Dimensional phase transition in superconductors with short coherence length

J. Guimpel, L. Civale, and F. de la Cruz
Centro Atómico Bariloche and Instituto Balseiro, 8400 Bariloche, Río Negro, Argentina

J. M. Murduck
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439

Ivan K. Schuller
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439
and Physics Department, University of California-San Diego, La Jolla, California 92093
(Received 19 November 1987)

We found a novel dimensional transition in the vortex lattice of a finite-sized superconductor. The low-field dependence (with field applied parallel to the film plane) of the magnetization exhibits two maxima, which signal the transition from one to two dimensions. In one dimension the vortices line up along the film, and with increasing field their arrangement changes into a two-dimensional array due to the competition between vortex-surface and vortex-vortex repulsion. Agreement is found with no adjustable parameter calculations using *a priori* known surface-barrier and vortex-repulsion effects.

The properties of anisotropic superconductors are particularly important in light of the recent discovery of high-T_c superconductivity in anisotropic metallic oxides.[1] In order to pinpoint the properties which distinguish the newly discovered oxides from ordinary anisotropic superconductors it is of importance to understand in detail the properties of the latter. Artificially engineered materials[2] provide a convenient test ground for ideas related to anisotropic superconductors, especially since the physical parameters (resistivity, coherence length, penetration depth, etc.) can be varied at will. In particular, layered superconductors that have a very short coherence length (ξ) compared to the penetration depth (λ) are similar to the oxide superconductors where also $\xi \ll \lambda$ (London limit). We present here a novel dimensional transition which occurs in superconductors with long-ranged vortex interactions in the London limit. The low-field dependence of the magnetization in Nb/Cu superlattices shows a transition from a one- (1D) to a two-dimensional (2D) array of vortices for a wide range of temperatures and sample properties. The results are in good agreement with a no adjustable parameter model in which only surface barrier and vortex repulsion effects have been included.

Nb/Cu superlattices have been prepared and characterized as described earlier.[3] Briefly, the samples were prepared using a multisource sputtering system by alternatively exposing the single-crystal sapphire substrate to the sputtering beams. The thicknesses were independently determined by the preparation condition and by x-ray diffraction measurements. The magnetic flux expulsion, $\Delta\phi$, was measured as a function of temperature, T, by cooling the sample through its transition temperature, T_c, at a constant applied magnetic field, H, following the method described in Ref. 4. For ease of presentation it is

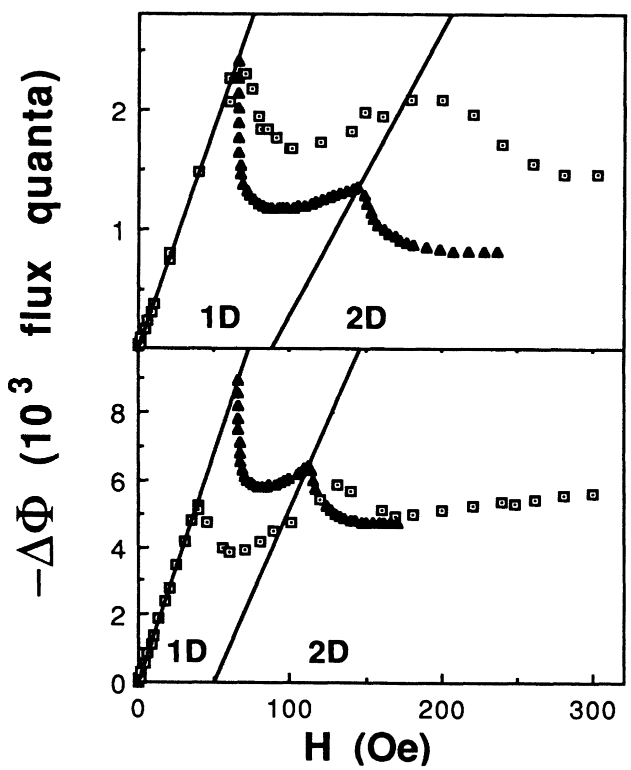

FIG. 1. Flux expulsion $\Delta\phi$ as a function of magnetic field, H, at $T=1.7$ K. Upper: Nb (16.5 Å)/Cu (16.5 Å) multilayer of total thickness $d=0.825$ μm, $T_c=2.61$ K, $\lambda(0)=0.5$ μm (Ref. 4). Lower: Nb (54 Å)/Cu(54 Å), $d=1.08$ μm, $T_c=3.71$ K, $\lambda(0)=0.33$ μm (Ref. 4). Squares: experimental data; solid triangles: calculated thermodynamic equilibrium expulsion; solid line: Meissner effect; dashed line: 1D-2D phase boundaries.

convenient to plot the flux expulsion as a function of H rather than T. The curves $\Delta\phi(H)$ are constructed from the measured $\Delta\phi(T)$. The resulting curves are equivalent to those that would have been obtained by decreasing H at constant T, because in these samples the bulk pinning can be considered to be zero.

Figure 1 shows the flux expulsion as a function of field for two different samples. Results for other samples with periods ranging from 10 to 200 Å are similar to those shown in Fig. 1. Two distinct features are present in these results. At low magnetic fields a Meissner effect, indicated by a constant $\Delta\phi/H$ is evident. At higher fields the flux explusion is not complete and the magnetization as a function of field shows two well-defined maxima. In contradistinction, the second maximum is absent in ordinary superconductors where $\Delta\phi(H)$ decreases monotonically above the lower critical field H_{c1}. Since the penetration depth, typically 0.5 μm, is comparable to sample thicknesses ($d \simeq 1$ μm) the vortices will interact strongly with the Meissner currents. This repulsive interaction tends to align the vortices along the center of the sample until the vortex-vortex repulsion induces a lateral displacement into a two-dimensional array. The experimental field dependence of the magnetization (Fig. 1) is indeed reminiscent of such a structural phase transition in the vortex lattice from one dimensions to two dimensions.[5,6]

The sample construction procedure suggests that these samples should behave as anisotropic superconductors. Nevertheless, upper critical-field measurements indicate that for multilayer periods smaller than 150 Å the system behaves as a *homogeneous* and *isotropic* superconductor.[7] The main reason for the isotropic behavior being the fact that the coupling is due to the proximity effect across a normal metal. Based on these results we use Tinkham's formalism[8] to calculate the Gibbs free energy of a regular array of N vortices in a slab[9] with H parallel to the sample surface

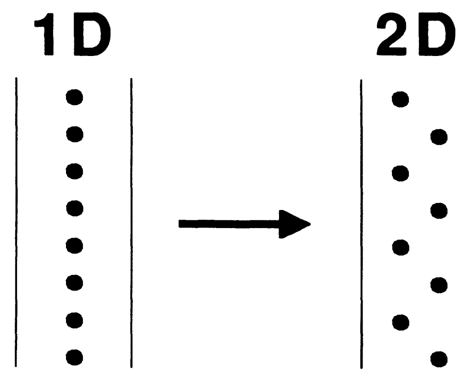

FIG. 2. 1D and 2D vortex array.

$$\frac{\Delta G}{L} = N\left[\frac{\phi_0}{8\pi N}\sum_{i=1}^{N} h(\mathbf{r}_i) - \frac{H}{8\pi}\left[\phi_0 + \frac{1}{N}\sum_{i=1}^{N}\phi_v(\mathbf{r}_i)\right]\right], \tag{1}$$

where L is the sample length along H, ϕ_0 is the flux quantum, $\mathbf{r}_i$ is the position of the ith vortex, $h(\mathbf{r}_i)$ is the local field, and $\phi_v(\mathbf{r}_i)$ is given by[10]

$$\phi_v(\mathbf{r}_i) = \phi_0\left[1 - \frac{\cosh(x_i/\lambda)}{\cosh(d/2\lambda)}\right] \tag{2}$$

with x_i being the distance from the vortex core to the film center.

For the N vortices distributed along a regular 1D or 2D array (see Fig. 2) Eq. (1) takes the form

$$\frac{\Delta G}{WL(\phi_0 H_{c1}/4\pi)} = \eta\left[\frac{H}{H_{c1}}\left[\frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)} - 1\right] + 1 + \frac{1}{\ln\kappa}\sum_{\substack{n,m=-\infty\\ n+m \text{ even}\\ (n,m)\neq(0,0)}}^{\infty}(-1)^n K_0\left[\frac{[(nd)^2+(m/\eta)^2]^{1/2}}{\lambda}\right] + \frac{1}{\ln\kappa}\sum_{\substack{n,m=-\infty\\ n+m \text{ odd}}}^{\infty}(-1)^n K_0\left[\frac{[(nd-2x)^2+(m/\eta)^2]^{1/2}}{\lambda}\right]\right] \tag{3}$$

with $H_{c1} = (\phi_0/4\pi\lambda^2)\ln\kappa$, η the vortex density per unit sample width W, $\kappa = \lambda/\xi$, and K_0 being a modified Bessel function of zeroth order.[11]

The equilibrium thermodynamic $\Delta\phi$ calculated by minimizing the Gibbs free energy [Eq. (3)] with respect to η and x is shown as the dotted line in Fig. 1. The field region between the two maxima corresponds to the 1D equilibrium solution ($x=0$) whereas at higher fields a 2D solution ($x\neq 0$) is encountered. It is quite satisfying to note that such a simple idea reproduces beautifully the general features of the experimental data.

The minimization of ΔG with respect to η implies that the number of vortices is determined by the thermodynamics. However, when cooling in a field, defects and surface barriers tend to lock the vortices in the sample so as to give a higher density than expected thermodynamically. To allow for a higher density of vortices the 1D–2D phase boundary, $H_{12}(T,\eta)$ is evaluated for an arbitrary η as the field at which the 1D vortex array becomes unstable

$$H_{12}(T,\eta)=8\frac{H_{c1}}{\ln\kappa}\cosh\left[\frac{d}{2\lambda}\right][A(T,\eta)-B(T,\eta)]$$

with

$$A(T,\eta)=\sum_{\substack{j=1\\ j\ \text{odd}}}^{\infty}\left[\frac{\lambda}{jd}K_1\left[\frac{jd}{\lambda}\right]+\frac{\eta\lambda}{j}K_1\left[\frac{j}{\eta\lambda}\right]+K_0\left[\frac{jd}{\lambda}\right]\right],$$

$$B(T,\eta)=2\sum_{\substack{j,k>0\\ j+k\ \text{odd}}}^{\infty}(-1)^j\lambda\left[\frac{(jd)^2-(k/\eta)^2}{[(jd)^2+(k/\eta)^2]^{3/2}}K_1\left[\frac{[(jd)^2+(k/\eta)^2]^{1/2}}{\lambda}\right]+\frac{(jd)^2}{(jd)^2+(k/\eta)^2}K_0\left[\frac{[(jd)^2+(k/\eta)^2]^{1/2}}{\lambda}\right]\right], \tag{4}$$

with K_1 the modified Bessel function of the first order.[11] The temperature dependence arises from $H_{c1}(T)$ and $\lambda(T)$.

In accordance with expectations, the solid line in Fig. 1 intersects the second maximum in the $\Delta\phi$ versus H curve, which signals the 1D to 2D transition. The facts that the 1D to 2D phase boundary is obtained only by adjusting the number of vortices and that agreement is obtained for the full $\Delta\phi$ versus H curve without adjustable parameters, are proof that the general ideas presented here are the essential ingredients governing the physics.[12]

In summary, we have observed for the first time a dimensional transition in the vortex lattice of a finite-size sample. The phase boundaries and detailed field dependence are in agreement with theoretical calculations based only on surface barriers and long-ranged vortex-vortex interactions.

Work partially supported by the Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET). One of us (J.G.) thanks the CONICET for financial support. We are thankful to H. Tutzauer, R. Scotti, and S. Trochine for technical assistance. This work was supported at Argonne National Laboratory by the U.S. Department of Energy, Office of Basic Energy Sciences, Division of Materials Sciences, under Contract No. 31-109-ENG-38 and at University of California, San Diego by the Department of Energy, under Contract No. DE-FG03-87ER45332.

[1]J. G. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986).
[2]*Synthetic Modulated Structures*, edited by L. L. Chang and B. C. Giessen (Academic, Orlando, 1985).
[3]I. K. Schuller, Phys. Rev. Lett. **44**, 1597 (1980).
[4]J. Guimpel, F. de la Cruz, J. Murduck, and I. K. Schuller, Phys. Rev. B **35**, 3655 (1987).
[5]J. Sutton, Proc. Phys. Soc. **87**, 971 (1966).
[6]B. Takacs, Czech. J. Phys. B **33**, 1248 (1983).
[7]I. Banerjee, Q. S. Yang, C. M. Falco, and I. K. Schuller, Phys. Rev. B **28**, 5037 (1983).
[8]*Introduction to Superconductivity*, edited by M. Tinkham (McGraw-Hill, New York, 1975), Chap. V.
[9]C. P. Bean and J. D. Livingston, Phys. Rev. Lett. **12**, 14 (1964).
[10]L. Civale and F. de la Cruz, Phys. Rev. B **36**, 3560 (1987).
[11]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1964), p. 374.
[12]The quantitative disagreement between experiment and calculation is not systematic from sample to sample and we believe it to be due to experimental errors in the determination of the absolute value of the penetration depth and sample thickness.