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Connection between spin-singlet and hierarchical wave functions in the fractional quantum Hall effect

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We propose trial expressions for the many-body wave functions of the incompressible fluid states responsible for the fractional quantum Hall effect at filling factor ν not equal to $1/m$. For $\nu=q/p$, the wave functions are related to the wave functions for the total $S=0$ states of objects with spin $(q-1)/2$. In particular, for $\nu=\frac{2}{5}$ the wave function is related to the total $S=0$, $\nu=\frac{1}{2}$ hollow-core-model ground state recently discovered by Haldane and Rezayi.

The quantum Hall effect[1,2] occurs when there is a discontinuity in the chemical potential of a two-dimensional electron gas in a strong perpendicular magnetic field[3] B at Landau-level filling factor $\nu \equiv 2\pi l^2 n$. [Here, n is the areal density of electrons and $l\equiv(\hbar c/eB)^{1/2}$).] For integral values of ν, this discontinuity evidently arises from the quantization of the electron's cyclotron motion. For fractional values of ν, however, the discontinuity was not anticipated in advance of the experimental discovery of the fractional quantum Hall effect[4] (FQHE). The way in which the fractional effect arises from electron-electron interactions became apparent only after the work of Laughlin[5] who proposed the following trial wave function for the incompressible state associated with the chemical potential discontinuity at $\nu=1/m$:

$$\Psi_m[Z]=\prod_{\substack{i,j\\ i<j}}(Z_i-Z_j)^m \exp\left(-\sum_k |Z_k|^2/4\right), \qquad (1)$$

where we have chosen the symmetric gauge, $Z\equiv x-iy$ is the electron position expressed as a complex number, l is the unit of length, and m is an odd integer. Subsequent numerical work[6] on small systems showed that the overlap between Laughlin's state and the exact ground state, $|\langle\Psi_m|\Psi_0\rangle|$ is nearly unity for sufficiently short-ranged repulsive interactions. Moreover, as the potential is weakened at short distances, the excitation gap associated with the chemical potential discontinuity and $|\langle\Psi_m|\Psi_0\rangle|$ vanish together proving that Ψ_m underlies the FQHE at $\nu=1/m$. The FQHE has been observed[7] to occur for many fractional values of ν less than (Ref. 8) $\frac{1}{2}$ ($\nu=\frac{2}{7}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{13}$), in addition to the Laughlin values, $\nu=\frac{1}{3}$ and $\frac{1}{5}$. It has been argued[9] that the incompressible states underlaying the FQHE at these additional fractions occur in a hierarchy in which the fractionally charged quasiparticles of the Laughlin state at one level form a Laughlin state themselves at the next level. To date, a quantitative test of these ideas has not proved possible although some important steps in this direction have been taken.[10] In this Rapid Communication, we present a set of trial wave functions for the so-called hierarchy states which may suggest a different picture.[11] For $\nu=q/p$ the wave functions are closely related to the total $S=0$ states of objects with spin $(q-1)/2$. Numerical calculations for small systems show that the overlap between our states and the exact ground states is nearly unity for Coulombic electron-electron interactions and that the excitation gaps and overlaps vanish together as the interaction is softened.

The studies leading to the wave functions we propose were originally motivated by experimental indications that the FQHE occurs at $\nu=\frac{1}{2}$ in the $n=1$ Landau level.[12,13] We reasoned that if the FQHE occurred at $\nu=\frac{2}{5}$ with all spins aligned, it should occur at $\nu=\frac{1}{2}$ in a total $S=0$ state.[14] Our argument was based on the observation that the incompressible spin-aligned state at $\nu=\frac{2}{5}$ can be written in the form

$$\Psi_{2/5}[Z]=Q_{2/5}[Z]\prod_{\substack{i,j\\ i<j}}(Z_i-Z_j)\prod_k e^{-|Z_k|^2/4}$$
$$=Q_{2/5}[Z]\Psi_S^F[Z], \qquad (2)$$

where Q is a completely symmetric polynomial of homogeneous order $3N(N-1)/4$. In Eq. (2), $\Psi_S^F[Z]$ is the Slater determinant for the full-Landau level ($\nu=1$) state of aligned spins. When the "correlation-factor" in Eq. (2), $Q_{2/5}[Z]$, is multiplied by the full Landau level ($\nu=2$) $S=0$ Slater determinant $\Psi_S^{S=0}(Z_1,\ldots,Z_n;\chi_1,\ldots,\chi_n)$ a simple angular momentum counting argument shows that it generates a spin-singlet state at $\nu=\frac{1}{2}$. (Here χ_i is a Pauli spinor.) Note that $Q_{2/5}[Z]$ cannot[15] have zeros at $Z_i=Z_j$. Then, using the Vandermonde determinant identity, we see that

$$\Psi_S^{S=0}=A\left[\Psi_{n,n,m}u_1\ldots u_{N/2}d_{N/2+1}\ldots d_N\right]. \qquad (3)$$

In Eq. (3) u_i and d_i are the up- and down-spin eigenstates of σ_{zi}. It follows that such a $\nu=\frac{1}{2}$ state will have the pair-correlation function $g_{\sigma\sigma'}(r)$ vanish as r^2 for like spins and approach a constant for unlike spins as $r\rightarrow 0$. In Eq. (3)

$$\Psi_{n,n,m}=\prod_{\substack{i,j\\ 1\le i<j\le N/2}}(Z_i-Z_j)^n(Z_{[i]}-Z_{[j]})^n \times\prod_{\substack{k,l\\ 1\le k\le N/2\\ 1\le l\le N/2}}(Z_l-Z_{[k]})^m, \qquad (4)$$

$[i]\equiv i+N/2$, and A is the antisymmetrization operator.

The occurrence of a strong FQHE at $\nu=\frac{2}{5}$ shows that a particularly favorable correlation factor for aligned spins is available at this filling factor and suggests that the same correlation factor might lead to a FQHE due to a total $S=0$ state at $\nu=\frac{1}{2}$ if the Zeeman energy were low enough. For example, the same argument leads from the strong $\nu=\frac{1}{3}$ FQHE for aligned spins to a $\nu=\frac{2}{5}$ FQHE, at sufficiently low Zeeman energy, due to a total $S=0$ state,[6,16] which has been confirmed by numerical calculations. In the case of $\nu=\frac{1}{2}$, $S=0$ states, as shown in a remarkable recent paper of Haldane and Rezayi[17] and discussed below, even better states can be obtained at least for some electron-electron interaction models. For spin-aligned states the antisymmetry requirement on fermion wave functions leads directly to the restriction that $\Psi_S^F[Z]$ be a factor (see, for example, Ref. 15). In contrast to the implicit assumption of our argument for the form of the spin-singlet wave function at $\nu=\frac{1}{2}$, antisymmetry does not require that $\Psi_S^{S=0}[Z:\chi]$ be a factor of $S=0$ wave functions and the possibility of better incompressible states at some filling factors in a direct consequence. For $S=0$, the antisymmetry requirements are succinctly stated by noting that any such wave function may be expressed in the "Greek-Roman" form

$$\Phi^{S=0}[Z:\chi]=\mathcal{A}[\Psi[Z]u_1\dots u_{N/2}d_{N/2+1}\dots d_N]\,, \tag{5}$$

where $\Psi[Z]$ satisfies the Fock conditions[18]

$$e(i,j)\Psi[Z]=e([i],[j])\Psi[Z]=-\Psi[Z],\ i,j\leq\frac{N}{2}\,,$$

and (6a)

$$\sum_{j=1}^{N/2}e(i,[j])\Psi[Z]=\Psi[Z]\,,\ i\leq\frac{N}{2}\,. \tag{6b}$$

Equation (6a) results from antisymmetrizing over those permutations which do not alter the spin state while Eq. (6b) emerges by requiring $\sum_{i=1}^{N}(\sigma_{xi}+i\sigma_{yi})\Phi^{S=0}[Z:\chi]\equiv 0$. Note that expectation values for any operator which is diagonal in the spin indices, and in particular $g_{\sigma\sigma'}(r)$, can be evaluated directly from $\Psi[Z]$ alone.

We can identify three classes of orbital wave functions for $S=0$ states which satisfy the Fock conditions and which are exact eigenstates of the Hamiltonian for appropriate model electron-electron interactions

$$\Psi_p^{\rm I}[Z]=\Psi_{2p,2p,2p}[Z]\Psi_{1,1,0}[Z]\,, \tag{7a}$$

$$\Psi_p^{\rm II}[Z]=\Psi_{2p,2p,2p}[Z]\Psi_{2,2,1}[Z]\ \text{per}\ |M|\,\Psi_{1,1,0}[Z]\,, \tag{7b}$$

$$\Psi_p^{\rm III}[Z]=\Psi_{2p,2p,2p}[Z]\Psi_{0,0,1}[Z]\ \text{per}\ |M'|\,\Psi_{1,1,0}[Z]\,. \tag{7c}$$

In Eqs. (7b) and (7c), M and M' are $N/2\times N/2$ matrices with matrix elements $M_{ij}=(Z_i-Z_{[j]})^{-1}$ and $M'_{ij}=(Z_i-Z_{[j]})$. In each of Eqs. (7a)-(7c), the factor on the left is

$$\Psi_{2p,2p,2p}[Z]=\prod_{\substack{i,j\\ i<j<N}}(Z_i-Z_j)^{2p}\,.$$

This factor is the same as the one used to generate the Laughlin states from the full Landau-level wave function for aligned spins. Note that if the Fock conditions[19] are satisfied in Eqs. (7) for $p=0$, they will be satisfied for all values of p. The factor on the right in each of Eqs. (7) is the $S=0$ full Landau-level orbital wave function so that the remaining factors in each case define the correlations in the system. {Note that Eq. (6a) may be used to prove that $\Psi_{1,1,0}[Z]$ must be a factor of $\Psi[Z]$.} As we have remarked above, the correlation factors need not be, and in the case of Eqs. (7b) and (7c) are not, symmetric under interchange of opposite spin particles. The wave functions of Eq. (7a) are the analogues of the Laughlin states mentioned above[6,16] and, for sufficiently low Zeeman energy, should form incompressible $S=0$ ground states at $\nu=2/(4p+1)=2,\frac{2}{5},\frac{2}{9},\dots$ $\Psi_p^{\rm I}$ is an exact nondegenerate zero-energy ground state for a hard-core model with pseudopotential parameters[6,9,17] $V_i=1$ for $i\leq 2p-1$ and $V_i=0$ otherwise. For $\Psi_p^{\rm I}[Z]$, $g_{\sigma\sigma'}(r)\sim r^{2+4p}$ for parallel spins and $\sim r^{4p}$ for opposite spins. The wave function of Eq. (7b) is (for $p=0$) the wave function recently discovered by Haldane and Rezayi.[17] It is an exact zero-energy incompressible $S=0$ ground state at $\nu=1/(2p+2)=\frac{1}{2},\frac{1}{4},\frac{1}{6},\dots$ for the hollow-core model with $V_i=1$ for $i\leq 2p-1$, $V_{2p}=0$, $V_{2p+1}=1$, and $V_i=0$ for $i\geq 2p+2$. The pair-correlation function vanishes as r^{6+4p} for parallel spins and as r^{4p} for opposite spins. Haldane and Rezayi[17] have proposed that this state is responsible for the $\upsilon=2+\frac{1}{2}$ FQHE. The wave functions of Eq. (7c) has not, to our knowledge, been suggested previously. It is an exact zero-energy $S=0$ ground state[20] at $\nu=1/(2p+1)=1,\frac{1}{3},\frac{1}{5},\dots$ for the hard-core model with $V_i=1$ for $i\leq 2p$ and $V_i=0$, otherwise. For this state, $g_{\sigma\sigma'}(r)\sim r^{2+4p}$ for both parallel spins and opposite spins. We remark that adding the factor per $|M'|$ or per $|M|$ changes a $S=N/2$ fermion wave function into a singlet fermion without changing the filling factor, while the factor $\det|M|$ per $|M|=\Psi_{1,1,-1}[Z]$ per $|M|$ changes a $S=N/2$ boson wave function into a fermion singlet wave function.

We have learned that at filling factors like $\nu=1/2$, $S=0$ states of electrons can take advantage of their two-spin-component nature to selectively reduce the probability of electrons being close together. This suggests the conjecture that at some filling factors spinless (or equivalently spin-aligned) electrons might form incompressible states by dividing into two components, i.e., that a spinless electron state, $\Psi^F[Z]$, may be generated from the orbital part of an $S=0$ state, $\Psi^{S=0}[Z]$, by the following prescription:

$$\frac{\Psi^F[Z]}{\Psi_S^F[Z]}\equiv Q^F[Z]=\mathcal{S}\left[\frac{\Psi^{S=0}[Z]}{\Psi_{1,1,0}[Z]}\right]\equiv\mathcal{S}[Q^{S=0}[Z]]\,. \tag{8}$$

In Eq. (8), $\mathcal{S}$ is the symmetrization operator which is required by the antisymmetry of $\Psi^F[Z]$. Noting that $\Psi_S^F[Z]=\Psi_{1,1,1}[Z]$, we see that Eq. (8) is equivalent to

$$\Psi^F[Z]=\mathcal{A}[\Psi_{0,0,1}[Z]\Psi^{S=0}[Z]]\,. \tag{9}$$

For $\Psi_p^I[Z]$ [Eq. (7a)] the correlation factor is completely symmetric, nothing is gained from forming two components, and the prescription of Eq. (9) generates the Laughlin states at $\nu=1/(2p+1)$. In the case of $\Psi_p^{\rm II}[Z]$, however, Eq. (9) produces

$$\Phi_p^{\rm II}[Z]=\prod_{\substack{i,j\\ i<j}}(Z_i-Z_j)^{2(p+1)}S[\Psi_{1,1,0}[Z]\ {\rm per}\ |M|]\,, \tag{10}$$

which, according to our conjecture should be the incompressible state responsible for the FQHE at $\nu=2/(4p+5)=\frac{2}{5}, \frac{2}{9}, \frac{2}{13}, \ldots$. Similarly,

$$\Phi_p^{\rm III}[Z]=\prod_{\substack{i,j\\ i<j}}(Z_i-Z_j)^{2p+1}S[\Psi_{0,0,1}[Z]\ {\rm per}\ |M'|] \tag{11}$$

should be the state responsible for the FQHE at $2/(4p+3)=\frac{2}{3}, \frac{2}{7}, \frac{2}{11}, \ldots$.

We have tested our conjecture by performing a series of numerical calculations for small systems on a sphere. $\Psi_p^{\rm II}[Z]$ and $\Psi_p^{\rm III}[Z]$ were generated by numerically solving the model systems for which they are exact eigenstates. The symbolic multiplication by $\Psi_{0,0,1}[Z]$ was performed by computer. The overlaps between the resulting states and the Coulomb interaction ground states for several filling factors are listed in Table I and compared with overlaps between Laughlin's state and Coulomb interaction ground states at $\nu=\frac{1}{3}$. In Fig. 1, we show how the overlap of the ground state with $\Phi_0^{\rm II}[Z]$, and the excitation energies vary as the pseudopotential parameter, V_1, varies near its Coulomb value. It is clear from these results that the excitation gap, and, hence, the FQHE at $\nu=\frac{2}{5}$, is associated with $\Phi_0^{\rm II}[Z]$.

Our results can be extended to filling factors $\nu=q/p$ for $q>2$. We conjecture that the incompressible states responsible for the FQHE's at such fractions are associated with singlet ground states for spin $(q-1)/2$ objects. For example, for spin-1 objects and the $\{V_1=1,\ V_i=0,\ i\neq 1\}$ hollow-core model, the orbital part of the $S=0$, zero-energy incompressible ground-state wave function at $\nu=\frac{3}{5}$ is

$$\Psi_{3/5}[Z]=\prod_{\substack{i,j\\ i<j\leq N/3}}(Z_i-Z_j)^3(Z_{[i]}-Z_{[j]})^3(Z_{\{i\}}-Z_{\{j\}})^3 \times\prod_{k,l=1}^{N/3}(Z_k-Z_{[l]})(Z_k-Z_{\{l\}})(Z_{[k]}-Z_{\{l\}}) \times{\rm per}\ |M^{(1)}|\ {\rm per}\ |M^{(2)}|\ {\rm per}\ |M^{(3)}|\,. \tag{12}$$

In Eq. (12) $M^{(1)}$, $M^{(2)}$, and $M^{(3)}$ are $N/3\times N/3$ matrices with matrix elements

$$M_{i,j}^{(1)}=(Z_i-Z_{[j]})^{-1}\,,\quad M_{i,j}^{(2)}=(Z_i-Z_{\{j\}})^{-1}\,,$$

and

$$M_{i,j}^{(3)}=(Z_{[i]}-Z_{\{j\}})^{-1}\,,$$

$[i]=i+N/3$, and $\{i\}=i+2N/3$. This wave function refers, in the sense of Eq. (5), to a spin configuration in which the first third of the electrons have spin up ($S_z=1$), the second third have spin down ($S_z=-1$), and the last third have spin sideways ($S_z=0$). Following earlier arguments, we conjecture in analogy with Eq. (9) that the incompressible state responsible for the spinless FQHE at $\nu=\frac{3}{7}$ is

$$\Psi_{3/7}=A\left\{\prod_{k,l=1}^{N/3}(Z_k-Z_{[l]})(Z_k-Z_{\{l\}}) \times(Z_{[k]}-Z_{\{l\}})\Psi_{3/5}[Z]\right\}. \tag{13}$$

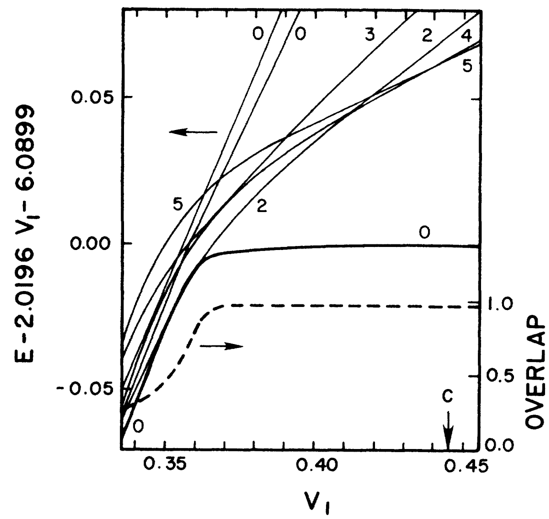

FIG. 1. Low-lying eigenenergies and ground-state overlap as a function of the $l=1$ pseudopotential parameter, V_1, at $\nu=\frac{2}{5}$ for 6 electrons. All other psdeudopotential parameters are fixed at their Coulomb potential values. (The Coulomb value for V_1 is indicated by the arrow labeled by C.) The dashed lines shows the overlap between the ground state and the incompressible state we propose (the scale for this curve is on the right). The solid lines show eigenenergies and are labeled by the angular momentum of the state. (The scale for these curves is on the left.)

TABLE I. Overlap between $\Phi_p^{\rm II}$, $\Phi_p^{\rm III}$ and the ground-state wave function with the Coulomb interaction Φ_c. N is the number of electrons, and d is the number of independent uniform ($L=0$) states.

	$\Phi_0^{\rm II}$ ($\nu=\frac{2}{5}$)	
N	$\langle\Phi_0^{\rm II}\mid\Phi_c\rangle$	d
6	0.9858	3
8	0.9771	8
	$\Phi_1^{\rm II}$ ($\nu=\frac{2}{9}$)	
N	$\langle\Phi_1^{\rm II}\mid\Phi_c\rangle$	d
4	0.9999	2
	$\Phi_1^{\rm III}$ ($\nu=\frac{2}{7}$)	
N	$\langle\Phi_1^{\rm III}\mid\Phi_c\rangle$	d
4	0.9999 •	2
6	0.9839	10
	Ψ_3 ($\nu=\frac{1}{3}$)	
N	$\langle\Psi_3\mid\Phi_c\rangle$	d
7	0.99636[a]	10

[a]G. Fano, F. Ortolani, and E. Colombo, Phys. Rev. B 34, 2670 (1986).

Numerical calculations show that the overlap between this state and the Coulomb interaction ground state for 6 electrons is 0.947.

As emphasized by Haldane and Rezayi,[17] the $\nu=\frac{1}{2}$ FQHE may be naively thought of as resulting from the pairing of opposite-spin electrons into charge-$2e$ bosons, which form a symmetric Laughlin state. In the hollow-core-model ground state, a tendency toward pairing is evident in the collapse of the correlation hole between opposite spin electrons [i.e., $g_{\uparrow\downarrow}(r)\sim r^0$]. The possibility that pairing of electrons may also play a role in the formation of hierarchy states was suggested very early on by Halperin.[21] Indeed, the trial wave functions based on Halperin's idea,[21] at least in the modified form appropriate for spherical geometry,[11] compare as well with the exact ground states for small systems as the wave functions studied here. The connection uncovered here, between spin-singlet and hierarchical wave functions provides another, perhaps more natural, point of view on pairing in the hierarchy states. The pairing suggests that the off-diagonal long-range order[22,23] for Laughlin's $\nu=1/m$ states, in which each particle binds m zeros of the analytic wave function, persists to hierarchy states with p zeros bound to a pair of electrons for $\nu=2/p$.

Note added in proof. F. D. M. Haldane and E. H. Rezayi have pointed out that $\Psi_p^{\rm III}$ [Eq. 7(c)] is formed from quasihole states of the $S=N/2, \nu=1/(2p+1)$ state.

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[20]$\Psi_{2p+1,2p+1,2p+1}[Z]$ is an $S=N/2$ exact zero-energy eigenstate of the same model Hamiltonian, which occurs for a system one flux quantum smaller in area. This state may be obtained by rotating the Laughlin state in spin space.

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