Kosterlitz-Thouless transition of fluxless solitons in superconducting $YBa_2Cu_3O_{7-\delta}$ single crystals

P. C. E. Stamp,* L. Forro,† and C. Ayache
Département de la Recherche Fondamentale, Service des Basses Temperatures, Centre d' Etudes Nucléaires de Grenoble, Boîte Postale No. 85X, 38041 Grenoble Cédex, France

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We present detailed evidence for a resistive Kosterlitz-Thouless transition in $YBa_2Cu_3O_{7-\delta}$ single crystals. The Kosterlitz-Thouless temperature T_{KT} is the onset temperature for zero-resistance superconductivity in the *ab* planes; three-dimensional superconductivity throughout the sample follows at a lower temperature. The objects which engage in the transition *do not carry flux*—they are not the flux vortices of conventional theory. They are more likely normal-state excitations that pair up at T_{KT} to form quasi-two-dimensional superconducting condensates.

Recent experiments on the new ceramic superconductors have excited some controversy over their *dimensionality*; although most authors favor two-dimensional (2D) behavior (at least above T_c), evidence has been presented for 3D behavior in both normal and superconducting states.[1,2]

Here we present rather striking evidence for 2D behavior, via systematic resistivity measurements on a high-quality single crystal of $YBa_1Cu_3O_{7-\delta}$. Previous measurements[3] of H_{c_2}, on this and other samples, showed a very anisotropic normal resistivity over a range of fields, and a highly anisotropic quasi-2D superconducting state, with ξ_c only 4.8 Å just below the superconducting transition.[4] Here we show that this transition shows all the characteristics of a Kosterlitz-Thouless (KT) transition.[5] This is startling, because in a layered superconductor of this type, the long-range magnetic-field coupling between the usual 2D flux vortices (fluxons) in each plane would link them together into 3D *flux lines*, so the fluxons ought never to show 2D behavior.[6] Thus, some rather unconventional ideas are necessary to explain our results (as discussed below). Indeed, rather stringent conditions are required to see KT behavior, and it is useful to start by clarifying these.

A set of *sufficient conditions* for KT behavior are (i) a set of "soliton" objects carrying a "soliton charge" $q=\pm 1$ (which may be topological, or electric, or etc.), moving in two dimensions; (ii) a soliton interaction energy $U(r)\sim qq'\ln(r/a)$ for charges q,q' separated by r, valid for length scales $a\ll r\ll L$; (iii) a soliton/current coupling $\propto qJr$, with current J acting oppositely on $\pm$ soliton charges. Conditions (i) and (ii) alone lead to the famous KT scaling equations[7] for the soliton dielectric function $\varepsilon^{-1}(r,T)=K(r,T)/K(a,T)$ and the soliton activity $y(r,T)$, viz.

$$K^{-1}(r,T)=K^{-1}(a,T)+4\pi^3\int_{\ln a}^{\ln r}\frac{dr'}{r'}y^2(r')\,,$$
$$y^2(r,T)=y^2(a,T)\exp\left[4\ln(r/a)-2\pi\int_{\ln a}^{\ln r}\frac{dr'}{r'}K(r')\right].\qquad(1)$$

For $L\neq\infty$, this scaling breaks down at a crossover temperature T_Λ, given by

$$T_\Lambda-T_{KT}\sim\alpha T_{KT}/\Lambda^2 \text{ for } \Lambda\gg 1\,,\qquad(2)$$

where $\Lambda=\ln(L/a)$, and T_{KT} is the true KT transition temperature. In general, the upper cutoff L smooths out the KT transition,[7,8] and moves it towards T_Λ; above T_Λ, the properties of the system are governed by dissociating $\pm$ soliton pairs, with correlation length $\xi_+(T)\sim a\exp(\pi/\beta\tau^{-1/2})$, where $\tau\equiv|(T-T_{KT})/T_{KT}|$, and β is a nonuniversal constant. Equation (2) is only approximate, and it has large corrections for small Λ; moreover, α is also nonuniversal, and varies widely between different systems [e.g., in superconducting films $\alpha\sim T_{KT}/(T_c^{(bulk)}-T_{KT})$, while in magnetic films $\alpha\sim 1$].

Thus, we can test conditions (i) and (ii) in the *resistive state* by measuring the electrical resistance $R_{ab}(T)$ in the *ab* planes; $\xi_+(T)$ implies a form $R_{ab}(T)\sim R_N\exp(\pi\beta\tau^{-1/2})$ above T_Λ.

Our measurements were made on a high-quality monocrystal of $YBa_1Cu_3O_{7-\delta}$ grown using a flux method, with subsequent oxygen treatment for several hours at 450 °C. The sample had dimensions $(L_a\times L_b\times L_c)=1.7\times 0.3\times 0.03$ mm^3, and a superconducting transition over the range 81.6–83.3 K (between 10% and 90% of R_N). As in all such samples, there were slight inhomogeneities in oxygen content, to which we return below. Resistivity measurements were made using both a four-probe and the Montgomery configuration;[9] contacts (resistance <0.3 Ω) were made using fired gold paste.[10]

Our interest in a possible KT behavior arose from the enormous anisotropy in both superconducting [cf. the $H_{c_2}(T)$ measurements[3,4]] and normal states. To test this idea, we examined the region of low resistivity $\rho_{ab}(T)$ in the *ab* plane (shown in the inset of Fig. 1), using a direct four-probe method, and tested for KT scaling in R_{ab} by plotting $\ln R_{ab}$ vs $(T-T_{KT})^{-1/2}$, with T_{KT} determined by the best fit. The fit is quite good as one approaches T_{KT}, but we were not able to go closer than $T-T_{KT}<0.8$ K, because the voltage V for which R_{ab} is Ohmic then becomes too small. We were not able to fit R_{ab} to Aslamazov-Larkin theory, in either its 2D or 3D forms, for any significant range of temperatures.[10] Our result

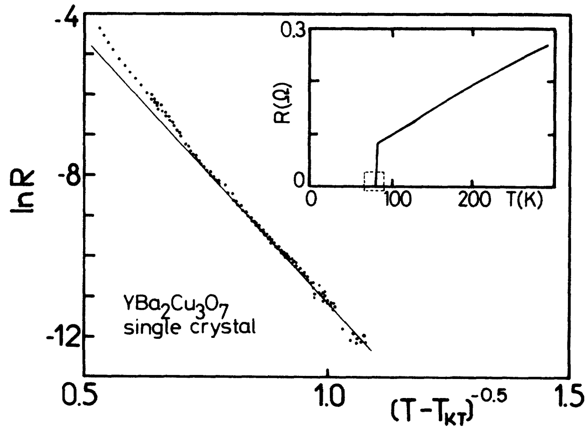

FIG. 1. Fit of R_{ab} to the KT scaling; the region covered is depicted in the inset. We extract a value of $T_{KT}=80.0\pm0.05$ from our fits.

then implies that $T_\Lambda - T_{KT} < 0.8$ K, so that $\Lambda > 10$; if we assume $L=L_b=0.3$ mm, and that $a\sim1$ (as we shall see later, we find no sign of a "$T_{c_{\rm bulk}}$"; the choice $a\sim1$ conforms with our conclusion, discussed below, that the solitons are normal-state excitations), then we get a rough upper bound for the "soliton core size" a, viz., $a < 120$ Å.

On the other hand, measurements of $R_c(T)$ on the same sample, using the same sample and the Montgomery method, show a rise to ~0.17 Ω around T_{KT}, followed by a steep fall; 3D superconductivity sets in around 76 K (these results are fully described in Refs. 3 and 10). The form of these results strongly indicated a *stratification* of oxygen content in the samples we used; the stratum with highest $T_{KT}=T_{KT}^{(\rm max)}$ will then totally dominate the sample conductivity as one approaches $T_{KT}^{(\rm max)}$ (this stratification has been confirmed by susceptibility measurements of Couach, Khoeder, and Barbara[11]). A similar situation obtains in superconducting films, except that there, there is a *continuous* distribution of $T_{KT}(\mathbf{r})$ throughout the film.[8] Our sample, incidentally, was chosen not to have the highest T_c possible, but the least stratification.

It is interesting to note that KT scaling apparently persists up to quite high fields,[10] but as T_{KT} falls, the transition rapidly broadens, and we are not able to follow $R_{ab}(T)$ to closer than ~2 K of $T_{KT}(H)$.

A much more stringent test of KT behavior is provided if our sufficient condition (iii) is added, since then a current can be used to dissociate soliton pairs below T_{KT}. If we further assume that the solitons couple to the charge carriers in the system (or carry electric charge themselves), then the resistance must have the very characteristic and unusual form described in detail in Ref. 12, viz.,

$$R_{ab}(T)\simeq R_N(2\pi K(\Lambda,T)-4)(I/I_0)^{1+\pi K(\Lambda,T)} \qquad (3)$$

below T_Λ (with logarithmic corrections near T_{KT} or if I is not $\ll I_0$). In a system without an upper cutoff ($L\to\infty$, $T_{KT}\equiv T_\Lambda$), $K(\Lambda,T)\to K_\infty(T)$, and is a direct measurement of the superfluid density ρ_s:

$$K_\infty(T)=\hbar^2\rho_s/4m^2k_BT\,, \qquad (4)$$

with the Nelson-Kosterlitz jump in $n(T)$ from 1 to 3 at T_{KT}. However, for finite L, things are more complicated (compare Refs. 7 and 8); in general, the square-root cusp in $\rho_s(T)$ is very difficult to see, and the transition appears smeared out, with a tail in $n(T)$ above T_Λ. This is essentially what we find: in Fig. 2, we see that the remarkable power-law behavior of Eq. (3) is accurately obeyed, with an exponent $n(T)$ shown in Fig. 3. $n(T)$ looks very much what is seen in superconducting films,[8] except that in films, the intercept of the expected straight-line behavior, well below T_{KT} and with the line $n(T)=1$, is expected at $T_c^{(\rm bulk)}$, whereas in our case, it seems to fall around 80.7 K; we are then led to infer from this that $T_\Lambda\sim80.7$ K, i.e., $\Lambda\sim11$. Thus we have no superfluid jump, and no $T_c^{(\rm bulk)}>T_{KT}$, only a continuous rise of $\rho_s(T)$ from T_Λ, with $\rho_s(T)\propto n(T)-1\sim(T_\Lambda-T)$ for $(T_\Lambda-T)\ll T_\Lambda$.

We thus seem to have rather strong evidence for KT behavior in $YBa_2Cu_3O_{7-\delta}$ monocrystals. However, the interpretation of this result is not straightforward. To see this, notice that we can make the following deductions from our data.

(i) The obvious apparent explanation of our data is that our solitons are fluxons, i.e., a conventional KT transition. We should then assume the clean limit ($T_\Lambda\to T_c^{\rm bulk}$) since we see no Nelson-Kosterlitz jump in $n(T)$. However, this explanation is incompatible with our data, because (a) then KT behavior would break down completely around T_Λ [$\xi_{ab}(T)$ and $\lambda_{ab}(T)$ would diverge, implying $L/a\to0$]; and (b) the length scale $\lambda_\perp(T)=\lambda_{ab}^2/d$ at which $\ln r$ interactions cross over to $1/r$ must obey $\lambda_\perp(T)/\xi_{ab}(T)\gg L/a$ [where now $a\equiv\xi_{ab}(T)$; d is the effective thickness of the superconducting wave function

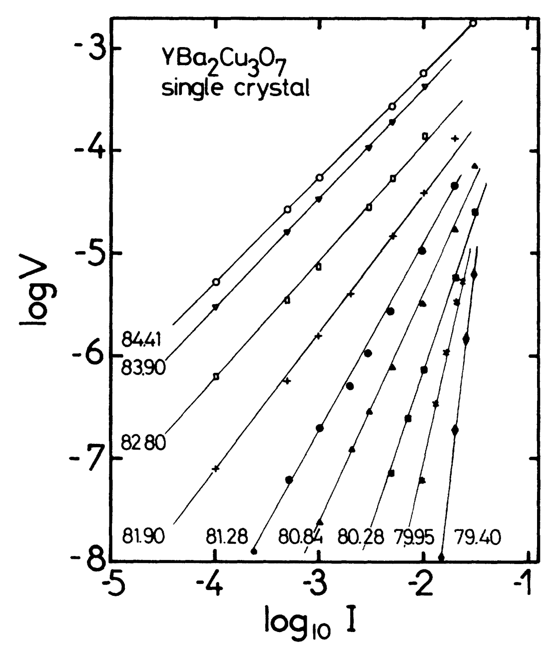

FIG. 2. Fit of our data to Eq. (3). Only a representative sample of the points is shown, for clarity. Errors around T_{KT} were rather larger than elsewhere, as were heating effects at the higher end of the current range, which caused points in this region to fall systematically above the lines; these points were ignored in the fits.

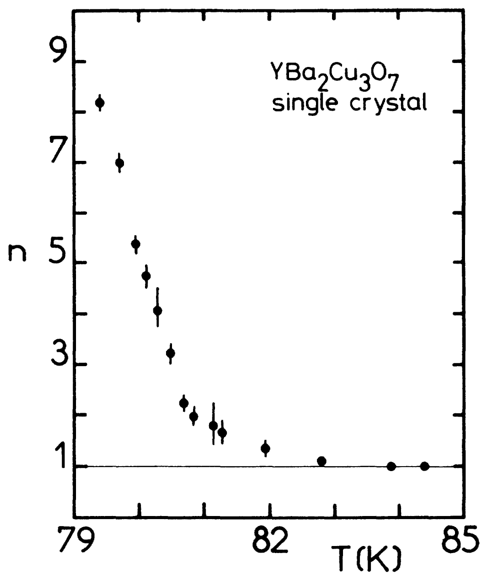

FIG. 3. Plot of the exponent $n = \pi K + 1$ vs T, with n taken from the straight-line fits of Fig. 2.

in the planes]. Assuming $\lambda_{ab}(0) = 260$ Å (Ref. 1), $\xi_{ab}(0) = 45$ Å (Ref. 3), and $d = \xi_c(T)$ [$\xi_c(0) = 4.8$ Å (Refs. 3 and 4)], then for $T_\Lambda - T \sim 2$ K (where KT is obeyed in our data), Bardeen-Cooper-Schrieffer (BCS) theory gives $\lambda_\perp \sim 6\times 10^4$ Å and $\lambda_\perp/\xi_{ab} \sim 300$, even in the absence of interplane Josephson coupling (which will considerably *reduce* this number!). However, our data give $L/a > 6\times 10^4$, a huge discrepancy. Thus, our solitons *cannot be fluxons*.

(ii) The sufficient conditions given above for KT behavior, in fact, describe all known examples of it (viz., He II and superconducting films, magnetic layer and films, and the 2D electron plasma). However, the necessary conditions are not the same as the sufficient ones; this is illustrated by the mapping of a 1D superconducting system at finite T onto KT theory.[13] However, we are clearly dealing with a layered quasi-2D system. If we thus assume that for such a system, sufficient conditions are equal to necessary ones, we may make the second deduction that *the solitons are not excitations of the superconducting wave function.* This follows because (a) even a very weak Josephson tunneling would immediately couple the solitons between planes, thus destroying KT behavior [compare Eq. (2)]; and (b) we would then expect a $T_c^{(\mathrm{bulk})} > T_\Lambda$, which is not seen. Thus, we can rule out any "internal gauge field" solitons that would arise from a nontrivial order parameter (such as the spin solitons in ^{3}He-B), as well as ordinary fluxons.[14]

Thus, we are forced to conclude that to explain our KT result, we need *normal-state excitations* satisfying the sufficient conditions given above. These conditions are very restrictive, and lead us to the following speculation.

Since we are apparently dealing with a "quantum spin fluid" of strong antiferromagnetic tendency[15] and very weak interplane magnetic coupling, and since such systems are well known to contain magnetic solitons, we conjecture that these are our solitons. These will clearly lower their energy by binding holes (removing bending and exchange energy). There are two kinds of them, and a logarithmic coupling would then give a KT transition by pairing solitons and antisolitons.[16]

One might also speculate that our solitons could be the spinons of RVB theory,[17] although it is not obvious to us that spinons interact logarithmically (or bind charge).

Note added. Since this paper was submitted, other papers relevant to our work have appeared. Reference 18 has similar data to ours, but on a polycrystal (as in Ref. 5); they interpret their results accordingly as intergrain tunneling effects. Our monocrystal results apparently rule this out; the question should be settled by measurements on samples free of twinning centers. Recent theoretical work[19] has investigated pairing mechanisms with bound holes on antiferromagnetic solitons. These ideas strongly resemble our conjecture above (see also Ref. 16).

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*Permanent address: Department of Physics, University of Lancaster, Bailrigg, Lancaster LA1 4YB, Lancashire, United Kingdom.

†Permanent address: Institute of Physics, University of Zagreb, P.O. Box 304, YU-41001 Zagreb, Croatia, Yugoslavia.

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[4]Our ξ_c agrees with that of Umezawa *et al.* (unpublished).

[5]Signs of KT behavior have also been seen recently by M. Suguhara *et al.*, Phys. Rev. A **125**, 426 (1987), in *polycrystalline samples*. This is slightly puzzling to us (since the grain size will set the upper cutoff L). However, they only follow the KT scaling to within ~ 1 K of T_{KT}; in our case, $T_\Lambda - T_{\mathrm{KT}}$ is certainly < 0.8 K, and may be < 0.7 K [compare discussion following Eq. (4)]. Since $L = a e^\Lambda$ varies extremely rapidly with $T_\Lambda - T_{\mathrm{KT}}$ in this region, their L may be several orders of magnitude smaller than our L_b.

[6]This point is easily demonstrated via model calculations [see P. C. E. Stamp (unpublished)].

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[10]A more detailed presentation of our results will be published elsewhere, including sample preparation (see also Ref. 3), and

high-field measurements.

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[14]S. Doniach (private communication) has pointed out to us that for sufficiently thin samples this argument may break down. In our sample $L_c = 3\times 10^5$ Å, but presumably the superconducting stratum is thinner. Only experiments on homogeneous thick samples will resolve this.

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