Response functions and collective modes of ³He in strong magnetic fields: Determination of material parameters from experiments

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This is the second of two papers on the response functions and collective modes of ³He-B in a strong magnetic field. Here we compare the theory developed in the first paper [R. S. Fishman and J. A. Sauls, Phys. Rev. B **33**, 6068 (1986)] with the susceptibility data of Scholz and Hoyt et al. and with the collective-mode data of Shivaram et al. at the lowest pressures, where strong-coupling corrections to the BCS free energy are known to be negligible. In principle this comparison yields new results for some of the material parameters of liquid ³He. Determinations of these material parameters are important for testing the consistency of quasiclassical theory and the interpretation of different measurable properties of ³He, and perhaps future microscopic theories of quantum liquids. The material parameters extracted from these two different data bases are in serious disagreement, and cannot be reconciled even with radically different empirical temperature scales. We do, however, find that our determination of the f-wave interaction at $p \approx 0$ bar agrees with that obtained by Meisel et al. who analyze the zero-field squashing-mode data.

INTRODUCTION

Liquid ³He is the only strongly interacting fermion system which is sufficiently simple (i.e., isotropic and homogeneous) so that Landau's Fermi liquid theory can be tested quantitatively. The predictions of Landau's theory depend on material parameters; the quasiparticle effective mass m^*, and other Landau parameters $(F_l^{s,a})$.[1] From the normal-phase experimental data only a limited amount of information can be obtained on the Landau parameters, unless one makes additional, and approximate, assumptions about the scattering amplitude between quasiparticles. From exact results of Landau theory one can obtain the effective mass from a measurement of the specific heat and density. Once the effective mass is determined the Landau parameter F_0^a is obtained from a measurement of the enhancement of the susceptibility, and the Landau parameter F_0^s is obtained from a measurement of the hydrodynamic sound velocity. To a high degree of precision, the Landau parameter F_2^s may be obtained from a measurement of the zero sound velocity. Even these determinations of the *low-order* Landau parameters are uncertain because they depend upon prior knowledge of the effective mass. There has been some controversy regarding interpretations of the measurements of the specific heat in normal ³He in different laboratories, and thus, one of the most basic material parameters of ³He, the quasiparticle effective mass, still remains uncertain. At zero pressure the effective mass reported by Wheatley[2] is $(m^*/m)_W = 3.01$, which is considerably larger than the value of $(m^*/m)_H = 2.12$ obtained by Haavasoja et al.[3,4] The more recent, and more

extensive, measurements of Greywall[5,6] yield the intermediate value $(m^*/m)_G = 2.76$, somewhat closer to that of Wheatley. The corresponding values of the Landau parameter F_0^a, obtained from the susceptibility measurements are: $F_0^a = -0.67(W)$, $-0.77(H)$, $-0.70(G)$. These small differences translate into large differences in the effective field acting on quasiparticles and Cooper pairs, and thus lead to distinctly different predictions for the magnetic susceptibility and collective modes in superfluid ³He. Measurements in the superfluid phases can provide new determinations of the Landau parameters, particularly F_0^a, that are independent of prior knowledge of the effective mass; however, uncertainties in the temperature scale complicate these determinations. Furthermore, Greywall has pointed out that the differences in m^*/m measured by different laboratories are quite probably related to differences in temperature scales.[6] Elimination of uncertainties in the temperature scale may also eliminate our uncertainty in m^*/m and F_0^a from normal-phase measurements.

There are several reasons why we would like to have more precise information on the material parameters of ³He. The B phase is the quantum fluid where we have the greatest hope of quantitatively testing the predictions of the quasiclassical (QC) theory, the extension of Landau's Fermi-liquid theory to the superfluid phases.[7] Such an extension is possible because the pairing correlations are weak; the characteristic pairing energy set by the transition temperature $T_c \approx 2$ mK, which is very small compared to the atomic scale set by the Fermi energy $T_F \approx 1$ K. In consequence, the Fermi liquid and pairing interactions are to high accuracy unaffected by the condensation

into the superfluid state. Of course the nature of the elementary excitations, the distribution of excitations in momentum space and the spectrum of collective modes is dramatically altered by the condensation. For this reason the superfluid phases provide a stringent test of the consistency of QC theory, and are the more useful phases for deducing the underlying quasiparticle interactions. It is worth mentioning that the quasiparticle interactions in the particle-particle (pairing) channel can only be obtained from comparison of the QC theory with observable properties of the superfluid phases. Liquid ³He is also one of the few quantum systems where microscopic many-body theory can be tested. Although quantitatively accurate predictions for the quasiparticle interactions are not yet possible, accurate determinations of the higher-order quasiparticle interactions using QC theory will provide a stringent test for future microscopic theories of quantum fluids.[8] Finally, the effective interactions in higher angular-momentum channels suggest the existence of novel correlations in the superfluid phases at very low temperatures. Sauls and Serene[9] have previously shown that an attractive f-wave pairing interaction implies the existence of an order-parameter collective mode in ³He-B with total angular momentum $J=4$, which should be observable in the ultrasound attenuation spectrum at sufficiently low temperatures. More recently, Israelsson, et al. reported a sizable shift in the clapping mode frequency of ³He-A, and attributed the shift to induced f-wave pairing fluctuations of the order parameter.[10] In addition to these dynamical effects, f-wave pairing correlations in ³He-A have an important effect on the

A-B transition near the melting pressure.[11] Determinations of the higher angular momentum quasiparticle interactions would be useful in suggesting where to look for these subtle correlations in the condensate.

In our earlier paper[12] (hereafter FS), we used the QC theory of superfluid ³He to calculate the nonlinear susceptibility and real squashing (RSQ) mode frequencies of ³He-B in strong magnetic fields. Although, the QC theory is powerful enough to handle the leading strong-coupling corrections to the BCS theory,[7] which are of course necessary for a quantitative theory of the superfluid phases at high pressures, the calculations in FS omit these strong-coupling corrections. But, we include all Landau parameters and higher-order pairing interactions that enter in weak-coupling BCS theory. Our expectation is that weak-coupling theory is a quantitative theory of superfluid ³He-B at low pressures, where it is known that the specific-heat discontinuity for the B phase is very close to the weak-coupling prediction. Given this assumption we extract, with the aid of our exact weak-coupling expressions for the nonlinear susceptibility and the RSQ mode frequencies, values for the Landau parameters F_0^a, F_2^a, and f-wave transition temperature, parametrized by $x_3 = \ln(T_{cf}/T_c)$, for pressures below 1 bar from the experimental data of Scholz,[13] Hoyt et al.,[14] and Shivaram, et al.[15,16]

SUSCEPTIBILITY ANALYSIS

Our result for the nonlinear susceptibility, given in FS, is

$$\mathcal{X}(H,T) = \mathcal{X}_1 + \mathcal{X}_3 ,$$

$$\mathcal{X}_1 = \mathcal{X}_N(1+F_0^a)[\tfrac{2}{3} + (\tfrac{1}{3} + F_2^a/5)y]/D ,$$

$$\mathcal{X}_3 = \frac{4}{3}\mathcal{X}_N(1+F_0^a)(1+F_2^a/5)^2 D^{-2} \left[\frac{\omega_L}{2\Delta_0}\right]^2 \left\{ (\Delta_0^2 Y_{3/2})[1 - A + \tfrac{3}{10}A^2 - \tfrac{1}{2}x_3^{-1}(\Delta_0^2 Y_{3/2})(1-A)^2] \right.$$

$$\left. + (\Delta_0^4 Y_{5/2}) \left[-1 + \frac{27}{40}\left[\frac{\Delta_0^4 Y_{5/2}}{\Delta_0^2 Y_{3/2}}\right] + \tfrac{1}{5}A + \tfrac{1}{5}A^2 \right] - \tfrac{3}{8}(\Delta_0^6 Y_{7/2}) \right\} ,$$

(1)

$$\omega_L = \gamma H(1+F_2^a/5)/D ,$$

$$D = 1 + (\tfrac{2}{3} + \tfrac{1}{3}y)F_0^a + (\tfrac{1}{3} + \tfrac{2}{3}y)F_2^a/5 + yF_0^a F_2^a/5 ,$$

$$A = (\Delta_0^2 Y_{3/2})(F_2^a/5)/(1+F_2^a/5) ,$$

where $\mathcal{X}_N$ is the normal phase susceptibility, $y=1 - \Delta_0^2 Y_{3/2}$ is the Yoshida function, and

$$Y_{m/2}(T/T_c) = \pi T \sum_n (\varepsilon_n^2 + \Delta_0^2)^{-m/2}$$

with $\varepsilon_n = (2n+1)\pi T$. The term $\mathcal{X}_1$ is the zero-field susceptibility for ³He-B, and the second term $\mathcal{X}_3$ is the leading nonlinear correction. Note that $\mathcal{X}$ is a function of the reduced temperature T/T_c, and the interaction parameters F_0^a F_2^a, and x_3^{-1}; *the f-wave interaction parameter enters only the nonlinear correction $\mathcal{X}_3$*. The expansion pa-

rameter is

$$\gamma H/\Delta_0 = 0.16[H(\text{kG})]/[\Delta_0(\text{mK})] , \qquad (2)$$

where $\Delta_0(T)$ is the zero-field, temperature-dependent BCS gap. The corrections to $\mathcal{X}(H,T)$ in Eqs. (1) are of order $(\gamma H/\Delta_0)^4$, and are expected to be small for fields less than 10 MHz, $\gamma[H(\text{MHz})] = 3.24[H(\text{kG})]$, except for temperatures very close to T_c. For the susceptibility data of Scholz[13] and Hoyt et al.[14] we restrict our analysis to the quadratic field region, where perturbation theory is valid. It is possible to derive an expression for the non-

linear susceptibility to all orders in $\gamma H /\Delta_0$, but the result is still an approximation since it is necessary to omit Fermi-liquid interactions with $l > 2$ and pairing interactions with $l > 3$.[17] The perturbation theory of FS for χ includes all interaction effects (except strong coupling) to order $(\gamma H /\Delta_0)^2$. Thus, for the purpose of extracting the material parameters from the data we prefer to restrict the data set to the region where our expressions are accurate, rather than fit an approximate theory over the complete range of magnetic fields. In the end it may not matter if it happens that the interaction parameters with large l are negligible.

In order to extract values for F_0^a, F_2^a, and x_3^{-1} from the susceptibility data we perform a fit of our theory to the data by minimizing a deviation function,

$$S = \sum_i [X_i - X(H, T_i)]^2 ,$$

with respect to the unknown parameters (F_2^a, x_3^{-1}), where X_i is the measured susceptibility at temperature T_i. There are a small number of input parameters and assumptions that are made in order to extract the material parameters from the data. They are as follows. (1) We assume that the ratios of T/T_c obtained from the data of Ref. (13) are the correct ratios of absolute temperatures. (2) The susceptibility and the nonlinear Zeeman shifts of the RSQ modes are most sensitive to the Fermi-liquid parameter F_0^a. This parameter can be extracted from the normal-state susceptibility, but its value is tied to the effective mass. We analyze the susceptibility data using F_0^a obtained from the effective mass determinations of Alvesalo, et al.[4] (AHMS) and Greywall.[5] (3) Since the field dependence of the susceptibility, as well as the RSQ mode frequencies discussed later in this paper, depend on the ratio $\gamma H /\Delta_0$, it is necessary to choose an empirically derived temperature scale. Based on Greywall's observation[6] that the differences in effective mass determinations by various laboratories may be explained in terms of the differences in the absolute temperature scales used by these laboratories, we use the AHMS scale for T_c for our
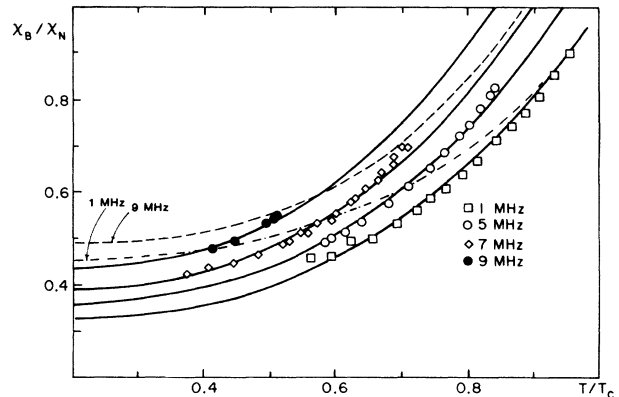

FIG. 1. The susceptibility data of Scholz (Ref. 13) and Hoyt et al., (Ref. 14) and the theoretical fit (solid curves) for 0 bar based on Greywall's input parameters: $T_c = 0.92$ mK and $F_0^a = -0.70$. The magnetic field is given in units of MHz. This fit gives $F_2^a = 1.04$ and $x_3^{-1} = -1.75$. The dash-dotted (dashed) curve is the theoretical calculation for a field of 1 MHz (9 MHz) with the same Greywall inputs, but with the parameters: $F_2^a = -1.37$ and $x_3^{-1} = +0.17$, which are the optimal parameters obtained from the fit of the RSQ mode splittings in a magnetic field.

fits based on the effective mass value reported in Ref. (4), and the Greywall scale[6] for our fit based on the Greywall determination of the effective mass.[5]

The fit of Eqs. (1) to the susceptibility data of Hoyt, et al. is shown in Fig. 1, and the resulting parameters x_3^{-1} and F_2^a are listed in Table I in the rows labeled by "χ." The important feature to note is that the f-wave pairing interaction is attractive and implies a rather large f-wave transition temperature for either set of input parameters; $x_3^{-1} = -1.75$ corresponds to $T_{cf} = 0.6$ mK, not far below the p-wave transition temperature of 1.04 mK. The value obtained for F_2^a is sensitive to the input parameters, in particular, F_2^a depends strongly on the choice of F_0^a, ranging from $F_2^a = -0.88$, based on the AHMS parameters, to $F_2^a = 1.04$ with Greywall's input parameters. As can

TABLE I. The parameters (F_2^a, x_3^{-1}) were obtained from the susceptibility data of Ref. (13) and the collective mode data of Refs. (20) and (16) for pressures $p < 1$ bar. The input parameters (T_c, F_0^a) are based on the temperature scales of Ref. (4) and Ref. (6); the number in parentheses in the $T_c(p)$ column is the pressure in bar. Rows 1 and 4 give results, with statistical errors, based on the susceptibility data; the values in parentheses refer to fits to the $X(H, T)$ data below $T/T_c = 0.6$. Results based on the RSQ mode data are given in rows 2 and 5, while the results obtained from the combined zero-field susceptibility and RSQ modes are given in rows 3 and 6. The labels "AHMS inputs" and "Greywall inputs" refer to values of T_c and F_0^a taken from Refs. 4 and 5, respectively.

T_c (p)	F_0^a	F_2^a	x_3^{-1}	fit
AHMS inputs				
1.04 (0)	−0.770	−0.88±0.10(−0.78)	−1.61±0.15(−1.2)	χ
1.18 (0.92)	−0.774	−4.09±0.10	0.63±0.02	RSQ
1.07 (0.2)	−0.771	−1.10±0.05	−0.20±0.02	$H = 0$
Greywall inputs				
0.92 (0)	−0.700	+1.04±0.10(0.60)	−1.75±0.15(−1.2)	χ
1.05 (0.92)	−0.710	−1.37±0.30	+0.17±0.08	RSQ
0.95 (0.2)	−0.703	+0.78±0.10	−0.35±0.15	$H = 0$

be seen in Fig. 1, there is a difference in the quality of the fit to the field-dependent susceptibility data as the temperature is varied. We generally found that the low-temperature data $(T/T_c < 0.6)$ gave better fits in the sense that the square deviation per datum was smaller in this limited temperature range. The error estimates σ_F and σ_x in Table I were obtained from standard regression theory using the formulas,

$$S(F_2^a + \sigma_F, x_3^{-1}) - S_{\min} = S(F_2^a, x_3^{-1} + \sigma_x) - S_{\min} = \sigma^2,$$

where $\sigma^2 = S_{\min}/(N - 2)$ is the estimate of the standard deviation of the susceptibility for a set of N measurements.[18] The parameters x_3^{-1} and F_2^a obtained from fitting the susceptibility data below $0.6T_c$ are also listed in Table I (shown in parentheses); the overall trends mentioned above are unchanged. To emphasize the sensitivity of the nonlinear susceptibility to the f-wave interaction we also show in Fig. 1 the susceptibility at $H = 1$ MHz (dash-dotted curve) and 9 MHz (dashed curve) for parameters $F_2^a = -1.37$, $x_3^{-1} = +0.17$, and Greywall's input parameters. The interactions F_2^a and x_3^{-1} produce nearly independent changes in the susceptibility. Note in particular that (i) the low-field susceptibility (e.g., at 1 MHz), which is essentially independent of x_3^{-1} [see Eq. (1)], is sensitive to the value of F_2^a, while (ii) the high-field, nonlinear correction (the spread in values between 1 and 9 MHz) is sharply reduced by the weakly repulsive f-wave interaction,[19] with the result that the field dependence of the susceptibility disagrees strongly with the data for these interaction parameters. This is important because, as we discuss below, these values of F_2^a and x_3^{-1} which give a poor representation of the susceptibility data are the optimum material parameters obtained from the fit of the theory of FS to the RSQ modes with Greywall's inputs. Finally, we remark that the sensitivity of the field-dependence of the susceptibility to the f-wave interaction is in sharp contrast to the *zero-field* collection mode frequencies in ^{3}He-*B*; in zero field the measured collective mode frequencies can be fit equally well by a family of possible values for F_2^a and x_3^{-1}.

RSQ MODE ANALYSIS

The analysis of the RSQ mode data of Mast *et al.*[20] and Avenel *et al.*[21] in zero field by Sauls and Serene[9] did not yield determinations of both F_2^a and x_3^{-1}. The temperature dependence and deviation of the measured RSQ mode frequency from the noninteracting result, $\omega = \sqrt{8/5}\Delta_0$, was explained equally well by an attractive f-wave pairing interaction or by a negative value for F_2^a. The analysis of the data of Avenel *et al.*[21] for the linear Zeeman shift of the RSQ modes by Sauls and Serene[22] was also unable to determine the values of F_2^a and x_3^{-1} given the small data base at the time and the uncertainties in F_0^a. However, the near degeneracy of the zero-field collective mode frequencies with respect to the family of parameters (F_2^a, x_3^{-1}) is lifted in a magnetic field; this allows us to make an independent determination of these two material parameters from a comparison of the theory of FS with the extensive data of Shivaram *et al.*[16] on the

nonlinear Zeeman shifts of the RSQ modes.

The analysis of the collective mode data is more difficult than the analysis of the susceptibility. A typical ultrasound experiment operates at a fixed frequency ω, and the collective modes are observed as anomalies in the absorption or group velocity at well-defined temperatures T_{m_J} satisfying

$$\omega = \omega_0(T_{m_J}) + m_J \alpha(T_{m_J}) \gamma H - \Gamma(T_{m_J}) \frac{\gamma^2 H^2}{\Delta_0(0)}$$
$$+ m_J^2 \beta(T_{m_J}) \frac{\gamma^2 H^2}{\Delta_0(0)} + a_q(T_{m_J}) \frac{(q v_F)^2}{\Delta_0(0)}$$
$$+ m_J^2 b_q(T_{m_J}) \frac{(q v_F)^2}{\Delta_0(0)}, \tag{3}$$

where $\omega = \omega_0(T^*) + a_q(T^*)(q v_F)^2/\Delta_0(0)$ is the frequency of sound, and T^* is the temperature at which the RSQ mode is observed in the attenuation spectrum for zero field. Equation (3) is the basic equation we use in analyzing the data of Shivaram *et al.* on the RSQ modes. In FS we give expressions for the coefficients ω_0, α, β, Γ, a_q, and b_q which parametrize the temperature and field dependences of the RSQ mode frequencies. We find that, because the order-parameter couples to the molecular field, β and Γ depend on the Fermi-liquid parameter F_4^a. However, a numerical study of the mode frequencies shows that the dependence on F_4^a is extremely weak; thus, we set $F_4^a = 0$ in our theoretical fits of the data.

In Eq. (3) we include the dispersion splitting on the modes calculated in FS through order $(q v_F/\Delta_0)^2$, where q is the wave vector of zero sound and v_F is the Fermi velocity. Although the dispersion splitting is small compared to the zero-field frequency and the Zeeman splitting at high fields, it is non-negligible compared to the nonlinear field dependence of the modes, and must be included. The dispersion splitting gives rise to an additional field dependence of the RSQ mode frequencies at low fields as the quantization axis rotates from the direction of sound propagation, $\hat{q}$, to the direction of the magnetic field. For fields above approximately 0.3 kG, which is the relevant case in this analysis, the quantization axis is fixed along the direction of the field. In FS we have shown that a_q and b_q depend on the material parameters F_1^a, F_2^a, F_3^a, x_2^{-1}, and x_3^{-1}.

To extract the material parameters from the collective mode data we fit out theory [Eq. (3) with the theoretical results for the coefficients] to the measured temperature shifts, $\Delta T = T_{m_J} - T^*$, and magnetic-field values for the RSQ modes. The numerical procedure is similar to that used to analyze the susceptibility data. Since the temperature shift ΔT is of order H, we can evaluate Γ, β, a_q, and b_q at T^*, but we must retain the temperature dependence of α through first order in ΔT and the temperature dependence of ω_0 through second order in ΔT. Higher-order corrections in ΔT are equivalent to higher-order corrections in the magnetic field, so we neglect them. In order to determine the maximum field for which our perturbation theory is reliable, the temperature dependences of β and Γ must be carefully considered; these coefficients

depend on the inverse of the gap, which diverges at T_c (see Figs. 5 and 6 of FS). At low pressures the mode crossing (not shown in Fig. 2) occurs at $\Delta T \approx -50 \, \mu$K, so $\beta(T^*+\Delta T)$ and $\Gamma(T^*+\Delta T)$ differ from $\beta(T^*)$ and $\Gamma(T^*)$ by approximately 25%. By including the temperature dependence of β and Γ the crossing field calculated from Eq. (3) changes by 40%. This means that Eq. (3), which includes only terms through second order in the field, fails near the mode crossing. The perturbation theory for the temperature shifts at fixed frequency is valid in a more restricted range than the pertubation theory applied to frequency shifts at fixed temperature. The functions β and Γ can be used to determine the magnetic field below which the quadratic theory is valid. In the temperature range $0.5 \leq T/T_c \leq 0.8$, which contains most of the data, $(T_c/\beta)(\partial\beta/\partial T) \approx (T_c/\Gamma)(\partial\Gamma/\partial T) \approx 5$. If we demand that β and Γ be constant to within 10% by imposing the conditions

$$\left| \frac{\Delta T}{\beta} \frac{\partial \beta}{\partial T} \right| \leq 0.10, \quad \left| \frac{\Delta T}{\Gamma} \frac{\partial \Gamma}{\partial T} \right| \leq 0.10 , \quad (4)$$

then we find that $|\Delta T| \leq 0.02 T_c$. At 0 bar this limits the temperature shifts to 20 μK and the maximum field to 1 kG. At higher pressures, where T_c is bigger, perturbation theory is applicable for larger temperature shifts and stronger fields. We note that the susceptibility is not as sensitive to higher-order corrections because the nonlinear corrections to $\mathcal{X}(T)$ do not contain odd powers of $\gamma H/\Delta_0$. Based on these considerations, we chose a maximum field of 1 kG. In the deviation function we assign 80% of the weight to the relative temperature shift $\Delta T_{m_J} = T_{m_J} - T_0$ from the central mode, and the remaining weight to the absolute temperature of the central mode $\Delta T_0 = T_0 - T^*$. This weighting of the data was chosen because the absolute temperature can drift as the field is increased but the shift of one mode relative to another is more stable.[16] The dispersion splitting, which is a small correction to begin with, depends very weakly on most of the interaction parameters, so we take $F_1^a = F_3^a = x_2^{-1} = 0$.

In Fig. 2 we show the fit of the RSQ mode data, for 0.92 bar, to our theory, and in Table I we list the results for F_2^a and x_3^{-1}, again for the AHMS and Greywall input parameters. In sharp contrast to the results obtained from the susceptibility fits, we find a large negative Landau parameter, $F_2^a = -4.1$, and a repulsive f-wave interaction parameter, $x_3^{-1} = 0.63$, based on the AHMS inputs. This value for F_2^a implies that the ^{3}He Fermi surface is nearly unstable to a spin-dependent quadrupole distortion. We know of no other analysis yielding such a large negative value of F_2^a, and view this result with skepticism. The value of F_2^a is sensitive to the input parameter F_0^a, and thus, the effective mass. For example with Greywall's input parameters we find $F_2^a \approx -1.4$. Greywall's value for T_c affects the analysis of the collective mode data more than it does the analysis of the susceptibility because the measured frequencies must be normalized by the gap in order to be compared with theory. With Greywall's values for T_c, the zero-field
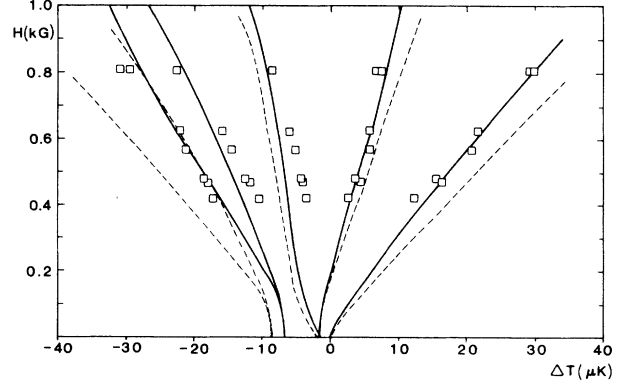

FIG. 2. The Zeeman splitting of the RSQ modes and the theoretical fit (solid lines) for 0.92 bar. With Greywall's inputs ($T_c = 1.05$ mK and $F_0^a = -0.71$) the fit gives $F_2^a = -1.37$ and $x_3^{-1} = +0.17$. The set of dashed curves is the theoretical calculation for the splittings obtained with the Greywall input parameters, $F_2^a = +1.04$ and $x_3^{-1} = -0.378$; the value for F_2^a is obtained from the theoretical fit to the susceptibility data, and the value of x_3^{-1} is chosen so that the calculated zero-field RSQ mode frequency agrees with the experimental value.

RSQ mode frequency is much closer to the noninteracting value $[\omega = \sqrt{8/5}\Delta_0(T)]$, so in general smaller interaction parameters are needed to account for the observed frequencies. The resulting material parameters obtained with Greywall's F_0^a and T_c, listed in Table I, are roughly a factor of three smaller in magnitude than those obtained with the Helsinki inputs.

The distinctive feature of these sets of calculations is that they disagree; there is no simple way to shift the empirical temperature scales and reconcile the material parameters obtained from the susceptibility data of Ref. (13) and those obtained from the RSQ mode data of Ref. (16). To further emphasize this point we have calculated the field splittings of the RSQ modes using Greywall's T_c and m^*, and the Landau parameter $F_2^a = 1.04$, taken from our fit of the susceptibility data to Eq. (1) at low pressures; the result is shown in Fig. 2 as the set of dashed curves. The disagreement with the collective mode data is clear. In fact this calculation *minimizes* the discrepancy between the susceptibility result and the collective mode data because the dashed curves were calculated with a value of $x_3^{-1} = -0.378$, chosen to guarantee that the zero-field RSQ mode frequency, ω_0, is correctly fit to the experimental value. If we were to use *both* material parameters obtained from the susceptibility analysis, $F_2^a = 1.04$ and $x_3^{-1} = +0.17$, then the set of dashed curves would be displaced off of the horizontal axis of Fig. 2 since the zero-field frequency would be shifted by approximately 10 MHz with respect to the experimental data. Thus, in contrast to the theoretical zero-field RSQ mode frequency, it is *not* possible to obtain equally good fits to the field splittings of the RSQ modes with a family of values for (x_3^{-1}, F_2^a); the field-dependent RSQ mode data, and the field-dependent susceptibility data separately select optimal values for these parameters that are incompatible with one another.

Since we expect weak-coupling theory to be accurate at $p \leq 1$ bar, this leaves us with an unresolved discrepancy between the two data bases.

ZERO-FIELD CALCULATIONS

Since we are unable to obtain consistency between the material parameters based on the susceptibility data and those based on the RSQ mode data, we consider the possibility that differences in magnetic-field calibration between the data sets are responsible for the discrepancies in material parameters. In order to extract the zero-field susceptibility from the susceptibility data at finite fields, we have used the formula, $X/X_N = (X/X_N)_0 + B(\gamma H /2\pi)^2$, and extract $(X/X_N)_0$ as a function of temperature.[23] We then fit the zero-field QC theory [Eq. (23) of FS] over the full temperature range to obtain the Landau parameter F_2^a, again with F_0^a an input parameter. Note that the theoretical zero-field susceptibility in Eq. (1) is independent of x_3^{-1}. The resulting values for F_2^a are comparable to the values obtained from the full susceptibility fits, and thus, are sensitive to the input parameter F_0^a. We then use these values of F_2^a as inputs in our fits of the QC theory to the RSQ mode frequencies in zero field.[20,21] The RSQ mode frequencies depend only on F_2^a, x_3^{-1}, and T_c, so we extract x_3^{-1}. The Greywall temperature scale implies an f-wave transition temperature, $T_{cf} \approx 0.06 T_c$, while the Helsinki scale implies a somewhat smaller value. This is the opposite trend we expect based on the differences in T_c between Greywall and Helsinki; the more attractive f-wave interaction obtained with the Greywall inputs is due to the positive value of F_2^a, obtained from the zero-field susceptibility fit with Greywall's value for F_0^a.

COMMENTS AND CONCLUSION

Meisel et al.[24] have recently analyzed the zero-field squashing (SQ) mode data of Avenel et al.[25] using the exact weak-coupling result of Ref. (9) for the SQ mode frequency, which depends on the material parameters, T_c, x_3^{-1}, and the spin-symmetric Landau parameter F_2^s.

Meisel et al. extract F_2^s from the normal-state data for the zero-sound velocity, using Greywall's values for the effective mass. With F_2^s and Greywall's scale for T_c they extract the f-wave interaction parameter from the SQ mode data, and find $x_3^{-1} = +0.2$ at $p = 0$ bar. This result is close to the value of $x_3^{-1} = +0.17$ we obtain from the RSQ mode analysis using the Greywall scale. Thus, there is internal consistency of the QC theory for the SQ and RSQ mode data provided that the Greywall scale is used for T_c.

In conclusion, we have used the weak-coupling result obtained in FS for the field-dependent susceptibility and RSQ modes of ^{3}He-B to extract the material parameters (F_2^a, x_3^{-1}) from the susceptibility data of Hoyt et al. and the RSQ mode data of Shivaram et al. The sets of parameters obtained from these different data bases are in serious disagreement. The discrepancy is large at the lowest pressure where strong-coupling effects are expected to be negligible, and cannot be removed even with radical changes in the empirical temperature scale. Since we cannot presently reconcile the differences in material parameters obtained from the field-dependent susceptibility and RSQ modes, a systematic study of the susceptibility and collective modes at the lowest pressures would be an important check on the weak-coupling response theory and perhaps help remove some remaining doubts about the temperature scale. We note, however, that the f-wave interaction parameter x_3^{-1} extracted from the RSQ mode data is in reasonable agreement with the result of Meisel et al., which is based on the SQ mode data.

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