PHYSICAL REVIEW B VOLUME 38, NUMBER 4 1 AUGUST 1988

Magnetic properties of amorphous mixed Ising spin systems in a transverse field

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(Received 5 January 1988)

The magnetic properties of amorphous mixed Ising ferrimagnetic (or ferromagnetic) systems with coordination numbers $z=3$ and $z=4$ consisting of spin $\frac{1}{2}$ and spin 1 with a transverse field are investigated within the framework of an effective-field approximation. The phase diagrams and the total longitudinal and transverse magnetizations are obtained. We find a number of interesting phenomena in these quantities, due to the fluctuation of exchange interaction and the applied transverse field.

I. INTRODUCTION

In the last decade there has been an increasing number of work dealing with magnetic quantities of amorphous and disordered solids. In particular, considerable interest has been directed to amorphous ferrimagnetic rare-earth and transition-metal alloys because of their potential for magneto-optical recording.[1]

In order to analyze the magnetic properties of amorphous magnets, it is necessary to introduce simplified yet not completely unrealistic models in which some types of disorder represent important ingredients.[2] Mean-field theory has been applied recently to amorphous ferrimagnets. However, its usefulness has been marred in the past by the existence of too many adjustable parameters and the neglect of the fluctuation of exchange interactions (or the structural fluctuation). In previous work,[3] one of the present authors (T.K.) has introduced the effective-field theory with correlations for amorphous ferrimagnets and discussed that the mean-field theory must be used with caution for amorphous ferrimagnetic alloys because of the serious effects of structural fluctuation on the magnetic properties.

On the other hand, some attention has recently been directed to two-sublattice mixed spin-$\frac{1}{2}$ and spin-1 Ising spin systems. The effect of single-ion anisotropy on the transition temperature has been investigated by exact[4,5] and approximate methods.[6,7] Moreover, one of the present authors (T.K.) has found that the tricritical point at which the phase transition changes from second order to first order exists in the system with $z>3$, where z is the coordination number.[8] As far as we know, however, the effects of an applied transverse field on the magnetic properties of amorphous ferro- and ferrimagnetic mixed alloys have not been studied both experimentally and theoretically.

The purpose of this work is to clarify the effects of a transverse field on phase diagrams (or transition temperatures) and the magnetization curves in the amorphization of ferrimagnetic (or ferromagnetic) mixed Ising spin systems. The problem is studied on the basis of the introduction of a differential operator[9] into a generalized but approximate Callen relation derived by SaBarreto *et al.*[10] for the transverse Ising model. The relation has been successfully applied to a great number of interesting transverse Ising systems.[11]

The outline of this work is as follows. In Sec. II, we briefly present the formalism and discuss the effective-field theory with correlations for the amorphization of a ferrimagnetic (or ferromagnetic) mixed Ising spin system. In Sec. III we study the phase diagrams. In Sec. IV the temperature dependences of total longitudinal and transverse magnetizations for the amorphization of ferrimagnetic mixed Ising systems in an applied transverse field are investigated. We find a number of interesting phenomena in these quantities discussed in Secs. III and IV, due to the structural fluctuation and the applied transverse field.

II. FORMULATION

We consider an amorphization of a mixed ferrimagnetic (or ferromagnetic) Ising spin system with a transverse field. The Hamiltonian of the system is given by

$$\mathcal{H}=-\sum_{\langle i,j\rangle} J_{ij}\mu_i^z s_j^z-\Omega\sum_i \mu_i^x-\Omega\sum s_j^x \ , \tag{1}$$

where μ_i^α and s_j^α $(\alpha=x,z)$ are components of spin-$\frac{1}{2}$ and spin-1 operators at sites i and j, respectively. Ω represents the transverse field. J_{ij} is the exchange interaction between neighboring sites, which is assumed to be randomly distributed according to the independent probability distribution function $P(J_{ij})$.

The starting point for the statistics of our spin system is the relation proposed by SaBarreto *et al.*,[10] in which the longitudinal and transverse site magnetizations for the mixed Ising systems[12] are given by

$$\sigma_i^z=\langle\mu_i^z\rangle=\frac{1}{2}\left\langle\frac{\theta_i}{E_i}\tanh\left[\frac{\beta}{2}E_i\right]\right\rangle , \tag{2}$$

$$\sigma_i^x=\langle\mu_i^x\rangle=\frac{1}{2}\left\langle\frac{\Omega}{E_i}\tanh\left[\frac{\beta}{2}E_i\right]\right\rangle , \tag{3}$$

and

$$m_j^z=\langle s_j^z\rangle=\left\langle\frac{\bar\theta_j}{\bar E_j}\frac{2\sinh(\beta\bar E_j)}{1+2\cosh(\beta\bar E_j)}\right\rangle , \tag{4}$$

$$m_j^x=\langle s_j^x\rangle=\left\langle\frac{\Omega}{\bar E_j}\frac{2\sinh(\beta\bar E_j)}{1+2\cosh(\beta\bar E_j)}\right\rangle , \tag{5}$$

with

$$\theta_i=\sum_j J_{ij}s_j^z , \tag{6}$$

$$\bar\theta_j=\sum_l J_{jl}\mu_l^z , \tag{7}$$

$$E_i=[\Omega^2+(\theta_i)^2]^{1/2} , \tag{8}$$

$$\bar E_i=[\Omega^2+(\bar\theta_i)^2]^{1/2} . \tag{9}$$

Here, $\langle\cdots\rangle$ indicates the canonical thermal average and $\beta=1/k_BT$. In the limit $\Omega=0$, $\sigma_i^x=m_j^x=0$ and Eqs. (2) and (4) reproduce the exact identities for the pure Ising model.[13] Expanding the right-hand side of Eqs. (2)–(5) as a formal series in the spin variables and neglecting correlations of E_i (or $\bar E_j$), the standard mean-field approximation results are recovered.[12,14]

Introducing the differential operator $D=\partial/\partial x$,[9] we may rewrite Eqs. (2), (3), (4), and (5) as

$$\sigma_i^z=\langle e^{D\theta_i}\rangle f(x)\,|_{x=0} , \tag{10}$$

$$\sigma_i^x=\langle e^{D\theta_i}\rangle g(x)\,|_{x=0} , \tag{11}$$

$$m_i^z=\langle e^{D\bar\theta_i}\rangle F(x)\,|_{x=0} , \tag{12}$$

$$m_j^x=\langle e^{D\bar\theta_i}\rangle G(x)\,|_{x=0} , \tag{13}$$

where functions $f(x)$, $g(x)$, $F(x)$, and $G(x)$ are defined by

$$f(x)=\frac{1}{2}\frac{x}{(\Omega^2+x^2)^{1/2}}\tanh\left[\frac{\beta}{2}(\Omega^2+x^2)^{1/2}\right] , \tag{14}$$

$$g(x)=\frac{1}{2}\frac{\Omega}{(\Omega^2+x^2)^{1/2}}\tanh\left[\frac{\beta}{2}(\Omega^2+x^2)^{1/2}\right] , \tag{15}$$

$$F(x)=\frac{x}{(\Omega^2+x^2)^{1/2}}\frac{2\sin[\beta(\Omega^2+x^2)^{1/2}]}{\{1+2\cosh[\beta(\Omega^2+x^2)^{1/2}]\}} , \tag{16}$$

$$G(x)=\frac{\Omega}{(\Omega^2+x^2)^{1/2}}\frac{2\sin[\beta(\Omega^2+x^2)^{1/2}]}{\{1+2\cosh[\beta(\Omega^2+x^2)^{1/2}]\}} . \tag{17}$$

Using the identities

$$\exp(\gamma\mu_i^z)=\cosh\left[\frac{\gamma}{2}\right]+2\mu_i^z\sinh\left[\frac{\gamma}{2}\right] \tag{18}$$

and

$$\exp(\gamma s_j^z)=(s_j^z)^2\cosh\gamma+s_j^z\sinh\gamma+1-(s_j^z)^2 , \tag{19}$$

the expectation values $\langle\exp(D\theta_i)\rangle$ and $\langle\exp(D\bar\theta_j)\rangle$ reduce to

$$\langle e^{D\theta_i}\rangle=\left\langle\prod_j[(s_j^z)^2\cosh(J_{ij}D)+s_j^z\sinh(J_{ij}D)+1-(s_j^z)^2]\right\rangle \tag{20}$$

and

$$\langle e^{D\bar\theta_j}\rangle=\left\langle\prod_l\left[\cosh\left[\frac{J_{jl}}{2}D\right]+2\mu_l^z\sinh\left[\frac{J_{jl}}{2}D\right]\right]\right\rangle . \tag{21}$$

The main purpose of the present work is to obtain from the above set of equations the phase diagrams and the behavior of the longitudinal as well as the transverse magnetizations as functions of the parameters T, Ω, and $P(J_{ij})$. For a disordered system with random bonds, we must perform the random configurational average. However, it is clear that if we try to treat exactly all the spin-spin correlations which appear through the expansion of the above equations, and to perform the configurational averages properly, which is still to be done, the problem becomes mathematically untractable. Therefore, some approximations are needed. As discussed in the previous works,[3,11] let us introduce the decoupling approximation

$$\begin{aligned}&\langle\langle\mu_i^z\mu_j^z\cdots\mu_k^z\rangle\rangle_r\cong\langle\langle\mu_i^z\rangle\rangle_r\langle\langle\mu_j^z\rangle\rangle_r\cdots\langle\langle\mu_k^z\rangle\rangle_r ,\\&\langle\langle s_m^z(s_n^z)^2\cdots s_l^z\rangle\rangle_r\cong\langle\langle s_l^z\rangle\rangle_r\langle\langle(S_n^z)^2\rangle\rangle_r\cdots\langle\langle S_l^z\rangle\rangle_r ,\end{aligned} \tag{22}$$

with $i\neq j\neq\cdots\neq k$ and $m\neq n\neq\cdots\neq l$, where $\langle\cdots\rangle_r$ denotes the random-bond average. It corresponds essentially to the Zernike approximation for $s=\frac{1}{2}$ Ising systems.[15] Nevertheless, the approximation procedure is quite superior to the standard mean-field theory, since within the present framework the relations like $(s_i^z)^2=1$ and 0 are taken exactly into account through the identities (18) and (19) (and, as a consequence, neglects only correlations between different spin variables). On the other hand, the standard mean-field theory neglects all correlations.

Using the decoupling approximation and taking account of the fact that the exchange interaction is given by the independent random variable, the averaged magnetizations can be written in compact forms, for the nearest-neighbor interactions,

$$\sigma_z=\langle\sigma_i^z\rangle_r=[q_z\bar c(1)+m_z\bar s(1)+1-q_z]^z f(x)\,|_{x=0} , \tag{23}$$

$$\sigma_x=\langle\sigma_i^x\rangle_r=[q_z\bar c(1)+m_z\bar s(1)+1-q_z]^z g(x)\,|_{x=0} , \tag{24}$$

$$m_z=\langle m_i^z\rangle_r=[\bar c(\tfrac{1}{2})+2\sigma_z\bar s(\tfrac{1}{2})]^z F(x)\,|_{x=0} , \tag{25}$$

$$m_x=\langle m_i^x\rangle_r=[\bar{c}(\tfrac{1}{2})+2\sigma_z\bar{s}(\tfrac{1}{2})]^z G(x)\,|_{x=0}\ , \tag{26}$$

where $q_z=\langle\langle(s_j^z)^2\rangle\rangle_r$ and z is the number of nearest neighbors. The parameters $\bar{c}(\gamma)$ and $\bar{s}(\gamma)$ are defined by

$$\bar{c}(\gamma)=\langle\cosh(\gamma J_{ij}D)\rangle_r$$
$$=\int P(J_{ij})\cosh(\gamma J_{ij}D)dJ_{ij}$$

and

$$\bar{s}(\gamma)=\langle\sinh(\gamma J_{ij}D)\rangle_r\ . \tag{27}$$

In order to evaluate the longitudinal as well as transverse magnetizations, it is necessary to calculate the parameter q_z. By the use of the relation proposed by SaBarreto *et al.*,[10] we can also obtain, in the same way as the evaluation of m_α and $\sigma_\alpha(\alpha=x,z)$,

$$q_z=[\bar{c}(\tfrac{1}{2})+2\sigma_z\bar{s}(\tfrac{1}{2})]^z H(x)\,|_{x=0}\ , \tag{28}$$

where the function $H(x)$ is defined by

$$H(x)=\frac{\Omega^2+(\Omega^2+2x^2)\cosh[\beta(\Omega^2+x^2)^{1/2}]}{(\Omega^2+x^2)\{1+2\cosh[\beta(\Omega^2+x^2)^{1/2}]\}}\ . \tag{29}$$

The averaged total longitudinal and transverse magnetizations of the mixed alloy are then given by

$$M_z=\frac{N}{2}(m_z+\sigma_z) \tag{30}$$

and

$$M_x=\frac{N}{2}(m_x+\sigma_x)\ , \tag{31}$$

where N is the number of magnetic atoms. In the above discussions we have not touched on the sign of J_{ij}. When J_{ij} is always positive, the ground state of the mixed alloy is ferromagnetic. On the other hand, when J_{ij} is always negative, it is given by the ferrimagnetic state. Thus, within the present framework, we can investigate the temperature (or transverse field) dependences of total and sublattice magnetizations for the ferromagnetic (or ferrimagnetic) mixed Ising spin systems. If $P(J_{ij})$ takes both sign and so called the effects of frustration appears, the ferro- (or ferri-) magnetic state becomes unstable and, as will be discussed in Sec. III, the transition temperature reduces to zero at some critical value of the fluctuation of J_{ij}, from which we can determine the ferromagnetic (or ferrimagnetic) phase stability limit.

III. PHASE DIAGRAM

In this section we investigate the phase diagram (or transition temperature) for the amorphization of the mixed Ising ferromagnetic (or ferrimagnetic) spin system in a transverse field. In a finite transverse field the s^z and μ^z components of this system are disordered at high temperatures, but below a transition temperature T_C they order so that $m_z\neq 0$ and $\sigma_z\neq 0$ and the directions of the moments change continuously, although there is an order with $m_x\neq 0$ and $\sigma_x\neq 0$ at all temperatures.

Here we are interested in studying the transition temperature (or the phase diagram) of the system. Expanding the right-hand side of Eqs. (23), (25), and (28) with respect to m_z (or σ_z), and retaining only terms linear in m_z (or σ_z), we find

$$m_z=zA_1\sigma_z+O(\sigma_z^3)\ , \tag{32}$$

$$\sigma_z=zB_1m_z+O(m_z^3)\ , \tag{33}$$

and

$$q_z=q_z^0+O(\sigma_z^2)\ , \tag{34}$$

where

$$A_1=2\bar{s}(\tfrac{1}{2})[\bar{c}(\tfrac{1}{2})]^{z-1}F(x)\,|_{x=0}\ , \tag{35}$$

$$B_1=\bar{s}(1)[q_z^0\bar{c}(1)+1-q_z^0]^{z-1}f(x)\,|_{x=0}\ , \tag{36}$$

$$q_z^0=[\bar{c}(\tfrac{1}{2})]^zH(x)\,|_{x=0}\ . \tag{37}$$

The second-order phase-transition line is then determined by

$$1=z^2A_1B_1$$

or

$$1=z\sqrt{A_1B_1}\ . \tag{38}$$

Here it is worth mentioning that the relation (38) can be used to evaluate both the ferromagnetic phase stability limit and the ferrimagnetic one, since it does not change even if the sign of J_{ij} is replaced by $-J_{ij}$.

Now, in order to evaluate the coefficients A_1 and B_1 and the parameter q_z^0, it is necessary to provide the actual form of the probability distribution function $P(J_{ij})$, describing the structural disorder in a simple way. In a series of works[3] we have used the probability distribution function $P(J_{ij})$ as follows:

$$P(J_{ij})=\tfrac{1}{2}[\delta(J_{ij}-J-\Delta J)+\delta(J_{ij}-J+\Delta J)]\ . \tag{39}$$

The random-bond averages of (27) are then given by

$$\begin{aligned}\bar{c}(\gamma)&=\cosh(\gamma\delta JD)\cosh(JD)\ ,\\ \bar{s}(\gamma)&=\cosh(\gamma\delta JD)\sinh(JD)\ ,\end{aligned} \tag{40}$$

with

$$\delta=\Delta J/J\ , \tag{41}$$

where δ is a dimensionless parameter which measures the amount of fluctuation of exchange interaction. The parameter δ is often called as the structural fluctuation in amorphous magnets. The result (40) can be also obtained by using the so called "lattice model" of amorphous magnets.[16] Then, the coefficients A_1 and B_1 and the parameter q_z^0 can easily be calculated by applying a mathematical relation $e^{\gamma D}\phi(x)=\phi(x+\gamma)$, when the coordination number z is given. In the following, two lattices, namely with $z=3$ and $z=4$, are investigated.

A. Amorphization of Honeycomb lattice ($z=3$)

Putting $z=3$ into (35), (36), and (37), and substituting (40) into them, we obtain the expressions of A_1, B_1, and q_z^0 (see the Appendix). By solving the relation (38) numerically, we can get the phase diagrams of the system as functions of parameters T_C, Ω, and δ.

In Fig. 1, the change of Curie temperature T_C with Ω is plotted for selected values of δ. With the increase of δ the phase region in which the ferromagnetic (or ferrimagnetic) state is realizable gradually becomes small. In particular, for the curve with $\delta=0.0$ (or the regular honeycomb lattice) the T_C value at $\Omega=0$ is given by $2k_B T_C/J=1.783$, which is to be compared with the exact value $2k_B T_C/J=1.320$,[4,8] and the Bethe-Peierls result $2k_B T_C/J=1.631$,[7] as well as the mean-field approximation result $2k_B T_C/J=2.449$.[12] As is seen from Fig. 1, on the other hand, when Ω increases from zero, in each curve T_C falls from its value in the mixed Ising system and reaches zero at a critical value Ω_c. The critical value for the curve with $\delta=0.0$ is given by $\Omega_c=1.42J$, which is compared with $\Omega_c=2.12J$ for the standard mean-field approximation.[12]

Figure 2 shows the behavior of T_C as a function of δ, when the value of Ω is changed. In particular, the curve labeled a with $\Omega=0$ is equivalent to that (curve a in Fig. 3) of Ref. 17. For each curve, T_C monotonically decreases and disappears at $\delta=1$ or at a certain value smaller than $\delta=1$; when the frustration effect ($\delta>1$) is set into the amorphous mixed Ising ferromagnetic (or ferrimagnetic) system with $z=3$, the ferromagnetic (or ferrimagnetic) state is easily broken and the system does not exhibit any reentrant phenomenon.

In Fig. 3, the critical value Ω_c obtained from Fig. 1 is plotted as a function of δ. It monotonically decreases with the value of δ and disappears at $\delta=1$.

B. Amorphization of square lattice ($z=4$)

For the mixed alloy with $z=4$, we can obtain the expressions of coefficients A_1 and B_1 and the parameter q_z^0, as in Sec. III A for $z=3$. Using these expressions, the behaviors of T_C as functions of Ω and δ can be studied by solving Eq. (38) numerically.

Figure 4 shows the behavior of the critical temperature as a function of δ for several values of Ω. In contrast with the case of $z=3$ (or Fig. 2), the curve labeled a with $\Omega=0$ exhibits a bulge for the region of $\delta>1$, which implies the occurrence of reentrant phenomenon due to the frustration effect of exchange interaction. In particular, the T_C value at $\delta=0$ for curve a corresponds to that of the regular mixed square lattice, which is given by $2k_B T_C/J=2.598$. It is to be compared with other results of the Bethe-Peierls approximation ($2k_B T_C/J=2.478$),[7] the real-space renormalization-group analysis ($2k_B T_C/J=2.372$, $2k_B T_C/J=2.748$),[18] and the standard mean-field approximation ($2k_B T_C/J=3.266$).[12] On the other hand, other results in Fig. 4 with $\Omega=J$ (curve b), $\Omega=1.5J$ (curve c), and

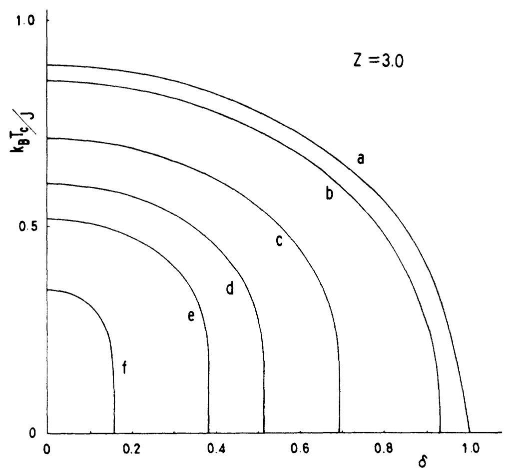

FIG. 2. Plots of T_C vs δ for an amorphous mixed Ising system with $z=3$, when Ω is changed as follows: (a) $\Omega=0$; (b) $\Omega=0.5J$; (c) $\Omega=J$; (d) $\Omega=1.2J$; (e) $\Omega=1.3J$; (f) $\Omega=1.4J$.

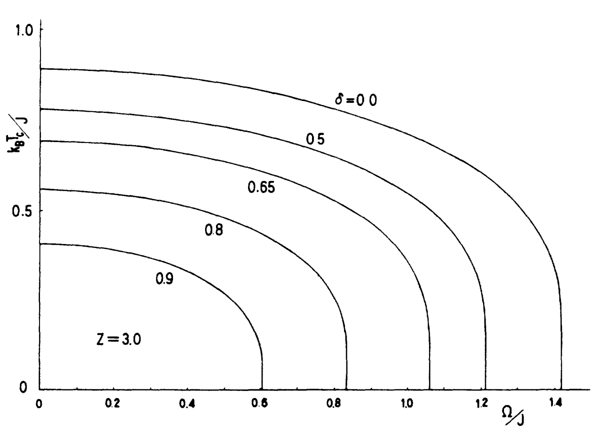

FIG. 1. Transverse field dependencies of Curie temperature for an amorphous mixed Ising spin system with $z=3$, when δ is changed.

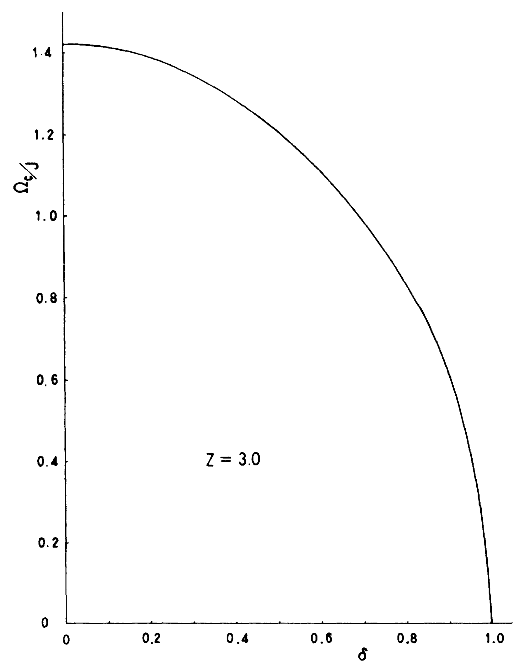

FIG. 3. The critical value Ω_c at which T_C reduces to zero in Fig. 2 is plotted as a function of δ for the mixed system with $z=3$.

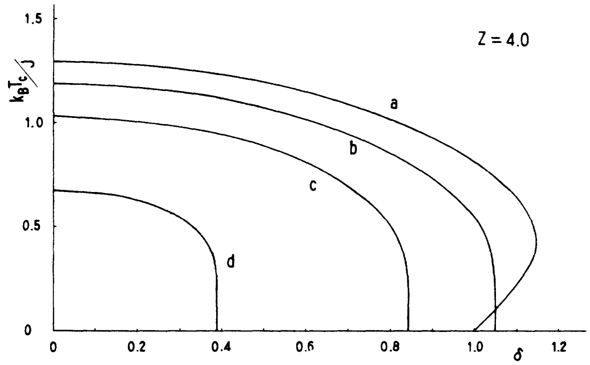

FIG. 4. Plots of T_C vs δ for an amorphous mixed Ising system with $z=4$, when Ω is changed as follows: (a) $\Omega=0$; (b) $\Omega=J$; (c) $\Omega=1.5J$; (d) $\Omega=2.0J$.

$\Omega=2.0J$ (curve d) express the behavior similar to those of Fig. 2.

Therefore, in Fig. 5, we investigate in detail the phase diagram of Fig. 4 for selected values of Ω in the region $0\le\Omega\le J$, especially focusing on the situation with $\delta\ge1$. Looking at Fig. 5, the critical lines exhibit some characteristic behaviors; the first is that the bulge (or the reentrant phenomenon) gradually disappears with the increase of Ω. The second is that the value of δ at which T_C goes to zero first increases for the value of Ω in the region $0\le\Omega\le0.65J$ and then decreases for other values of Ω.

Figure 6 shows the changes of T_C with Ω for several values of δ. For the values of δ smaller than unity, the T_C versus Ω curves exhibit the behavior similar to that of Fig. 1. On the other hand, when δ becomes larger than unity, the important difference may appear in the plot because of the frustration effect of exchange interaction; for instance, the curve labeled $\delta=1.1$ separates into two parts and expresses a weak bulge near $\Omega=0.7J$. Particularly, only one value of T_C is found in a restricted region

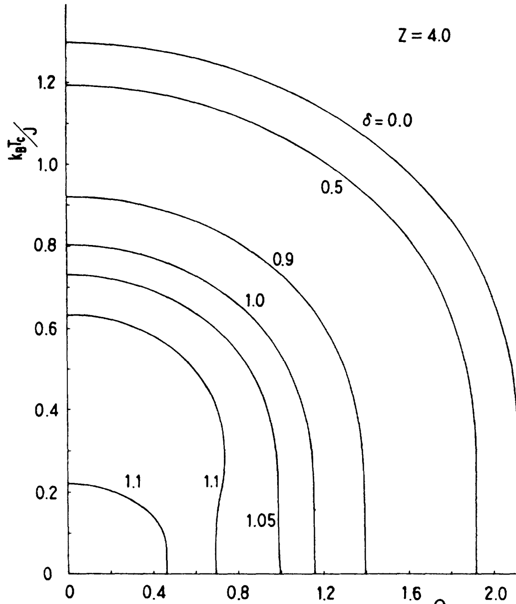

FIG. 6. Transverse field dependencies of T_C for an amorphous mixed Ising system with $z=4$, when δ is changed. For $\delta=1.1$, T_C curve splits into two boundaries and a bulge is observed.

of Ω, namely $0.46<\Omega/J<0.69$ for the curve of $\delta=1.1$. The characteristic behaviors come from the peculiar results found in Fig. 5 and are consistent with those of Fig. 5.

In Fig. 7, the critical value of Ω at which T_C goes to zero in Fig. 6 is plotted as a function of δ, like Fig. 3. The critical value Ω_c at $\delta=0$ is given by $\Omega_c=2.12J$, which is to be compared with the standard mean-field approximation result $\Omega_c=2.83J$.[12] In contrast with the

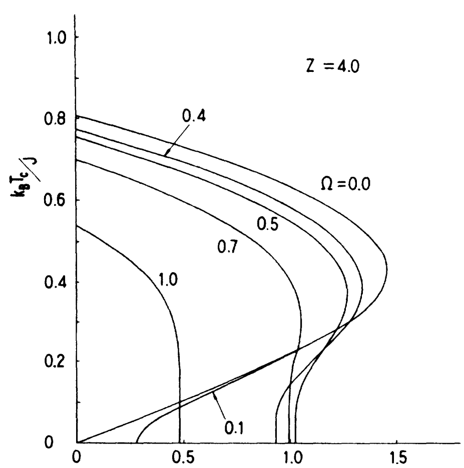

FIG. 5. The change of T_C with δ for the mixed system with $z=4$, especially focussing the situation with $\delta\ge1$. The value of Ω is selected as follows: $\Omega=0$; $\Omega=0.1J$; $\Omega=0.4J$; $\Omega=0.5J$; $\Omega=0.7J$; $\Omega=J$.

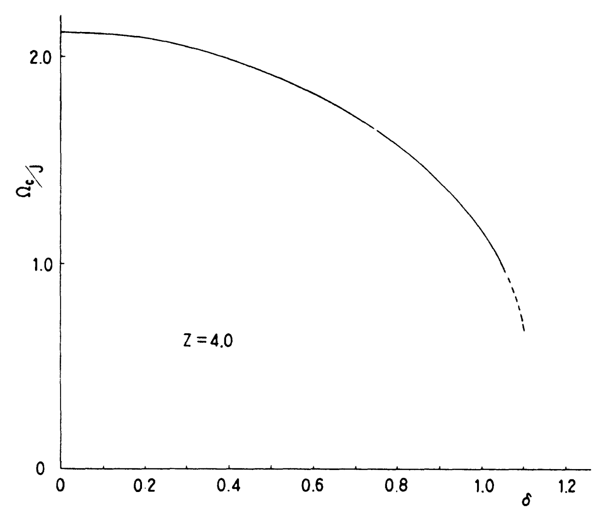

FIG. 7. The critical value Ω_c determined from Fig. 6 is plotted as a function of δ. For $\delta>1.0$, the plot cannot be determined definitely because of the appearance of two Ω_c, like the curve labeled $\delta=1.1$ in Fig. 6.

case of Fig. 3, the critical value Ω_c does not monotonically go to zero. Moreover, we can not determine it definitely, when the value of δ becomes larger than $\delta=1.0$, since, as shown in Fig. 6, the two values of Ω may appear like the curve with $\delta=1.1$.

IV. AMORPHOUS MIXED FERRIMAGNETS

In Sec. III we have discussed the phase diagrams for amorphous mixed Ising spin systems with $z=3$ and $z=4$ in a transverse field. In this section the temperature (or transverse field) dependencies of total magnetizations (M_z and M_x) in the mixed ferrimagnetic alloys with the structural disorder are investigated within the framework given in Sec. II, since from the experimental point of view the study may be very important.

A. Magnetization curves for $z=3$

As discussed in Sec. III A, let us at first study the temperature (or transverse field) dependencies of magnetizations for the amorphous mixed ferrimagnetic alloy with $z=3$.

Using Eqs. (23)–(26) and (28) and putting $J_{ij}=-J_{ij}$ ($J_{ij}>0$) into the factors $\bar{c}(\gamma)$ and $\bar{s}(\gamma)$, we obtain a set of equations for the system as follows:

$$m_z=-3A_1\sigma_z-A_2(\sigma_z)^3\ , \tag{42}$$

$$\sigma_z=-3B_1m_z-B_2(m_z)^3\ , \tag{43}$$

$$q_z=q_z^0+12q_z^1(\sigma_z)^2\ , \tag{44}$$

and

$$m_x=c_1+12c_2(\sigma_z)^2\ , \tag{45}$$

$$\begin{aligned}\sigma_x=&D_4(q_z)^3+3D_3(q_z)^2(1-q_z)\\&+3D_2q_z(1-q_z)^2+D_1(1-q_z)^3\\&+3E_2q_z(m_z)^2+3E_1(1-q_z)(m_z)^2\ ,\end{aligned} \tag{46}$$

where the coefficients A_1 and B_1 and the parameter q_z^0 are given by the same functions as those in Sec. III A, except that the parameter q_z^0 in the B_1 of Sec. III is replaced by q_z (see the Appendix). The other coefficients A_2,B_2, C_i ($i=1,2$), D_m ($m=1-4$), and E_n ($n=1,2$) are defined by

$$\begin{aligned}&A_2=8[s(\tfrac{1}{2})]^3F(x)\,|_{x=0}\ ,\\&B_2=[\bar{s}(1)]^3f(x)\,|_{x=0}\ ,\\&C_1=[\bar{c}(\tfrac{1}{2})]^3G(x)\,|_{x=0}\ ,\\&C_2=\bar{c}(\tfrac{1}{2})[\bar{s}(\tfrac{1}{2})]^2G(x)\,|_{x=0}\ ,\\&D_m=[\bar{c}(1)]^{m-1}g(x)\,|_{x=0}\ ,\\&E_n=[\bar{s}(1)]^2[\bar{c}(1)]^{n-1}g(x)\,|_{x=0}\ .\end{aligned} \tag{47}$$

The parameter q_z^1 is given by

$$q_z^1=[\bar{s}(\tfrac{1}{2})]^2\bar{c}(\tfrac{1}{2})H(x)\,|_{x=0}\ . \tag{48}$$

The coefficients (47) and the parameter q_z^1 can also be calculated by using (40) and a mathematical relation $e^{\gamma D}\phi(x)=\phi(x+\gamma)$, as done in Sec. III A. In this way, by solving the set of coupled equations numerically, we can obtain the temperature (or transverse field) dependencies of total magnetizations (M_z and M_x) for the mixed alloy.

In Fig. 8 we show the temperature dependencies of sublattice magnetizations (m_z, σ_z, m_x, and σ_x) and the parameter q_z for the regular ($\delta=0.0$) honeycomb lattice, when the value of Ω is taken as $\Omega=0$ and $\Omega=J$. For $\Omega=0$, the transverse sublattice magnetizations are always given by $m_x=0$ and $\sigma_x=0$ and the values of m_z and σ_z at $T=0$ K are $m_z=-1.0$ and $\sigma_z=0.5$. At $T=T_C$, m_z reduces to $m_z=0$ and q_z expresses the discontinuity for its derivative which is similar to that known for the spin-1 Ising model.[19]

For $\Omega=J$, on the other hand, m_z and σ_z fall below the corresponding curves for $\Omega=0$, and m_x and σ_x have finite values in the whole temperature range; the role of the transverse field is essentially to inhibit the ordering of the s^z and μ^z components. In the ordered phase m_x and σ_x weakly depend on temperature and at $T=T_C$ their derivatives show discontinuities. The results are very similar to those found for the spin-$\frac{1}{2}$ transverse Ising model.[11,20]

One of the temperature dependencies of M_z and M_x in a transverse field is shown in Fig. 9. The results are obtained by selecting three values of δ under $\Omega=J$. As is understood from Fig. 1, M_z for the curve c with $\delta=0.9$ is always given by $M_z=0$. The figure clearly shows that with the increase of δ, the value of M_x increases from that of $\delta=0$ in the ordered phase (or $M_z\neq0$), although the value of M_z decreases. Moreover, the compensation point does not appear in the present system, since a uniform transverse field is applied.[21]

In Fig. 10, the transverse field dependencies of M_z and M_x at a fixed temperature ($T=0.1J$) are depicted, changing the value of δ. The M_z curve labeled a with $\delta=0$ takes $M_z=-0.25N$ at $\Omega=0$, decreases monotonically

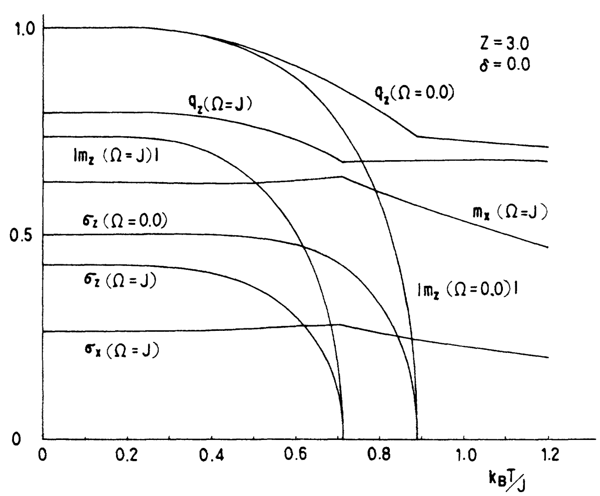

FIG. 8. Temperature dependencies of sublattice magnetizations (m_z, σ_z, m_x, and σ_x) and the parameter q_z for the pure ($\delta=0.0$) ferrimagnetic honeycomb lattice ($z=3$), when Ω is taken as $\Omega=0$ and $\Omega=J$.

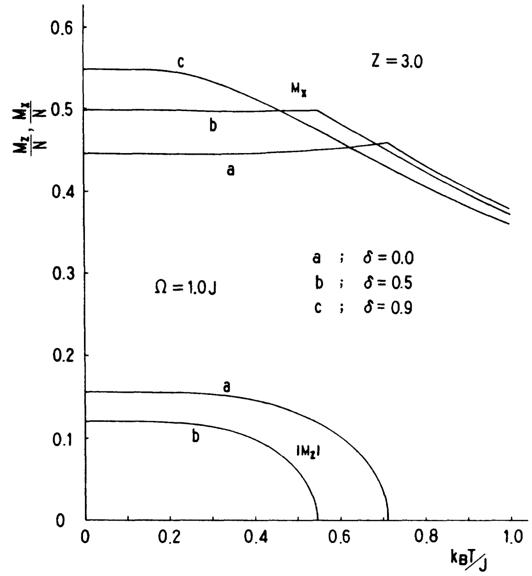

FIG. 9. Temperature dependencies of M_z and M_x for the mixed ferrimagnetic system with $z=3$, when Ω is fixed as $\Omega=J$. The value of δ is then changed as follows: (a) $\delta=0.0$; (b) $\delta=0.5$; (c) $\delta=0.9$.

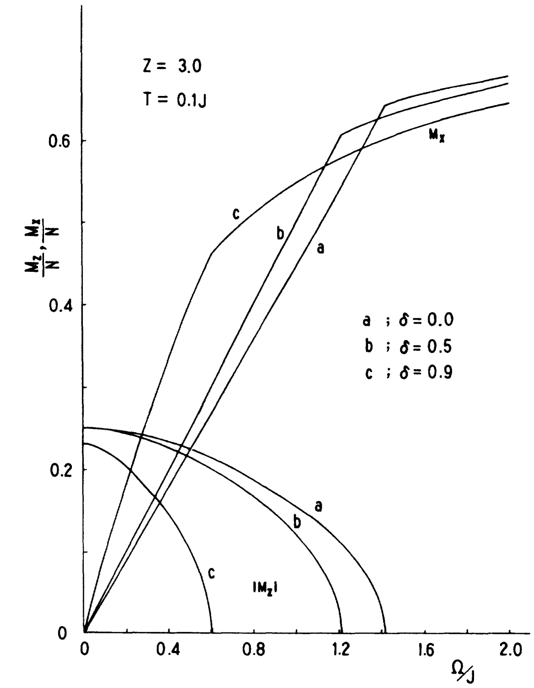

FIG. 10. Transverse field dependencies of M_z and M_x at a fixed temperature ($T=0.1J$) for the mixed ferrimagnetic system with $z=3$, when δ is changed as follows: (a) $\delta=0.0$; (b) $\delta=0.5$; (c) $\delta=0.9$.

and disappears at the values of Ω near $\Omega_c=1.42J$. On the other hand, the M_x curve for $\delta=0$ increases linearly with Ω and changes the inclination at the value of Ω which M_z reduces to zero. The behavior of M_x is similar to the dependence of magnetization of a spin-$\frac{1}{2}$ isotropic antiferromagnet on the value of the applied transverse field at zero temperature.[22,23]

As is seen from the Fig. 10, the introduction of the structural disorder into the mixed honeycomb lattice affects severely the inclination of M_x in the ordered phase. In particular, the M_x curve labeled c with $\delta=0.9$ exhibits a downward curvature in the disordered phase with $M_z=0$. On the other hand, the M_z curve for a finite value of δ decreases more rapidly than that for $\delta=0$; it means that when the M_z curve is plotted in the reduced units $(M_z(T)/M_z(0),\ \Omega/\Omega_c)$, it falls below the corresponding curve for $\delta=0$. The phenomenon found for the M_z curve in the reduced plots is similar to that generally found in amorphous ferromagnets, when the magnetization M_z is plotted in the reduced units $(M_z(T)/M_z(0),\ T/T_C)$.[2]

B. Magnetization curves for $z=4$

In Sec. III B we have examined the phase diagrams of the amorphous mixed Ising ferrimagnetic (or ferromagnetic) square lattice ($z=4$) and found that in contrast with the system with $z=3$, a number of interesting results appear in the phase diagrams due to the frustration effects of exchange interactions. In this part, therefore, we investigate the temperature (or transverse field) dependencies of M_z and M_x for the ferrimagnetic system with $z=4$, especially focusing the situation with the effects ($\delta>1$). They can be evaluated by solving the set of equations with $z=4$ [or (23)–(26) and (28) in Sec. II] numerically, as in Sec. IV A.

In Fig. 11, the behaviors of M_z and M_x as a function of T are shown for several values of δ, when the value of Ω is fixed at $\Omega=0.4J$. As is predicted from Fig. 6, the reentrant phenomenon is observed in the magnetization curve of M_z, namely curve e with $\delta=1.1$ in Fig. 11(b). In particular, the M_z curve labeled d with $\delta=1.05$ expresses a characteristic behavior; with the increase of T the magnetization first increases and then decreases to zero. The initial increase may come from the release of the frustrated spins due to the thermal agitation, like the reentrant phenomenon of curve e in Fig. 11(b). On the other hand, the value of M_x increases in the ordered phase with the increase of δ.

Figure 12 shows the behaviors of M_z and M_x as a function of T for the system with a fixed value of δ ($\delta=0.5$), when the value of Ω is changed. As is understood from the phase diagram of Fig. 4, any characteristic behaviors, like those found in Fig. 11, do not appear for the system with $\delta=0.5$. With the increase of Ω, the values of M_z simply decrease from those for $\Omega=0$ in the ordered phase and instead the values of M_x increase.

As shown in Figs. 5 and 6, when the value of δ becomes larger than unity, some characteristic behaviors may appear in the phase diagrams because of the frustration effects of exchange interactions. In Fig. 13, therefore, the temperature dependences of M_z and M_x are examined in detail for the system with $\delta=1.1$, changing the value of Ω. For $\Omega=0$, the system shows the reentrant phenomenon for the M_z curve, in accordance with the phase diagrams of Sec. III B. As Fig. 5 clearly indicates, the reentrant behavior for the M_z curve is observed in the region $0 \leq \Omega/J < 0.46$, like the curve with $\Omega = 0.4$ in Fig. 13, but in the region $0.46 \leq \Omega/J \leq 0.695$ the behavior of the M_z curve changes like the curves labeled as $\delta = 0.5$ and $\delta = 0.6$ in Fig. 13. In the very narrow region given by $0.695 < \Omega/J < 0.74$, the reentrant behavior is once again observed in the M_z curve, like the curve labeled $\delta = 0.7$ in Fig. 13. For $\Omega > 0.74J$, M_z is always given by $M_z = 0$, so that the M_x curve monotonically decreases from the value at $T = 0$ K with the increase of T, as shown in the curve labeled $\Omega = 0.8J$ of Fig. 13.

In Fig. 14, the transverse field dependencies of M_z and M_x at a fixed temperature ($T = 0.1J$) are depicted, changing the value of δ. For the system with $\delta = 0$ and $\delta = 0.5$ in Fig. 14, the behaviors of M_z and M_x as a function of Ω are very similar to those (or the curves labeled b and c) in Fig. 10. On the other hand, when δ is equal to or larger than $\delta = 1.0$, some clear differences arise in M_z and M_x curves; the M_z curve exhibits the reentrant phenomenon,

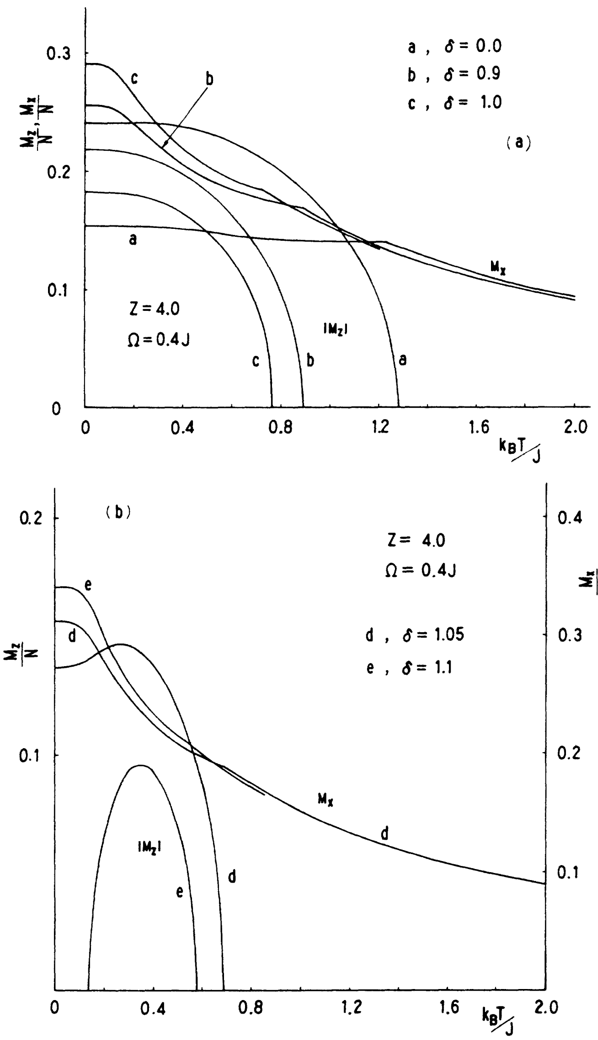

FIG. 11. (a) Magnetization curves of M_z and M_x for the mixed ferrimagnetic system with $z = 4.0$, when Ω is fixed at $\Omega = 0.4J$. The value of δ is then changed as follows: (a) $\delta = 0.0$; (b) $\delta = 0.9$; (c) $\delta = 1.0$. (b) The value of δ is taken as $\delta = 1.05$ (curve d) and $\delta = 1.1$ (curve e).

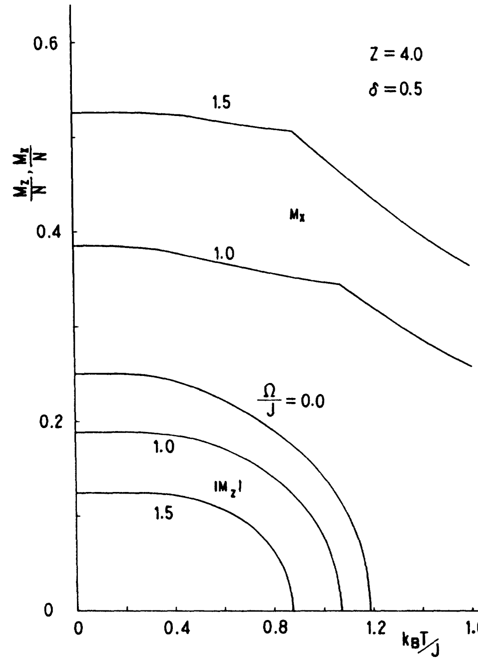

FIG. 12. Temperature dependencies of M_z and M_x for the mixed ferrimagnetic system with $z = 4.0$ and $\delta = 0.5$, when the value of Ω is changed as $\Omega = 0.0$, $\Omega = J$, and $\Omega = 1.5J$.

like the curves labeled d and e. The M_x curves show the downward curvature even in their ordered phase.

V. CONCLUSIONS

In this work, we have studied amorphous mixed Ising spin systems in a transverse field by the use of the effective-field theory with correlations. The phase dia-

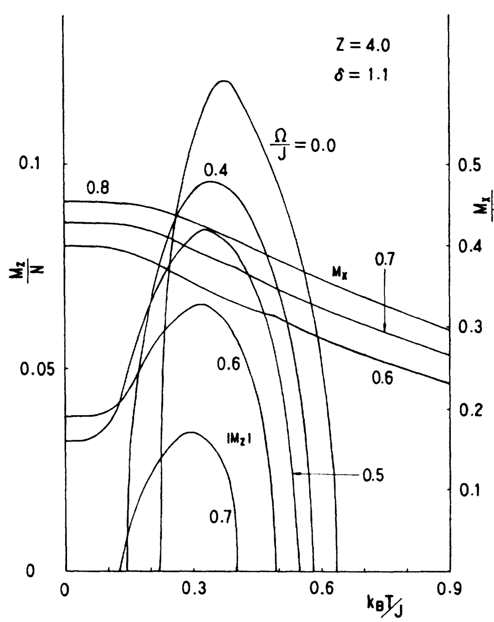

FIG. 13. Temperature dependencies of M_z and M_x for the mixed ferrimagnetic system with $z = 4$ and $\delta = 1.1$, when the value of Ω is changed as $\Omega = 0$, $\Omega = 0.4J$, $\Omega = 0.5J$, $\Omega = 0.6J$, and $\Omega = 0.7J$.

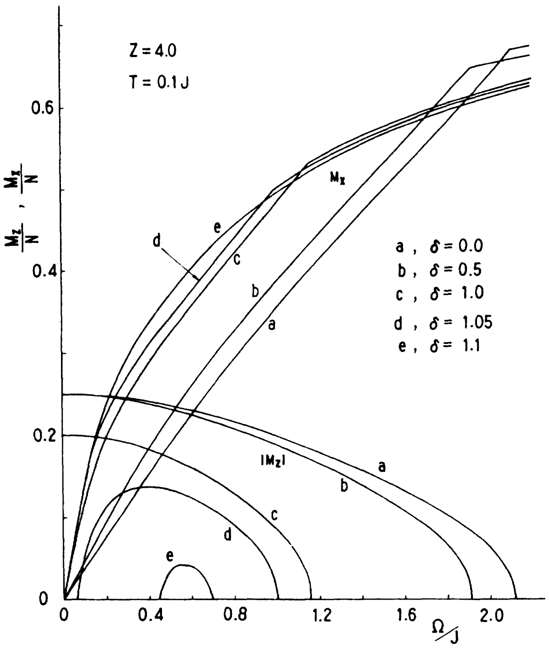

FIG. 14. Transverse field dependencies of M_z and M_x at a fixed temperature ($T=0.1J$) for the mixed ferrimagnetic system with $z=4$, when δ is changed as follows: (*a*) $\delta=0.0$; (*b*) $\delta=0.5J$; (*c*) $\delta=1.0$; (*d*) $\delta=1.05$; (*e*) $\delta=1.1$.

grams have been calculated in Sec. III. The temperature (or transverse field) dependencies of total longitudinal and transverse magnetizations for the amorphous mixed ferrimagnetic alloys have been examined in Sec. IV. As discussed in Secs. III and IV, a number of interesting phenomena have been found in the physical quantities, which are due to the structural fluctuation and the transverse field.

The present effective-field approach is based on a generalization of the Callen relation for the Ising model in the presence of a transverse field. The obtained results are quite remarkable considering that the approximation used within this simple effective-field approach neglects spin-spin correlations. In the previous works,[24,25] on the other hand, we have improved the decoupling approximation (22) (or the effective-field theory with correlations) by introducing the concept of correlated effective field[26] and a new decoupling approximation[25] into multispin correlation functions. The introduction of the correlated effective field into the multispin correlation function is closely related to the reaction field of Onsager in dielectrics,[27] which theory is equivalent to the Bethe-Peierls approximation. Furthermore, the new decoupling approximation improves the Bethe-Peierls approximation. It has also predicted the possibility of the reentrant phenomenon, when it is applied to the spin-$\frac{1}{2}$ Ising model with random bonds. The results obtained in the previous works have indicated that the decoupling (22) gives reasonable results for various physical quantities, although the approximation is the simplest one that can be done. Thus, as previous works on other models have indicated, we find that the results obtained herein can be given qualitative, and to a certain extent, quantitative liability.

In this work, we have studied a simplified mixed alloy system, in comparison with real existed amorphous ferromagnetic and ferrimagnetic alloys. However, some phenomena found in this work may be observed in real systems, when a transverse field is applied. We wish that this work could stimulate further experimental and theoretical works on amorphous magnetism.

ACKNOWLEDGMENT

Two of the present authors (F.S and I.P) were partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (Brazilian agency).

APPENDIX

The expressions of the parameter q_z^0 and the coefficients A_1 and B_1 in Sec. III A are given by

$$\begin{aligned} q_z^0 &= \cosh^3\left[\frac{J}{2}\delta D\right]\cosh^3\left[\frac{J}{2}D\right]H(x)\big|_{x=0} \\ &= (\tfrac{1}{2})^5\left\{H(\tfrac{3}{2}J+\tfrac{3}{2}J\delta)+3H(\tfrac{1}{2}+\tfrac{3}{2}J\delta)+3H\left[\frac{J}{2}-\frac{3}{2}J\delta\right]+H(\tfrac{3}{2}J-\tfrac{3}{2}J\delta)\right. \\ &\qquad \left.+3\left[H\left[\frac{3}{2}J+\frac{J}{2}\delta\right]+3H\left[\frac{J}{2}+\frac{J}{2}\delta\right]+3H\left[\frac{J}{2}-\frac{J}{2}\delta\right]+H\left[\frac{3}{2}J-\frac{J}{2}\delta\right]\right]\right\}, \end{aligned} \tag{A1}$$

$$\begin{aligned} A_1 &= 2\cosh^3\left[\frac{J}{2}\delta D\right]\sinh\left[\frac{J}{2}D\right]\cosh^2\left[\frac{J}{2}D\right]F(x)\big|_{x=0} \\ &= (\tfrac{1}{2})^4\left\{F(\tfrac{3}{2}J+\tfrac{3}{2}J)+F\left[\frac{J}{2}+\frac{3}{2}J\delta\right]+F\left[\frac{J}{2}-\frac{3}{2}J\delta\right]+F(\tfrac{3}{2}J-\tfrac{3}{2}J\delta)\right. \\ &\qquad \left.+3\left[F\left[\frac{3}{2}J+\frac{J}{2}\delta\right]+F\left[\frac{J}{2}+\frac{J}{2}\delta\right]+F\left[\frac{J}{2}-\frac{J}{2}\delta\right]+F\left[\frac{3}{2}J-\frac{J}{2}\delta\right]\right]\right\}, \end{aligned} \tag{A2}$$

and

$$B_1=(q_z^0)^2b_1+2q_z^0(1-q_z^0)b_2+(1-q_z^0)^2b_3 \tag{A3}$$

with

$$\begin{aligned}b_1&=\cosh^3(J\delta D)\sinh(JD)\cosh^2(JD)f(x)\,|_{x=0}\\&=(\tfrac{1}{2})^5\{f(3J+3J\delta)+f(J+3J\delta)+f(J-3J\delta)+f(3J-3J\delta)\\&\qquad+3[f(3J+J\delta)+f(J+J\delta)+f(J-J\delta)+f(3J-J\delta)]\}\ ,\end{aligned} \tag{A4}$$

$$b_2=\cosh^2(J\delta D)\sinh(JD)\cosh(JD)f(x)\,|_{x=0}=(\tfrac{1}{2})^3[f(2J+2J\delta)+f(2J-2J\delta)+2f(2J)]\ , \tag{A5}$$

$$b_3=\cosh(J\delta D)\sinh(JD)f(x)\,|_{x=0}=\tfrac{1}{2}[f(J+J\delta)+f(J-J\delta)]\ . \tag{A6}$$

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