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Experimental studies of the $\nu = 1/5$ hierarchy in the fractional quantum Hall effect

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We present an experimental study of the $\nu = 1/5$ ground state of the fractional quantum Hall effect. Weak ρ_{xx} minima at 2/9 and 2/11 are observed, consistent with the predicted "daughter" states of a 1/5 hierarchical "parent" state. The 1/5 activation energy $\Delta_{1/5}$ is shown to be significantly smaller than $\Delta_{2/5}$. Empirical fits for Coulomb constants $C(\nu)$ confirm that $C(1/3) > C(2/5) > C(3/7) > C(1/5)$. The data are consistent with a separate hierarchy emanating from $\nu = 1/5$.

The fractional quantum Hall effect (FQHE) is a many-body phenomenon occurring in high-mobility two-dimensional electron systems at mK temperatures in high magnetic fields.[1] It is characterized by minima in the longitudinal resistivity ρ_{xx} and quantized Hall plateaus $\rho_{xy} = h/\nu e^2$ at Landau-level filling factors $\nu = p/q$, where q is odd in the $N = 0$ level. The Coulomb interaction between electrons leads to new collective ground states[2] at $\nu = 1/q$, whose excited states are fractionally charged quasiparticles[2-5] separated from the ground state by an energy gap Δ. The hierarchical model[4-6] describes sequences of fractional states emanating from "parent" states $1/q$, in which "daughter" states are formed by correlation of the quasiparticles of the preceding state. The parent state at $\nu = 1/3$ has been extensively studied and high-order daughter states up to $\nu = 6/13$ have been reported.[7] However, the 1/5 hierarchy has proved elusive, with only recent confirmation of the FQHE at $\nu = 1/5$ (Refs. 8 and 9) and 4/5 (Refs. 7 and 10) through observation of quantized Hall plateaus. Corresponding ρ_{xx} minima have been weak, precluding quantitative activation studies.

We report an experimental study of the $\nu = 1/5$ ground state, for which the activation energy $\Delta_{1/5}$ is determined for the first time. $\Delta_{1/5}$ is compared directly with the energy gaps of the parent 1/3 (2/3) and its daughter 2/5, 3/7, (3/5,4/7) states in the same sample. We also report observation of weak ρ_{xx} minima at $\nu = 2/9$, 2/11 providing experimental confirmation of the 1/5 hierarchy, which is quite separate from the sequence of states derived from $\nu = 1/3$. Additionally, the ρ_{xx} minimum at $\nu = 2/11$ sets a lower limit to which the FQHE has been shown to persist into the extreme quantum limit.

The predicted sequences of fractional states deriving from the 1/3 and 1/5 parent states are shown schematically below and fractions observed in this experiment are denoted by an asterisk.

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                  1/3*                                  1/5*
                  / \                                   / \
              2/5*   2/7*                           2/9*   2/11*
              / \    / \                            / \    / \
         3/7*  5/13  5/17  3/11*                3/13          3/17
         / \                / \
     4/9*                      4/15
     /
 5/11
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The work was carried out using high-mobility modulation-doped GaAs-$Ga_{0.68}Al_{0.32}$As heterojunctions, sample G139 A, B, and L.[3] The Hall bar samples had 50 and 75 μm channel widths and Ni-Au-Ge contacts. Following photoexcitation of carriers using a red LED, the 1600 Å spacer layer structure had an electron concentration $n = 0.95\times10^{11}$ cm^{-2} and mobility $\mu = 1.0\times10^6$ cm^2/Vs (100 mK–4 K). Data for $\nu \leq 1/2$ were obtained for temperatures down to 160 mK using a dilution refrigerator in a 30-T hybrid magnet and down to 50 mK using an 11-T resistive solenoid. The samples were mounted in the dilute phase of the ^{3}He/^{4}He mixture in both cases and the temperature was measured using a calibrated Speer resistor mounted at field center and corrected for magnetoresistance. The ρ_{xx}, ρ_{xy} measurements were made using 23 Hz ac currents of typical density below 4×10^{-4} A m^{-1}.

Development of the minimum at $\nu = 1/5$ for three temperatures down to 175 mK is shown for sample B in Fig. 1(a). The depth of the 1/5 minimum represents a significant improvement on previous data. Daughter states of the $\nu = 1/5$ ground state develop as weak ρ_{xx} minima at $\nu = 2/9$ and 2/11, and were most clearly observed in sample A, as shown in Fig. 1(b). In addition the 1/3 (2/3) hierarchy[3] is observed up to 4/9 (6/11) and the 3/11 state is also reported for the first time. The 2/11

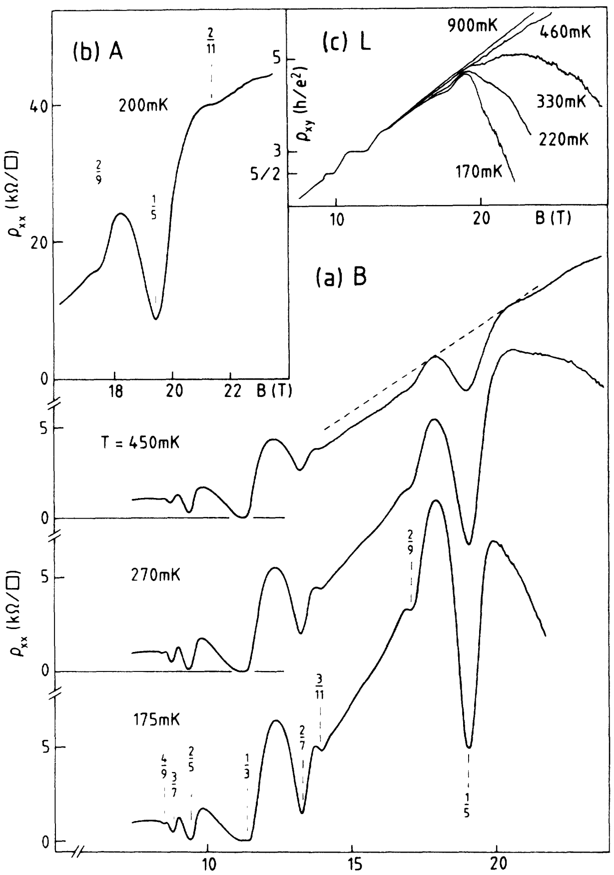

FIG. 1. (a) Temperature dependence of ρ_{xx} vs B for sample B, (b) sample A at 200 mK showing the 2/9 and 2/11 ρ_{xx} structure, and (c) temperature dependence of ρ_{xy} in the extreme quantum limit for sample L.

structure is best resolved above base temperature due to experimental difficulties in the extreme quantum limit, where the phases of both ρ_{xx} and ρ_{xy} in the ac measurements change as a function of magnetic field below a threshold ν value, which is sample/contact and electron-concentration dependent. This effect prevents observation of the FQHE in the extreme quantum limit as the temperature decreases and is most obvious in the Hall voltage, as shown in Fig. 1(c) for sample L.

The progressive strengthening of the 1/5 structure as the temperature is lowered is not reflected by a decrease in the value of ρ_{xx} at the minimum ($\rho_{xx}^{\min}$) except at very low temperatures due to an increase in the steep background, as can be seen from Fig. 1(a). This makes analysis of the ρ_{xx} temperature dependence nontrivial and we therefore consider the activation of the ratio of $\rho_{xx}^{\min}$ to the underlying background at $\nu=1/5$ (ρ_{xx}^{b}). This is justified as follows: Assuming that the only conduction process is due to quasiparticles, the conductivity is given by

$$\sigma_{xx}(B,T)=n_{\text{qp}}(B,T)e^{*}\mu_{\text{qp}}(B,T)\,, \tag{1}$$

where n_{qp}, μ_{qp}, and e^{*} are the quasiparticle concentration, mobility, and charge, respectively. The effect of $n_{\text{qp}}(B,T)$ on the temperature dependence of $\sigma_{xx}(B,T)$ is small except at exactly $\nu=1/5$ where the quasiparticles result from thermal excitation and n_{qp} is exponentially activated at temperatures comparable to, or below, the energy gap. At nearby filling factors n_{qp} is mostly determined by the deviation from exact 1/5 occupancy. The rising background and sharp peaks either side of $\nu=1/5$ are therefore due to $\mu_{\text{qp}}(B,T)$. Assuming that the number of quasiparticles at nonexact occupancy is constant, we can eliminate the temperature dependence of $\mu_{\text{qp}}(B,T)$ from $\rho_{xx}^{\min}$ by taking the ratio

$$\frac{\rho_{xx}^{\min}}{\rho_{xx}^{b}}\approx\frac{\sigma_{xx}^{\min}}{\sigma_{xx}^{b}}\propto n_{\text{qp}}(B,T)\propto e^{-\Delta/kT}\,. \tag{2}$$

The background at $\nu=1/5$ is estimated by interpolation between the ρ_{xx} "peaks" immediately at either side of the 1/5 minimum, as shown by the dashed line in the 450 mK trace of Fig. 1(a). At low temperatures the high-field peak is progressively distorted, as discussed above. We note, however, that the ρ_{xx} gradient at $\nu\sim1/4$ is parallel to the dashed line and can be used to extrapolate from the lower-field peak as shown by the inset to Fig. 2.

Figure 2 shows the "normalized" plot of $\ln\rho_{xx}$ against $1/T$ at $\nu=1/5$ for two different samples, giving good straight-line fits at intermediate temperatures. The activation energies are $\Delta_{1/5}=215$ and 305 mK $\pm$ 30 mK, broadly consistent with the temperature at which the

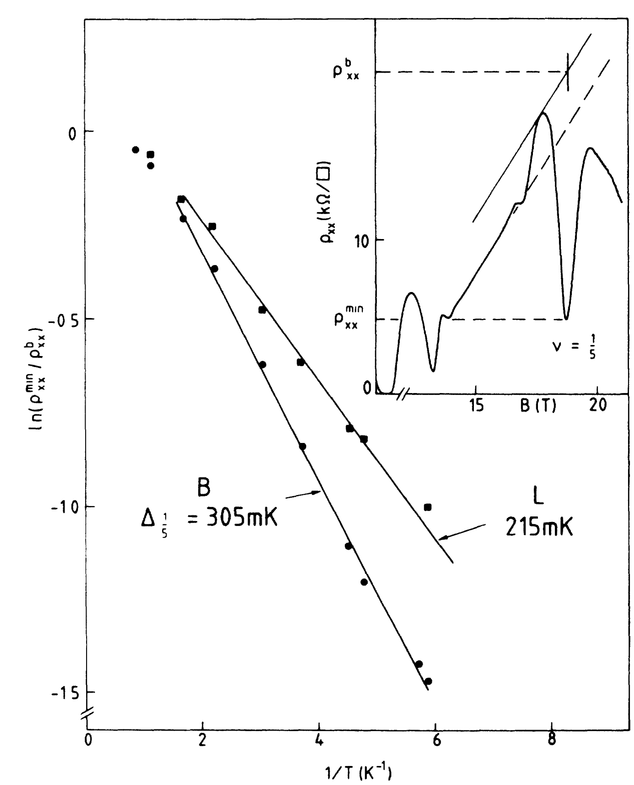

FIG. 2. Activation plot for the $\nu=1/5$ ρ_{xx} minimum at 19 T, using the ratio technique. The inset shows how the background ρ_{xx}^{b} is obtained at low temperatures.

minima are first visible. The difference in these two values is attributed to the strong dependence of $\Delta_{1/5}$ on the level of disorder in the samples. A weaker ρ_{xx} minimum at $\nu=1/5$ was also observed at 9.8 T in sample B at lower electron concentration. Its depth of 60% of the background at 50 mK, when scaled by a functional temperature dependence assumed to be the same as for $\nu=1/5$ at 19 T, gives $\Delta_{1/5}\sim 50$ mK at this field.

A comparison of the activation energies[3] of the 1/3 family of states (1/3,2/5,3/7) to the 1/5 state is shown in Fig. 3. The energy gaps are obtained from the slopes of the Arrhenius plots and we have made no attempt to subtract the small model-dependent hopping contribution to the activated conductivity, as in Ref. 11, which would increase the Δ values slightly. It can be seen from Fig. 3 that the $\nu=1/5$ energy gap, even at 19 T, is substantially lower than that for $\nu=2/5$ (3/5) at less than 10 T.

Theoretically, the FQHE energy gap in an ideal system is determined solely by the scale of the Coulomb interaction

$$2\Delta=\frac{C(\nu)e^2}{4\pi\epsilon_0\epsilon l_0}, \qquad (3)$$

where ϵ is the background dielectric constant ($\epsilon_{\mathrm{GaAs}}=12.9$) and $l_0=(\hbar/eB)^{1/2}$ is the magnetic length. Theoretical values for the constants $C(\nu)$ are summarized in Table I. The consensus of Refs. 12-16 is $C(1/3)\simeq 0.1$, $C(2/5)\simeq 0.05$, $C(1/5)\simeq 0.025-0.03$. Reference 17 and the early hypernetted-chain calculations of Ref. 18 give somewhat lower results, although the ratios of their $C(\nu)$ values are similar. Experimental data[11] give systematically lower activation energies than theory and show evidence of saturation at high field. The reduction of the gap in real systems is attributed to the finite z extent of the wave function, the mixing of Landau levels, and disorder. The effect of the latter on reducing the quasielectron-quasihole correlation may result in the measured Δ corresponding to a finite k value, closer to the roton minimum of the dispersion curve.[19]

TABLE I. Theoretical $C(\nu)$ values.

Reference	$C(1/3)$	$C(2/5)$	$C(3/7)$	$C(1/5)$
12	0.099			
13	0.114			0.031
14	0.104			0.024
15	0.105			
16	0.11	0.0527		
17	0.08	0.05	0.035	0.01
18	0.053			0.014

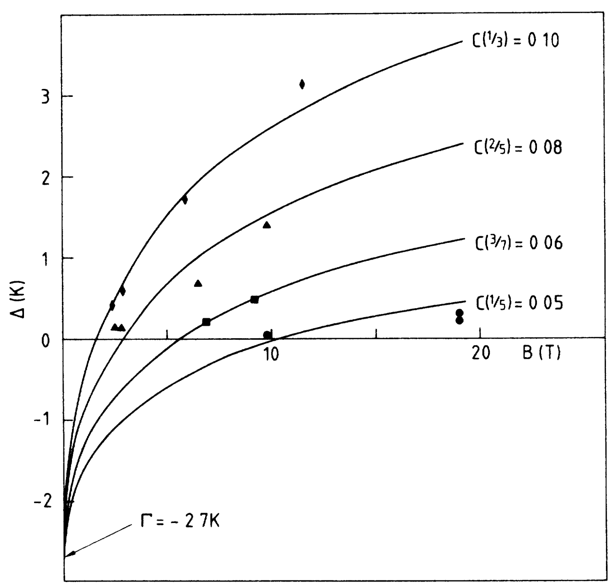

FIG. 3. Δ vs B results ($n=0.95\times10^{11}$ cm^{-2}, $\mu=1\times10^6$ cm^2/V s). One point is also shown for $\nu=1/5$ at $n=0.47\times10^{11}$ cm^{-2}. The 1/3, 2/3, 4/3, 5/3 data are denoted ◆; 2/5, 3/5, 7/5, 8/5 data by ▲; 3/7, 4/7, 10/7 data by ■; and 1/5 data by ●. The solid curves show fits using the finite z-extent correction for $\nu=1/3$ of Ref. 20 as a correction to the $B^{1/2}$ dependence for all fractional states, and a constant Δ offset $\Gamma=-2.7$ K to account for disorder.

Fits for $C(\nu)$ to measured energy gaps Δ are shown in Fig. 3 for the groups of states p/q $(1-p/q,\ 1+p/q,\ 2-p/q)$ at fixed electron concentration. The calculations of Ref. 20 are used for the finite z-extent correction, with the experimentally determined depletion charge[21] $N_d=0.25\times10^{-11}$ cm^{-2} giving the wave function parameter $b^{-1}=48$ Å. The same correction as for $q=3$ is assumed for higher-order states in the absence of exact calculations. We phenomenologically take account of disorder by offsetting Δ by a fixed amount Γ, assuming a ν-independent reduction of the gap due to finite broadening of the quasiparticle band. Curves representing the average $C(\nu)$ value for each p/q group were compared for a range of Γ offsets. The best overall fit is obtained for $\Gamma=-2.7$ K, as shown in Fig. 3. This value is consistent with inhomogeneous broadening dominating over homogeneous broadening, as expected for wide spacer layer structures, where the latter is shown by cyclotron resonance measurements to be $\sim$0.1 K or less in the G139 samples.[22] The fits also lead to magnetic field thresholds B_c that are somewhat larger than calculated values obtained from Ref. 23 with the spacer layer parameter $d_i=1600$ Å and more than an order of magnitude greater than the predictions of Ref. 24, in which B_c is strongly dependent on d_i. The Coulomb constants from the above analysis are $C(1/3)=0.10$, $C(2/5)=0.08$, $C(3/7)=0.06$, and $C(1/5)=0.05$ with error ±0.005, and hence, in agreement with theory, $C(1/3)>C(2/5)>C(3/7)>C(1/5)$. The magnitude of $C(1/3)$ is in excellent agreement with the calculated values of Refs. 12-16. However, our values of $C(2/5)$, $C(1/5)$ are larger by a factor of 1.5-2. While the exact $C(\nu)$ values deduced from experiment are sensitive to the disorder offset when this is comparable in magnitude to the activation energy [with $\Gamma=-1$ K, $C(1/3)=0.055$ for example], the ratio $C(1/3){:}C(2/5){:}C(3/7){:}C(1/5)$ is relatively insensitive to Γ, and varies from 1:0.81:0.62:0.50 for $\Gamma=-2.7$ K to 1:0.66:0.48:0.36 for $\Gamma=-1$ K. These are somewhat larger than predicted by any of the references of Table I, indicating that the reduction of the Coulomb energy due to the finite z-extent correction may be less important for the higher-order states, which would then lead to lower $C(\nu)$ values.

We conclude that the energy gap of the $\nu=1/5$ FQHE ground state is substantially smaller than either the parent $\nu=1/3$ ground state or its daughter 2/5 state. Our $C(\nu)$ analysis with the assumption of a fixed reduction of the energy gap with disorder in our wide spacer layer samples, combined with allowance for a finite z extent of the wave function, leads to quantitative agreement with theory for $C(1/3)$. Although a rigorous fit for $C(\nu)$ of 2/5, 3/7, and 1/5 fractions requires detailed z-extent calculations, our analysis indicates that $C(1/3)>C(2/5)>C(3/7)>C(1/5)$ in agreement with theoretical predictions. This conclusion is not sensitive to the precise value of Γ and demonstrates a qualitative difference between a strong 2/5 state formed by recondensing $e/3$ charged quasiparticles and a new but weaker 1/5 parent state associated with $e/5$ charged excitations. The alternative view is that the difference in $\Delta_{1/5}$ and $\Delta_{2/5}$ is due principally to stronger disorder in the single-electron Landau level tails (leading to an increased Γ offset for the 1/5 data). However, the observation of weak ρ_{xx} minima at 2/9 and 2/11, the predicted first daughter states of the $\nu=1/5$ hierarchy supports the description of two separate FQHE hierarchies emanating from $\nu=1/3$ and $\nu=1/5$.

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