Nonreciprocal propagation and localization of plasmons by a magnetic field in finite semiconductor *n-i-p-i* superlattices

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We consider the effects of an external magnetic field on plasmons propagating in finite *n-i-p-i* (*n*-doped material–insulator–*p*-doped material–insulator) superlattices. The magnetic field is applied parallel to the interfaces and we consider propagation perpendicular to the applied field. We find that the magnetic field can have two effects: (1) the dispersion relations can become nonreciprocal, or (2) the excitations can become (nonreciprocally) localized. The particular effect depends critically upon the number of layers; for a structure composed of an unequal number of *n*- and *p*-doped layers, the modes can be strongly localized, while in a structure composed of an equal number of *n*- and *p*-doped layers the dispersion curves become nonreciprocal.

I. INTRODUCTION

Collective excitations of artificially layered media have been the subject of a wide range of theoretical and experimental studies in recent years. These investigations have included magnons,[1-3] plasmons,[4-8] and phonons,[9-11] to name a few.

Many of these studies have been aimed at uncovering the effects of various symmetry lowerings on the collective-mode spectra. In previous papers, we have demonstrated that (1) surface plasmons propagating in a symmetric structure consisting of metallic films separated by (initially) vacuum-filled gaps, with the whole structure surrounded by vacuum, will be dramatically localized by a small perturbation in the dielectric strength of the gaps, and (2) plasmons propagating in the same symmetric structure (vacuum-filled gaps) are basically unaffected by severe variations in the thickness of the constituent layers. Other investigators have discussed collective plasmon modes in quasiperiodic structures.[12,13]

In addition to the lowering of symmetry caused by physical perturbations in the layering of superlattice, one may also lower the symmetry by the application of a magnetic field. Both magnetic fields parallel[14] and perpendicular[15] to the surfaces of the superlattice have been considered. The case where the field is parallel to the surface and propagation is perpendicular to the field is particularly interesting since nonreciprocal surface wave propagation $[\omega(k)\neq\omega(-k)$, i.e., a reversal of the wave vector k can lead to a different frequency $\omega]$ is possible, a very dramatic consequence of the lowering of symmetry due to a magnetic field.[16]

In this paper, we present a brief theoretical study demonstrating the effects of an external magnetic field on the collective plasmon modes in *n-i-p-i* (*n*-doped-material–insulator–*p*-doped-material–insulator) superlattices with different symmetries. Both nonreciprocal propagation as well as nonreciprocal localization are discussed. An explicit dispersion relation is not derived; however, a theoretical development is presented which is valid for any number of layers (or layer thicknesses), including the magnetic field, and some numerical studies are presented which illustrate the symmetry-lowering effects of the applied field.

Precedent for this paper comes from the study of applied magnetic fields in antiferromagnetic films[17] and in superlattices.[18] It has been shown that the effect of an external magnetic field on thin-film antiferromagnets depends upon the symmetry of the sublattices. If we assume the film consists of alternating layers of oppositely directed spins, labeled A and B, it turns out that for an equal number of layers A and B ($ABABAB$, for instance) the magnon dispersion is nonreciprocal (i.e., the allowed energy for a given wave vector is not the same for $+k$ as for $-k$). On the other hand, if there is one more layer of A spins than B spins ($ABABABA$), the propagation remains reciprocal. Similar results have been found for superlattices composed of ferromagnetic films, where the magnetizations of alternate films are oppositely directed.

To understand these effects, consider the following symmetry argument. In order for the propagation to be reciprocal, there must exist a set of symmetry operations which will take the wave vector from $+k$ to $-k$ while leaving the structure in its original configuration. If no set of operations can be found, then the propagation need not be reciprocal. Therefore, the symmetry of the structure becomes a critical feature. In the first case mentioned above, $ABABAB$ with the propagation direction defined by the x axis, and with a magnetic field applied along the z axis, there is no symmetry operation which takes k to $-k$ and leaves the structure and field direction unchanged. In contrast, the structure $ABABA$ can be taken into itself and take k into $-k$ by a simple rotation of 180° about an axis parallel to z centered in the midpoint of the central film. Thus this structure has reciprocal propagation.

In addition to the nonreciprocity induced in the dispersion relations, the symmetry-lowering effects of the magnetic field can also serve to *localize* certain spin waves. If one examines the amplitude of the excitations for surface magnons as a function of depth, two distinct cases again emerge, depending upon the symmetry of the structure.

For the even case, $ABABAB$, there is reflection symmetry about the midplane—even though at first glance this does not appear to be the case. Recall that in this geometry, when spins are reflected about a mirror plane parallel to the axis of ths spins, not only does the position go from y to $-y$, but the directions of the spins are reversed as well. Because of this, the excitations which had a definite parity in the absence of an applied field retain a definite parity with an applied field, independent of propagation direction. For the odd case, $ABABA$, there is no midplane symmetry, and as a result the surface modes which had definite parity without a field can be significantly localized by an applied field. The localization, either to the top surface or the bottom surface, depends upon the direction of propagation.

Based on this, one might expect that a superlattice consisting of alternating films composed of materials containing charge carriers of opposite sign would display a similar character, since the sense of rotation of positively charged particles about a magnetic field is opposite to that of negatively charged particles. Such a structure could be realized with a so-called n-i-p-i superlattice, a structure with a unit cell of an n-doped material, an insulating material, a p-doped material, and an insulating material. In this paper, we will consider an n-i-p-i superlattice consisting of layers of GaAs. We find that a superlattice of this type displays the nonreciprocal behavior discussed above.

II. THEORY

In this section we only outline the theoretical development of the dispersion relations for magnetoplasmons propagating in a finite n-i-p-i superlattice since the results are a straightforward generalization of earlier work. At first we assume a two-constituent superlattice with alternating layers of material 1 and material 2. Material 1 is assumed to carry free charges and to occupy layers $n=1,3,5,\ldots$, while material 2 is assumed to be an insulator occupying layers $2,4,6,\ldots$. Furthermore, the free charges in material 1 are assumed to change sign from one region to the next. Thus the unit cell consists of four layers. Throughout the paper, we assume the thickness of the films with free charges ($n=1,3,5,\ldots$) is twice that of the insulating layers ($n=2,4,6,\ldots$). The geometry is shown in Fig. 1.

It is possible to solve this problem analytically and obtain an implicit dispersion relation for the finite structure. Recently Djafari-Rouhani and Dobrzynski have developed methods for dealing with superlattices with unit cells containing N layers.[19] However, for the small number of layers we deal with here, numerical methods are much simpler and provide all the necessary information.

In the films containing free charge carriers ($n=1,3,5,\ldots$), the application of a magnetic field along z gives us the following dielectric tensor:

$$\epsilon=\begin{bmatrix}\epsilon_1 & -i\epsilon_2 & 0\\ i\epsilon_2 & \epsilon_1 & 0\\ 0 & 0 & \epsilon_3\end{bmatrix}, \qquad (1)$$

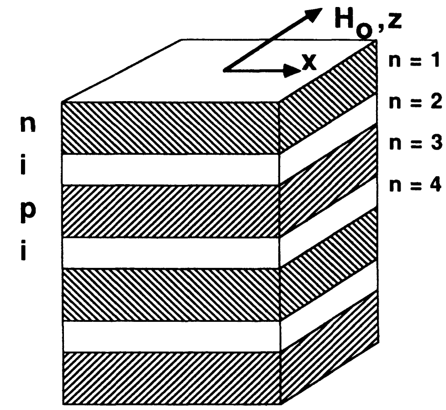

FIG. 1. The geometry considered in this paper. The odd-numbered layers are doped alternately n and p type, while the even-numbered layers are insulating. We consider only propagation along the $+x$ and $-x$ directions.

where

$$\epsilon_1=\epsilon_\infty[1+\omega_p^2/(\omega_c^2-\omega^2)]\ , \qquad (2)$$

$$\epsilon_2=\epsilon_\infty\omega_c\omega_p^2/\omega(\omega_c^2-\omega^2)\ , \qquad (3)$$

$$\epsilon_3=\epsilon_\infty[1-\omega_p^2/\omega^2]\ , \qquad (4)$$

and where ϵ_∞ is the background dielectric constant, ω_p is the plasma frequency, and ω_c is the cyclotron frequency eB/mc. We note a sign change in the gyrotropic (off-diagonal) elements depending upon the sign of the charge carriers in the film. In the insulating layers, the dielectric tensor is isotropic and given by $\epsilon=\epsilon_\infty I$.

If we assume the long-wavelength static limit, we can use the static form of Maxwell's equations. This allows the introduction of the electrostatic scalar potential $\mathbf{E}=-\nabla\phi$. We use the constitutive relation $\mathbf{D}=\epsilon\mathbf{E}$, and find the following equation for the scalar potential ϕ:

$$\epsilon_1(\partial^2/\partial x^2+\partial^2/\partial y^2)\phi+\epsilon_3(\partial^2/\partial z^2)\phi=0\ . \qquad (5)$$

This is the analog of the Walker equation for magnetic systems. If we now assume propagation only along the x direction (perpendicular to the applied field), we then have nothing which depends on z and we reduce Eq. (5) to the simpler two-dimensional Laplace's equation which is valid in all regions:

$$(\partial^2/\partial x^2+\partial^2/\partial y^2)\phi=0\ . \qquad (6)$$

This equation has solutions

$$\Phi(\mathbf{x},t)=(A_n e^{k\delta_{ny}}+A_n' e^{-k\delta_{ny}})e^{i(kx-\omega t)} \qquad (7)$$

in film n,

$$\Phi(\mathbf{x},t)=A_0 e^{|k|y}e^{i(kx-\omega t)} \qquad (8)$$

above the superlattice ($y<0$),

$$\Phi(\mathbf{x},t)=A_F e^{-|k|y}e^{i(kx-\omega t)} \qquad (9)$$

below the superlattice, where δ_{ny} measures the distance along the y axis inside a single film n and A_n, A'_n, A_0, and A_F are, as yet, unknown amplitudes.

We are now ready to impose boundary conditions. These demand that ϕ and the normal component of **D** be continuous at each interface. Matching the potentials across each boundary yields

$$A_0 = A_1 + A'_1 , \tag{10}$$

$$A_1 e^{kd_1} + A'_1 e^{-kd_1} = A_2 + A'_2 , \tag{11}$$

$$A_2 e^{kd_2} + A'_2 e^{-kd_2} = A_3 + A'_2 , \tag{12}$$

$$A_N e^{kd_N} + A'_N e^{-kd_N} = A_F , \tag{13}$$

etc., d_n is the thickness of the nth film and there are a total of N films. The normal component of **D** is given by

$$D_y = -i\epsilon_2 \frac{\partial\phi}{\partial x} - \epsilon_1 \frac{\partial\phi}{\partial y} . \tag{14}$$

Matching normal **D** gives us

$$\epsilon_{\text{out}} A_0 = \sigma[(-\epsilon_2+\epsilon_1)A_1 + (-\epsilon_2-\epsilon_1)A'_1] , \tag{15}$$

$$(-\epsilon_2+\epsilon_1)e^{kd_1} A_1 + (-\epsilon_2-\epsilon_1)e^{-kd_1} A'_1 = \epsilon_{\text{film}}(A_2 - A'_2) , \tag{16}$$

$$\epsilon_{\text{film}}(A_2 e^{kd_2} - A'_2 e^{-kd_2}) = (+\epsilon_2+\epsilon_1)A_3 + (+\epsilon_2-\epsilon_1)A'_3 , \tag{17}$$

$$\vdots$$

$$(\epsilon_2+\epsilon_1)e^{kd_N} A_N + (\epsilon_2-\epsilon_1)e^{-kd_N} A'_N = \frac{\epsilon_{\text{out}}}{-\sigma} A_F , \tag{18}$$

etc., where ϵ_{out} is the dielectric constant valid outside the structure and ϵ_{film} is the dielectric constant valid inside the gaps. σ is $|k|/k$ and takes into account the match of the equations on the interior of the structure to those on the exterior. Note that Eq. (18) is written with the assumption that the final film contains charge carriers of opposite type to the first film and is in contact with the same material as the first film. Changes to include a substrate or different final films are straightforward.

At this stage we may put the entire set of boundary condition equations into matrix form. Setting the determinant of the matrix of coefficients of A_0, A_F, A, and A' to zero yields the dispersion relations. In Sec. III, we solve these equations numerically and obtain the dispersion curves for a few example geometries.

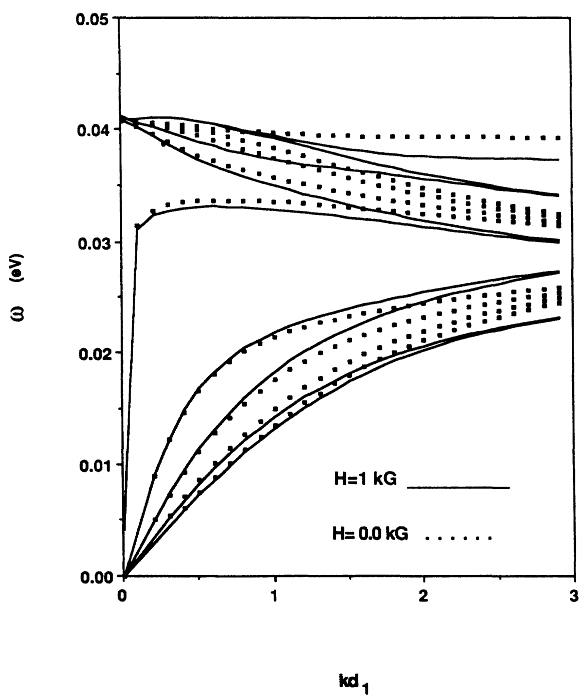

FIG. 2. The dispersion curves for a superlattice composed of five layers of doped GaAs separated by insulating GaAs gaps. The dotted lines represent the dispersion without a magnetic field, while the solid lines are for the same structure with a 1-kG field applied along the z axis. Note the frequency shifts with the field.

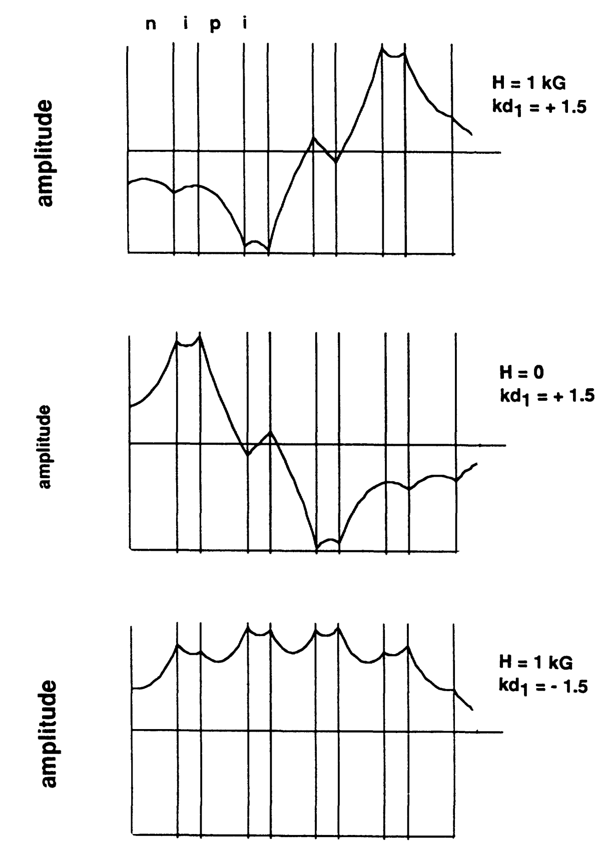

FIG. 3. The electrostatic potential as a function of depth for the five-layer structure of Fig. 2. The bottom picture represents a bulk mode with $kd_1 = 1.5$, $\omega = 0.033\,35$ eV. The upper two pictures are the same mode with an applied field ($\omega = 0.0327$ eV for the $+k$ case and $\omega = 0.0323$ eV for the $-k$ case). The top picture represents propagation along $+k$ while the center picture represents propagation along $-k$. Note that the localization depends upon propagation direction.

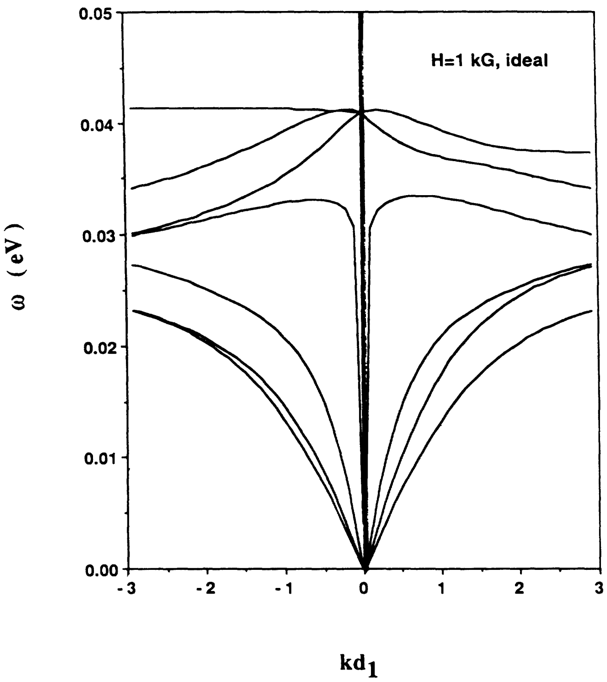

FIG. 4. The dispersion curves for a four-layer ideal structure with an applied field of 1 kG. Note the nonreciprocal behavior produced by the field. The upper two modes on both sides ($+k$ and $-k$) are too close to be resolved on this scale, as are the second-highest modes on the $+k$ side.

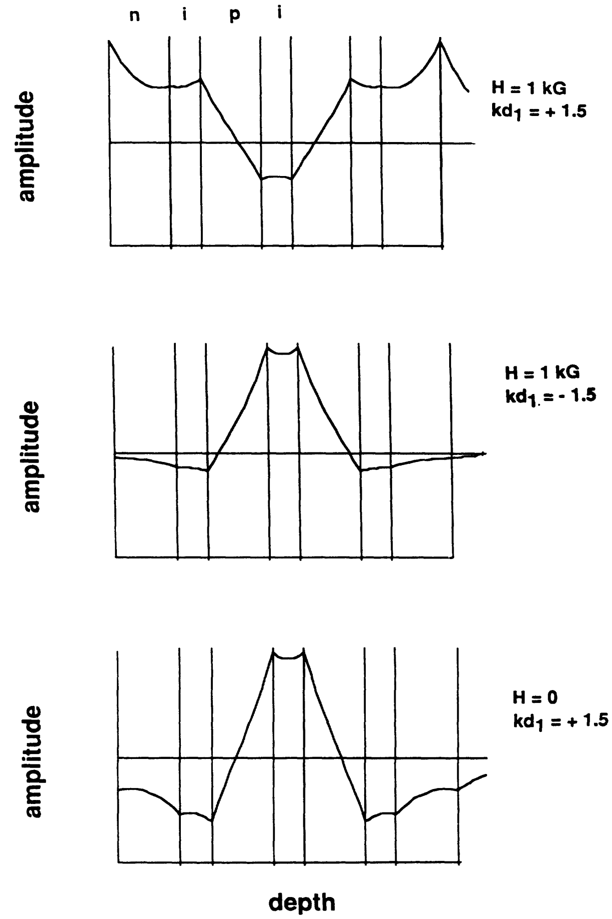

FIG. 5. The electrostatic potential of a bulk mode for the four-layer geometry of Fig. 4. Again the bottom picture represents the mode in zero field ($\omega=0.03633$ eV), while the upper two pictures represent the mode with an applied field ($\omega=0.03657$ eV for the $+k$ case and $\omega=0.03753$ eV for the $-k$ case). Note that the mode retains its definite parity regardless of the field or propagation direction.

III. RESULTS AND DISCUSSION

In this section, we will focus on numerical results of the previous section, demonstrating the symmetry-lowering effects of applied magnetic fields on plasmons propagating in finite semiconductor *n-i-p-i* superlattices. We consider two broad types of *n-i-p-i* geometries: (1) an "ideal" GaAs superlattice, which is surrounded on all sides by vacuum, and (2) a GaAs superlattice which rests on a semi-infinite insulating GaAs substrate. In all cases, we take $\epsilon_\infty=13.13$, and $\epsilon_{out}=1.0$. We assume further that the doping concentrations of the conductive layers is the same for both *n*- and *p*-type layers, and that these active layers respond according to Eq. (1). The assumed doping level is $n=10^{18}$ cm^{-3}, a fairly high value, which gives a plasma frequency $\omega_p=0.04075$ eV.

We begin by examining the ideal geometry, consisting of five active layers (three *n* type, two *p* type) separated by four insulating gaps. Under a field applied along the *z* axis, this geometry is analogous to the *ABABA* odd-spin geometry discussed in Sec. I, and therefore the dispersion relations are reciprocal. In Fig. 2 we have interposed the dispersion curves in zero field (dotted lines) and the dispersion curves for $H_0=1$ kG (solid lines). We note that significant shifts can be obtained by application of a magnetic field with the largest frequency shifts on the order of the cyclotron frequency, $\omega_c=0.004075$ eV, a relative shift of about 10%. Lower doping levels (with a resulting smaller ω_p) would allow for significantly larger relative shifts.

Figure 3 shows the evolution of the electrostatic potential for a bulk mode in the five-layer ideal geometry. The lower picture shows the mode in zero field, while the two upper pictures show the mode in an applied field of 1 kG. The top picture corresponds to $kd_1=+1.5$ while the middle picture corresponds to $kd_1=-1.5$. The zero-field case has a definite parity (even) while the modes for $+k$ and $-k$ show localization to the opposite surfaces of the superlattice, as discussed in Sec. I.

The next set of figures are studies of the ideal geometry with four layers of doped GaAs (two each of *n* and *p* doping) separated by three insulating gaps. This corresponds to the even-spin geometry of Sec. I, *ABAB*, and as a result the dispersion relations are nonreciprocal while the potentials retain a definite parity. Figure 4 displays the dispersion curves, for positive and negative *k*, for the four-layer geometry with a field of 1 kG. We note the nonreciprocity, particularly for the surface modes, the lowest- and highest-frequency bulk modes. The uppermost modes are surface modes, and show the highest degree of nonreciprocity.

Figure 5 shows a typical bulk-mode potential in the four-layer geometry; again the lowest picture gives the zero-field results, and the upper pictures are with a field. While the upper pictures illustrate the expected retention of definite parity under an applied field for the four-layer ideal geometry, the application of the field does dramatically alter the scalar potential as a function of depth for $+k$. Thus even though there is a definite parity in all cases, there is still a clear nonreciprocity in that the scalar potential is clearly different for $+k$ and $-k$.

We now turn our attention to the more realizable case of a GaAs n-i-p-i superlattice resting on a GaAs substrate. In this case, the existence of the substrate is itself a lowering of the structure's symmetry, and therefore we expect the results to be slightly less simple than for the ideal geometry. For example, the rotation-symmetry operations presented in Sec. I no longer apply; if we rotate the structure 180° about the z axis ($k \to -k$), this brings the substrate above the superlattice, and there is no way to return the structure to its original configuration. Therefore, the dispersion curves need not be reciprocal for any layer geometry. Furthermore, the presence of the substrate changes the symmetry of the boundary conditions—the top surface is in contact with vacuum and the bottom surface is in contact with an insulator with high background dielectric constant—and therefore the potentials need not display definite parity.

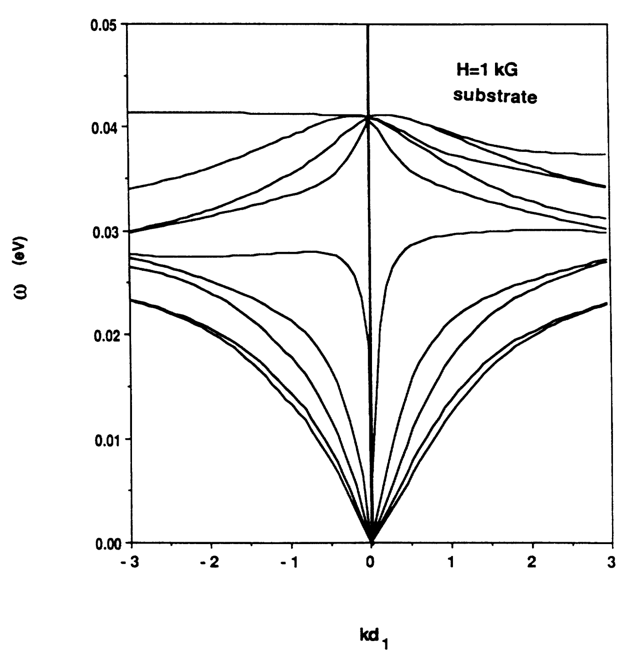

FIG. 6. The dispersion curves for the five-layer geometry including a GaAs substrate and an applied field. The substrate lowers the symmetry and therefore the dispersion relations become nonreciprocal for this geometry. We note that the second-highest mode on the $-k$ side is really an unresolved pair of modes.

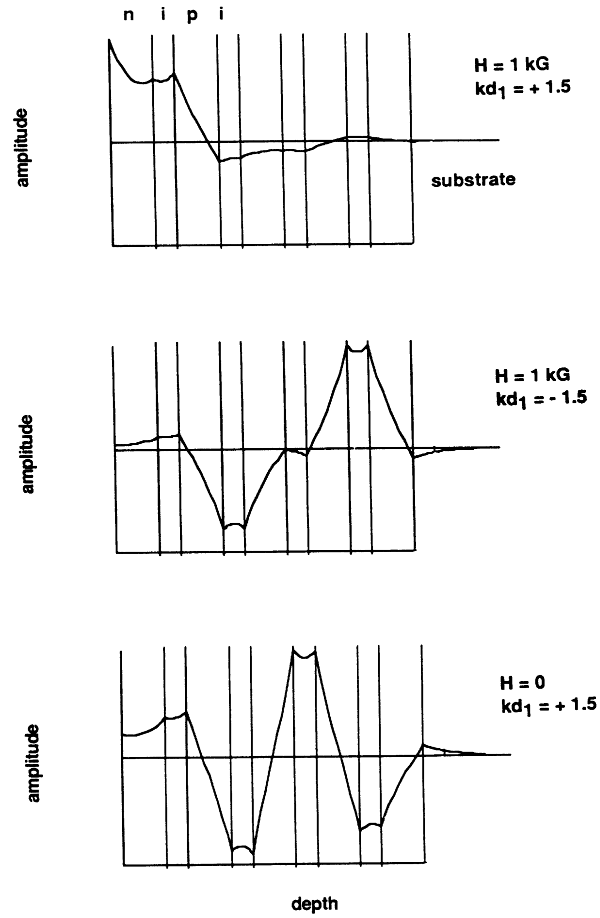

FIG. 7. The evolution of the electrostatic potential for the five-layer–substrate geometry. The bottom picture is again zero field ($\omega = 0.036\,56$ eV), the absence of definite parity is caused by the substrate. Here only the $+k$ propagation ($\omega = 0.036\,40$ eV) direction is strongly localized by the combination of field and substrate (top picture). The $-k$ case has a frequency of $\omega = 0.037\,40$ eV.

As a first example, consider the five-layer geometry with a substrate. Figure 6 shows the dispersion curves (positive and negative k) for the five-layer structure with an applied field. Note the nonreciprocity, caused by the combination of the field and the substrate, which did not exist for the five-layer ideal structure. Figure 7 shows the behavior of a typical bulk mode as a function of applied field. The bottom picture is the mode for zero field; the absence of definite parity here is a consequence of the substrate. The middle picture ($kd_1 = +1.5$), although not clearly localized at either the top or bottom surface, shows a distinct difference from the zero-field case. The top picture ($kd_1 = -1.5$), however, displays a strong localization of the potential to the upper surface.

In Fig. 8, we see the effect of the magnetic field and the substrate on the four-layer structure. Once again, there is relatively strong nonreciprocity, particularly for the lowest-frequency pair and the highest-frequency pair, similar to Fig. 4. Figure 9 shows a surface mode for the

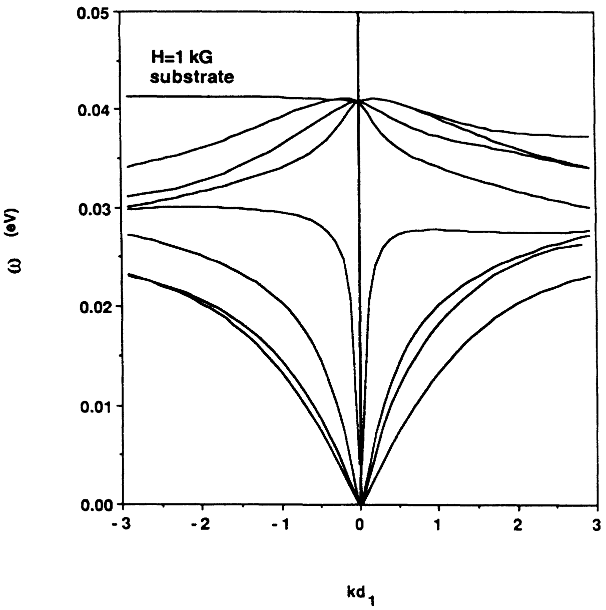

FIG. 8. The dispersion relations for the four-layer geometry with a substrate and an applied field. Once again, strong nonreciprocity is observed. We note that the upper modes on the $-k$ side may be resolved, in contrast to Fig. 4.

four-layer–substrate structure. By noting the zero-field picture (bottom), we see that the symmetry lowering of the substrate destroys the definite parity of the mode. The other pictures demonstrate that the field has little effect. We find this result to be a general trend for the four-layer geometry—the substrate has a dramatic effect while the applied field does not.

In summary, the *n-i-p-i* superlattice structure, consisting of oppositely-doped active layers separated by insulating gaps, responds to an applied magnetic field in a way analogous to antiferromagnetic thin films and magnetic superlattices. Depending upon the symmetry of the structure, the applied field can cause (1) significant nonreciprocity in the dispersion relations, or (2) strong localizations of the excitations themselves. The presence of a substrate serves to further lower the symmetry, and the combination of applied field and substrate has significant effects on the propagation of plasma oscillations.

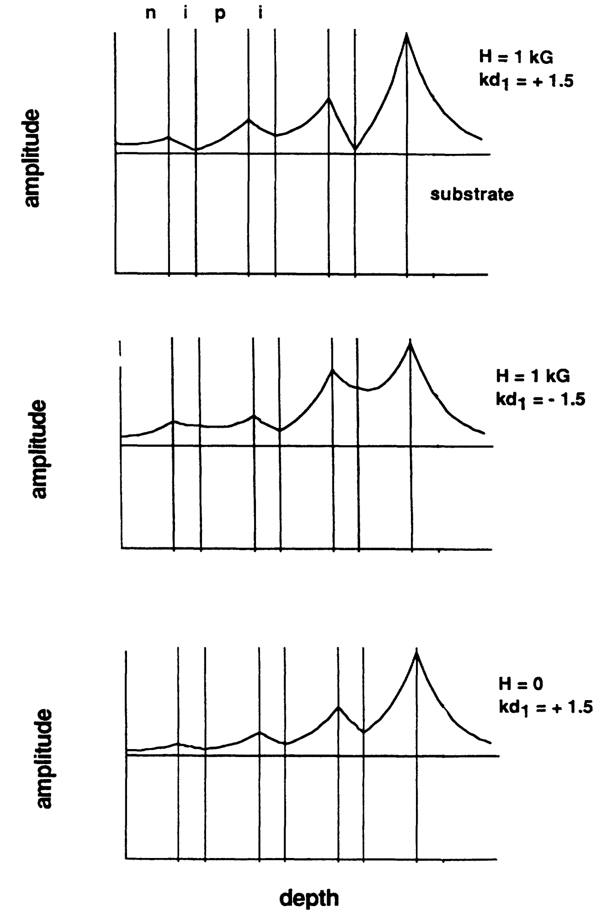

FIG. 9. The electrostatic potential for a surface mode in the four-layer–substrate geometry. Here the parity of the mode is destroyed by the presence of the substrate and the applied field has little effect. The frequencies of the modes are $+k$ case, $\omega=0.028\,807$ eV; $-k$ case, $\omega=0.029\,939$ eV; and zero-field case, $\omega=0.027\,674$ eV.

ACKNOWLEDGMENT

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