Upper critical magnetic field of superconductors with a dielectric gap on the Fermi-surface sections

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In the framework of the model describing a superconductor with a dielectric gap on the Fermi-surface sections, the equation for the upper critical magnetic field $H_{c2}(T)$ in the dirty limit is obtained and solved. It is shown that due to the electron spectrum dielectrization, the quantities $H_{c2}(0)$ and $[\,|\,dH_{c2}/dT\,|\,]_{T=T_c}$ increase and the curvature $[d^2H_{c2}/dT^2]_{T=T_c}$ may become positive. These results are in good agreement with experimental data for a number of substances with charge- and spin-density waves, e.g., Chevrel phases, layered transition-metal dichalcogenides, antiferromagnetic organic metals $(TMTSF)_2X$, $Cr_{1-x}Re_x$ alloys (where TMTSF is an abbreviation for tetramethyltetraselenafulvalene), and the heavy-fermion superconductor URu_2Si_2.

I. INTRODUCTION

Recently it became clear that for a number of important (especially high-field) superconducting materials the temperature dependence of upper critical magnetic field H_{c2} is substantially different from the dependence of Werthamer, Helfand, and Hohenberg[1] obtained by them in the framework of BCS theory.[2] Namely, in a wide-temperature region starting from $T=T_c$ (the latter being the temperature of superconducting transition when the magnetic field $H=0$) the curve $H_{c2}(T)$, contrary to Ref. 1, possesses zero or even positive curvature. Here are some superconductors with $d^2H_{c2}/dT^2\geq 0$: ternary molybdenum chalcogenides (Chevrel phases)[3,4] layered group-V transition-metal dichalcogenides[5,6]; $NbSe_3$ (Ref. 7); oxide systems Rb_xWO_3 (Ref. 8), and $BaPbO_3$-$BaBiO_3$ (Ref. 9); $A15$ compounds Nb_3Sn and V_3Si (Ref. 10); pseudoternary boride $Ho(Ir_{0.7}Rh_{0.3})_4B_4$ (Ref. 11); heavy-fermion superconductor URu_2Si_2 (Refs. 12 and 13); low-dimensional organic superconductors $(TMTSF)_2X$ (Ref. 14), and β-$(BEDT\text{-}TTF)_2I_3$.[15]

Various explanations of this fact have been proposed. But their applicability was confined in every case to a definite class of superconducting materials. In particular, anomalies of $H_{c2}(T)$ may be caused by macrostructural distortion in superconductors with low dimensionality,[5,7] Fermi surface (FS), and order-parameter anisotropy,[16,17] compensation of the external magnetic field by localized magnetic moments,[18] size effect in layered or granular systems with Josephson coupling between constituents,[19,20] enhancement of the Coulomb pseudopotential μ^* in weakly[21(a)] and strongly[21(b)] disordered metals due to Altshuler-Aronov effect,[22] magnetic-field influence on the diffusion coefficient in the vicinity of the Anderson transition,[23] fluctuation renormalization of the coherence length,[23] and bipolaronic mechanism of superconductivity.[24]

At the same time most superconductors with deviations of $H_{c2}(T)$ from the BCS behavior[1] show a partial dielectrization of the electron spectrum. Namely, the FS of such superconductors involves sections with nesting. There electron and hole branches of the quasiparticle spectrum are degenerate [see Eq. (7)]. The degeneracy disappears when a structural or an antiferromagnetic phase transition occurs, so that a dielectric gap Σ emerges on the formly degenerate FS sections.

The coexistence on the FS sections of a superconducting gap and a spin-singlet dielectric gap Σ was discovered in Chevrel phases,[25,26] layered transition-metal dichalcogenides,[5] the quasi-one-dimensional metal $NbSe_3$,[27] the bronze Rb_xWO_3,[8] solid solutions $BaPb_{1-x}Bi_xO_3$,[28,29] Laves phases, and $A15$ compounds.[30] Unlike Chevrel and Laves phases where the dielectrization is accompanied[25,30] by a negative-temperature coefficient of resistance R, the curves $R(T)$ for the compounds Nb_3Sn and V_3Si with $A15$ structure involve only cusps[30] when T is equal to the temperature T_d of the structural transition. The latter is the Peierls-type transition in $A15$ structures and leads to the easily measured tetragonal distortion ϵ_t of the crystal lattice, the quantity ϵ_t being proportional to the dielectric gap Σ. Weak manifestation of the gapping effects here is due to the small portion of the dielectrized FS sections. This conclusion is testified by a small enhancement of T_c (≈ 0.3 K) in V_3Si after the structural transition is suppressed technologically.

On the other hand, superconducting transition in metals with spin-triplet dielectric gap Σ is observed in $Ho(Ir_xRh_{1-x})_4B_4$ ($x\geq 0.6$),[31] $Cr_{1-x}Re_x$ alloys ($x>0.18$),[32] organic conductors $(TMTSF)_2X$ ($X=AsF_6$, PF_6, ClO_4),[14,33,34] and URu_2Si_2.[12,13]

No doubt, the development of the dielectric gap on the FS sections should influence the electromagnetic properties of superconductors. The effect of electron-spectrum degeneracy on $H_{c2}(T)$ in a simplified quasi-one-

dimensional model with a complete dielectrization of electron spectrum was considered earlier for spin-density waves (SDW) (Ref. 35) and charge-density waves (CDW).[36] Unfortunately, in our view the main results of these papers are erroneous. This statement can be easily proved if one notes that the corrections to the electromagnetic kernel obtained there are proportional to (Σ/E_F) (SDW case[35]) or $(\Sigma/E_F)^2$ (CDW case[36]), where E_F is the Fermi energy. They cannot be taken into account in principle by the BCS-type theory.[35,36] The correct calculation of $H_{c2}(T)$ in the same quasi-one-dimensional model for a SDW superconductor with Neél temperature $T_N < T_c$ was carried out in Ref. 37. In contrast to the work of Ro and Levin,[37] the present work involves the model of more realistic partial dielectrization rather than the full one. Moreover, we study the case $T_N > T_c$, whereas in Ref. 37, $T_c > T_N$. Finally, the upper critical field of CDW superconductors is considered here as well as the critical field of antiferromagnetic superconductors. Only the latter case was studied in Ref. 37.

In this paper the dependence $H_{c2}(T)$ is calculated analytically in the limiting T regions [$T \to 0$ and $(T-T_c)/T_c \ll 1$] for the model of partial gapping[28,38–41] for superconducting anisotropic metal with CDW or SDW in a most realistic case when Σ is much greater than T_c in agreement with the experiment. It is shown that electron-spectrum dielectrization always leads to increase of $H_{c2}(0)$ and the slope $|dH_{c2}/dT|$ near T_c in comparison to the BCS theory.[1] For certain values of the model parameters the quantity d^2H_{c2}/dT^2 can change its sign. Our treatment is quite different from those (Refs. 5, 7, 16, 17, 19–22, and 24) described above and has the advantage of making possible the consideration of electromagnetic properties of various types at first glance, unlike superconductors from the unified point of view.

The plan of the paper is the following. In Sec. II, using the anisotropic metal model due to Bilbro and McMillan,[38] we obtain the self-consistency equations for a superconductor with nesting sections on the FS in the presence of the magnetic field. The electromagnetic kernel taking into account various impurity scattering processes in the "ladder" approximation is calculated in Sec. III. The dependence H_{c2} versus T provided that $T \to 0$ or $T \to T_c$, is obtained in Sec. IV. Finally, Sec. V contains a discussion of the results and comparison to the experimental data. Short letters with some results of this paper were published previously.[42,43]

II. SELF-CONSISTENCY EQUATIONS

The Hamiltonian of the electron subsystem in the presence of impurities and external magnetic field $\mathbf{H}$ has the form ($\hbar = k_B = 1$):

$$\hat{\mathcal{H}} = \sum_{\alpha,\beta,i} \int d\mathbf{r} [\psi^\dagger_{i\alpha}(\mathbf{r}) \epsilon_i(\hat{\mathcal{P}}) \psi_{i\alpha}(\mathbf{r}) - \mu_B^* (\sigma \cdot \mathbf{H})_{\alpha\beta} \psi^\dagger_{i\alpha}(\mathbf{r}) \psi_{i\beta}(\mathbf{r})] + \sum_{\substack{\alpha,\beta \\ i,j,l,m}} V_{ijlm} \int d\mathbf{r}\, \psi^\dagger_{i\alpha}(\mathbf{r}) \psi^\dagger_{j\beta}(\mathbf{r}) \psi_{m\beta}(\mathbf{r}) \psi_{l\alpha}(\mathbf{r})$$
$$+ \sum_{\substack{\alpha,\beta \\ i,j}} \int d\mathbf{r} \int d\mathbf{r}'\, \psi^\dagger_{i\alpha}(\mathbf{r}) W^{\alpha\beta}_{ij}(\mathbf{r},\mathbf{r}') \psi_{j\beta}(\mathbf{r}') \,. \tag{1}$$

Here $\psi^\dagger_{i\alpha}(\mathbf{r})$ and $\psi_{i\alpha}(\mathbf{r})$ are the creation and annihilation operators of an electron in the ith band with spin projection $\alpha = \pm\frac{1}{2}$ at the point $\mathbf{r}$, μ_B^* is the effective Bohr magneton, σ are Pauli matrices, V_{ijlm} and $W_{ij}(\mathbf{r},\mathbf{r}')$ are matrix elements of the contact interaction (including electron-phonon as well as Coulomb contributions) and the electron-impurity interaction, respectively. The quantity

$$\epsilon_i(\hat{\mathcal{P}}) = \frac{\hat{\mathcal{P}}^2}{2m_i} - \mu \tag{2}$$

is the electron energy in the ith band. The following notations were introduced in Eqs. (1) and (2): μ is the chemical potential, $\hat{\mathcal{P}} = \hat{\mathbf{p}} - (e/c)\mathbf{A}(\mathbf{r})$ is the generalized momentum operator, $\mathbf{A}$ is the vector potential, e is the electron charge, c is the light velocity, and m_i is the quasiparticle mass in the ith band.

The Dyson-Gor'kov equations for the normal $G_{ij}(\mathbf{r},\mathbf{r}';\omega_n)$ and anomalous $F_{ij}(\mathbf{r},\mathbf{r}';\omega_n)$ (nonzero-) temperature Green's functions of a superconductor in a magnetic field,[2] have the form [$\omega_n = (2n+1)\pi T$]:

$$[i\omega_n - \epsilon_i(\hat{\mathcal{P}})] G^{\alpha\beta}_{ij}(\mathbf{r},\mathbf{r}';\omega_n) + \sum_\gamma \mu_B^* (\sigma \cdot \mathbf{H})_{\alpha\gamma} G^{\gamma\beta}_{ij}(\mathbf{r},\mathbf{r}';\omega_n)$$
$$- \sum_{m,\gamma} \int d\mathbf{r}'' [\Sigma^{\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'') + W^{\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'')] G^{\gamma\beta}_{mj}(\mathbf{r}'',\mathbf{r}';\omega_n) + \sum_{m,\gamma} \int d\mathbf{r}''\, \Delta^{\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'') F^{\dagger\gamma\beta}_{mj}(\mathbf{r}'',\mathbf{r}';\omega_n) = \delta(\mathbf{r}-\mathbf{r}')\delta_{ij}\delta_{\alpha\beta} \,, \tag{3}$$

$$[i\omega_n + \epsilon_i(\hat{\mathcal{P}}^\dagger)] F^{\dagger\alpha\beta}_{ij}(\mathbf{r},\mathbf{r}';\omega_n) - \sum_\gamma \mu_B^* (\sigma \cdot \mathbf{H})_{\alpha\gamma} F^{\dagger\gamma\beta}_{ij}(\mathbf{r},\mathbf{r}';\omega_n)$$
$$+ \sum_{m,\gamma} \int d\mathbf{r}'' [\Sigma^{*\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'') + W^{*\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'')] F^{\dagger\gamma\beta}_{mj}(\mathbf{r}'',\mathbf{r}';\omega_n) - \sum_{m,\gamma} \int d\mathbf{r}''\, \Delta^{\dagger\alpha\gamma}_{im}(\mathbf{r},\mathbf{r}'') G^{\gamma\beta}_{mj}(\mathbf{r}'',\mathbf{r}';\omega_n) = 0 \,. \tag{4}$$

The normal $\Sigma^{\alpha\beta}_{ij}(\mathbf{r},\mathbf{r}')$ and anomalous $\Delta^{\alpha\beta}_{ij}(\mathbf{r},\mathbf{r}')$ self-energy parts are determined by the self-consistency conditions, which in the considered case of contact interaction take the form

$$\Sigma_{ij}^{\alpha\beta}(\mathbf{r})=T\sum_{\omega_n}\sum_{l,m}\left[V_{imlj}G_{lm}^{\alpha\beta}(\mathbf{r},\mathbf{r};\omega_n)-\delta_{\alpha\beta}V_{imjl}\sum_{\gamma}G_{lm}^{\gamma\gamma}(\mathbf{r},\mathbf{r};\omega_n)\right], \tag{5}$$

$$\Delta_{ij}^{\alpha\beta}(\mathbf{r})=T\sum_{\omega_n}\sum_{l,m}V_{ijlm}F_{lm}^{\alpha\beta}(\mathbf{r},\mathbf{r};\omega_n). \tag{6}$$

To describe the superconductors with a partial dielectrization of the electron spectrum, we use the Bilbro-McMillan model,[38–41] for which the dielectric gap $\Sigma_{12}^{\alpha\beta}$ appears on that FS section, where the nesting condition for the electron- and hole-spectrum branches is fulfilled:

$$\epsilon_1(\mathbf{p})=-\epsilon_2(\mathbf{p}+\mathbf{Q})\equiv\epsilon(\mathbf{p}). \tag{7}$$

The vector $\mathbf{Q}$ determines the period of the emerging CDW or SDW. On the rest of the FS the quasiparticle spectrum $\epsilon_3(\mathbf{p})$ is nondegenerate. The following treatment will be carried out in the approximation of strong mixing of states from the different FS sections, when

$$V_{iiii}=V_{iijj}=V_{ii33}\equiv -V<0 \quad (i,j=1,2).$$

As a consequence, a single superconducting order parameter Δ develops on the whole FS. We regard its spin structure as singlet in agreement with the experimental data available so far. On the other hand, the spin structure of the dielectric order parameter matrix $\hat{\Sigma}_{12}$ may be either singlet or triplet for the existing substances, as was specified in Sec. I. Then

$$\hat{\Sigma}_{12}=\Sigma[\hat{\sigma}_0(1-\alpha)+\alpha\hat{\sigma}_3],$$

where $\alpha=0$ for the spin-singlet dielectric gap (CDW) and $\alpha=1$ for the spin-triplet one (SDW).

In order to determine the upper critical magnetic field $H_{c2}(T)$ in the self-consistency equation (6), it is enough to retain only the terms linear in Δ, so that the equation for Δ, taking into account Eq. (4), reads

$$\Delta(\mathbf{r})=VT\sum_{\omega_n}\int d\mathbf{r}'[2\tilde{G}_{11}^{\beta\beta}(\mathbf{r},\mathbf{r}';\omega_n)\tilde{G}_{11}^{-\beta,-\beta}(\mathbf{r},\mathbf{r}';-\omega_n)+2\tilde{G}_{12}^{\beta\beta}(\mathbf{r},\mathbf{r}';\omega_n)\tilde{G}_{21}^{-\beta,-\beta}(\mathbf{r},\mathbf{r}';-\omega_n)$$
$$+\tilde{G}_{33}^{\beta\beta}(\mathbf{r},\mathbf{r}';\omega_n)\tilde{G}_{33}^{-\beta,-\beta}(\mathbf{r},\mathbf{r}';-\omega_n)]\Delta(\mathbf{r}'). \tag{8}$$

Here $\tilde{G}_{ij}^{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega_n)$ is the normal metal Green's function in a magnetic field.

After the extraction of the standard exponential factor, the expression for the function $\tilde{G}_{ij}^{\alpha\beta}$ takes the form

$$\tilde{G}_{ij}^{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega_n)=G_{ij0}^{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega_n)\exp\left[i\frac{e}{c}\int_{\mathbf{r}'}^{\mathbf{r}}\mathbf{A}(\mathbf{s})\cdot d\mathbf{s}\right], \tag{9}$$

where $G_{ij0}^{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega_n)$, according to Eq. (3), satisfies the equation

$$\left[i\omega_n-\frac{1}{2m_i}\left[\hat{\mathbf{p}}+\frac{e}{c}(\mathbf{r}-\mathbf{r}')\times\mathbf{H}\right]^2+\mu\right]G_{ij0}^{\alpha\beta}(\mathbf{r},\mathbf{r}';\omega_n)$$
$$+\sum_{\gamma}\mu_B^*(\boldsymbol{\sigma}\cdot\mathbf{H})_{\alpha\gamma}G_{ij0}^{\gamma\beta}(\mathbf{r},\mathbf{r}';\omega_n)-\sum_{m\gamma}\int d\mathbf{r}''[\Sigma_{im}^{\alpha\gamma}(\mathbf{r},\mathbf{r}'')+W_{im}^{\alpha\gamma}(\mathbf{r},\mathbf{r}'')]G_{mj0}^{\gamma\beta}(\mathbf{r}'',\mathbf{r}';\omega_n)=\delta(\mathbf{r}-\mathbf{r}')\delta_{\alpha\beta}\delta_{ij}. \tag{9a}$$

The quasiclassical approximation consists in dropping the diamagnetic term in braces, which is suggested to be small. It is valid until the penetration depth $\lambda(T)$ of the magnetic field and the coherence length $\xi(T)$ are much less than the cyclotron radius r_c. All the materials of interest for us are type-II superconductors. Thus in order to estimate the validity of the quasiclassical approximation for CDW and SDW superconductors it is enough to look at the relationship between r_c and $\lambda(T)$. First of all, note that even for the highest-field superconductors with $H_{c2}\lesssim 600$ kOe (Ref. 30) the diamagnetic corrections to the equations for Green's functions, which lead to the Landau quantization, are insignificant, because $eH_{c2}/m^*c\ll v_F/l$. Here m^* is the effective electron mass, v_F is the Fermi velocity, and l is the mean free path. For example, for $PbMo_6S_8$, where $H_{c2}=600$ kOe, $l=20$ Å, $v_F=1.6\times10^8$ cm/s, and $m^*=1.3m_0$ (m_0 is the free electron mass) we have $eH_{c2}l/m^*cv_F\approx 10^{-2}\ll 1$. On the other hand, as was shown in Ref. 44 in the case of highest possible contaminated or disordered superconductors, the quasiclassical approximation for large fields should be used with caution. This remark does not concern the most interesting for us Ginzburg-Landau temperature region near T_c, where $H_{c2}(T)\cong(1-T/T_c)$ [see below, Eq. (37)], and $\lambda(T)\simeq(1-T/T_c)^{-1/2}$. Then

$$\lambda(T)/r_c(H_{c2})=e\lambda(T)H_{c2}(T)/m^*v_Fc\to 0.$$

At the same time, when $T\ll T_c$ the required relationship between r_c and $\lambda(0)$ is fulfilled with difficulty, e.g., for the above-mentioned ternary compound $PbMo_6S_8$ with $T_c\simeq 12.6$ K we have $r_c\approx 2\times10^{-5}$ cm, and $\lambda(0)\approx 1.3\times10^{-5}$ cm.

But fortunately, just in this temperature region the anomalies of the curve $H_{c2}(T)$ that we want to explain disappear altogether. This behavior is reproduced in our theory as well [see Eq. (38)].

After the substitution of Eq. (9), Eq. (8) is transformed to

$$\Delta(\mathbf{r})=VT\sum_{\omega_n}\int d\mathbf{r}'\,K_0^{\beta,-\beta}(\mathbf{r}-\mathbf{r}';\omega_n)\exp[i(\mathbf{r}-\mathbf{r}')\hat{\mathbf{\Pi}}]\Delta(\mathbf{r})\ . \tag{10}$$

Here

$$\hat{\mathbf{\Pi}}=i\nabla+\frac{2e}{c}\mathbf{A}(\mathbf{r})\ , \tag{11}$$

$$\begin{aligned}K_0^{\beta,-\beta}(\mathbf{r}-\mathbf{r}';\omega_n)&=2[G_{110}^{\beta\beta}(\mathbf{r}-\mathbf{r}';\omega_n)G_{110}^{-\beta,-\beta}(\mathbf{r}-\mathbf{r}';-\omega_n)+G_{120}^{\beta\beta}(\mathbf{r}-\mathbf{r}';\omega_n)G_{210}^{-\beta,-\beta}(\mathbf{r}-\mathbf{r}';-\omega_n)]\\&\quad+G_{330}^{\beta\beta}(\mathbf{r}-\mathbf{r}';\omega_n)G_{330}^{-\beta,-\beta}(\mathbf{r}-\mathbf{r}';-\omega_n)\\&\equiv 2K_{di0}^{\beta,-\beta}(\mathbf{r}-\mathbf{r}';\omega_n)+K_{nd0}^{\beta,-\beta}(\mathbf{r}-\mathbf{r}';\omega_n)\ .\end{aligned} \tag{12}$$

The quantity $K_0^{\beta,-\beta}$ is the integral equation kernel in which the correlations inside an electron pair due to the multiple scattering by the same impurity are not yet taken into account. Provided the Fourier transform of the kernel $K_0^{\beta,-\beta}$ is introduced, we obtain from Eq. (10) the following differential equation:

$$\left[1-VT\sum_{\omega_n}K_0^{\beta,-\beta}(\mathbf{q}=-\hat{\mathbf{\Pi}};\omega_n)\right]\Delta(\mathbf{r})=0\ . \tag{13}$$

III. CALCULATION OF COOPER PAIR PROPAGATOR WITH THE ALLOWANCE FOR MULTIPLE IMPURITY SCATTERING

The unrenormalized pair propagator $K_0(\mathbf{q})$ actually has to be replaced by the full kernel $K(\mathbf{q})$ including contributions from the multiple scattering by one impurity.[1] Our model is three dimensional, so we treat the impurity scattering in a "ladder" approximation[2] and neglect the localization corrections from maximally crossed diagrams[45] and corrections due to the interference of the electron-electron interaction and elastic impurity scattering.[21(a),22] Furthermore, we ignore also the interband impurity scattering and obtain

$$K^{\beta,-\beta}(\mathbf{q};\omega_n)=2K_{di}^{\beta,-\beta}(\mathbf{q};\omega_n)+K_{nd}^{\beta,-\beta}(\mathbf{q};\omega_n)\ , \tag{14}$$

$$K_{di\,(nd)}^{\beta,-\beta}(\mathbf{q};\omega_n)=K_{di0(nd0)}^{\beta,-\beta}(\mathbf{q};\omega_n)[1+a_{d\,(nd)}K_{di\,(nd)}^{\beta,-\beta}(\mathbf{q};\omega_n)+b_{d\,(nd)}K_{di\,(nd)}^{-\beta,\beta}(\mathbf{q};\omega_n)]\ , \tag{15}$$

where

$$\begin{aligned}a_d&=\frac{1}{\pi N_d(0)}\left[\frac{1}{\tau_d^0}+\frac{1}{3\tau_d^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^d}\right],\quad b_d=\frac{2}{3\pi N_d(0)}\left[\frac{1}{\tau_d^{\text{s.o.}}}-\frac{1}{\tau_{\text{ex}}^d}\right],\\ a_{nd}&=\frac{1}{2\pi N_{nd}(0)}\left[\frac{1}{\tau_{nd}^0}+\frac{1}{3\tau_{nd}^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^{nd}}\right],\quad b_{nd}=\frac{1}{3\pi N_{nd}(0)}\left[\frac{1}{\tau_{nd}^{\text{s.o.}}}-\frac{1}{\tau_{\text{ex}}^{nd}}\right].\end{aligned} \tag{16}$$

Here $N_d(0)$ and $N_{nd}(0)$ are the electron densities of states on the degenerate and nondegenerate FS sections; $\tau_{d\,(nd)}^0$, $\tau_{d\,(nd)}^{\text{s.o.}}$, $\tau_{\text{ex}}^{d\,(nd)}$ are the elastic relaxation times due to the nonmagnetic, spin-orbit (s.o.), and magnetic scatterers for the same sections. (The explicit forms for $\tau_{d\,(nd)}$ can be found in Ref. 41.) In Eq. (16) a possible magnetic scattering anisotropy was not taken into account. This approximation is valid for CDW superconductors ($\alpha=0$), if the paramagnetic limitation of H_{c2} is unessential, i.e., the paramagnetic limit H_p is much larger than H_{c2}. As to the antiferromagnetic superconductor ($\alpha=1$), it is really characterized by four magnetic relaxation times $\tau_{\text{ex}\parallel}^{d\,(nd)}$, $\tau_{\text{ex}\perp}^{d\,(nd)}$, but in the case when $H_p\gg H_{c2}$ the final results of the theory contain only isotropic quantities $1/\tau_{\text{ex}}^{d\,(nd)}=1/\tau_{\text{ex}\parallel}^{d\,(nd)}+1/\tau_{\text{ex}\perp}^{d\,(nd)}$.

In order to calculate the kernels $K_0(\mathbf{q};\omega_n)$ and $K(\mathbf{q};\omega_n)$, we make use of the electron normal state Green's functions[41] for a Bilbro-McMillan model[38]:

$$\begin{aligned}G_{110}^{\uparrow\uparrow\,(\downarrow\downarrow)}&=-\frac{i\tilde{\omega}_n^{\pm}+\epsilon(\mathbf{p})}{(\tilde{\omega}_n^{\pm})^2+\epsilon^2(\mathbf{p})+\Sigma^2},\quad G_{120}^{\uparrow\uparrow(\downarrow\downarrow)}=\frac{R^{\uparrow\uparrow\,(\downarrow\downarrow)}\Sigma}{(\tilde{\omega}_n^{\pm})^2+\epsilon^2(\mathbf{p})+\Sigma^2}\ ,\\ R^{\uparrow\uparrow\,(\downarrow\downarrow)}&=\{1,1-2\alpha\},\quad G_{330}^{\uparrow\uparrow\,(\downarrow\downarrow)}=\frac{1}{i\tilde{\Omega}_n^{\pm}-\epsilon_3(\mathbf{p})}\ .\end{aligned} \tag{17}$$

Here $\tilde{\omega}_n^{\pm}$ and $\tilde{\Omega}_n^{\pm}$ are renormalized by the impurity scattering and the paramagnetic effect Matsubara "frequencies" for electrons from degenerate and nondegenerate FS sections, respectively. In the most important and realistic case when $\Sigma\gg T_c$ and, moreover, Σ is the largest energy parameter of the problem, the quantities $\tilde{\omega}_n^{\pm}$ and $\tilde{\Omega}_n^{\pm}$ have the form ($\alpha=0;1$):

$$\tilde{\omega}_n^{\pm}=\omega_n\left[1+\left[\frac{1}{\tau_d^0}+\frac{1}{\tau_d^{\text{s.o.}}}+\frac{1}{\tau_{\text{ex}}^d}\right]\frac{1}{2\Omega_n}\right]\pm ih\left[1+\left[\frac{1}{\tau_d^0}-\frac{1}{3\tau_d^{\text{s.o}}}\right]\frac{1}{2\Omega_n}\right]\equiv\tilde{\omega}_n\pm i\tilde{h}\ , \tag{18}$$

$$\tilde{\Omega}_n^{\pm}=\left[|\omega_n|+\frac{1}{2}\left[\frac{1}{\tau_{nd}^0}+\frac{1}{\tau_{nd}^{\text{s.o}}}+\frac{1}{\tau_{\text{ex}}^{nd}}\right]\right]\text{sgn}\omega_n\pm ih\equiv\tilde{\Omega}_n\pm ih\ , \tag{19}$$

$$\Omega_n=(\omega_n^2+\Sigma^2)^{1/2},\quad h=\mu_B^* H\ . \tag{20}$$

We substitute into Eq. (12) the explicit form of Green's functions (17) with allowance for Eqs. (18)–(20) and carry out the expansion in small parameter q/p_F. So, taking into account that the typical nonlocality scale in BCS theory is much larger than an atomic one $\sim 1/p_F$, where p_F is the Fermi momentum, we obtain the expressions for $K_{nd0}^{\beta,-\beta}$ and $K_{di0}^{\beta,-\beta}$. Since all the superconductors suitable for application of our theory are type-II dirty superconductors, the following calculations will be carried out in the so-called "dirty" limit, when $qv_{d\,(nd)}\tau_{d\,(nd)}\ll 1$. Here $v_{nd}(v_d)$ are the Fermi electron velocities for nondegenerate (degenerate) FS sections, and

$$\frac{1}{\tau_{d\,(nd)}}=\frac{1}{\tau_{d\,(nd)}^0}+\frac{1}{\tau_{d\,(nd)}^{\text{s.o.}}}+\frac{1}{\tau_{\text{ex}}^{d\,(nd)}}\ . \tag{21}$$

In this case the solution of Eq. (15) for the kernels K_d, K_{nd} taking into account the expressions for K_{nd0} and K_{d0} has the form

$$K_{nd}^{\beta,-\beta}=\pi N_{nd}(0)\left[|\omega_n|+\frac{2}{3\tau_{nd}^{\text{s.o.}}}+\frac{1}{3\tau_{\text{ex}}^{nd}}+\frac{D_{nd}q^2}{2}-2i\beta h\,\text{sgn}\omega_n\right]$$
$$\times\left[\left[\left[|\omega_n|+\frac{1}{3\tau_{nd}^{\text{s.o.}}}+\frac{2}{3\tau_{\text{ex}}^{nd}}+\frac{D_{nd}q^2}{2}\right]^2+h^2-\left[\frac{1}{3\tau_{nd}^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^{nd}}\right]^2\right]^{-1}\right., \tag{22}$$

$$K_{di}^{\beta,-\beta}=\frac{\pi N_d(0)}{2}\left[1-\frac{\alpha\Sigma^2\phi_{n\beta}^2}{|\phi_{n\beta}|^4}\right]\left[\phi_n-\frac{1}{2}\left[\frac{1}{\tau_d^0}-\frac{1}{3\tau_d^{\text{s.o.}}}+\frac{1}{3\tau_{\text{ex}}^d}\right]\left[1-\frac{\alpha\Sigma^2\phi_{n,-\beta}^2}{|\phi_{n\beta}|^4}\right]+\frac{D_d q^2}{4\phi_n\tau_d}-2i\beta\gamma_n\right]$$
$$\times\left\{\left[\phi_n-\frac{1}{2}\left[\frac{1}{\tau_d^0}+\frac{1}{3\tau_d^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^d}\right]+\frac{D_d q^2}{4\phi_n\tau_d}\right]^2\right.$$
$$\times\left[1+\alpha\Sigma^2\frac{\left[\frac{1}{\tau_d^0}+\frac{1}{3\tau_d^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^d}\right](\phi_n^2-\gamma_n^2)}{\left[\phi_n-\frac{1}{2\tau_d^0}-\frac{1}{6\tau_d^{\text{s.o.}}}+\frac{1}{6\tau_{\text{ex}}^d}+\frac{D_d q^2}{4\phi_n\tau_d}\right]|\phi_{n\beta}|^4}\right]$$
$$-\left[\frac{1}{3\tau_d^{\text{s.o.}}}-\frac{1}{3\tau_{\text{ex}}^d}\right]^2\left[1-2\alpha\Sigma^2\frac{(\phi_n^2-\gamma_n^2)}{|\phi_{n\beta}|^4}\right]$$
$$+\frac{1}{4}\alpha\Sigma^4\left[\frac{1}{\tau_d^0}-\frac{1}{3\tau_d^{\text{s.o.}}}+\frac{1}{3\tau_{\text{ex}}^d}\right]\left[\frac{1}{\tau_d^0}+\frac{1}{\tau_d^{\text{s.o.}}}-\frac{1}{\tau_{\text{ex}}^d}\right]|\phi_{n\beta}|^{-4}$$
$$\left.\left.+\gamma_n^2\left[1+2\alpha\Sigma^2\left[\frac{1}{\tau_d^0}+\frac{1}{3\tau_d^{\text{s.o.}}}-\frac{1}{\tau_{\text{ex}}^d}\right]\phi_n|\phi_{n\beta}|^{-4}\right]\right\}^{-1}\right. . \tag{23}$$

Here

$$\phi_{n\beta}=\phi_n+2i\beta\gamma_n,\quad \phi_n=(\tilde{\omega}_n^2+\Sigma^2)^{1/2}\left[1-\frac{\tilde{h}^2\Sigma^2}{2(\tilde{\omega}_n^2+\Sigma^2)^2}\right],$$
$$\gamma_n=\frac{\tilde{h}\tilde{\omega}_n}{(\tilde{\omega}_n^2+\Sigma^2)^{1/2}}\left[1+\frac{\tilde{h}^2\Sigma^2}{2(\tilde{\omega}_n^2+\Sigma^2)^2}\right]. \tag{24}$$

$D_{nd\,(d)}=\frac{1}{3}v_{nd\,(d)}^2\tau_{nd\,(d)}$ are the diffusion coefficients for electrons from the proper FS sections.

IV. TEMPERATURE DEPENDENCE OF THE UPPER CRITICAL MAGNETIC FIELD

After the substitution of kernels (22) and (23) into the self-consistency equation (13), the summation over discrete "frequencies" for the kernel K_{nd} can be carried out as usual,[46] whereas for the kernel K_d the summation over ω_n can be replaced by integration over the continuous variable ω due to the inequality $\Sigma \gg T_c$:

$$\sum_{\omega_n} f(\omega_n,\Sigma)=\frac{1}{\pi T}\int_0^{\omega_D} f(\omega,\Sigma)d\omega\ , \tag{25}$$

where ω_D is the limiting (Debye) phonon frequency.

Thus, the self-consistency equation takes the form

$$\hat{\mathbf{R}}(\mathbf{q}=-\hat{\mathbf{\Pi}},T)\Delta(\mathbf{r})=E\Delta(\mathbf{r})\ , \tag{26}$$

$$\hat{\mathbf{R}}(\mathbf{q},T)=\frac{1}{2}\left[1-\frac{\kappa}{(\kappa^2-h^2)^{1/2}}\right]\psi\left[\frac{1}{2}+\frac{a-(\kappa^2-h^2)^{1/2}}{2\pi T}\right]$$
$$+\frac{1}{2}\left[1+\frac{\kappa}{(\kappa^2-h^2)^{1/2}}\right]\psi\left[\frac{1}{2}+\frac{a+(\kappa^2-h^2)^{1/2}}{2\pi T}\right]-\psi\left[\frac{1}{2}\right]$$
$$+\frac{D_d q^2}{4\nu\tau_d\Sigma^2(2\alpha+1)}+\frac{8\alpha h^2}{15\Sigma^2\nu}+(1-\alpha)\left[\frac{2h^4}{105\Sigma^4\nu}+\frac{\pi h^2}{4\Sigma^3\nu}\left[\frac{1}{2\tau_d^0}-\frac{1}{3\tau_d^{s.o.}}\right]\right], \tag{27}$$

$$E=\ln\left[\frac{\tilde{T}_c}{T}\right],\quad \kappa\equiv\frac{1}{3}\left[\frac{1}{\tau_{ex}^{nd}}-\frac{1}{\tau_{nd}^{s.o.}}\right],\quad a\equiv\frac{D_{nd}q^2}{2}+\frac{1}{3\tau_{nd}^{s.o.}}+\frac{2}{3\tau_{ex}^{nd}}\ . \tag{28}$$

Here $\psi(x)$ is the digamma function. Note that the expression (27) was obtained using the expansion in small parameters $(\Sigma\tau_d)^{-1}$, $(v_d q/\Sigma)^2$, h/Σ. In Eq. (27) $\nu=N_{nd}(0)/N_d(0)$ and the quantity $\tilde{T}_c$ is expressed with the help of a critical temperature T_c^* of the pure Bilbro-McMillan metal[38,40] as follows[41]:

$$\tilde{T}_c\simeq T_c^*\left[1+\frac{\pi}{8\nu\Sigma(\alpha+1)}\left[\frac{1}{\tau_d^0}+\frac{1}{\tau_d^{s.o.}}\right]-\frac{\pi}{4T_c^*\tau_{ex}^{nd}}\right.$$
$$\left.-3\pi/[8\nu\Sigma\tau_{ex}^d(2\alpha+1)]\right], \tag{29}$$

$$T_c^*=T_{c0}\left[\frac{\pi T_{c0}}{\gamma\Sigma[(1-\alpha)+\tilde{e}\alpha]}\right]^{1/\nu}. \tag{30}$$

Here $\tilde{e}$ is the base of the natural logarithm, $\gamma=1.78\ldots$ is the Euler constant, and

$$T_{c0}=\frac{2\omega_D\gamma}{\pi}\exp\left[-\frac{1}{V[N_{nd}(0)+N_d(0)]}\right] \tag{31}$$

is the system's critical temperature in the absence of dielectrization. Note that according to Eq. (29), as was shown earlier,[41] the superconducting transition critical temperature in metals with CDW and SDW grows when the nonmagnetic impurity concentration increases. This phenomenon can be regarded as the consequence of the renormalization of the electron density of states $N(E)$ due to the peculiar quantum-mechanical interference between two factors. One is the electron spectrum dielectrization owing to the electron-electron interaction and the other is the elastic impurity scattering.[41,47] This effect resembles one[22] when the density of states $N(E)$ of a nonsuperconducting disordered metal is changed due to the interference of the electron-electron interaction and elastic impurity scattering.

Note, that the above-mentioned similarity of the two effects has little to do with the role of the Altshuler-Aronov interference in the enhancement of the Coulomb pseudopotential μ^*. This enhancement can lead to the reduction of T_c and the distortion of the dependence $H_{c2}(T)$ similar to ones obtained in our paper (see, e.g., Refs. 21 and 48).

From the formal point of view Eq. (26) reduces to the Schrödinger equation for a particle with a dispersion law $\hat{R}(-\hat{\mathbf{\Pi}},T)$ and "energy" E. The conventional method to calculate $H_{c2}(T)$ consists in finding the minimal eigenval-

ue $\epsilon_0(H,T)$ of the operator $\hat{R}(-\hat{\Pi},T)$ and solving the equation[46]

$$\ln(\tilde{T}_c/T)=\epsilon_0(H_{c2},T)\ . \tag{32}$$

Equation (32) with the allowance for Eq. (27) gives full information about the critical magnetic field of a superconductor with CDW or SDW. However, explicit calculations outlined below show that the dependence $H_{c2}(T)$ changes qualitatively relative to one for the nondielectrized superconductors even in the simplest case when only nonmagnetic impurities are taken into consideration. Therefore, we shall consider below only the solutions of Eq. (32) when

$$1/\tau_d^{s,o.}{}_{(nd)}=1/\tau_{ex}^{d\ (nd)}=h=0.$$

In this case the operator $\hat{\mathbf{R}}(\mathbf{q},T)$ takes the simpler form

$$\hat{\mathbf{R}}(\mathbf{q},T)=\psi\left[\frac{1}{2}+\frac{D_{nd}q^2}{4\pi T}\right]-\psi\left[\frac{1}{2}\right]+\frac{D_dq^2}{4\nu\tau_d\Sigma^2(2\alpha+1)}\ . \tag{33}$$

We choose the magnetic field direction to be along $0Z$ axis and the gauge is determined by the relation $\mathbf{A}=0$, so that $A_x=A_z=0$, $A_y=Hx$. Then eigenvalues ϵ_H of the operator $\hat{\mathbf{R}}(-\hat{\mathbf{\Pi}},T)$ are determined by the equation

$$\left\{-\psi(\tfrac{1}{2})+\psi\left[\frac{1}{2}+\frac{D_{nd}}{4\pi T}\left[-\frac{d^2}{dx^2}+\frac{4e^2}{c^2}H^2x^2\right]\right]+\frac{D_d}{4\nu\tau_d\Sigma^2(2\alpha+1)}\left[-\frac{d^2}{dx^2}+\frac{4e^2}{c^2}H^2x^2\right]\right\}\Delta(x)=\ln\frac{T_c}{T}\Delta(x)\ . \tag{34}$$

From Eq. (34) it follows that $\epsilon_H\neq0$ if $\Delta(x)$ is the eigenfunction of the operator

$$-\frac{d^2}{dx^2}+\frac{4e^2}{c^2}H^2x^2\ .$$

Its eigenvalues coincide with eigenvalues of linear harmonic oscillator Hamiltonian with angular frequency $\omega_H=2eH/c$ and unit mass. Thus eigenvalues of the operator $\hat{\mathbf{R}}(-\hat{\mathbf{\Pi}},T)$ are determined from the transcendent relationship

$$-\psi(\tfrac{1}{2})+\psi\left[\frac{1}{2}+\frac{D_{nd}eH}{c\pi T}(n+\tfrac{1}{2})\right]+\frac{D_deH}{\nu\tau_d\Sigma^2(2\alpha+1)c}(n+\tfrac{1}{2})=\epsilon_H\quad(n=0,1,2,\ldots)\ . \tag{35}$$

Minimal value corresponds to $n=0$ and according to Eqs. (32) and (35) the upper critical field $H_{c2}(T)$ in various temperature regions has the form

$$H_{c2}=\frac{4cT_c(1-T/T_c)}{\pi eD_{nd}[1+2D_dT_c/(\pi\nu\Sigma^2D_{nd}\tau_d\{2\alpha+1\})]}\left[1-\left[1-\frac{T}{T_c}\right]\frac{\frac{1}{2}-\left[\frac{28\zeta(3)}{\pi^4}+2\left[\frac{D_dT_c}{\pi\nu\Sigma^2D_{nd}\tau_d(2\alpha+1)}\right]^2\right]}{\left[1+\frac{2D_dT_c}{\pi\nu\Sigma^2D_{nd}\tau_d(2\alpha+1)}\right]^2}\right]\quad(T\leq T_c)\ , \tag{36}$$

$$H_{c2}=\frac{\pi cT_c}{2\gamma eD_{nd}}\left[1-\frac{\pi D_dT_c}{4\gamma\nu\Sigma^2D_{nd}\tau_d(2\alpha+1)}\right]\left[1-\frac{2}{3}\left[\frac{\gamma T}{T_c}\right]^2\right]\quad(T\to0)\ , \tag{37}$$

where $\zeta(x)$ is the Riemann ζ function. The critical temperature of the system T_c is determined now by Eq. (29) with $1/\tau_d^{s,o.}=1/\tau_{ex}^d=1/\tau_{ex}^{nd}=0$.

It should be mentioned once more that unlike Eq. (36) the formula (37) is applicable only to materials satisfying the quasiclassical condition when $T=0$.

V. DISCUSSION AND COMPARISON WITH EXPERIMENT

Equations (36) and (37) are the main results of this paper. From them one concludes that the appearance of the dielectric gap on the FS sections leads to the reduction of the effective electron-diffusion coefficient:

$$D_{\text{eff}}\approx D_{nd}\left[1+A\frac{D_dT_c}{D_{nd}\tau_d\Sigma^2}\right]\ ,$$

where A is a numerical factor of the order of unity. Such a renormalization to a certain extent is analogous to the diffusion coefficient reduction in the "dirty" superconductor due to the weak Anderson localization.[21] From Eqs. (36) and (37) we see that in metals with nesting parts of the FS large values of $H_{c2}(0)$ and $|dH_{c2}/dT|_{T=T_c}$ should be observed. Furthermore, it follows from Eq. (36) that near T_c, provided the ratio $\nu=N_{nd}(0)/N_d(0)$ is small, the curvature positiveness for the $H_{c2}(T)$ dependence is possible, although not inevitable.

These conclusions agree well with the experimental data for several classes of substances reviewed in the Introduction. So, e.g., in agreement with Eq. (36), in the tetragonal (partially dielectrized) phase of Nb_3Sn with $A15$ structure the slope $|dH_{c2}/dT|_{T=T_c}$ is always larger than in the cubic one.[30] Moreover, in this compound the reduction of sample purity accompanied by a suppression of the structural transition results in the change of sign of d^2H_{c2}/dT^2 from positive to negative.[10] By the way, the observed[10] dependence of d^2H_{c2}/dT^2 on sample resistivity seems to rule out the earlier interpretation[2] of the experimental data for Nb_3Sn in the framework of the theory taking into account the influence of strong localization on the Coulomb pseudopotential μ^*.

Note that a change of sign may be achieved by an external effect on the FS, e.g., by pressure or variation of composition. The latter was realized for solid solutions $BaPb_{1-x}Bi_xO_3$.[9] The following fact is of principle importance here: the positive curvature of the critical field exists here only for compounds with $x \geq 0.20$ (close enough to metal-semiconductor transition when $x \approx 0.4$) for which the numerous experimental data[28] show the dielectric-gap appearance on the FS section. Moreover, the reflection of light from the samples of $BaPb_{1-x}Bi_xO_3$ with metallic conductivity[49] show that long before the metal-insulator transition takes place one should change the Drude-like expression for the dielectric function $\epsilon(\omega)$ in order to describe the experimental data successfully. Namely, when $x > 0.15$ it is necessary to suggest that $\epsilon(\omega)$ together with metallic contribution involves a contribution from the excitations across the semiconducting gap ω_G, although the substance transport properties remain metallic. The composition dependence rules out another explanation[50] of the inequality $d^2H_{c2}/dT^2 > 0$ by the bipolaronic mechanism of superconductivity in $BaPb_{1-x}Bi_xO_3$. The inapplicability of this mechanism here is supported by the relative smallness[9] of the electron-phonon coupling constant $\lambda_{e\text{-ph}} < 1$.

The other superconducting oxide showing the partial electron-spectrum gapping for some compositions is the hexagonal tungsten bronze Rb_xWO_3.[8] Again, precisely for these compositions the positive curvature of $H_{c2}(T)$ and large values of $H_{c2}(0)$ are observed. This tendency correlates well to the theory outlined in this paper.

Among the superconductors with $d^2H_{c2}/dT^2 > 0$ there are many intercalated layered dichalcogenides of Ta and Nb,[5,6] where the structural transition exists at some temperature much higher than T_c. Usually the superconducting properties of such systems are described in the framework of Klemm, Luther, and Beasley (KLB) theory[19] based on the idea of the Josephson coupling between the layers. However, KLB theory does not explain the positive curvature $H_{c2\perp}(T)$ for the field normal to layers which is practically always observed in addition to the positive curvature of $H_{c2\parallel}(T)$.[5,6,51] As to the dependence $H_{c2\parallel}(T)$, the experimentally determined inflection point T^* does not necessarily coincide[51] with T^*_{KLB}, calculated from the equality condition between the vortex core radius $\xi_\perp(T)$ and the interlayer distance. It raises doubts for the applicability of the KLB theory also in the cases when the analysis of inflection-point location was not carried out.

The dielectric gap $\Sigma \gg T_c$ on the FS sections is observed in the quasi-one-dimensional metal $NbSe_3$.[27] In agreement with our theory, $d^2H_{c2}/dT^2 > 0$ there.[7] Normal state properties of Laves phases $Hf_{1-x}Zr_xV_2$ are quite similar[30] to those of $NbSe_3$. Unfortunately, comprehensive measurements of $H_{c2}(T)$ in these systems have not been carried out so far. But it is known[30] that in $Hf_{1-x}Zr_xV_2$ a small excess of the paramagnetic limit is observed in agreement with the theory[52] of the paramagnetic properties of the superconductors with partial electron-spectrum gapping. The existence of the latter in Laves phases is confirmed also by the anomalously small[53] heat capacity jump when $T = T_c$, which was predicted theoretically.[28,40]

Experimental situation for Chevrel phases where the anomalies of $H_{c2}(T)$ were observed[3,4] is rather complicated. For compounds $SnMo_6S_8$ and $PbMo_xS_8$ (Ref. 3) without rare-earth ions our interpretation seems unique. As to the compounds with Eu *ion*,[4] the crucial role plays the compensative effect of Jaccarino and Peter.[18] It was proved in famous experiments,[54] where the magnetic field-induced superconductivity of $Eu_{0.75}Sn_{0.25}Mo_6S_{7.2}Se_{0.8}$ was discovered.

Recently[55] the coexistence of superconductivity with $T_c = 1.9$ K and CDW emerging when $T \approx 25$ K was clearly demonstrated for quasi-two-dimensional purple bronze $Li_{0.9}Mo_6O_{17}$. For this substance, in agreement with our point of view, the curvature of $H_{c2}(T)$ is positive.

In all compounds listed above superconductivity manifests itself against the CDW background. At the same time there are substances where at $T = T_N$ above T_c a phase transition into the antiferromagnetic state occurs. For antiferromagnetic superconductors ($\alpha = 1$) Eq. (37) as well as in the CDW case ($\alpha = 0$) forecasts the possibility of the positive $H_{c2}(T)$ curvature. Such a behavior is observed indeed in heavy-fermion superconductor URu_2Si_2,[12,13] $Ho(Ir_{0.7}Rh_{0.3})_4B_4$,[11] $Cr_{1-x}Re_x$,[56] and organic superconductors $(TMTSF)_2X$.[14] It is significant that in the alloy $Cr_{78}Re_{22}$ the positive sign of the curvature d^2H_{c2}/dT^2 after annealing transforms into the negative one.[56] The width of the superconducting transition in this case does not change, and $H_{c2}(0)$ decreases. The decrease of $H_{c2}(0)$ indicates the reduction of the microscopic defect concentration n_i in the sample. These facts are properly described by our theory. Really, the diffusion coefficients D_d and D_{nd} rise as n_i^{-1} and, according to Eq. (36), the quantity $2D_dT_c/3\pi\nu\Sigma^2D_{nd}\tau_d$ (which determines the sign of d^2H_{c2}/dT^2) decreases proportionally to the reduction of n_i.

To summarize, we should note that despite a good qualitative agreement of the presented theory with the numerous experimental data, a quantitative comparison of the former with the properties of CDW and SDW superconductors is now quite difficult. The main obstacle is the lack of measured or calculated values of diffusion coefficients, dielectric gaps, and densities of states $N_{nd}(0), N_d(0)$. The only exceptions are alloys $Cr_{1-x}Re_x$ (Ref. 32) and the compound URu_2Si_2,[12,13,57–59] for which

the degree ν of the electron spectrum dielectrization is determined. In the case of URu_2Si_2 the gap Σ is also known. Unfortunately, the data obtained by different authors are contradictory. For instance, for URu_2Si_2, according to Ref. 13, we have $\nu=1.5$ and $\Sigma=129$ K, whereas, according to Ref. 12, $\nu=0.4$ and $\Sigma=115$K.

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