However, so far, there has not been any theory which gives a rigorous definition of the diffusion function for relativistic channeled particles.

Recently, we have developed a quantum theory of dechanneling and obtained a definition of the local diffusion function for nonrelativistic particles[9,10] which is given by

$$D_i(\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp} e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{2v}\int\frac{d^3q}{(2\pi)^3}\hbar^2(q_i+g_i/2)(q_i-g_i/2)\times\sum_{n(\neq 0)}S_n(\mathbf{q}+\mathbf{g}_\perp/2,\mathbf{q}-\mathbf{g}_\perp/2)\,, \tag{1.1}$$

where v is the particle velocity, $\mathbf{q}$ is the momentum transfer, and $\mathbf{g}_\perp$ is the reciprocal lattice vector of the target crystal. $S_n(\mathbf{Q},\mathbf{Q}')$ represents the generalized inelastic scattering factor defined by

$$S_n(\mathbf{Q},\mathbf{Q}')=\frac{2\pi}{\hbar}\frac{1}{V_c}H'_{0n}(\mathbf{Q})H'_{n0}(-\mathbf{Q}')\delta(\Delta\varepsilon-E_{n0})\,, \tag{1.2}$$

where H' is the interaction Hamiltonian between the projectile and the crystal. Using Eq. (1.1), we derived a set of quantum diffusion functions and succeeded in explaining the phenomenological phonon diffusion function[9] and in revealing the quantum effect of target electron states for the electronic diffusion function.[10,11] We also succeeded in proving that Eq. (1.1) reduces to the general definition of the classical diffusion function if we neglect the quantum effect of the target crystal.[12] Thus, we now consider that our quantum formulas are well-defined, and that they are strong tools for solving dechanneling problems.

It should be noted that the definitions of the diffusion function and stopping power should be consistently included in the kinetic equation. Equation (1.1) was obtained through the derivation of the Fokker-Planck equation starting with the Schrödinger equation. In the process of the derivation, we used the Wigner distribution function to link the quantum equation with the classical stochastic equation.

Here we extend our quantum theory of dechanneling to relativistic case. To do this, we should return to the basic relativistic wave equation. If relativistic corrections in quantum theory were only the mass correction, we would easily get the relativistic quantum formula for the local diffusion function from Eq. (1.1) with simple modifications. However, as we know, in contrast to the classical equation of motion, the relativistic quantum equation is not as simple as the Schrödinger equation. Indeed, the negative-energy solution appears and, for Dirac particle, the spinor effects. Therefore to obtain the relativisitic quantum diffusion function we should start again with the basic quantum wave equation. Besides these problems, we should include the radiation field in our relativistic theory because the effect of photon emission is very important for electron/positron channeling, and also because retardation effect becomes important for electronic excitation.

In Sec. II, we derive a Fokker-Planck equation for relativistic channeled particles starting with the Dirac equation. Definitions of the local diffusion function $D(\mathbf{R}_\perp)$ and the stopping power $S(\mathbf{R}_\perp)$ are obtained as coefficients in the Fokker-Planck equation. In Sec. III, we introduce the generalized inelastic scattering factor $S_a^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$ in its relativistic form and connect it with the diffusion function and the stopping power obtained in Sec. II. The reader who is not interested in the rather complex and tiresome derivation of relativistic Fokker-Planck equation can skip Sec. II and begin to read from Sec. III. Our final result of the relativistic diffusion function, Eq. (3.5), will be intuitively accepted because it has a form of a natural extension of the nonrelativistic formula, Eq. (1.1).

As an example of the calculations of our formula, we derive an analytical expression of the local diffusion function due to phonon excitation, $D^{(n)}(\mathbf{R}_\perp)$. We show that the phenomenological $D^{(n)}(\mathbf{R}_\perp)$ introduced by Beloshitsky and Kumakhov[7] is clearly accounted for in our present result.

The effects of photon emission are considered in Sec. IV. The channeled GeV electrons and positrons radiate photons due to the potential caused by thermally displaced target nuclei, which we call "incoherent bremsstrahlung." The method developed here enables us to deal with the local effects of incoherent bremsstrahlung which one had been able to calculate only by the method of virtual quanta.[13]

Incoherent bremsstrahlung is the origin of the background of channeling radiation spectra, which can be explained by the "local bremsstrahlung probability" $p^{(b)}(\mathbf{k}_\lambda,\mathbf{R}_\perp)$ introduced here. The diffusion function and the stopping power due to incoherent bremsstrahlung are also derived.

II. DERIVATION OF A FOKKER-PLANCK EQUATION

In this section we derive a Fokker-Planck equation that describes the kinematics of relativistic channeled particles. Though our purpose is to derive a well-defined formula of the local diffusion function and stopping power, derivation of the Fokker-Planck equation is not avoidable because the definition of the diffusion function and stopping power should be consistently included in the Fokker-Planck equation.

The theoretical method is an extension of the nonrelativistic theory of dechanneling recently developed by one of the authors.[9] In our present article, the channeled particles are treated as Dirac particles and the target crystal is regarded as a nonrelativistic system. The radiation field is considered because the interaction with the radiation field becomes very important for the case of relativistic electron-position channeling.

A. Preliminary formalism

We consider a system composed of a channeled particle, a target crystal, and a radiation field. The Hamiltonian for the system is given as follows:

$$H_{\text{tot}}=H_p+H_{\text{cry}}+H_{\text{rad}}+H_{p\text{-}c}+H_{p\text{-}r}+H_{c\text{-}r}\,. \tag{2.1}$$

H_p is the Hamiltonian of the free particle ($\hbar=1$),

$$H_p=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m\,,$$

where $\boldsymbol{\alpha}$ and β are the Dirac matrices,[14] $\mathbf{p}$ and m are the momentum and the rest mass of the particle. H_{cry} and H_{rad} are the total Hamiltonian of the crystal and the Hamiltonian of the free radiation field, which satisfy the following equations:

$$H_{\text{cry}}\,|\phi_n\rangle=E_n\,|\phi_n\rangle\,, \tag{2.2}$$

$$H_{\text{rad}}\,|N_\lambda\rangle=\omega_\lambda N_\lambda\,|N_\lambda\rangle\,, \tag{2.3}$$

where ϕ_n is the eigenfunction of the crystal and E_n is its energy, and $|N_\lambda\rangle$ is the eigenvector of the radiation field with energy $\omega_\lambda N_\lambda$ in the number representations. The interaction Hamiltonians $H_{p\text{-}c}$ (particle-crystal interaction) and $H_{p\text{-}r}$ (particle-radiation field interaction) are given by

$$H_{p\text{-}c}=\pm\sum_i\frac{e^2}{|\mathbf{r}-\mathbf{r}_i|}\mp\sum_l\frac{Ze^2}{|\mathbf{r}-\mathbf{R}_l|}\,, \tag{2.4}$$

$$H_{p\text{-}c}=\pm e\boldsymbol{\alpha}\cdot\mathbf{A}(\mathbf{r})\,, \tag{2.5}$$

where $\mathbf{r}$, $\mathbf{r}_i$, and $\mathbf{R}_l$ are the position of the channeled particle, of the ith target electron and of the lth target nucleus, respectively. Z is the atomic number of the target nuclei. We assumed that the charge number of the incident particle is ± 1, which corresponds to the sign of Eqs. (2.4) and (2.5). In this paper we use the Coulomb gauge and the vector potential $\mathbf{A}(\mathbf{r})$ is written as[14]

$$\mathbf{A}(\mathbf{r})=\sum_\lambda[(2\pi/k_\lambda)^{1/2}\mathbf{e}_\lambda a_\lambda e^{i\mathbf{k}_\lambda\cdot\mathbf{r}}+\text{c.c.}]\,,$$

where $\mathbf{k}_\lambda$, $\mathbf{e}_\lambda$, and a_λ are the wave vector, the polarization vector, and the annihilation operator for the photon λ. For simplicity, we neglect the crystal-radiation field interaction Hamiltonian $H_{c\text{-}r}$. $H_{c\text{-}r}$ is mainly related to the retardation effects of the excitation of target electrons. These effects will be extrapolated to our formalism in Sec. III. Neglecting $H_{c\text{-}r}$, we can consider a system composed of the crystal and the radiation field as a "reservoir" of which the Hamiltonian H_R is given by $H_R=H_{\text{cry}}+H_{\text{rad}}$. Then Eqs. (2.2) and (2.3) are conveniently written as

$$H_R\,|\alpha\rangle=\mathcal{E}_\alpha\,|\alpha\rangle\,,$$

where $|\alpha\rangle$ and $\mathcal{E}_\alpha$ are the eigenvector and the energy of the reservoir in α state. Further, defining

$$H_0=H_p+V,\;\; V=\langle\phi_0\,|H_{p\text{-}c}|\,\phi_0\rangle,$$

$$H'=(H_{p\text{-}c}-V)+H_{p\text{-}r}\,,$$

we can rewrite Eq. (2.1) as

$$H_{\text{tot}}=H_0+H_R+H'\,. \tag{2.6}$$

Since V represents the interaction between the channeled particle and the crystal without the crystal excitations, V can be regarded as the crystal potential for the channeled particle. Thus H_0 denotes the elastic motion of the channeled particle and H' generates the stochastic forces which perturb the channeling motion.

Let us now consider the wave function $\Phi(t)$ for the entire system as a whole which satisfies the wave equation

$$i\frac{\partial}{\partial t}\Phi(t)=H_{\text{tot}}\Phi(t)\,. \tag{2.7}$$

As mentioned above, we have assumed that the crystal is nonrelativistic and use the Coulomb gauge. Thus our theory does not have a Lorentz covariant form. This is not serious since the dechanneling process should be described in the coordinate fixed on the crystal. If we decompose $\Phi(t)$ according to the Yoshioka theory[15] as

$$\Phi(t)=\sum_{\alpha}\psi_\alpha(x)\,|\,\alpha\rangle e^{-i\mathcal{E}_\alpha t}, \tag{2.8}$$

then we get from Eqs. (2.6)-(2.8)

$$i\frac{\partial}{\partial t}\psi_\alpha(x)=H_0\psi_\alpha(x)+\sum_\beta H'_{\alpha\beta}(x)\psi_\beta(x), \tag{2.9}$$

with

$$H'_{\alpha\beta}(x)=\langle\alpha\,|\,H'\,|\,\beta\rangle e^{i\mathcal{E}_{\alpha\beta}t},\ \ \mathcal{E}_{\alpha\beta}=\mathcal{E}_\alpha-\mathcal{E}_\beta .$$

Here we have written the particle coordinate $x=(t,\mathbf{r})$ explicitly. In the above, we took the normalization condition for our reservoir as $\langle\alpha\,|\,\beta\rangle=\delta_{\alpha\beta}$. Equation (2.9) describes the time development of the wave function of the channeled particle which is exciting the reservoir into α state. It should be noted that $\psi_\alpha(x)$ is a four-component spinor.

B. Distribution function

Now we introduce a one-particle distribution function for the channeled particle. In this paper we assume that the elastic motion of the channeled paticle is well described by classical mechanics which is the case for protons, GeV electrons, positrons, etc. In this case the statistical distribution of the channeled particle should be characterized both by the position and by the momentum. Thus we have to introduce the appropriate distribution function.

First, we define a function by

$$g(x,x')=\sum_\alpha\psi_\alpha^\dagger(x')\psi_\alpha(x) .$$

Using $g(x,x')$, we next define a distribution function $g(\mathbf{p},\varepsilon;\mathbf{R},T)$ by

$$g(\mathbf{p},\varepsilon;\mathbf{R},T)=\int d^4x\, e^{ip\cdot x}g(X+x/2,X-x/2), \tag{2.10}$$

where we used short-hand (collective) notations $X=(T,\mathbf{R})$ and $p=(\varepsilon,\mathbf{p})$. The scalar product $p\cdot x$ is defined by $p\cdot x=\varepsilon t-\mathbf{p}\cdot\mathbf{r}$.

Under channeling conditions, the wave function for the channeled particle can be assumed to be transversely bound in the field of the continuum potential and to be free along the channeling direction. Thus we may take an approximated expression as[16]

$$\psi_\alpha(x)\cong\frac{1}{\sqrt{L}}u_0\varphi_\alpha(\mathbf{r}_\perp,t)e^{i(p_{z0}z-\varepsilon_0 t)}, \tag{2.11}$$

where L and p_{z0} are the crystal length and the particle momentum along the z direction; $\varepsilon_0=(p_{z0}^2+m^2)^{1/2}$. $u_0=u_0(\mathbf{p}_0,s_0)$ is a spinor for the incident particle with spin state s_0, which includes a momentum operator $\hat{\mathbf{p}}_{\perp 0}$ acting on the transverse coordinate $\mathbf{r}_\perp$. However, since the transverse momentum $|\mathbf{p}_\perp|$ is very small compared to p_{z0}, we may neglect $\hat{\mathbf{p}}_{\perp 0}$. Hence u_0 is determined by the initial condition and we should average it over s_0. Inserting Eq. (2.11) into Eq. (2.10), we get

$$g(\mathbf{p},\varepsilon;\mathbf{R},T)=\int dt\int d^2r_\perp e^{-i(\varepsilon_0-\varepsilon)t}e^{-i\mathbf{p}_\perp\cdot\mathbf{r}_\perp}\sum_\alpha\varphi_\alpha^*(\mathbf{R}_\perp-\mathbf{r}_\perp/2,T-t/2)\varphi_\alpha(\mathbf{R}_\perp+\mathbf{r}_\perp/2,T+t/2)\delta_{p_z,p_{z0}} . \tag{2.12}$$

Because of the plane-wave approximation along the z axis, $g(\mathbf{p},\varepsilon;\mathbf{R},T)$ is actually not applicable to describe the energy-loss distribution for the channeled particles. It is not so serious because we concentrate ourselves on dechanneling problem and particle distribution in transverse phase space. It is well known that the dechanneling length is much smaller than the stopping length, and so we can regard the energy-loss distribution as another problem.

If we expand $\varphi_\alpha(\mathbf{r}_\perp,t)$ as $\varphi_\alpha(\mathbf{r}_\perp,t)=\sum_j c_j\exp(-i\varepsilon_{\perp j}t)$ and perform the integration over t, we get

$$\delta(\varepsilon-\varepsilon_0-(\varepsilon_{\perp j}+\varepsilon_{\perp j'})/2),$$

where $\varepsilon_{\perp j}$ are the transverse eigenenergy. Since $\varepsilon_0\gg\varepsilon_{\perp j}$, we may neglect the last term in the parentheses. Then the t dependence of $\varphi_\alpha^*(\mathbf{R}_\perp-\mathbf{r}_\perp/2,T-t/2)\varphi_\alpha(\mathbf{R}_\perp+\mathbf{r}_\perp/2,T+t/2)$ disappears. Hence we obtain from Eq. (2.12)

$$g(\mathbf{p},\varepsilon;\mathbf{R},T)=f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)[2\pi\delta(\varepsilon-\varepsilon_0)]\delta_{p_z,p_{z0}}, \tag{2.13}$$

where

$$f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)=\int d^2\mathbf{r}_\perp e^{-i\mathbf{p}_\perp\cdot\mathbf{r}_\perp}\sum_\alpha\varphi_\alpha^*(\mathbf{R}_\perp-\mathbf{r}_\perp/2,T)\varphi_\alpha(\mathbf{R}_\perp+\mathbf{r}_\perp/2,T) \tag{2.14}$$

is the Wigner distribution function[17] in the transverse phase space.

It is well known that the Wigner distribution function turns out to be negative in some regions of phase space. However, it does not affect our problem because we do not use $f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)$ directly for the calculation of dechanneled fractions. To obtain the dechanneled fraction, the distribution function in the transverse energy space $F(E_\perp,T)$ is used. The Fokker-Planck equation for $F(E_\perp,T)$ can be derived from the Fokker-Planck equation for $f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)$ [see Eq. (2.24)] by averaging it over $\mathbf{R}_\perp$, according to the theory by Beloshitsky and Kumakhov.[7]

C. Fokker-Planck equation

Let us now try to derive a Fokker-Planck equation for the distribution function given by Eq. (2.13) or Eq. (2.14). From Eq. (2.9) we obtain

$$\sum_{a}\psi_a^\dagger(x_2)\left[i\left(\frac{\vec{\partial}}{\partial t_1}+\frac{\overleftarrow{\partial}}{\partial t_2}\right)-[\vec{H}_0(\mathbf{r}_1)-\overleftarrow{H}_0(\mathbf{r}_2)]\right]\psi_a(x_1)=\sum_{a}\sum_{a'}[\psi_a^\dagger(x_2)H'_{aa'}(x_1)\psi_{a'}(x_1)-\psi_{a'}^\dagger(x_2)H'_{a'a}(x_2)\psi_a(x_1)]\,, \tag{2.15}$$

where the arrows indicate the direction in which the differential operates. Now we introduce the variables x and X by

$$X=\frac{x_1+x_2}{2}\,,\quad x=x_1-x_2\,,$$

and perform integrations and a spin-average operation

$$\tfrac{1}{2}\sum_{s_0}\frac{1}{L}\int dZ\int d^4x\, e^{ip\cdot x}\ldots \tag{2.16}$$

We get from the left-hand side of Eq. (2.15),

$$i\left[\frac{\partial}{\partial T}+\frac{\mathbf{p}_\perp}{\gamma m}\cdot\frac{\partial}{\partial \mathbf{R}_\perp}-\frac{\partial U}{\partial \mathbf{R}_\perp}\cdot\frac{\partial}{\partial \mathbf{p}_\perp}\right] \times f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)\delta_{p_z p_{z0}}\cdot 2\pi\delta(\varepsilon-\varepsilon_0)\,, \tag{2.17}$$

where $\gamma=\varepsilon_0/m$ is the Lorentz factor and $U(\mathbf{R}_\perp)$ is the (thermal averaged) continuum potential;[5]

$$U(\mathbf{R}_\perp)=\frac{1}{L}\int dZ\, V(\mathbf{R}_\perp,Z)\,.$$

To derive Eq. (2.17), we used the identity $u^\dagger \boldsymbol{\alpha} u \cong (\mathbf{p}/\gamma m)u^\dagger u$ and assumed that the potential $V(\mathbf{r})$ is slowly varying in the $\mathbf{r}_\perp$ plane.

Next we consider the right-hand side of Eq. (2.15), which represents the inelastic collisional effects between the channeled particle and the reservoir. Since, in general, the coupling between the channeled particle and the reservoir is weak, we can use the condition

$$|\psi_a|\ll|\psi_0|\quad\text{for } a\neq 0\,. \tag{2.18}$$

Then Eq. (2.9) can be approximated by

$$\left[i\frac{\partial}{\partial t}-(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m+V)\right]\psi_a(x)=H'_{a0}(x)\psi_0(x)\,. \tag{2.19}$$

Further, assuming that the channeled particle can be well described by a plane wave at each individual collision, we can approximately express a formal solution of Eq. (2.19) in terms of the relativistic free Green's function $G_0(x)$;

$$\psi_a(x)=\int d^4x\, G_0(x-x')H'_{a0}(x')\psi_0(x')\,, \tag{2.20}$$

with

$$G_0(x)=\int\frac{d^4p}{(2\pi)^4}e^{ip\cdot x}\frac{\varepsilon+\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m}{\varepsilon^2-\mathbf{p}^2-m^2+i0}\,. \tag{2.21}$$

By making use of Eqs. (2.18) and (2.20), the right-hand side of Eq. (2.15) leads to

$$\sum_{a(\neq 0)}\int d^4x_3\,\psi_0^\dagger(x_3)H'_{0a}(x_3)G_0^\dagger(x_2-x_3)[H'_{a0}(x_2)-H'_{a0}(x_1)]\psi_0(x_1) + \sum_{a(\neq 0)}\int d^4x_3\,\psi_0^\dagger(x_2)[H'_{0a}(x_1)-H'_{0a}(x_2)]G_0(x_1-x_3)H'_{a0}(x_3)\psi_0(x_3)\,, \tag{2.22}$$

where $G_0^\dagger(x)$ is the complex conjugate of $G_0(x)$. Performing the operating Eqs. (2.16) to (2.22), we obtain after some manipulations (see Appendix A)

$$i\left[\mathcal{F}_\mu\frac{\partial}{\partial p_\mu}+\mathcal{D}_{\mu\nu}\frac{\partial^2}{\partial p_\mu\partial p_\nu}\right]g_0(\mathbf{p},\varepsilon,\mathbf{R},T)\,, \tag{2.23}$$

where $p_\mu=(\varepsilon,-\mathbf{p})$ is a covariant four-vector, and the repeated indices μ,ν are summed. $g_0(\mathbf{p},\varepsilon,\mathbf{R},T)$ is a function which corresponds to Eq. (2.12) but is composed only of the coherent wave ψ_0. The coefficients $\mathcal{F}_\mu,\mathcal{D}_{\mu\nu}$ are defined as

$$\mathcal{F}_\mu=\tfrac{1}{2}\sum_{s_0}\frac{1}{L}\int dz\int d^4x\, e^{ip\cdot x}\sum_{a(\neq 0)}u_0^\dagger\left[\frac{\partial H'_{0a}(X+x/2)}{\partial X^\mu}G_0(x)H'_{a0}(X-x/2)+H'_{0a}(X+x/2)G_0^\dagger(-x)\frac{\partial H'_{a0}(X-x/2)}{\partial X^\mu}\right]u_0\,,$$

$$\mathcal{D}_{\mu\nu}=-\frac{i}{2}\sum_{s_0}\frac{1}{L}\int dz\int d^4x\, e^{ip\cdot x}\sum_{a(\neq 0)}u_0^\dagger\frac{\partial H'_{0a}(X+x/2)}{\partial X^\mu}[G_0(x)-G_0^\dagger(-x)]\frac{\partial H'_{a0}(X-x/2)}{\partial X^\nu}u_0\,,$$

where $X^\mu=(T,X,Y,Z)$. $\mathcal{F}_\mu$ and $\mathcal{D}_{\mu\nu}$ can be interpreted as a four-momentum damping vector and diffusion functions. If we further compute $\mathcal{F}_\mu$ and $\mathcal{D}_{\mu\nu}$, we get local stopping power ($\mathcal{F}_0$), local diffusion functions ($\mathcal{D}_{ii}$), and so on (see Appendix A). Though we have used the plane-wave approximation along the z axis [Eq. (2.11)], Eq. (2.23) formally includes the energy loss. It is not strange, however, because $\mathcal{F}_0$ describes the mean energy gain of the reservoir. As the consequence of the crystal excitation or photon emission, we can get the stopping power in spite of the assumption of constant velocity of the channeled particle (ε_0,p_{0z} are constant). This situation is very similar to the calculation of the local stopping power by the impact parameter method. Indeed, we can show that our stopping power formula is equivalent to the formula by the impact parameter method.[18] It should be noted that the difference between these relativistic kinetic coefficients and the nonrelativistic ones[9,10] is only the spinor factors.

Now we derive a Fokker-Planck equation which describes the multiple scattering of the relativistic channeled particles.

Combining Eqs. (2.17) and (2.23), and integrating over ε and p_z, we obtain

$$\left[\frac{\partial}{\partial T}+\frac{\mathbf{p}_\perp}{\gamma m}\cdot\frac{\partial}{\partial \mathbf{R}_\perp}-\frac{\partial}{\partial \mathbf{R}_\perp}[U(\mathbf{R}_\perp)+C^r(\mathbf{R}_\perp)]\cdot\frac{\partial}{\partial \mathbf{p}_\perp}\right]f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)=\sum_i\left\langle\frac{\Delta p_i^2}{2\Delta T}\right\rangle\frac{\partial^2}{\partial p_i^2}f(\mathbf{p}_\perp,\mathbf{R}_\perp,T)\ \ (i=x,y)\,,$$

where $\langle\Delta p_i^2/2\Delta T\rangle$ is the relativistic local diffusion function per unit time. In the above, we neglected the damping terms and the cross terms ($\mathcal{D}_{xy}$, etc.) and approximated that $g_0(\mathbf{p},\varepsilon,\mathbf{R},T)\cong g(\mathbf{p},\varepsilon,\mathbf{R},T)$. $C^r(\mathbf{R}_\perp)$ is the polarization potential due to virtual excitation of the reservoir. For MeV electrons and positrons, of which their quantum levels are well defined, the polarization potential will become important because it will give the line shift of channeling radiation spectra. For GeV electrons and positrons, it will give small corrections to the continuum potential and may be neglected.

Equation (2.24) is the same Fokker-Planck equation from which Beloshitsky and Kumakhov[7] developed a classical theory of dechanneling for relativistic electrons. However, as we pointed out in Sec. I, their theory did not give diffusion functions from fundamental approach.

III. DIFFUSION FUNCTION AND STOPPING POWER

A. General formulas

We shall now discuss general definitions of the local diffusion functions and the stopping power.

The local stopping power per unit length $S(\mathbf{R}_\perp)$ can be obtained from $\mathcal{F}_0$ (see Appendix A);

$$S(\mathbf{R}_\perp)=\frac{1}{v}\left\langle\frac{\Delta\varepsilon}{\Delta T}\right\rangle=\frac{\pi}{vL}\sum_{s_0}\sum_{s}\int dZ\int d^3\mathbf{r}\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}$$
$$\times\sum_{\alpha(\neq o)}\mathcal{E}_{\alpha o}u_0^\dagger H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)u_0\delta(\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0})\,, \tag{3.1}$$

where $v=p_{0z}/\gamma m$ is the velocity of the channeled particle. For reasons of convenience we introduce the generalized inelastic scattering factor $S_\alpha^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$ defined as follows:

$$S_\alpha^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)=\frac{\pi}{V_c}\sum_{s_0}\sum_{s}u_0^\dagger H'_{0\alpha}(\mathbf{Q})uu^\dagger H'_{\alpha 0}(-\mathbf{Q}')u_0\delta(\Delta\varepsilon-\mathcal{E}_{\alpha 0})\,, \tag{3.2}$$

where

$$H'_{0\alpha}(\mathbf{Q})=\int d^3R\,e^{-i\mathbf{Q}\cdot\mathbf{R}}H'_{0\alpha}(\mathbf{R})\,,$$

$\delta(\Delta\varepsilon-\mathcal{E}_{\alpha 0})$ means the energy conservation for the reservoir excitation energy $\mathcal{E}_{\alpha 0}$, and V_c is the volume of the unit cell. Equation (3.2) is just an extended form of nonrelativistic generalized inelastic scattering factor $S(\mathbf{Q},\mathbf{Q}')$.[5,9,10] Using Eq. (3.2), Eq. (3.1) can be easily transformed to

$$S(\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\sum_{\alpha(\neq 0)}\mathcal{E}_{\alpha 0}S_\alpha^{(\mathrm{rel})}(\mathbf{q}+\mathbf{g}_\perp/2,\mathbf{q}-\mathbf{g}_\perp/2;\varepsilon_0)\,, \tag{3.3}$$

where $\mathbf{g}_\perp$ is a two-dimensional reciprocal-lattice vector and $\mathbf{q}$ is the momentum transfer. It is worth noting that the random stopping power formula can be exactly obtained from Eq. (3.3) by only taking the term $\mathbf{g}_\perp=\mathbf{0}$ (i.e., the average value over $\mathbf{R}_\perp$). The nonrelativistic formula similar to Eq. (3.3) has been already presented in Refs. 9 and 18 which gives the same expression as the electronic stopping power by Esbensen and Golovchenko.[19] Using Eq. (3.3), we can get a local relativistic stopping power due to electronic excitation. It should be noted that the impact parameter method can hardly give such a local relativistic stopping power, because this method is not able to include the spinor effects.

As shown in Appendix A, the local diffusion function Eq. (2.26) is explicitly given as follows:

$$D_i(\mathbf{R}_\perp)=\frac{1}{v}\left\langle\frac{\Delta p_i^2}{2\Delta T}\right\rangle=\frac{\pi}{2vL}\sum_{s_0}\sum_{s}\int dZ\int d^3r\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}$$
$$\times\sum_{\alpha(\neq 0)}u_0^\dagger[\nabla_i H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)]uu^\dagger[\nabla_i H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)]u_0\delta(\varepsilon_0-\varepsilon\mathbf{p}-\mathcal{E}_{\alpha o})\,. \tag{3.4}$$

In the classical theory of dechanneling, the diffusion functions are proportional to the mean-square fluctuation of force.[20] Equation (3.4) corresponds to such classical expressions since $D_i(\mathbf{R}_\perp)$ is determined by $\nabla_i H'_{0\alpha}\nabla_i H'_{\alpha 0}$ which can be interpreted as a transition-matrix element due to "force fluctuation." Equation (3.4) can also be expressed in terms of $S_\alpha^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$;

$$D_i(\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{2v}\int\frac{d^3q}{(2\pi)^3}(q_i+g_i/2)(q_i-g_i/2)\sum_{\alpha(\neq 0)}S_\alpha^{(\mathrm{rel})}(\mathbf{q}+\mathbf{g}_\perp/2,\mathbf{q}-\mathbf{g}_\perp/2;\varepsilon_0)\,. \tag{3.5}$$

Therefore, to obtain a diffusion function and/or stopping power for a certain inelastic scattering process, all we have to do is to calculate the corresponding inelastic scattering factor.

Typical fundamental processes which cause to inelastic scattering under relativistic channeling conditions are diagrammed as in Fig. 1. Diagram (a) shows the static interaction between the channeled particle and the crystal which corresponds to the phonon excitation and the electron excitation. The phonon excitation process contributes only to diffusion and does not contribute to energy loss, because the phonon energy is negligibly small in our problem. Diagram (b) shows the excitation of the target electrons including retardation effects. The retardation effects partly come from $H_{c\text{-}r}$, which was neglected in our formalism. However, as mentioned in Sec. II, we should also consider these effects for ultrarelativistic electron/positron channeling. Diagram (c) represents incoherent bremsstrahlung due to the thermally displaced potential, which will be important both for diffusion and energy loss of electron/positron channeling. Naturally, there are other processes which can be represented by higher-order diagrams, such as bremsstrahlung by target electrons. Such higher-order processes may increase their importance when more delicately designed experiments are performed. However, at present, it is sufficient to consider just the processes shown in Fig. 1 for our dechanneling and/or energy-loss problems.

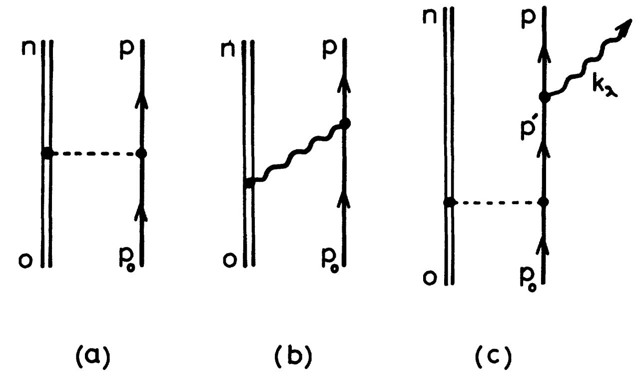

FIG. 1. Inelastic scattering processes of relativistic channeled particles. The solid lines correspond to the incident particle. The twin-solid lines show the crystal state. The dotted lines and the wavy lines represent the static Coulomb interactions and photons, respectively.

Since $S_a^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$ given by Eq. (3.2) only describes the first-order perturbation, this expression is not applicable to the processes shown in Figs. 1(b) and 1(c). To calculate these processes, we have to extend $S_a^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$ up to the second-order perturbation. The second-order formula, $S_a'^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$, can be expressed by a similar formula to Eq. (3.2) if we introduce the "compound matrix element" $K_{\alpha 0}(\mathbf{Q})$ (see Appendix B);

$$K_{\alpha 0}(\mathbf{Q})=\int\frac{d^3p'}{(2\pi)^3}\sum_{\pm s'}\sum_{\beta(\neq 0)}\frac{H'_{\alpha\beta}(\mathbf{p}-\mathbf{p}')u'u'^{\dagger}H'_{\beta 0}(\mathbf{p}'-\mathbf{p}_0)}{\varepsilon_0-\varepsilon_{\mathbf{p}'}-\mathscr{E}_{\beta 0}},\quad(\mathbf{Q}=\mathbf{p}_0-\mathbf{p})\,. \tag{3.6}$$

Then $S_a'^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)$ is given by the formula

$$S_a'^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q}';\varepsilon_0)=\frac{\pi}{V_c}\sum_{s_0}\sum_{s}u_0^{\dagger}K_{0\alpha}(\mathbf{Q})uu^{\dagger}K_{\alpha 0}(-\mathbf{Q}')u_0\delta(\Delta\varepsilon-\mathscr{E}_{\alpha 0})\,. \tag{3.7}$$

It should be also noted that $S'^{(\mathrm{rel})}(\mathbf{Q},\mathbf{Q};\varepsilon_0)$ becomes the usual second-order transition probability per unit time. A detailed calculation of Eq. (3.7) for the incoherent bremsstrahlung process will be given in the next section.

B. Phonon diffusion function

Here we show a calculation of phonon (nuclear) diffusion function $D^{(n)}(\mathbf{R}_\perp)$ using the general formula derived in Sec. III A. Within the Einstein model, the interaction Hamiltonian H' is given as

$$H'^{(\mathrm{phonon})}=V_a(\mathbf{r}-\delta\mathbf{R})-\langle\langle V_a(\mathbf{r}-\delta\mathbf{R})\rangle\rangle\,, \tag{3.8}$$

where $V_a(\mathbf{r})$ is the atomic potential and $\delta\mathbf{R}$ is the thermal displacement of the target atom. In the above, we replaced the ground-state average $\langle\phi_0|(\cdots)|\phi_0\rangle$ by the thermal average $\langle\langle\cdots\rangle\rangle$. Inserting Eq. (3.8) into Eq. (3.2), we obtain

$$\begin{aligned}S_a^{(n)}(\mathbf{q}+\mathbf{g}_\perp/2,\mathbf{q}-\mathbf{g}_\perp/2;\varepsilon_0)&=2\pi N V_a(\mathbf{q}+\mathbf{g}_\perp/2)V_a(\mathbf{q}-\mathbf{g}_\perp/2)\\&\quad\times\left(e^{-M(\mathbf{g}_\perp)}-e^{-M(\mathbf{q}+\mathbf{g}_\perp/2)}e^{-M(\mathbf{q}-\mathbf{g}_\perp/2)}\right)\tfrac{1}{2}\sum_{s_0}\sum_{s}(u_0^{\dagger}uu^{\dagger}u_0)\delta(\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{q}})\,,\end{aligned} \tag{3.9}$$

where $V_a(\mathbf{Q})$ is the Fourier component of $V_a(\mathbf{r})$, $\exp[-M(\mathbf{Q})]$ is the Debye-Waller factor, and N is the atomic density of the crystal. As can be seen, the difference between the present form and the corresponding nonrelativistic form is just the spinor factor. This spinor factor is easily calculated as

$$\tfrac{1}{2}\sum_{s_0}\sum_{s}(u_0^{\dagger}uu^{\dagger}u_0)\cong 1-q^2/4\varepsilon_0^2\,.$$

The term $-q^2/4\varepsilon_0^2$ represents the spinor effects. However, under channeling conditions, we may take as $q^2\ll\varepsilon_0^2$ because the momentum transfer $|\mathbf{q}|$ is much smaller than the incident momentum. Thus the phonon diffusion function becomes the same form as the nonrelativistic formula and can again be expressed by the simple formula[10]

$$D^{(n)}(R_\perp)\cong D_{K-0}(R_\perp)+\beta D_L(R_\perp)\,, \tag{3.10}$$

where D_{K-0} and D_L are the formula of Kitagawa-Ohtsuki[21] and the formula of Lindhard:[22]

$$D_{K-0}(R_\perp)=D_{\mathrm{random}}P(R_\perp)\,,\quad D_L(R_\perp)=\frac{d}{v^2}\rho_\perp^2\left[\frac{1}{R_\perp^2}U'(R_\perp)^2+U''(R_\perp)^2\right],$$

where

$$P(R_\perp)=\frac{r_0^2}{\rho_\perp^2}\exp\left[-\frac{R_\perp^2}{\rho_\perp^2}\right]$$

is the distribution of the thermally vibrated target atom in the transverse plane. $\rho_\perp^2$, d, and r_0 are the two-dimensional mean-square amplitude of the thermal vibrations, the interatomic distance of a channel string, and the channel radius, respectively. D_{random} is the diffusion function for a random material which should be suitably chosen for relativistic case.[7] Equation (3.10) includes a parameter β which will be approximately taken as unity. Our result Eq. (3.10) is considered as an extended form of the expression proposed by Beloshitsky and Kumakhov for relativistic channeled electrons[2,7] on the basis of phenomenological discussions, as mentioned in Sec. I.

IV. DIFFUSION FUNCTION AND STOPPING POWER DUE TO INCOHERENT BREMSSTRAHLUNG

In this section we calculate the diffusion function and the stopping power due to incoherent bremsstrahlung in detail.[23] The incoherent bremsstrahlung processes [Fig. 1(c)] becomes important for high-energy electron-positron channeling. Indeed, this process gives the background of channeling radiation spectra, and contributes to the energy loss of electrons and positrons. In the present article we do not consider bremsstrahlung due to electronic collisions but concentrate ourselves on bremsstrahlung by thermally vibrated target nuclei.

For the incoherent bremsstrahlung process, the compound matrix element $K_{0\alpha}(\mathbf{Q})$ defined by Eq. (3.6) is composed of Hamiltonians Eqs. (2.5) and (3.8). Hence it becomes

$$K_{0\alpha}(\mathbf{q}+\mathbf{k}_\lambda)=\pm e(2\pi/k_\lambda)^{1/2}H_{0n}'(\mathbf{q}+\mathbf{k}_\lambda)\sum_{\pm s'}\left[\frac{u'u'^{\dagger}\mathbf{e}_\lambda\cdot\boldsymbol{\alpha}}{\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{k}_\lambda}-E_{n0}}+\frac{\mathbf{e}_\lambda\cdot\boldsymbol{\alpha}u'u'^{\dagger}}{\varepsilon_0-\varepsilon_{\mathbf{p}+\mathbf{k}_\lambda}-k_\lambda}\right], \tag{4.1}$$

where the initial state 0 represents that the crystal is in its ground state without photons, while the final state α is that the crystal is excited in n state with a photon $\mathbf{k}_\lambda$. It should be noted that, in addition to the process in Fig. 1(c), Eq. (4.1) includes another process that the particle emits a photon before a collision with a target nucleus. Neglecting the recoil energy of the nucleus E_{n0}, we get the inelastic scattering factor for incoherent bremsstrahlung:

$$S_{n,\mathbf{k}_\lambda}^{(\mathrm{br})}(\mathbf{q}_{\mathbf{k}_\lambda}+\mathbf{g}_\perp/2,\mathbf{q}_{\mathbf{k}_\lambda}-\mathbf{g}_\perp/2;\varepsilon_0)=\frac{(2\pi e)^2}{V_c k_\lambda}H_{0n}'(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)H_{n0}'(-\mathbf{q}-\mathbf{k}_\lambda+\mathbf{g}_\perp/2)\times[\text{spinor part}]\,\delta(\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{q}-\mathbf{k}_\lambda}-k_\lambda)\,, \tag{4.2}$$

where

$$[\text{spinor part}]=\tfrac{1}{2}\sum_{s_0}\sum_{s}\left|\sum_{\pm s'}\frac{(u_0^\dagger u')(u'^{\dagger}\mathbf{e}_\lambda\cdot\boldsymbol{\alpha}u)}{\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{k}_\lambda}}+\sum_{\pm s'}\frac{(u_0^\dagger\mathbf{e}_\lambda\cdot\boldsymbol{\alpha}u')(u'^{\dagger}u)}{\varepsilon_0-\varepsilon_{\mathbf{p}+\mathbf{k}_\lambda}-k_\lambda}\right|^2$$

is identical to the ordinary spinor factor in the celebrated Bethe-Heitler bremsstrahlung cross section.[14] Then the stopping power and the diffusion function can be obtained by using Eqs. (3.3) and (3.5);

$$S^{(\mathrm{br})}(\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\sum_\lambda\int\frac{d^3k_\lambda}{(2\pi)^3}k_\lambda\sum_{n(\neq0)}S_{n,\mathbf{k}_\lambda}^{(\mathrm{br})}(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2,\mathbf{q}+\mathbf{k}_\lambda-\mathbf{g}_\perp/2;\varepsilon_0)\,, \tag{4.3}$$

$$D_i^{(\mathrm{br})}(\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\sum_\lambda\int\frac{d^3\mathbf{k}_\lambda}{(2\pi)^3}(q_i^2-g_i^2/4)\sum_{n(\neq0)}S_{n,\mathbf{k}_\lambda}^{(\mathrm{br})}(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2,\mathbf{q}+\mathbf{k}_\lambda-\mathbf{q}_\perp/2;\varepsilon_0)\,. \tag{4.4}$$

We can also obtain a "local incoherent bremsstrahlung probability,"

$$p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)\frac{d^3\mathbf{k}_\lambda}{(2\pi)^3}=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\sum_{n(\neq0)}S_{n,\mathbf{k}_\lambda}^{(\mathrm{br})}(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2,\mathbf{q}+\mathbf{k}_\lambda-\mathbf{g}_\perp/2;\varepsilon_0)\frac{d^3\mathbf{k}_\lambda}{(2\pi)^3}\,, \tag{4.5}$$

which is a probability for the emission of a photon $\mathbf{k}_\lambda$ per unit length at position $\mathbf{R}_\perp$. Equation (4.5) includes a factor

$$H'_{0n}(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)H'_{n0}(-\mathbf{q}-\mathbf{k}_\lambda+\mathbf{g}_\perp/2)\,,$$

which is just the same factor appeared in $S^{(n)}(Q,Q';\varepsilon_0)$ [see Eq. (3.9)], and becomes

$$V_a(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)V_a(\mathbf{q}+\mathbf{k}_\lambda-\mathbf{g}_\perp/2)(e^{-M(\mathbf{g}_\perp)}-e^{-M(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)}e^{-M(\mathbf{q}+\mathbf{k}_\lambda-\mathbf{g}_\perp/2)})\,.$$

Inserting this factor into Eq. (4.5), we get

$$p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)=\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\frac{(2\pi e)^2}{k_\lambda}V_a(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)V_a(\mathbf{q}+\mathbf{k}_\lambda-\mathbf{g}_\perp/2)$$
$$\times(e^{-M(\mathbf{g}_\perp)}-e^{-M(\mathbf{q}+\mathbf{k}_\lambda+\mathbf{g}_\perp/2)}e^{-M(\mathbf{q}+\mathbf{k}_\lambda-\mathbf{q}_\perp/2)})[\text{spinor part}]\delta(\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{q}-\mathbf{k}_\lambda}-k_\lambda)\,. \quad (4.6)$$

Since the analytic form of [spinor] is given in textbooks,[14] the numerical calculation of Eq. (4.6) is straightforward. However, here we try to derive a simple analytical form of Eq. (4.6). Since the Debye-Waller factors in Eq. (4.6) become small when $|\mathbf{g}_\perp|>1/\rho_\perp$, the main contribution to the summation over $\mathbf{g}_\perp$ comes from $|\mathbf{g}_\perp|<1/\rho_\perp$. Then, for the large momentum-transfer region, $|\mathbf{g}_\perp|>1/\rho_\perp$ or $|\mathbf{R}_\perp|<\rho_\perp$, we can neglect the second term in the parentheses and the $\mathbf{g}_\perp$ dependence of $V_a(\mathbf{Q})$ in Eq. (4.6). Thus we approximately get from Eq. (4.6),

$$p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)\cong\sum_{\mathbf{g}_\perp}e^{i\mathbf{g}_\perp\cdot\mathbf{R}_\perp}e^{-M(\mathbf{g}_\perp)}\frac{1}{v}\int\frac{d^3q}{(2\pi)^3}\frac{(2\pi e)^2}{k_\lambda}|V_a(\mathbf{q}+\mathbf{k}_\lambda)|^2[\text{spinor part}]\delta(\varepsilon_0-\varepsilon_{\mathbf{p}_0-\mathbf{q}-\mathbf{k}_\lambda}-k_\lambda)=p^{(\mathrm{br})}_{\mathrm{random}}P(R_\perp)\,, \quad (4.7)$$

where $p^{(\mathrm{br})}_{\mathrm{random}}$ is identical with the well-known Bethe-Heitler bremsstrahlung probability per unit length.[14] From this formula we can see that the spectra, which correspond to the background of channeling radiation spectra, have similar energy profiles to the random incidence. Such a behavior of $p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)$ agrees well with experimental results.[24] It is worth noting that the incoherent bremsstrahlung probability has temperature dependence through the Debye-Waller factors [Eq. (4.6)] or $P(R_\perp)$ [Eq. (4.7)]. For an estimation of background spectra, we introduce a ratio of $p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)$ to $p^{(\mathrm{br})}(\mathbf{k}_\lambda)_{\mathrm{random}}$ by

$$\Gamma=\frac{p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)}{p^{(\mathrm{br})}(\mathbf{k}_\lambda)_{\mathrm{random}}}\,.$$

In general, Γ depends both on $\mathbf{k}_\lambda$ and on $\mathbf{R}_\perp$. However, if we use the approximated formula, Eq. (4.7), Γ becomes equal to $P(R_\perp)$. For plannar channeling cases, $P(y)$ has a value ~ 10 near the atomic planes. Thus the background spectra for electron channeling radiation become larger than the radiation spectra for random incidence because the distribution of impact parameters of channeled electrons is peaked at the atomic planes. These effects have been also observed by experiments.[24] On the contrary, the background spectra for channeled positrons are greatly suppressed because their impact parameter distribution is peaked at the middle of the channel.

Next we derive a simple formula of the local stopping power due to the incoherent bremsstrahlung. Substituting Eq. (4.5) for Eq. (4.1) we can rewrite $S^{(\mathrm{br})}(\mathbf{R}_\perp)$ as

$$S^{(\mathrm{br})}(\mathbf{R}_\perp)=\sum_\lambda\int\frac{d^3\mathbf{k}_\lambda}{(2\pi)^3}k_\lambda p^{(\mathrm{br})}(\mathbf{k}_\lambda,\mathbf{R}_\perp)\,. \quad (4.8)$$

If we substitute Eq. (4.7) into Eq. (4.8), we obtain

$$S^{(\mathrm{br})}(R_\perp)=S^{(\mathrm{br})}_{\mathrm{random}}P(R_\perp)\,, \quad (4.9)$$

where S_{random} is the Bethe-Heitler bremsstrahlung stopping power for random materials. For the extreme relativistic particles, $S^{(\mathrm{br})}_{\mathrm{random}}$ becomes[14]

$$S^{(\mathrm{br})}_{\mathrm{random}}=4N\varepsilon_0[\ln(183Z^{-1/3})+2/9]\bar{\phi}\,,$$

where

$$\bar{\phi}=\frac{Z^2}{137}\left(\frac{e^2}{m}\right)^2\,.$$

Equation (4.9) will be useful for the estimation of the energy loss of channeled electrons. It should be noted that the diffusion function $D^{(\mathrm{br})}(\mathbf{R}_\perp)$ will also be expressed by a formula like that of Eq. (4.9);

$$D^{(\mathrm{br})}(R_\perp)=D^{(\mathrm{br})}_{\mathrm{random}}P(R_\perp)\,. \quad (4.10)$$

Finally, we point out that the exact numerical value of Eq. (4.9) will be similar to that of the phonon diffusion function $D^{(n)}(R_\perp)$. Thus the more accurate form of the approximated formula of $S^{(\mathrm{br})}(R_\perp)$ should include the "one phonon part" as in Eq. (3.10), which becomes important in the region $R_\perp\gg\rho_\perp$. Since, however, in the region $R_\perp\gg\rho_\perp$, the real screened atomic potential cannot give a sufficient momentum for electrons, its contribution to $S^{(\mathrm{br})}(R_\perp)$ will be negligibly small compared with the contribution from the region $R_\perp\lesssim\rho_\perp$. Thus we consider that the simple formula Eq. (4.9) will be sufficient for qualitative discussions.

V. CONCLUDING REMARKS

We have developed a dechanneling theory for relativistic Dirac particles. General definitions of the local diffusion function and the local stopping power are obtained which enable us to calculate various types of local diffusion function and/or local stopping power from the first principle (without phenomenological assumptions which have been used previously). As an example of calculation, we derived the nuclear (phonon) diffusion function. Our result gave a theoretical basis for the phenomenological estimate by Beloshitsky and Kumakhov.[7]

The present theory also includes the interaction between the channeled particles and the radiation field. The local stopping power and the diffusion function due to incoherent bremsstrahlung are derived, and a simple formula for the stopping power is obtained. Our formula expresses the intuitive prediction by Beloshitsky and Trikalinos:[6] "The bremsstrahlung energy loss on nuclei may be calculated with the Bethe-Heitler formula averaged over the atomic density distribution in the channel analogous to the multiple scattering from the thermal vibrations."

There still remain to be considered some local diffusion functions and local stopping power. In particular, the electronic diffusion function including the retardation effects should be derived because the electronic diffusion function has an important role in the dechanneling problem for all kinds of relativistic charged particles.

Finally, we note that our analytical expression of the local bremsstrahlung probability [Eq. (4.7)] and the stopping power [Eq. (4.9)] are simple estimates; therefore to obtain additional quantitative information on the local stopping power and the radiation probability, we should perform rigorous (numerical) calculations of Eqs. (4.6) and (4.8).

ACKNOWLEDGMENT

The authors thank E. Tamura and S. Namiki for valuable discussions.

APPENDIX A

Here we derive Eq. (2.23). First, we compute the second term of Eq. (2.22). For simplicity, henceforth we omit the four-dimensional notation from the variables. Using Eq. (2.11) and expanding $H'_{0n}(x_2)$ around x_1, we get

$$\sum_{a(\neq 0)}\int dx_3\, \psi_0^\dagger(x_2)[H'_{0a}(x_1)-H'_{0a}(x_2)]G_0(x_1-x_3)H'_{a0}(x_3)\psi_0(x_3)$$
$$=\sum_{m=1}^{\infty}\frac{1}{m!}(x_2-x_1)^m\int dx_3\sum_{a(\neq 0)}u_0^\dagger\frac{\partial^m H'_{0a}(x_1)}{\partial x_1^m}G_0(x_1-x_3)H'_{a0}(x_3)u_0 g_o(x_3,x_2)\,. \tag{A1}$$

If we denote that

$$A^{(m)}(x_1,x_3)=\sum_{a(\neq 0)}u_0^\dagger\frac{\partial^m H'_{0a}(x_1)}{\partial x_1^m}G_0(x_1-x_3)H'_{a0}(x_3)u_0$$

and rewrite functions of x and x' as

$$f(x,x')=f(x-x'\,|\,(x+x')/2)$$

and then multiply (A1) by $\exp(ipx)$ and integrate it over x, we obtain

$$\int dx\, e^{ipx}\mathcal{T}_{(A1)}=\int dx\, e^{ipx}\sum_{m=1}^{\infty}\frac{1}{m!}(-x)^m\int dx_3 A^{(m)}\left(x_1-x_3\left|\frac{x_1+x_3}{2}\right.\right)g_0\left(x_3-x_2\left|\frac{x_3+x_2}{2}\right.\right), \tag{A2}$$

where $\mathcal{T}_{(A1)}$ represents Eq. (A1). By using an identity

$$\int dx\, e^{ip\cdot x}x^m f(x)=\left(-i\frac{\partial}{\partial p}\right)^m\int dx\, e^{ip\cdot x}f(x)\,,$$

and the Taylor expansion, Eq. (A2) becomes

$$\mathcal{T}_{(A2)}=\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{i^m}{m!k!l!}\left(\frac{\partial}{\partial p}\right)^m\int dx\, e^{ip\cdot x}\int dx_3\left(\frac{x_3-x_2}{2}\right)^k\left(\frac{x_3-x_1}{2}\right)^l$$
$$\times\left(\frac{\partial^k}{\partial X^k}A^{(m)}(x_1-x_3\,|\,X)\right)\left(\frac{\partial^l}{\partial X^l}g_0(x_3-x_2\,|\,X)\right). \tag{A3}$$

Since $g_0(x\,|\,X)$ is a very slowly varing function of X, we may neglect the terms which satisfy $l\geqq 1$. Further, taking the low-order terms which satisfy $k+m\leqq 2$ (this approximation corresponds to an assumption that the space-time correlation of the interaction between the particle and the reservoir is swiftly damped), Eq. (A3) becomes convolutional integrals over x_3. Hence, using Fourier transformation, we obtain

$$\mathcal{T}_{(\mathrm{A3})} \cong i\frac{\partial}{\partial p}\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial H'_{0\alpha}(X+x/2)}{\partial X} G_0(x) H'_{\alpha 0}(X-x/2)u_0\right] g_0(p;X)$$
$$+\frac{1}{2}\frac{\partial}{\partial p}\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial^2 H'_{0\alpha}(X+x/2)}{\partial X^2} G_0(x) H'_{\alpha 0}(X-x/2)u_0\right]\frac{\partial}{\partial p} g_0(p;X)$$
$$+\frac{1}{2}\frac{\partial}{\partial p}\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial H'_{0\alpha}(X+x/2)}{\partial X} G_0(x) \frac{\partial H'_{\alpha 0}(X-x/2)}{\partial X}u_0\right]\frac{\partial}{\partial p} g_0(p;X)$$
$$-\frac{1}{2}\frac{\partial^2}{\partial p^2}\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial^2 H'_{0\alpha}(X+x/2)}{\partial X^2} G_0(x) H'_{\alpha 0}(X-x/2)u_0\right] g_0(p;X)$$
$$\cong i\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial H'_{0\alpha}(X+x/2)}{\partial X} G_0(x) H'_{\alpha 0}(X-x/2)u_0\right]\frac{\partial}{\partial p} g_0(p;X)$$
$$+\frac{1}{2}\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial H'_{0\alpha}(X+x/2)}{\partial X} G_0(x) \frac{\partial H'_{\alpha 0}(X-x/2)}{\partial X}u_0\right]\frac{\partial^2}{\partial p^2} g_0(p;X)\,.$$

In the above, since $G_0(x)$ has a sharp peak at $x=0$, we assumed that the factors which include $G_0(x)$ to be almost independent of p. Performing the same manipulations to the first term of Eq. (2.22) and combining it with the result in the above, we get

$$i\left[\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \left\{\frac{\partial H'_{0\alpha}(X+x/2)}{\partial X} G_0(x) H'_{\alpha 0}(X-x/2) + H'_{0\alpha}(X+x/2)G_0^\dagger(-x)\frac{\partial H'_{\alpha 0}(X-x/2)}{\partial X}\right\}u_0\right]\frac{\partial}{\partial p} g_0(p;X)$$
$$+\left[-\frac{i}{2}\int dx\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger \frac{\partial H'_{0\alpha}(X+x/2)}{\partial X}[G_0(x)-G_0^\dagger(-x)]\frac{\partial H'_{\alpha 0}(X-x/2)}{\partial X}u_0\right]\frac{\partial^2}{\partial p^2} g_0(p;X)\,. \tag{A4}$$

Averaging Eq. (A4) over Z and the initial spin state s_0, we obtain Eq. (2.23):

$$\frac{1}{L}\int dZ\, \tfrac{1}{2}\sum_{s_0}\cdot\mathcal{T}_{(\mathrm{A4})} = i\left\{\mathcal{F}_\mu \frac{\partial}{\partial p_\mu} + \mathcal{D}_{\mu\nu}\frac{\partial^2}{\partial p_\mu \partial p_\nu}\right\} g_0(p;X)\,.$$

To calculate $\mathcal{F}_\mu$ and $\mathcal{D}_{\mu\nu}$ further, we use the expression

$$G_0(x) = -i\theta(t)\int \frac{d^3p}{(2\pi)^3}\sum_s uu^\dagger e^{-ip\cdot x} + i\theta(-t)\int \frac{d^3p}{(2\pi)^3}\sum_s vv^\dagger e^{ip\cdot x}, \tag{A5}$$

where $v=v(\mathbf{p},s)$ denotes a negative energy solution. Because of our assumption that the coupling between the incident particle and the reservoir is weak, the final state of the particle will be in a positive energy state. Thus we neglect the negative energy solutions of $G_0(x)$.

For example, let us compute $\mathcal{F}_0$:

$$\mathcal{F}_0 = \tfrac{1}{2}\sum_{s_0}\frac{1}{L}\int dZ \int d^4x\, e^{ip\cdot x}\sum_{\alpha(\neq 0)} u_0^\dagger\left\{\frac{\partial H'_{0\alpha}(X+x/2)}{\partial T} G_0(x) H'_{\alpha 0}(X-x/2) + H'_{0\alpha}(X+x/2) G_0^\dagger(-x)\frac{\partial H'_{\alpha 0}(X-x/2)}{\partial T}\right\}u_0\,.$$

Noting that

$$\frac{\partial H'_{0\alpha}(X+x/2)}{\partial T} = i\mathcal{E}_{0\alpha} H'_{0\alpha}(\mathbf{R}+r/2)e^{i\mathcal{E}_{0\alpha}(T+t/2)}\,,$$

and an identity that

$$\int_{-\infty}^{\infty} dt\, e^{i\varepsilon t}\theta(\pm t) = \pm iP\frac{1}{\varepsilon} + \pi\delta(\varepsilon)\,,$$

we obtain

$$\mathcal{F}_0 = \tfrac{1}{2}\sum_{s_0}\sum_{s}\frac{1}{L}\int dZ \int d^3r \int \frac{d^3p}{(2\pi)^3} e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}}\sum_{\alpha(\neq 0)}\mathcal{E}_{\alpha 0} u_0^\dagger H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)u_0 \cdot 2\pi\delta(\varepsilon - \varepsilon_{\mathbf{p}'} - \mathcal{E}_{\alpha 0})\,.$$

Computing other coefficients, we finally obtain

$$\mathcal{T}_{(2.23)} = i\left\{\frac{\partial}{\partial \mathbf{R}_\perp}C'(\mathbf{R}_\perp)\cdot\frac{\partial}{\partial \mathbf{p}} - \left\langle\frac{\Delta\varepsilon}{\Delta T}\right\rangle\frac{\partial}{\partial \varepsilon} - \left\langle\frac{\Delta \mathbf{p}}{\Delta T}\right\rangle\cdot\frac{\partial}{\partial \mathbf{p}} + \left\langle\frac{\Delta\varepsilon^2}{2\Delta T}\right\rangle\frac{\partial^2}{\partial\varepsilon^2} + \left\langle\frac{\Delta p_i \Delta p_j}{2\Delta T}\right\rangle\frac{\partial^2}{\partial p_i \partial p_j}\right\} g_0(p;X)\,,$$

where

$$C'(\mathbf{R}_\perp)=-\tfrac{1}{2}\sum_{s_0}\sum_{s}\frac{1}{L}\int dz\int d^3r\,\mathcal{P}\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}\left[\sum_{\alpha(\neq 0)}\frac{u_0^\dagger H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)u_0}{\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0}}\right],$$

$$\left\langle\frac{\Delta\varepsilon}{\Delta T}\right\rangle=\tfrac{1}{2}\sum_{s_0}\sum_{s}\frac{1}{L}\int dz\int d^3r\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}\sum_{\alpha(\neq 0)}\mathcal{E}_{\alpha 0}u_0^\dagger H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)u_0\cdot 2\pi\delta(\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0}),$$

$$\left\langle\frac{\Delta\mathbf{p}}{\Delta T}\right\rangle=-\frac{i}{2}\sum_{s_0}\sum_{s}\frac{1}{L}\int dz\int d^3r\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}\sum_{\alpha(\neq 0)}u_0^\dagger\left[\frac{\partial H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)}{\partial\mathbf{R}}uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)-H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger\frac{\partial H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)}{\partial\mathbf{R}}\right]\times u_0\cdot 2\pi\delta(\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0}),$$

$$\left\langle\frac{\Delta\varepsilon^2}{2\Delta T}\right\rangle=\tfrac{1}{2}\sum_{s_0}\sum_{s}\frac{1}{L}\int dZ\int d^3r\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}\sum_{\alpha(\neq 0)}\mathcal{E}_{\alpha 0}^2u_0^\dagger H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)uu^\dagger H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)u_0\cdot 2\pi\delta(\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0}),$$

$$\left\langle\frac{\Delta p_i\Delta p_j}{2\Delta T}\right\rangle=\tfrac{1}{4}\sum_{s_0}\sum_{s}\frac{1}{L}\int dZ\int d^3r\int\frac{d^3p}{(2\pi)^3}e^{-i(\mathbf{p}_0-\mathbf{p})\cdot\mathbf{r}}\times\sum_{\alpha(\neq 0)}u_0^\dagger\left[\frac{\partial H'_{0\alpha}(\mathbf{R}+\mathbf{r}/2)}{\partial R_i}uu^\dagger\frac{\partial H'_{\alpha 0}(\mathbf{R}-\mathbf{r}/2)}{\partial R_j}\right]u_0\cdot 2\pi\delta(\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\alpha 0}).$$

In the above, we used an identity of δ function;

$$f(x)\delta(x-x_0)=f(x_0)\delta(x-x_0),$$

and neglected the coefficients $\mathcal{D}_{0i}$ and $\mathcal{D}_{i0}$. As explained in Sec. III, $\langle\Delta\varepsilon/\Delta T\rangle$ and $\langle\Delta p_i^2/2\Delta T\rangle$ represent the stopping power and the diffusion function per unit time, respectively.

APPENDIX B

The derivation of the compound matrix element $K_{0\alpha}$. The iterated expression of $\psi_\alpha(x)$ up to the second order is

$$\psi_\alpha(x)\cong\int d^4x'G_0(x-x')H'_{\alpha 0}(x')\psi_0(x')+\int d^4x'\int d^4x''G_0(x-x')\sum_{\beta(\neq 0)}H'_{\alpha\beta}(x')G_0(x'-x'')H'_{\beta 0}(x'')\psi_0(x''). \quad \text{(B1)}$$

The second term of the right-hand side of Eq. (B1) represents the second-order interaction. Let us consider the factor

$$\int d^4x''\sum_{\beta(\neq 0)}H'_{\alpha\beta}(x')G_0(x'-x'')H'_{\beta 0}(x'')\psi_0(x''). \quad \text{(B2)}$$

The relativistic Green's function $G_0(x)$ given in Eq. (A5) can be rewritten in the form

$$G_0=\pm i\theta(\pm t)\sum_{\pm s}\int\frac{d^3p}{(2\pi)^3}uu^\dagger e^{-i\varepsilon_{\mathbf{p}}t}e^{i\mathbf{p}\cdot\mathbf{r}}. \quad \text{(B3)}$$

It should be noted that $\varepsilon_{\mathbf{p}}$ and $\mathbf{p}$ take negative values for the negative energy states. By using Eqs. (2.11) and (B3), Eq. (B2) becomes

$$\int d^4x''\sum_{\beta(\neq 0)}H'_{\alpha\beta}(x')\{\pm i\theta[\pm(t'-t'')]\}\int\frac{d^3p}{(2\pi)^3}\sum_{\pm s}uu^\dagger e^{-i\varepsilon_{\mathbf{p}}(t'-t'')}e^{i\mathbf{p}\cdot(\mathbf{r}'-\mathbf{r}'')}H'_{\beta 0}(\mathbf{r}'')e^{i\mathcal{E}_{\beta 0}t''}\times\left[\frac{1}{\sqrt{L}}u_0e^{i\mathbf{p}_0\cdot\mathbf{r}''}e^{-i\varepsilon_0t''}\varphi_0(\mathbf{r}''_\perp,t'')\right]. \quad \text{(B4)}$$

Since $G_0(x'-x'')$ has a sharp peak around $t'=t''$, we can take $\varphi_0(t'')$ out of the integral over t'' in Eq. (B4) and get

$$\int d^3r''\int\frac{d^3p}{(2\pi)^3}\sum_{\pm s'}\sum_{\beta(\neq 0)}\frac{H'_{\alpha\beta}(\mathbf{r}')u'u'^\dagger H'_{\beta 0}(\mathbf{r}'')}{\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\beta 0}}e^{i\mathcal{E}_{\alpha 0}t'}e^{i\mathbf{p}\cdot(\mathbf{r}'-\mathbf{r}'')}\frac{1}{\sqrt{L}}u_0e^{i\mathbf{p}_0\cdot\mathbf{r}'}e^{-i\varepsilon_0t'}\varphi_0(\mathbf{r}'_\perp,t')=K_{\alpha 0}(x')\psi_0(x'), \quad \text{(B5)}$$

where we defined

$$K_{\alpha 0}(x)=\int d^3r'\int\frac{d^3p}{(2\pi)^3}\sum_{\pm s'}\sum_{\beta(\neq 0)}\frac{H'_{\alpha\beta}(\mathbf{r})u'u'^\dagger H'_{\beta 0}(\mathbf{r}')}{\varepsilon_0-\varepsilon_{\mathbf{p}}-\mathcal{E}_{\beta 0}}e^{i(\mathbf{p}-\mathbf{p}_0)\cdot(\mathbf{r}-\mathbf{r}')}e^{i\mathcal{E}_{\alpha 0}t}. \quad \text{(B6)}$$

If we insert Eq. (B6) into Eq. (B1), we obtain

$$\psi_\alpha(x) = \int d^4x' G_0(x-x')H'_{\alpha 0}(x')\psi_0(x') + \int d^4x' G_0(x-x')K_{\alpha 0}(x')\psi_0(x') .$$

Thus we can conclude that if we want to take account of the second-order inelastic scattering effects, we only have to substitute $K_{\alpha 0}$ for $H'_{\alpha 0}$ in the first-order expressions. Since $K_{\alpha 0}(x) = K_{\alpha 0}(\mathbf{r})e^{i\mathcal{E}_{\alpha 0}t}$ we get $K_{\alpha 0}(\mathbf{q})$ by the Fourier transformation.

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