# Magnetoplasmon-phonon coupling in a $Ga_xIn_{1-x}As$ heterostructure

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The coupling between magnetoplasmons and phonon modes in a $Ga_xIn_{1-x}As$ heterostructure is investigated theoretically. It is found that the magnetoplasmons of the quasi-two-dimensional electron gas strongly interact with two LO-phonon modes of the supporting lattice system. The frequencies of the coupled modes and their corresponding oscillator strengths are studied as a function of the wave vector, of the magnetic field strength, and of the electron density of the system. Based on our numerical results, it seems that the magnetoplasmon-phonon coupling will not result in a splitting of the cyclotron resonance mass at TO-phonon frequencies as suggested by R. J. Nicholas *et al.* [Phys. Rev. Lett. **55**, 883 (1985)].

## I. INTRODUCTION

There are many theoretical and experimental studies of the magnetoplasmon in two-dimensional (2D) electron systems.[1–16] The dispersion relation of the magnetoplasmon predicted by theories[1–10] is confirmed by experiments[11–16] on Si inversion layers and on GaAs-$Al_xGa_{1-x}As$ heterostructures and superlattices. In those semiconductor heterostructures made of polar materials like GaAs, $Ga_xIn_{1-x}As$, the magnetoplasmons are expected to interact with the electric field associated with the longitudinal-optical (LO) phonons. This interaction should become stronger when the magnetoplasmon frequency is comparable with the LO-phonon frequency. In the present paper we will investigate the coupling between magnetoplasmons and phonon modes in a $Ga_xIn_{1-x}As$ heterostructure.

In the presence of a strong perpendicular magnetic field, the collective excitation of a quasi-2D electron gas, the so-called magnetoplasmon, has a rather different dispersion relation compared to that in the case of zero magnetic field. In the long-wavelength limit, which is most accessible experimentally, the plasmon frequency approaches zero as the wave vector approaches zero if there is no magnetic field present. Thus it is far below the LO-phonon frequencies of the lattice system. The magnetoplasmon frequency, however, approaches the cyclotron frequency in the long-wavelength limit.[1–10] Therefore one can, by changing the strength of the magnetic field, sweep the collective excitation frequencies through the LO-phonon frequencies and observe a strong coupling between the magnetoplasmon and phonons.

The coupling between the plasmons and phonons has been studied in detail both experimentally and theoretically for systems without an external magnetic field. We refer to the review papers by Theis[17] and by Heitmann[18] for recent developments in this subject. In the case that a strong perpendicular magnetic field is applied to the 2D electron gas, the magnetoplasmon-phonon coupling has been studied theoretically,[19–22] but, to the best of our knowledge, there are no experimental data available.

In Ref. 19 Oji and MacDonald studied the magnetoplasmon-phonon coupling in a GaAs-$Al_xGa_{1-x}As$ heterostructure. The dispersion relation of the coupled modes was calculated where the electron gas was treated as ideally two dimensional. The polarization of the 2D electron gas was considered within different approximations.[19] The coupling was also studied by Tselis and Quinn[21] and by Das Sarma[20] for semiconductor superlattice structures. In Ref. 22 Lassnig briefly discussed the coupling in the context that the coupling between the magnetoplasmons and LO phonons may lead to a splitting of the cyclotron resonance mass at TO-phonon frequencies instead of at LO-phonon frequencies, which was recently observed experimentally by Nicholas *et al.*[23] In the present paper we will also investigate this problem.

The differences between the present work and that mentioned above[19–22] are first, that we will consider magnetoplasmon-phonon coupling in a $Ga_xIn_{1-x}As$ heterostructure where there are two LO-phonon modes, namely the InAs- and GaAs-like LO phonons. Secondly, not only the dispersion relation of the magnetoplasmon-phonon modes but also its corresponding oscillator strength will be calculated. The latter was not calculated in Refs. 19–22. The oscillator strength of the coupled modes is important in analyzing the experimental results, since, as will be shown later, some modes may carry a very small oscillator strength which implies that those modes will not be observable experimentally.

## II. MAGNETOPLASMON-PHONON COUPLING

The coupled magnetoplasmon-phonon modes are determined by the equation

$$\mathrm{Re}\,\epsilon(\mathbf{k},\omega)=0\ , \tag{1}$$

where $\epsilon(\mathbf{k},\omega)$ is the total dielectric function of the system which consists of contributions from the polarizations of the 2D electron gas and of LO phonons. In the present paper the contribution to the total dielectric function from the quasi-2D electron gas will be calculated within the random-phase approximation (RPA). For simplicity, the broadening of Landau levels will be neglected. The calculation will be limited to zero temperature, since usually the experiments are performed at liquid-helium temperature.[11–16] The 2D electron gas part of the dielectric function has been calculated by several researchers for an ideal 2D electron gas and for superlattice structures.[1–10] In this paper a single $Ga_xIn_{1-x}As$ heterostructure will be studied and the finite extent of the 2D electron gas in the direction perpendicular to the 2D plane will be included by considering the lowest subband with the Stern-Fang-Howard variational wave function.[24]

The contribution to the total dielectric function from the LO phonons will be taken as[25]

$$\epsilon_\infty\left[1+\frac{\omega_{\rm LO}^2-\omega_{\rm TO}^2}{\omega_{\rm TO}^2-\omega^2}+\frac{\omega_{\rm LO}^2}{\omega_{\rm TO}^2}\frac{\omega_{\rm LO'}^2-\omega_{\rm TO'}^2}{\omega_{\rm TO'}^2-\omega^2}\right], \tag{2}$$

where $\omega_{\rm LO}$ ($\omega_{\rm LO'}$) is the InAs- (GaAs-) like LO-phonon frequency, and $\omega_{\rm TO}$ ($\omega_{\rm TO'}$) the InAs- (GaAs-) like TO-phonon frequency. The values of the phonon frequencies are taken to be $\omega_{\rm LO}=233$, $\omega_{\rm LO'}=272$, $\omega_{\rm TO}=226$, and $\omega_{\rm TO'}=256$ cm$^{-1}$.[26–28] Note that the coupling strength with the GaAs-like LO phonon is about twice as strong as the InAs-like LO phonon.[26–28] The lifetime of phonons will be neglected for simplicity.

In the RPA the total dielectric function of the system is just the sum of contributions from the quasi-2D electron gas and from the polarization of LO phonons.[29] The total dielectric function can be written as

$$\begin{aligned}{\rm Re}\epsilon(\mathbf{k},\omega)/\epsilon_\infty=&1+\frac{\omega_{\rm LO}^2-\omega_{\rm TO}^2}{\omega_{\rm TO}^2-\omega^2}+\frac{\omega_{\rm LO}^2}{\omega_{\rm TO}^2}\frac{\omega_{\rm LO'}^2-\omega_{\rm TO'}^2}{\omega_{\rm TO'}^2-\omega^2}\\&+\frac{4m_be^2}{\hbar^2\epsilon_\infty}\frac{1}{k}f(k,b)\frac{1}{\sin\left[\pi\frac{\omega}{\omega_c}\right]}\int_0^\pi dt\,\sin\left[\frac{\omega}{\omega_c}t\right]\sin\left[\frac{a}{2}\sin t\right]e^{-a/2(1+\cos t)}G(a(1+\cos t))\,,\end{aligned} \tag{3}$$

where $m_b$ is the effective mass of the electron, $\epsilon_\infty$ the high-frequency dielectric constant, $\omega_c$ the cyclotron resonance frequency, $a=\hbar k^2/2m_b$, and $G(x)=L_{n-1}^1(x)+g(\nu)L_n^0(x)$ with $L_n^1(x)$ the Laguerre polynomial, $n=[\nu]$ the integer part of the filling factor, $g(\nu)=\nu-n$, and $f(k,b)$ the form factor[24] with $b$ the variational parameter which is inversely proportional to the width of the 2D electron layer. The filling factor $\nu$ is defined as $\nu=(n_e/2)(2\pi\hbar/m_b\omega_c)$ with $n_e$ the electron density of the system. The last term on the right-hand side of Eq. (3) is the polarization function of a noninteracting quasi-2D electron gas which can be easily obtained using standard diagrammatic method.[29] In Eq. (3) this polarization function is given in the form of an integral[5] which is more convenient for numerical calculations. In the present paper the magnetoplasmon-phonon modes $\omega_n$ $(n=1,2,\ldots)$ will be labeled in such a way that the frequencies are in increasing order, i.e., $\omega_n<\omega_{n+1}$. The oscillator strength corresponding to each magnetoplasmon-phonon mode is given by

$$A_n=\pi\left|\frac{\partial}{\partial\omega}{\rm Re}\epsilon(\mathbf{k},\omega)\right|_{\omega=\omega_n}^{-1}, \tag{4}$$

which has the dimension of frequency and will be scaled by the GaAs-like LO-phonon frequency in the following.

## III. NUMERICAL RESULTS AND DISCUSSION

In our numerical calculations all physical parameters are taken corresponding to a $Ga_xIn_{1-x}As$ heterostructure ($x=0.47$). First, the wave-vector dependence of the magnetoplasmon-phonon modes will be studied. In Fig. 1 the frequencies of coupled modes and their oscillator strength are plotted as a function of the wave vector for the three values of the magnetic field. For convenience, the cyclotron frequency instead of the magnetic field is used in the calculations. For $Ga_xIn_{1-x}As$ the magnetic field strength $B$ (in T) $=0.4\hbar\omega_c$ (in meV). The small effective mass of the electron in a $Ga_xIn_{1-x}As$ heterostructure ($m_b=0.047m_e$ in comparison with $m_b=0.07m_e$ in a GaAs-$Al_xGa_{1-x}As$ heterostructure) enables one to obtain a relatively large cyclotron frequency with a small magnetic field. The values of the magnetic field are chosen as (a) $\hbar\omega_c=25$ meV below the InAs-like TO-phonon frequency, (b) $\hbar\omega_c=30$ meV between the InAs-like LO-phonon and GaAs-like TO-phonon frequency, and (c) $\hbar\omega_c=35$ meV above the GaAs-like LO-phonon frequency. The electron density is $4\times10^{11}$ cm$^{-2}$. In Fig. 1 the InAs- (GaAs-) like LO- and TO-phonon frequencies are indicated with dashed lines together with the symbols LO (LO′) and TO (TO′).

When the cyclotron frequency is smaller than but close to the InAs-like TO-phonon frequency, the magnetoplasmon-phonon modes consist of two plasmon-like ($n=1,4$) modes and two phononlike ($n=2,3$) modes [see Fig. 1(a)]. Most of the oscillator strength is contained in the GaAs phononlike mode ($n=3$). As the magnetic field increases, the coupling between the magnetoplasmon and the InAs-like LO phonon becomes strong [see Fig. 1(b)]. However, the first two modes ($n=1,2$) contain a small portion of the oscillator strength. This is due to the fact that the coupling strength with the InAs-like phonon is much weaker than with the GaAs-like phonon as mentioned before.[26–28] The further increase of the magnetic field results in a strong mixing of the magnetoplasmon and GaAs-like LO-phonon mode as shown in Fig. 1(c). The first mode is almost dispersionless and carries a very small oscillator strength. The oscillator strength contained in the $n=2$ and $n=3$ modes is transferred first from the second mode to the third one and then for larger wave vectors from the third mode to the second one.

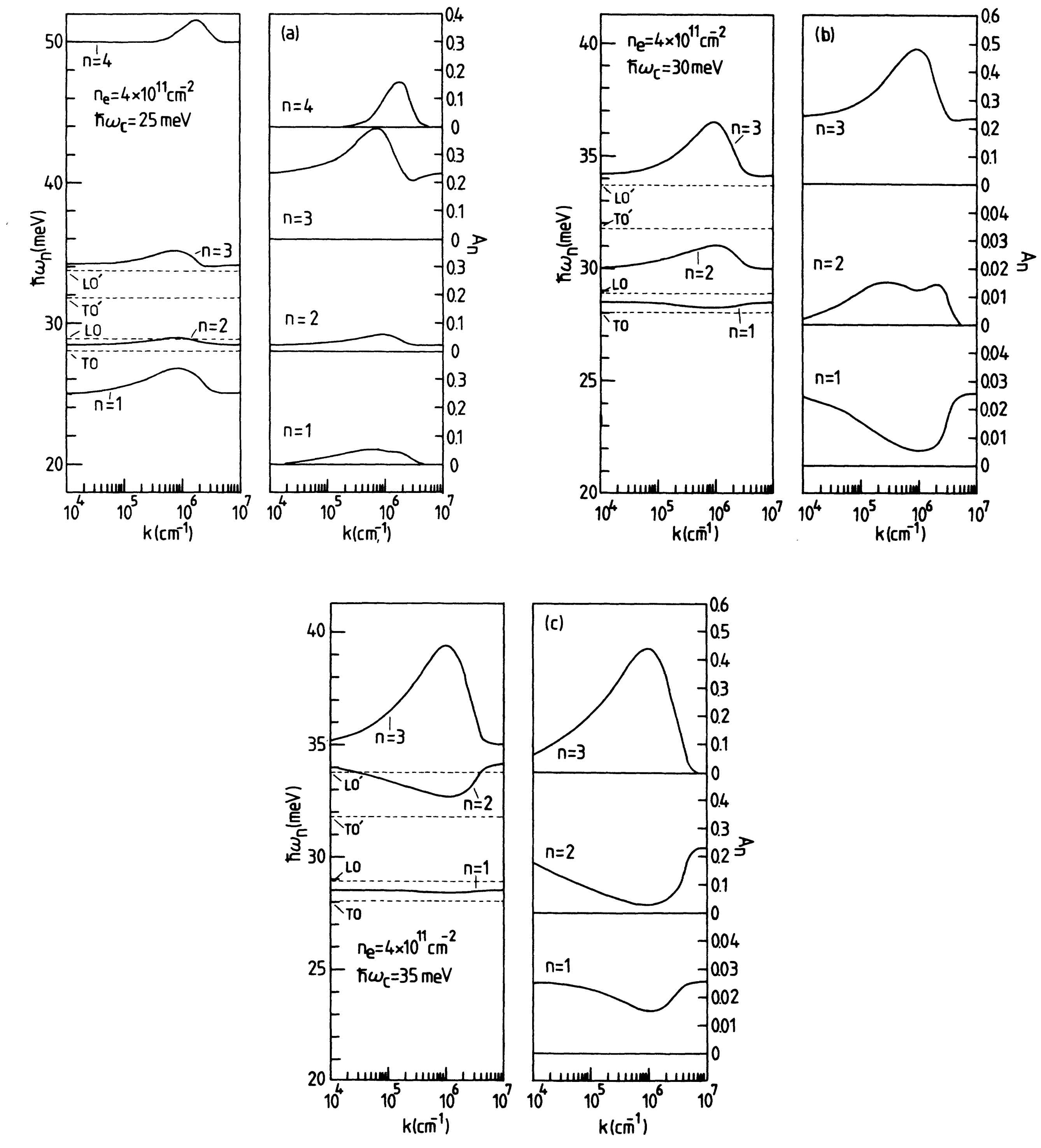


FIG. 1. The frequencies of coupled magnetoplasmon-phonon modes and their corresponding oscillator strength are plotted as a function of the wave vector for three values of the magnetic field: (a) $\hbar\omega_c=25$ meV, (b) $\hbar\omega_c=30$ meV, and (c) $\hbar\omega_c=35$ meV. All physical parameters are taken corresponding to a $Ga_xIn_{1-x}As$ heterostructure. Dashed lines indicate the positions of the GaAs- and InAs-like LO- and TO-phonon frequencies.

In a recent Letter, Nicholas *et al.*[23] found that the cyclotron resonance mass as a function of the laser frequency is discontinuous at TO-phonon frequencies instead of at LO-phonon frequencies. It seems that the electrons are coupled to TO phonons, while theoretically it is expected that electrons interact with LO phonons.[30] This was attributed to the magnetoplasmon-phonon coupling in Ref. 23 and later in Ref. 22. From the present study, it seems that the magnetoplasmon-phonon coupling will not result in a mode whose frequency is close to the TO-phonon frequency and at the same time it contains a large portion of the oscillator strength. It is known that a magnetoplasmon mode must have a relatively large oscillator strength in order to contribute significantly to the correction to the polaron cyclotron resonance mass.[31] It may be said that the magnetoplasmon-phonon coupling is not the reason for what was observed in Ref. 23. We rather believe that electrons are interacted with phonons whose frequencies are very close to the TO-phonon frequencies as suggested by Das Sarma.[32]

In the experiment of Batke *et al.*[15,16] the values of the wave vector are fixed while the strength of the magnetic field is used as a tunable parameter. In Fig. 2 the magnetic field dependence of the magnetoplasmon-phonon modes is investigated. In Fig. 2(a) the coupled mode frequencies are plotted for a fixed value of wave vector and

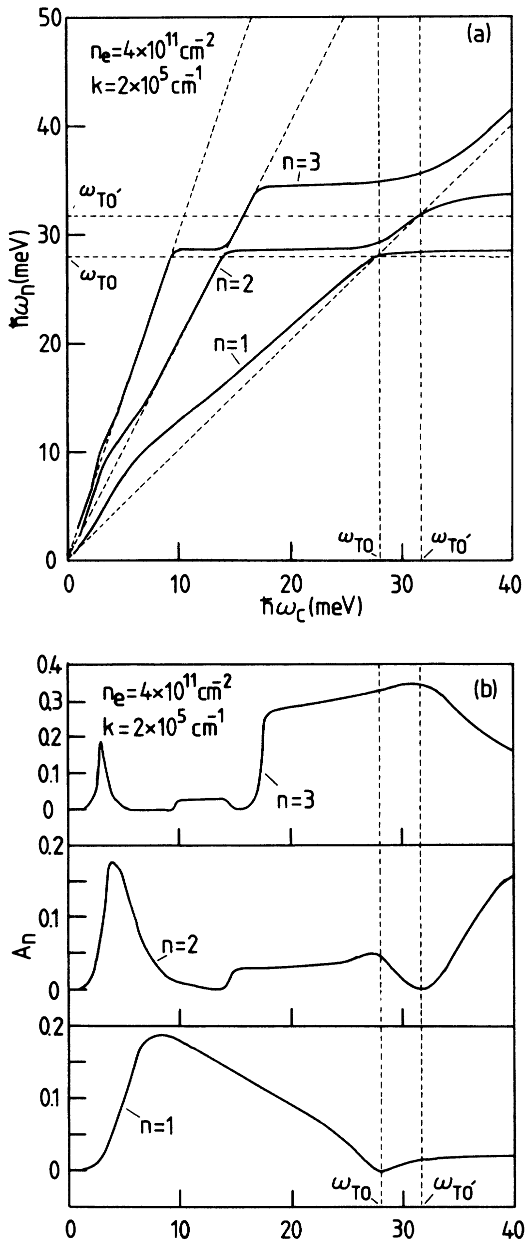


FIG. 2. Magnetoplasmon-phonon mode frequencies (a) and their oscillator strengths (b) are plotted as a function of the magnetic field strength for a fixed value of the wave vector and electron density. The GaAs- and InAs-like TO-phonon frequencies are indicated with the vertical and horizontal dashed line.

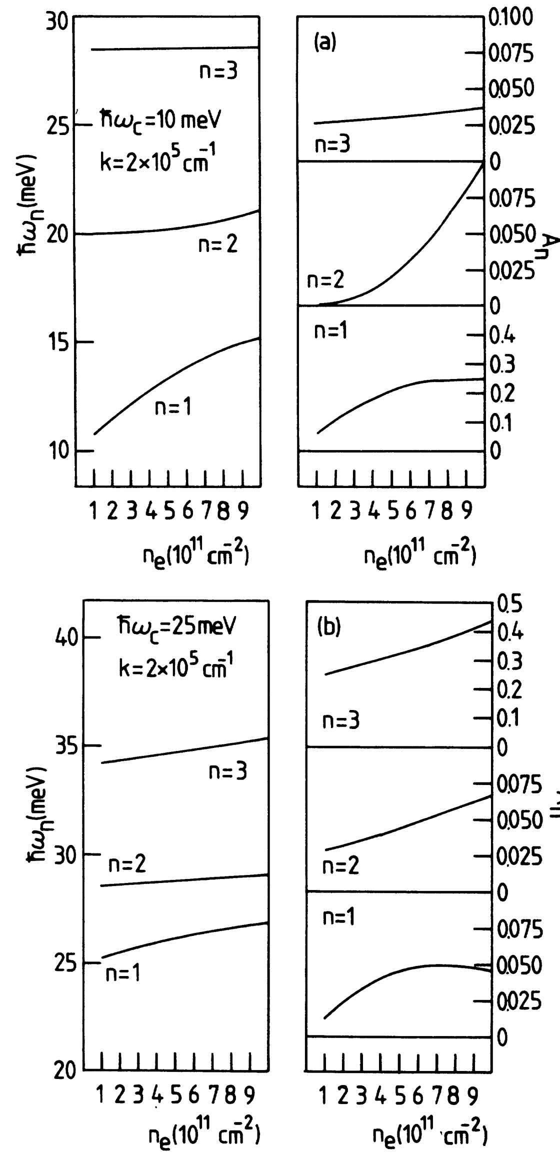


FIG. 3. Magnetoplasmon-phonon mode frequencies and oscillator strengths are plotted vs the electron density of the system for a fixed wave vector and for two values of the magnetic field: (a) $\hbar\omega_c=10$ meV and (b) $\hbar\omega_c=25$ meV.

of electron density. Figure 2(b) shows the oscillator strengths of the corresponding modes. The wave vector is chosen to be $2\times10^5$ cm$^{-1}$, which is experimentally accessible.[16] In Fig. 2 the vertical and horizontal dashed lines indicate the position of InAs- and GaAs-like TO-phonon frequencies. In Fig. 2(a) three dashed lines starting from the origin are $\omega_n = n\omega_c$ ($n=1,2,3$).

For small magnetic fields ($\hbar\omega_c < 15$ meV), the magnetoplasmon-phonon modes are essentially plasmonlike [see Fig. 2(a)]. The frequencies of the coupled modes lie slightly above the cyclotron resonance frequencies and the corresponding oscillator strengths transfer from the second mode to the first one as the magnetic field increases [Fig. 2(b)]. This behavior is qualitatively the same as found experimentally in a GaAs-$Al_xGa_{1-x}As$ heterostructure.[16] As the magnetic field increases but $\hbar\omega_c < 18$ meV, most of the oscillator strength is contained in the first mode with its frequency close to the cyclotron frequency. When the magnetic field further increases so that $\hbar\omega_c > 18$ meV, there is an abrupt transferring of the oscillator strength. The mode most likely to be observed becomes the third mode with its frequency being almost independent of the magnetic field [see Figs. 2(a) and 2(b)]. Our numerical calculations show that this magnetic field ($\hbar\omega_c \approx 18$ meV), at which the abrupt transferring of the oscillator strength occurs, is almost independent of the electron density ($10^{11}$–$10^{12}$ cm$^{-2}$) for wave vectors $10^5 < k < 4\times10^5$ cm$^{-1}$. For still larger magnetic fields ($\hbar\omega_c > 30$ meV), two modes ($n=2,3$) will be observable with their frequencies lying below and above the cyclotron frequency, respectively. The oscillator strength of the $n=2$ mode increases while the oscillatory strength of the $n=3$ mode decreases as the magnetic field increases. Note that at LO-phonon frequencies there is no structure appearing either in the coupled mode frequencies or in the corresponding oscillator strength.

Finally, the electron density dependence of the magnetoplasmon-phonon modes will be studied. In Fig. 3 the coupled mode frequencies and their corresponding oscillator strength are plotted versus the electron density of the system for two values of the magnetic field (a) $\hbar\omega_c = 10$ meV and (b) $\hbar\omega_c = 25$ meV. The frequencies of the magnetoplasmon-phonon modes increase with increasing electron density. At a low magnetic field [Fig. 3(a)], the frequency of the third mode, which is phononlike, is almost independent of the electron density. Most of the oscillator strength is contained in the first mode, which is plasmonlike. The frequency of the $n=2$ mode increases slightly but its oscillator strength increases rapidly as the electron density becomes larger. At a larger value of the magnetic field [Fig. 3(b)] the slope of the coupled mode frequencies versus the electron density is small and most of the oscillator strength is contained in the third mode.

In conclusion, we have studied the magnetoplasmon-phonon coupling in a $Ga_xIn_{1-x}As$ heterostructure. The dispersion and oscillator strengths of the magnetoplasmon-phonon modes are studied as a function of the wave vector, the magnetic field strength, and the electron density. A strong mixing of the magnetoplasmons and phonons is found and is expected to be observable experimentally. We ruled out that the magnetoplasmon-phonon coupling will result in a splitting of the cyclotron resonance mass at TO-phonon frequencies.


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