Surface magnetoplasmon polaritons in truncated semiconductor superlattices

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(Received 26 January 1988)

A theoretical investigation has been made of surface magnetoplasmon modes in truncated semiconductor superlattices. The thicknesses of the superlattice layers are assumed sufficiently large that quantum-well effects can be neglected. The materials are characterized by their macroscopic dielectric tensors, and a local theory with retardation is employed. The external magnetic field is taken to be parallel to the surface and perpendicular to the direction of propagation of the polariton. Numerical results are presented for the specific case of a GaAs-$Ga_{1-x}Al_xAs$ superlattice. The effect of superlattice structure on the gap in the dispersion curve is examined in detail.

I. INTRODUCTION

Interest in the properties of artificially created periodic structures or superlattices has steadily increased ever since their introduction.[1] Especially intriguing are the electronic properties of semiconductor superlattices composed of materials such as GaAs and $Ga_{1-x}Al_xAs$ in which the individual layers are only a few atomic layers thick and where the conduction electrons in the GaAs layers behave as two-dimensional systems confined in quantum wells.[2] Extensive theoretical investigations of the plasmonlike excitations of such systems have been made for both infinite periodic arrays[3] and arrays truncated by a vacuum or insulating half-space.[4] In such investigations the conducting layers cannot be characterized by macroscopic dielectric functions because of their quantum-well nature. The theoretical predictions have found confirmation in experimental data.[5]

It is possible, however, to consider superlattices in which the individual layers are sufficiently thick (≥ 1000 Å) that they *can* be characterized by macroscopic dielectric functions. For example, plasmonlike excitations have been investigated in the nonretarded limit by Camley and Mills[6] for both infinite and truncated superlattices in which the conducting layers are metallic. The effect of a magnetic field in the nonretarded limit has been considered by Kushwaha.[7] The inclusion of retardation has been carried out by Giuliani, Quinn, and Wallis[8] and by Szenics *et al.*[9] for these systems in the absence of an external magnetic field, and by Wallis *et al.*[10] for systems in the presence of an external magnetic field. The effect of damping on the polariton modes of infinite superlattices has been investigated by Haupt and Wendler.[11]

The qualitative results obtained in the investigations mentioned above are the following. In infinite superlattices one obtains plasmon polariton bands in accordance with Bloch's theorem for a periodic structure. The truncation of the superlattice leads to surface-plasmon polariton branches that appear in regions of the spectrum forbidden to polariton excitations of the infinite superlattice. The application of an external magnetic field parallel to the surface of the truncated superlattice and perpendicular to the direction of propagation of the surface polariton produces a splitting of the surface-polariton branches as a consequence of the nonequivalence of the two opposite directions of propagation when the magnetic field is present.

The effects of experimentally available magnetic fields on superlattices containing metallic conductors are typically small because the effective mass of conduction electrons in metals is relatively large, i.e., on the order of the free-electron mass m_0. In semiconductors such as GaAs, on the other hand, the effective mass m^* of conduction electrons is quite small,[12] on the order of $0.07m_0$, and the effect of experimentally available magnetic fields should be relatively large. In the present paper, we report the results of a theoretical investigation of magnetic field effects in GaAs-$Ga_{1-x}Al_xAs$ superlattices. The magnetic field is taken to be parallel to the surface and perpendicular to the direction of propagation of the surface polaritons. Numerical results are presented for representative cases and the possibilities for experimental observation of the effects predicted are discussed.

II. THEORETICAL DEVELOPMENT AND NUMERICAL RESULTS

In previous papers the derivations of the dispersion relations for plasmon polaritons[8,9] and for magnetoplasmon polaritons[10] in infinite and truncated superlattices are presented in detail. For the case where an external magnetic field is present, the magnetic field is assumed to be parallel to the interfaces and perpendicular to the direction of propagation of the polariton. The electromagnetic field amplitudes are taken to be localized at each interface. Retardation is included, but both damping and spatial dispersion are neglected.

The geometry of the systems we treat is shown in Fig. 1. Alternating layers of materials A and B extend until a terminating layer, in the case of a truncated superlattice, is reached. The material beyond the terminating layer is assumed to have a frequency-independent dielectric constant ϵ_C.

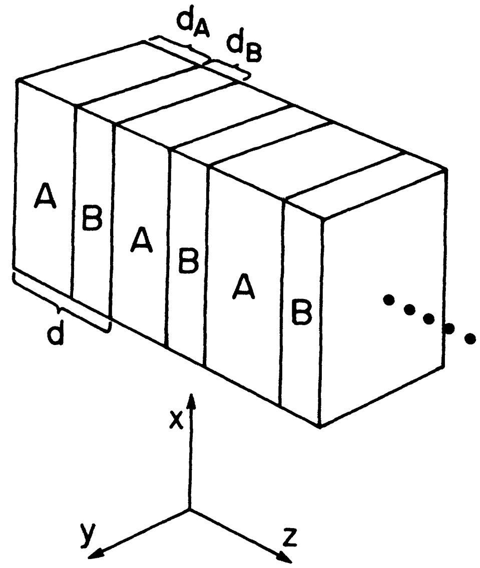

FIG. 1. Schematic representation of a periodic superlattice consisting of two different types of dielectric slabs.

Let us focus our attention first on the zero magnetic-field case. The dispersion relation for the infinite superlattice can be written as[8,9]

$$\left[\frac{\epsilon_A^2}{\alpha_A^2}+\frac{\epsilon_B^2}{\alpha_B^2}\right]\sinh\alpha_A d_A \sinh\alpha_B d_B + 2\frac{\epsilon_A\epsilon_B}{\alpha_A\alpha_B}(\cosh\alpha_A d_A\cosh\alpha_B d_B-\cos 2Q)=0\ , \qquad (1)$$

where ϵ_A and ϵ_B are the dielectric constants of media A and B, respectively, α_A and α_B are the decay constants of the field amplitudes in media A and B, d_A and d_B are the layer thicknesses, and Q is the Bloch wave vector. For the truncated superlattice, the dispersion relation for surface-plasmon polaritons, localized at the terminating interface, takes the form[9]

$$(\alpha_C\epsilon_B-\alpha_B\epsilon_C)(1-e^{-2\alpha_B d_B}) + (P_+ - e^{-2\alpha_B d_B}P_-)\tanh\alpha_A d_A = 0\ , \qquad (2)$$

where α_C is the decay constant in medium C specified by $\alpha_C^2=q^2-(\omega^2/c^2)\epsilon_C$, q is the wave vector parallel to the interfaces, ω is the frequency, and

$$P_+=\frac{(\alpha_A^2\epsilon_B\epsilon_C+\alpha_B\alpha_C\epsilon_A^2)(\alpha_C\epsilon_B-\alpha_B\epsilon_C)}{\alpha_A\epsilon_A(\alpha_C\epsilon_B+\alpha_B\epsilon_C)}\ , \qquad (3)$$

$$P_-=\frac{\alpha_A^2\epsilon_B\epsilon_C-\alpha_B\alpha_C\epsilon_A^2}{\alpha_A\epsilon_A}\ . \qquad (4)$$

Let us turn our attention to the case where the external magnetic field is parallel to the interfaces and perpendicular to the direction of propagation (x direction). For general purposes we assume that media A and B contain free charge carriers and that their dielectric tensors have the form

$$\overleftrightarrow{\epsilon}(\omega)=\epsilon_\infty \begin{vmatrix} \epsilon_3 & 0 & 0\\ 0 & \epsilon_1 & i\epsilon_2\\ 0 & -i\epsilon_2 & \epsilon_1\end{vmatrix}\ , \qquad (5)$$

where $\epsilon_1=1-[\omega_p^2/(\omega^2-\omega_c^2)]$, $\epsilon_2=\omega_c\omega_p^2/\omega(\omega^2-\omega_c^2)$, $\epsilon_3=1-(\omega_p^2/\omega^2)$, and ω_p, ω_c, and ϵ_∞ are the plasma frequency, cyclotron frequency, and background dielectric constant of the medium of interest. For the infinite superlattice, the dispersion relation can be expressed as[10]

$$2(T_A^+-T_A^-)(T_B^+-T_B^-)e^{-\alpha_A d_A-\alpha_B d_B}\cos(2Q)+(T_A^+T_A^-+T_B^+T_B^-)(1-e^{-2\alpha_A d_A})(1-e^{-2\alpha_B d_B})$$
$$=(T_A^+-T_A^-e^{-2\alpha_A d_A})(T_B^+-T_B^-e^{-2\alpha_B d_B})+(T_A^--T_A^+e^{-2\alpha_A d_A})(T_B^--T_B^+e^{-2\alpha_B d_B})\ , \qquad (6)$$

where

$$T_j^\pm=\epsilon_1^{(j)}S_j^\pm-\epsilon_2^{(j)}\ ,$$

$$S_j^\pm=(q\epsilon_1^{(j)}\mp\alpha_j\epsilon_2^{(j)})/(q\epsilon_2^{(j)}\mp\alpha_j\epsilon_1^{(j)})\ ,$$

$$\alpha_j^2=q^2-(\omega^2/c^2)\epsilon_{Vj}\ ,$$

and

$$\epsilon_{Vj}=\epsilon_1^{(j)}-(\epsilon_2^{(j)})^2/\epsilon_1^{(j)}$$

for $j=A,B$. The quantities α_j are the decay constants for the field amplitudes in the presence of the external magnetic field.

For the truncated superlattice in the presence of an external magnetic field oriented as discussed above, the dispersion relation for surface-magnetoplasmon polaritons has the form[10]

$$Z_1Z_4e^{-2\alpha_B d_B}=Z_2Z_3\ , \qquad (7)$$

where

$$Z_1=X_-(1+V_A^+)e^{\alpha_A d_A}-X_+(1+V_A^-)e^{-\alpha_A d_A}\ , \qquad (8a)$$

$$Z_2=2X_-U_{++}-2X_+U_{+-}+e^{-2\alpha_B d_B}(1+V_B^+)(X_-U_{-+}-X_+U_{--})\ , \qquad (8b)$$

$$Z_3=X_-(1-V_A^+)e^{\alpha_A d_A}-X_+(1-V_A^-)e^{-\alpha_A d_A} , \quad (8c)$$

$$Z_4=(1-V_B^+)(X_-U_{-+}-X_+U_{--}) , \quad (8d)$$

$$U_{+\pm}=(T_B^+-T_A^\pm)/(T_B^+-T_B^-) , \quad (9a)$$

$$U_{-\pm}=(T_B^--T_A^\pm)/(T_B^--T_B^+) , \quad (9b)$$

$$V_j^\pm=T_j^\pm/T_B^-, \quad j=A,B \quad (10)$$

$$X_\pm=(q\epsilon_C/\alpha_C)+T_A^\pm , \quad (11)$$

The attenuation of the field amplitudes from unit cell to unit cell in the direction normal to and away from the surface is described by a parameter λ which must be positive for a surface polariton as we define it. The surface polariton will cease to exist if any of the parameters $\lambda,\alpha_A,\alpha_B,\alpha_C$ become zero or pure imaginary.

III. NUMERICAL RESULTS

Calculations have been made of the dispersion curves for bulk and surface polaritons for several cases of the system GaAs-$Ga_{1-x}Al_xAs$ with $x=0.25$. The GaAs was assumed to contain conduction electrons with effective mass $m^*=0.07m_0$, where m_0 is the free-electron mass, and to have a background dielectric constant[13] $\epsilon_\infty=13.1$. $Ga_{1-x}Al_xAs$ was assumed to be intrinsic with dielectric constant[13] $\epsilon=12.4$. We introduce a dimensionless layer thickness $\delta=d\omega_p/c$, and we measure frequency in units of ω_p. The cyclotron frequency ω_c for all calculations involving an external magnetic field was taken to be $0.25\omega_p$. We examined situations where either GaAs or the $Ga_{1-x}Al_xAs$ constitutes the surface layer (constituent A) and situations where the GaAs layer thickness is twice that of the $Ga_{1-x}Al_xAs$ layer thickness or vice versa. We now present the results for the various cases considered.

A. $Ga_{1-x}Al_xAs$ surface layer, $\delta_A=0.5$, $\delta_B=1.0$

In Figs. 2 and 3 are shown the dispersion curves for bulk and surface polaritons in the absence of and in the presence of an external magnetic field, respectively, when the GaAs layers are twice as thick as the $Ga_{1-x}Al_xAs$ layers. There is a single, doubly degenerate surface-polariton branch for $\mathbf{B}=0$ that lies in the gap between the bulk bands in the region of large wave vector and terminates at a minimum or cutoff wave vector corresponding to the vanishing of the decay constant in $Ga_{1-x}Al_xAs$. In the presence of an external magnetic field, the gap widens, particularly at large wave vector, and the surface branch splits into two branches. For the branch marked +, the external magnetic field, the direction of propagation, and the normal to the surface form a right-handed triple, whereas for the branch marked − they form a left-handed triple. We can view these two cases as involving opposite directions of propagation of the surface polariton, and the fact that they are inequivalent is an indication of the nonreciprocity produced

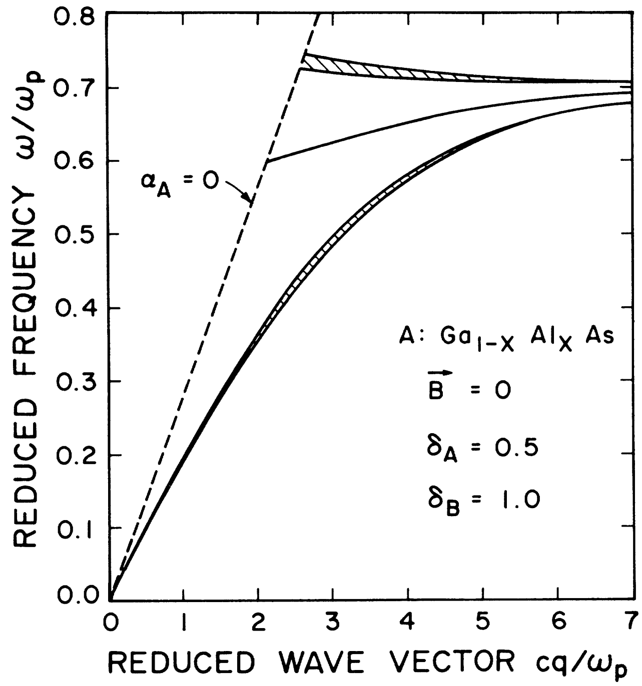

FIG. 2. Dispersion curves for surface (solid curve) and bulk (cross-hatched areas) polaritons in zero magnetic field when the GaAs layers are twice as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is $Ga_{1-x}Al_xAs$.

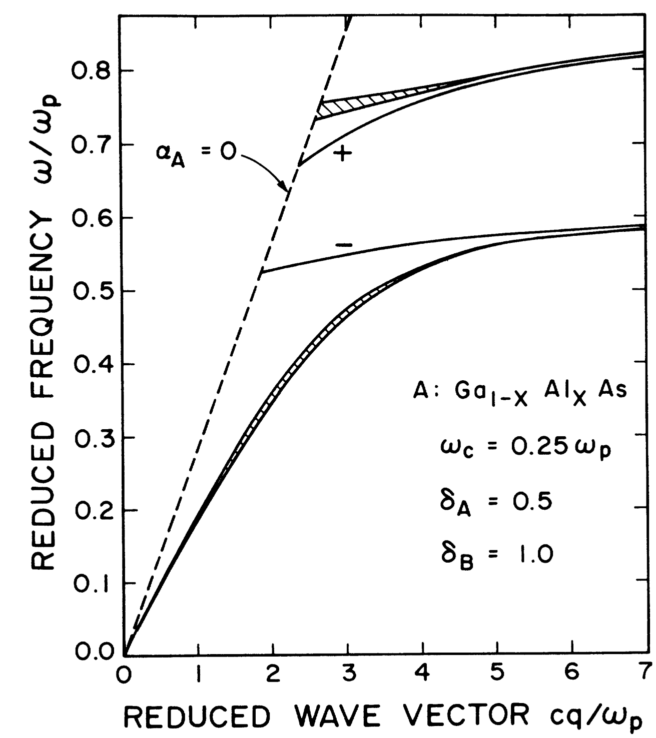

FIG. 3. Dispersion curves for surface (solid curves) and bulks (cross-hatched areas) polaritons in an external magnetic field when the GaAs layers are twice as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is $Ga_{1-x}Al_xAs$.

by the external magnetic field in the configuration employed. The positive and negative surface branches asymptotically approach the upper and lower bulk bands, respectively.

B. $Ga_{1-x}Al_xAs$ surface layer, $\delta_A = 1.0$, $\delta_B = 0.5$

We now turn to the case where $Ga_{1-x}Al_xAs$ is still the surface layer, but the GaAs layers are half as thick as the $Ga_{1-x}Al_xAs$ layers. In the absence of an external magnetic field as shown in Fig. 4, there is a gap between the bulk bands, except at the value of the reduced wave vector, cq/ω_p, equal to ~ 2.912. The existence of touching points of bulk bands has also been noted by Haupt and Wendler.[11] A doubly degenerate surface polariton exists in the region in the gap between the touching point of the bulk bands and the light line ($\alpha_A = 0$) for $Ga_{1-x}Al_xAs$. When an external magnetic field is applied, one sees in Fig. 5 that the gap widens, and the touching point of the bulk bands disappears. The surface branch splits into the + and − branches. These branches extend from the $\alpha_A = 0$ line toward larger wave vectors until the positive branch merges into the upper bulk band at $cq/\omega_p \simeq 3.03$. There are no surface polaritons in the nonretarded limit in either the presence or absence of an external magnetic field.

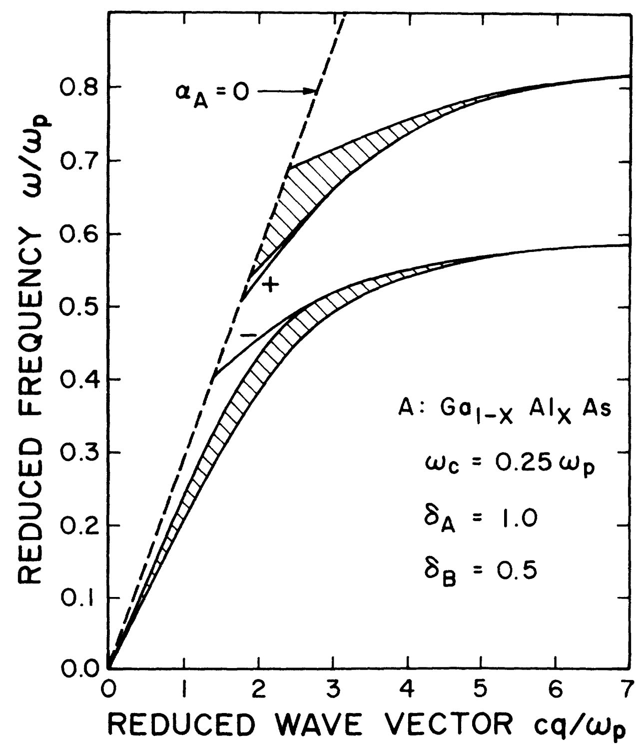

FIG. 5. Dispersion curves for surface (solid curves) and bulk (cross-hatched areas) polaritons in an external magnetic field when the GaAs layers are half as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is $Ga_{1-x}Al_xAs$.

C. GaAs surface layer, $\delta_A = 0.5$, $\delta_B = 1.0$

If we make the surface layer GaAs and take the thickness of its layers to be half that of the $Ga_{1-x}Al_xAs$ layers, we find that the bulk bands are exactly the same as those in Sec. III B, both with and without an external magnetic field. The surface modes are quite different, however. In the absence of a magnetic field, there are two doubly degenerate surface-polariton branches, as shown in Fig. 6. One branch lies in the gap between the bulk bands, starting at the point of contact of the band edges and extending out to large values of the wave vector. The second branch lies above the upper bulk band and extends from the light line of the $Ga_{1-x}Al_xAs$ out to large wave vector.

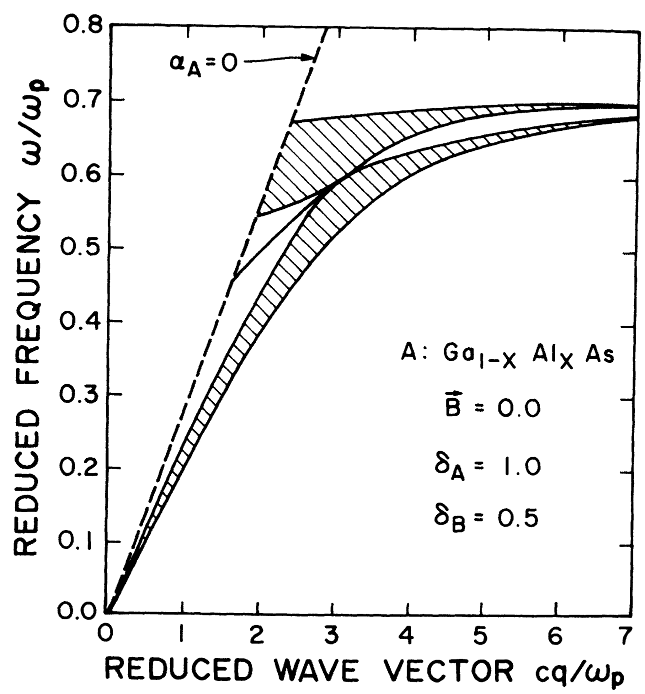

FIG. 4. Dispersion curves for surface (solid curve) and bulk (cross-hatched areas) polaritons in zero magnetic field when the GaAs layers are half as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is $Ga_{1-x}Al_xAs$.

Application of an external magnetic field causes each surface branch to split into two branches as shown in Fig. 7. The positive and negative surface branches in the gap emerge from the lower and upper bulk bands, respectively, and extend out within the gap to large wave vector. The reduced wave vectors at which the positive and negative branches emerge from the bulk bands are ~ 3.15 and ~ 2.42, respectively. For the branches above the upper bulk band, the positive branch has the higher frequency. Both branches start at the light line for $Ga_{1-x}Al_xAs$ and extend out to large wave vector.

D. GaAs surface layer, $\delta_A = 1.0$, $\delta_B = 0.5$

In this case we see from Fig. 8 that there is no surface branch in the gap between the bulk bands in the absence of an external magnetic field. Instead, there is a surface branch that lies above the upper bulk band, starting at

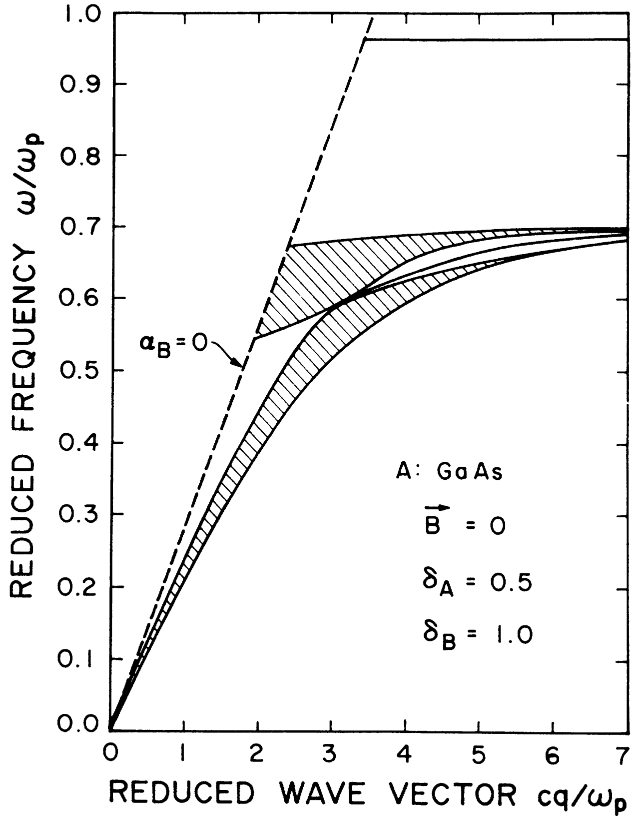

FIG. 6. Dispersion curves for surface (solid curves) and bulk (cross-hatched areas) polaritons in zero magnetic field when the GaAs layers are half as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is GaAs.

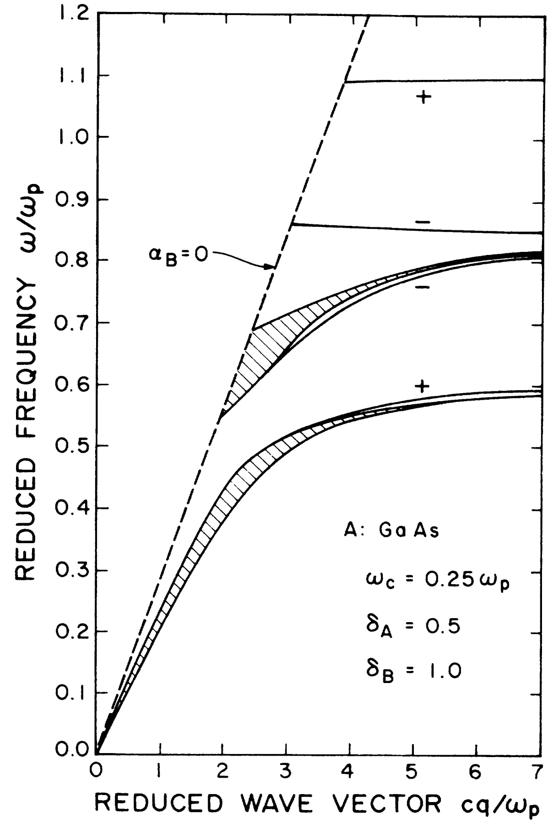

FIG. 7. Dispersion curves for surface (solid curves) and bulk (cross-hatched areas) polaritons in an external magnetic field when the GaAs layers are half as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is GaAs.

the light line for $Ga_{1-x}Al_xAs$ and extending out to large wave vector. If an external magnetic field is applied, the surface branch splits as shown in Fig. 9, the positive branch having a higher frequency than the negative branch.

IV. DISCUSSION

The diverse phenomena exhibited by the numerical results presented in Sec. III can be understood in terms of some simple considerations. Let us first take the situation provided by Figs. 2 and 8, where GaAs is the material with the thicker layers. When the surface layer is GaAs, the surface branch lies above the upper bulk band, whereas when the surface layer is $Ga_{1-x}Al_xAs$, the surface branch lies in the gap between the bulk bands. In the former case, when the wave vector is large, the surface mode has fields strongly localized at the GaAs-vacuum interface, and its frequency is closely approximated by that for the surface mode of a single GaAs-vacuum interface in the nonretarded limit—i.e., $0.964\omega_p$. The bulk bands in the large-wave-vector limit approach the frequency $0.717\omega_p$ characteristic of a single GaAs-$Ga_{1-x}Al_xAs$ interface, so the surface branch lies above the bulk bands. When the surface layer is $Ga_{1-x}Al_xAs$, on the other hand, the surface mode has fields strongly localized at a GaAs-$Ga_{1-x}Al_xAs$ interface in the large-wave-vector limit. It therefore has the frequency $0.717\omega_p$ and is sandwiched between the two bulk bands. When an external magnetic field is applied in either of these two cases, the surface mode undergoes a Zeeman-like splitting into an upper positive mode and a lower negative mode, as shown in Figs. 3 and 9.

Let us now consider the situation presented in Figs. 4 and 6, where GaAs is the material with the thinner layers. When the surface layer is $Ga_{1-x}Al_xAs$, all of the layers of the active medium (GaAs) are surrounded by layers of the inactive medium ($Ga_{1-x}Al_xAs$), which has essentially the same background dielectric constant as the active medium. Under these circumstances there is a surface mode only between the light line for $Ga_{1-x}Al_xAs$ and the touching point of the bulk bands. No surface mode exists in the nonretarded limit. When the surface layer is GaAs, however, it is now the active medium and is bounded by media of two highly different background dielectric constants, i.e., vacuum ($\epsilon_\infty=1$) and $Ga_{1-x}Al_xAs$ ($\epsilon_\infty=12.4$). Under these circumstances two surface modes appear, one above the upper bulk band and the other in the gap to the large-wave-vector side of the touching point. As the ratio of active-medium-layer thickness to inactive-medium-layer thickness increases, the touching point of the bulk bands

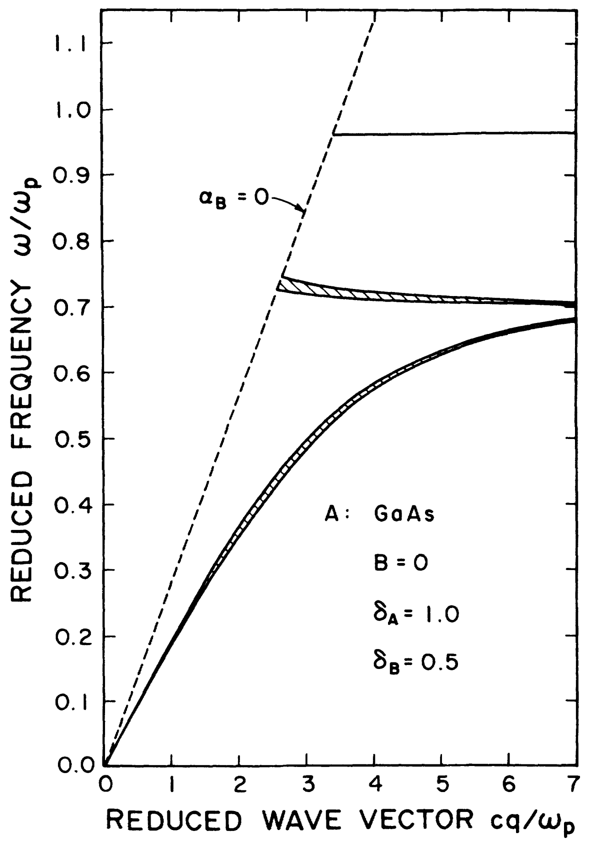

FIG. 8. Dispersion curves for surface (solid curve) and bulk (cross-hatched areas) polaritons in zero magnetic field when the GaAs layers are twice as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is GaAs.

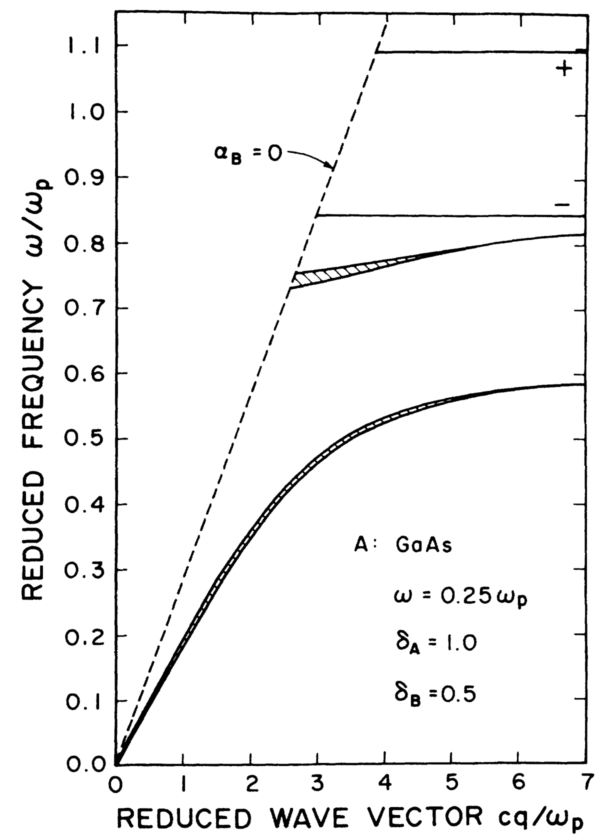

FIG. 9. Dispersion curves for surface (solid curves) and bulk (cross-hatched areas) polaritons in an external magnetic field when the GaAs layers are twice as thick as the $Ga_{1-x}Al_xAs$ layers and the surface layer is GaAs.

moves to larger wave vector, carrying the surface mode in the gap with it. This explains the absence of the gap surface mode for the case where the active-inactive-thickness ratio is greater than unity. When an external magnetic field is applied, the surface modes split as discussed in Sec. III.

Application of an external magnetic field leads to an interesting contrast in the ordering of the positive and negative surface modes in the gap as seen in Figs. 5 and 7. One may view a given surface branch as split off from a particular bulk band edge and maintaining this association on each side of the crossing of the band edges at the touching point of the bands.

Attractive possibilities exist for the experimental observation of surface-magnetoplasmon polaritons in truncated semiconductor superlattices. Either attenuated total reflection (ATR) or Raman scattering may be used. If the active medium is n-type GaAs with a conduction-electron concentration of 1×10^{16} cm^{-3} and an effective mass of $0.07m_0$, then it requires only a modest magnetic field of $\sim$6000 G to achieve $\omega_c=0.25\omega_p$, as assumed in our calculations.

In our treatment we have ignored the interaction of the current carriers with the optical phonons. This procedure is justified for carrier concentrations on the order of 10^{16} cm^{-3} because the corresponding plasma frequency, $\sim 6\times10^{12}$ rad/s is nearly an order of magnitude smaller than the long-wavelength transverse-optical-phonon frequency, $\sim 5\times10^{13}$ rad/s.

ACKNOWLEDGMENTS

The work of one of the authors (R.F.W.) was supported by the U.S. National Science Foundation (NSF) under Grant No. DMR 85-17634 and that of the other author (J.J.Q.) by the U.S. Office of Naval Research and the Laboratory for Interface Science and Engineering at Brown University funded by the IBM Corporation.

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