Penetration of a magnetic field in a very strong coupling superconductor

J. Blezius,* R. Akis, F. Marsiglio, and J. P. Carbotte
Physics Department, McMaster University, Hamilton, Ontario, Canada L8S4M1
(Received 1 February 1988; revised manuscript received 3 May 1988)

We have calculated the absolute value and temperature dependence of the London penetration depth for several values of the strong-coupling index T_c/ω_{ln} and find large characteristic differences between the weak coupling $T_c/\omega_{ln} \sim 0$, conventional strong coupling $T_c/\omega_{ln} \lesssim 0.25$, and very strong coupling $T_c/\omega_{ln} \sim 1$ regimes. Our results do not depend sensitively on the value of Coulomb pseudopotential, nor on the shape of the electron-boson exchange spectral density, but depend mainly on the characteristic boson energy ω_{ln}.

I. INTRODUCTION

The discovery of superconductors with critical temperature (T_c) near 100 K in the oxides has led to the study of many possible novel mechanisms.[1] Some of the theories can be classified as weak-correlation theories which are based on conventional band-structure models with the idea of Cooper-pair formation and a conventional Bardeen-Cooper Schrieffer (BCS) wave function. In these cases, the Eliashberg equations remain, at least in a first approximation, although the attractive electron-electron interaction is usually mediated throughout the exchange of bosons[2-5] other than phonons or a combination of the two.[3,5,6] Popular examples are plasmons[2,3] and excitons,[4-7] although in some cases, the exact nature of the exciton involved is not clear.

Other more radical, competing theories, which we will refer to as strong-correlation theories, are based on the Hubbard model.[8] They have been put forward on the ground that the parent substance La_2CuO_4 (Ref. 9) is an insulator and is magnetic. The resonating-valence-bond model (RVB)[10-12] is perhaps the best known of these theories and has received the most attention. In these approaches, the nature of the condensing bosons can be very different from ordinary Cooper pairs and their relation to conventional superconductivity is far from clear. In this paper, we consider only the first class of theories.

Within the Eliashberg formulation[13,14] of superconductivity, we calculate the magnetic-field penetration depth in the London limit as a function of temperature and impurity content for various values of the strong-coupling parameter T_c/ω_{ln} where ω_{ln} is a characteristic frequency associated with the electron-boson exchange spectral density. Our work is related to the recent study of Rammer,[15] but our aims are very different. Rammer[15] is interested in comparing recent experimental results obtained by muon-spin relaxation in La-Sr-Cu-O (Refs. 16-18) and Y-Ba-Cu-O (Ref. 19) with results for the temperature variation of the penetration depth, calculated within Eliashberg theory for a specific model electron-phonon spectral density. In this paper, we will instead sweep over all values of T_c/ω_{ln} so as to include the weak-doubling BCS limit $T_c/\omega_{ln} \sim 0$, the conventional strong-coupling range $T_c/\omega_{ln} \lesssim 0.25$ (which includes Rammer's work) and the very strong coupling case $T_c/\omega_{ln} \cong 1.0$. Also, we are interested in the T_c/ω_{ln} dependence of the strong-coupling index[14] $\eta_{\lambda_L}(T) \equiv \lambda_L(T)/\lambda_L^{BCS}(T)$ for zero temperature and T_c as well as in the dimensionless ratio $y_L(0)/[T_c\,|y'(T_c)|]$ where $y_L(T) \equiv 1/\lambda_L^2(T)$.

In Sec. II some of the necessary formulas are summarized while results are presented and discussed in Sec. III. Short conclusions can be found in Sec. IV.

II. FORMALISM

Within Eliashberg theory,[13,14] the fundamental microscopic parameters for superconductivity are an electron-boson spectral density $\alpha^2F(\omega)$, which leads to an attractive interaction between two electrons at the Fermi surface and a Coulomb pseudopotential μ^*. While the equations were originally derived for an electron-phonon superconductor and have built into them Migdal's theorem which may not be valid for other boson exchange mechanisms, we can, nevertheless, retain the same equations as phenomenological equations that should be valid as a first approximation. It may not be possible to calculate accurately the appropriate effective kernels in a particular nonphonon case, but this is of no significance here, since we intend to model them.

The necessary equations for the pairing energy $\tilde{\Delta}(i\omega_n)$ and renormalized frequency $\tilde{\omega}(i\omega_m)$ at the Matsubara frequencies $(i\omega_n) \equiv i\pi T(2n-1)$ with T the temperature and $n=0, \pm 1, \pm 2, \ldots$ are [13,14]

$$\tilde{\Delta}(i\omega_n) = \pi T \sum_m [\lambda(n-m) - \mu^*] \frac{\tilde{\Delta}(i\omega_m)}{\sqrt{\tilde{\Delta}^2(i\omega_m) + \tilde{\omega}^2(i\omega_m)}} , \tag{1}$$

and

$$\tilde{\omega}(i\omega_n) = \omega_n + \pi T \sum_m \lambda(n-m) \frac{\tilde{\omega}(i\omega_m)}{\sqrt{\tilde{\Delta}^2(i\omega_m) + \tilde{\omega}^2(i\omega_m)}} , \tag{2}$$

where

$$\lambda(n-m) = 2\int_0^\infty \frac{\Omega \alpha^2 F(\Omega)\, d\Omega}{\Omega^2 + (\omega_n - \omega_m)^2} . \tag{3}$$

The magnetic-field penetration depth at temperature T in the London limit $\lambda_L(T)$ follows from the above equations and the prescription[20-24]

$$\lambda_L(T)=\left[\Lambda^{-1}2\pi T\mu_0\sum_{n=1}^{\infty}\frac{\tilde{\Delta}_n^2}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{3/2}}\right]^{-1/2} \tag{4}$$

with the London parameter $\Lambda^{-1}=ne^2/m=\frac{2}{3}N(0)e^2v_F^2$. In the above formulas, μ_0 is the permeability, e is the charge on the electron, m is its mass, n is the free-electron density, v_F is the Fermi velocity, and $N(0)$ is the single-spin density of electronic states at the Fermi energy.

To introduce impurity scattering into the problem, we need to add onto the right-hand side of Eqs. (1) and (2) the terms

$$\pi t^+\frac{\tilde{\Delta}(i\omega_n)}{\sqrt{\tilde{\omega}^2(i\omega_n)+\tilde{\Delta}^2(i\omega_n)}}$$

and

$$\pi t^+\frac{\tilde{\omega}(i\omega_n)}{\sqrt{\tilde{\omega}^2(i\omega_n)+\tilde{\Delta}^2(i\omega_n)}}, \tag{5}$$

respectively, where $\pi t^+=1/(2\tau)$ with τ being an impurity scattering time. This completes the list of formulas that are needed.

III. CALCULATION AND DISCUSSION

To calculate the London penetration depth at any temperature in units of $\Lambda^{1/2}$, it is only necessary to specify the electron-boson spectral density $\alpha^2F(\omega)$ and the Coulomb pseudopotential μ^*. This latter quantity could be larger in the low electron density oxides than it is in the conventional case where it is known to be of order 0.1. For the spectral density, we will start by using the form calculated by Weber[25] for the electron-phonon interaction in $La_{1.85}Sr_{0.15}CuO_4$. Of course, other forms could be used, but we will see that the choice does not matter much. This is fortunate since, in fact, we do not want to commit ourselves to a phonon mechanism. In the end, it is sufficient to characterize the spectral density by a single boson exchange frequency denoted by ω_{ln} and defined by[26]

$$\omega_{ln}=\exp\left[\frac{2}{\lambda}\int_0^{\infty}\frac{\alpha^2F(\Omega)ln(\Omega)}{\Omega}\right]. \tag{6}$$

Formula (6) gives more weight to the lower frequency bosons than to the higher frequency ones, and it can differ considerably from an arithmetic average. This particular weighting of $\alpha^2F(\omega)$ has been very successful in describing the critical temperature by a single moment of this distribution other than the mass enhancement parameter

$$\lambda=2\int_0^{\infty}\frac{\alpha^2F(\omega)}{\omega}d\omega,$$

which is the first inverse moment.[26] The same parameter describes remarkably well results of exact Eliashberg equation solutions for thermodynamic and other properties[27,28] of conventional superconductors.

For the results given below we take $\alpha^2F(\omega)$ to be related to a model $\alpha^2F_{\text{mod}}(\omega)$ (La-Sr-Cu-O) according to[29]

$$\alpha^2F(\omega)=B\alpha^2F_{\text{mod}}(\gamma\omega), \tag{7}$$

where B and γ are constants. The factor γ is varied at will and changes ω_{ln} through the relationship $\omega_{ln}=\omega_{ln}^{\text{mod}}/\gamma$. For a given choice of ω_{ln}, the factor B is chosen to get a T_c of 96 K characteristic of $YBa_2Cu_3O_{7-\delta}$. In Fig. 1, we show the relationship between λ and T_c/ω_{ln} for two spectral shapes, that of La-Sr-Cu-O and Pb. Note that as T_c/ω_{ln} exceeds the conventional strong-coupling limit ($\sim$0.25), λ attains very large values which are probably not consistent with lattice stability if they have their origin in a phonon mechanism.

Results for the temperature variation of $\lambda_L(T)$ for four different values of T_c/ω_{ln} are given in Fig. 2. The quantity $[\lambda_L(0)/\lambda_L(T)]^2-(1-t^4)$ is plotted as a function of reduced temperature $t=T/T_c$. To appreciate properly the values of T_c/ω_{ln} chosen, it is important to know that for conventional electron-phonon superconductors, T_c/ω_{ln} ranges from near zero (BCS limit) to about 0.25. Pb,[30] which is the prototype strong-coupling superconductor, falls near the center of this range. For comparison, it should be kept in mind that the curve for Al (BCS limit) calculated by Blezius and Carbotte[24] would fall, at minimum, slightly below -0.2. When the coupling is increased, the minimum becomes shallower as is the case for the solid curve with $T_c/\omega_{ln}=0.074$. This value of T_c/ω_{ln} is intermediate between Sn and Nb.[27,28] As T_c/ω_{ln} is increased to 0.296, slightly beyond the limit of the conventional range, the curve (dotted) has continued its upward trend and has developed a small positive part at low temperatures followed by a minimum above -0.05. As the coupling is increased further, however, the situation reverses and the curve goes back towards a BCS behavior (dashed line with $T_c/\omega_{ln}=0.661$). For the very strong coupling regime with $T_c/\omega_{ln}=1.175$, the minimum in the curve falls much lower than the BCS value. This temperature variation is distinctive and was not expected. In

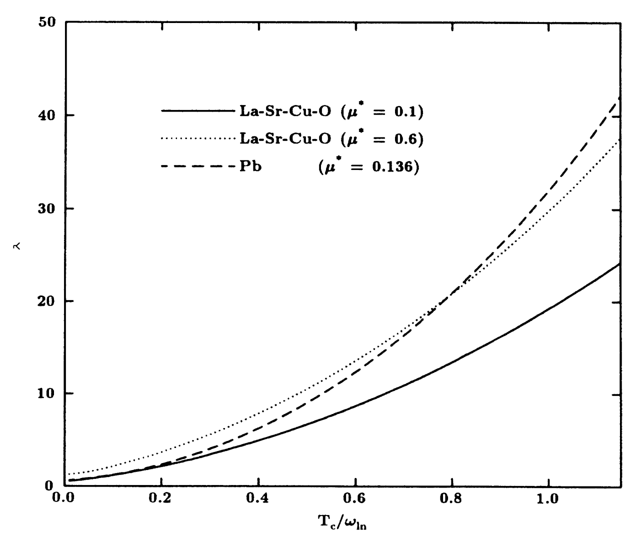

FIG. 1. Plot of λ vs T_c/ω_{ln} for La-Sr-Cu-O and Pb shapes. Note the u^* dependence for the former shape.

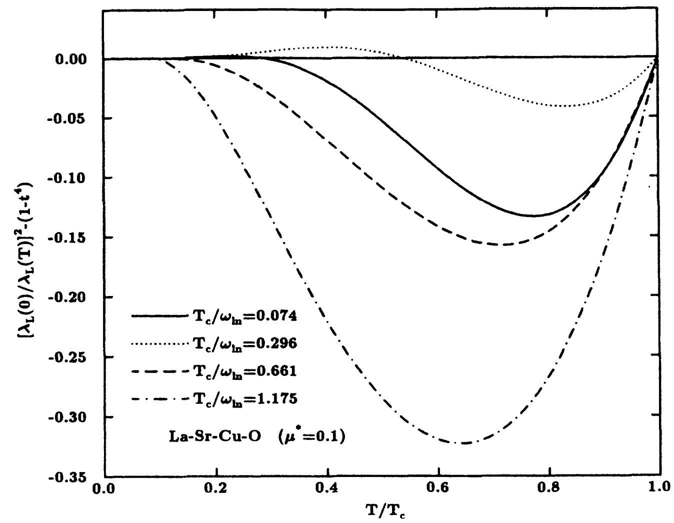

FIG. 2. The temperature dependence ($t = T/T_c$) of $[\lambda_L(0)/\lambda_L(T)]^2-(1-t^4)$ for four different values of T_c/ω_{ln}, namely 0.074 (intermediate coupling), 0.296 (strong), and 0.661 and 1.175 (both in the very strong coupling regime).

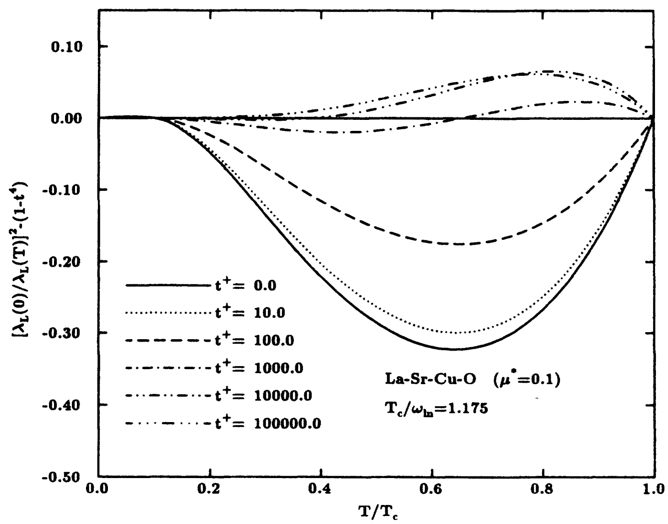

FIG. 4. Same as for Fig. 2 except that now $T_c/\omega_{ln} = 1.175$ and higher values of t^+ are also included.

principle, it could be used to get some information about the typical boson frequency involved in the pairing interaction.

The effect of impurities on the temperature variation of the London penetration depth is given in Figs. 3 and 4 for $T_c/\omega_{ln} = 0.296$ and $T_c/\omega_{ln} = 1.175$, respectively. We have obtained many other results; the ones given cover well the possibilities obtained. The first of these two figures applies for a case which corresponds to the limit of the conventional strong-coupling regime; the second corresponds to the very strong coupling case. In both figures, the effect of impurities is to make the deviation function positive definite. Note, however, the different scales used, as well as the different range of impurity concentrations required. Part of the reason for the higher t^+ values in the latter figure is that $(1+\lambda)$ is much higher in this case, and it is $t^{+*} \equiv t^+/1+\lambda$ which enters the equations.

The temperature variation of $\lambda_L(T)$ has been measured in $YBa_2Cu_3O_{7-\delta}$ by muon-spin-resonance (μSR) techniques, and Rammer[15] has given a detailed comparison of its temperature variation with results of strong-coupling calculations. The data certainly show departures from the conventional BCS (Al) behavior towards the limit of the conventional strong-coupling range. In Fig. 5, we compare results (solid curve) for the case $T_c/\omega_{ln} = 0.6$ (which is an appropriate value for Y-Ba-Cu-O) with the results of

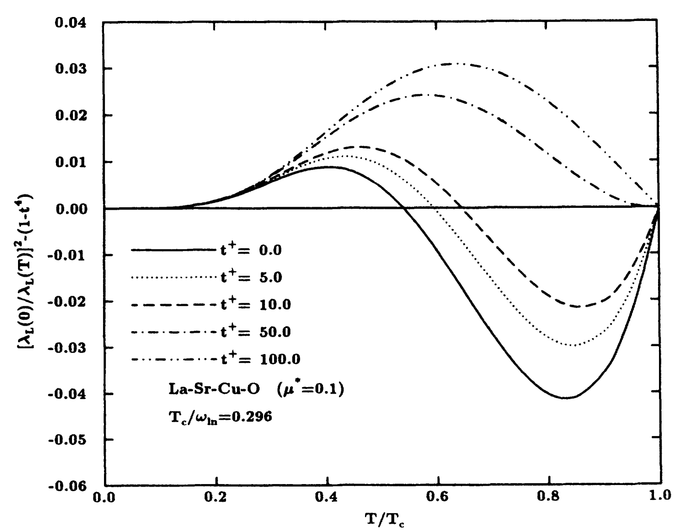

FIG. 3. Same as for Fig. 1 but for a single value of $T_c/\omega_{ln} = 0.296$ and different impurity content, namely $t^+ = 0.0$ (pure), 5.0, 10.0, 50.0, and 100.0 (meV).

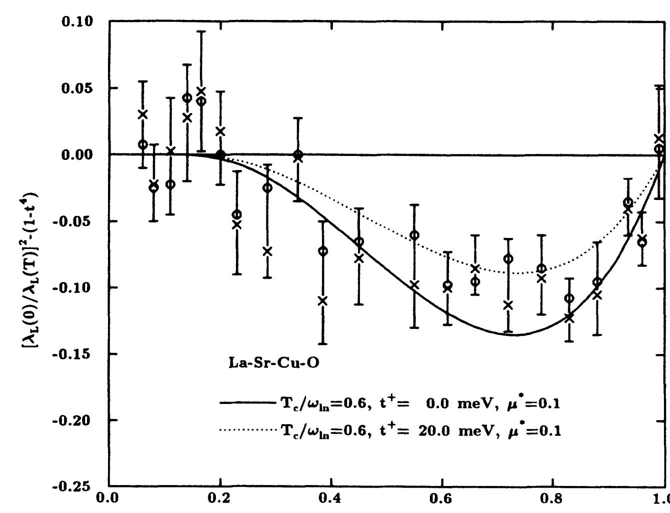

FIG. 5. The temperature dependence of $[\lambda_L(0)/\lambda_L(T)]^2-(1-t^4)$ for $T_c/\omega_{ln} = 0.6$. The solid curve applies to the pure case while the dotted curve is for $t^+ = 20.0$ meV. Also shown is the muon-spin-relaxation data of Kiefl *et al.* (Ref. 30).

μSR experiments by R. Kiefl *et al.*[30] The data certainly agree with the general trend shown and the fit could be improved by adding some impurities as is clear from the dotted curve which applied in the case of $t^+=20.0$ meV. While it is clear that the data on the temperature variation of the London penetration depth is consistent with a strong-coupling model, we wish to stress that we do not favor such a model and believe that an equally acceptable fit could be obtained with a combined exciton-phonon model.

It is interesting to define a strong-coupling parameter $\eta_{\lambda_L}(T)$ which compares results of a full calculation to those of BCS theory, for the penetration depth. We define

$$\lambda_L(T) \equiv \eta_{\lambda_L}(T)\lambda_L^{\mathrm{BCS}}(T) , \tag{8}$$

where λ_L^{BCS} is the BCS limit of the London penetration depth. At zero temperature and zero impurities it is

$$\lambda_L^{\mathrm{BCS}}(0) = \Lambda^{1/2}\sqrt{1+\lambda} . \tag{9}$$

In Fig. 6, we show $\eta_{\lambda_L}(T)$ for zero temperature as a function of T_c/ω_{ln} for three different cases, namely with $\mu^*=0.1$ and $\mu^*=0.6$ using a La-Sr-Cu-O spectrum and with $\mu^*=0.136$ with a Pb spectrum.[31] All curves are similar, showing that the results are not sensitive to the value of μ^* used even if it should be very much larger in the oxides than it is for conventional superconductors. Also, the shape of the phonon spectrum used is not important since the same qualitative results are obtained for a Pb shape which itself represents reasonably well a δ function for many purposes. Due to the lack of sensitivity of our results, it is clear that, even though both the shapes considered for the spectral density come from phonon work, they should, at least, simulate the results expected for the plasmon or exciton exchange case. In this latter instance, ω_{ln} is likely to be very high and T_c/ω_{ln} close to zero so that the BCS limit is recovered and no modifications apply. As T_c/ω_{ln} increases $\eta_{\lambda_L}(0)$ can be slightly larger than one before starting to drop after $T_c/\omega_{ln} \lesssim 0.1$. We see that in the very strong coupling limit for which T_c/ω_{ln} is of order 1.0, $\eta_{\lambda_L}(0)$ has fallen to 0.6 which is a large correction to the BCS result.

Figure 7 is concerned with results for $\eta_{\lambda_L}(T)$ at $T=T_c$ instead of at zero. Again, the shape of the spectrum used and the value of μ^* is not central to the qualitative behavior of the curves obtained. Note that, as T_c/ω_{ln} increases from zero, the curves now drop rapidly before leveling off around 0.4 with a much more gradual decline after that. This is in contrast to the $T=0$ case. In the very strong coupling limit, we find values around 0.6 as for the $T=0$ case.

Another interesting quantity is the ratio $y_L(0)/T_c|y_L'(T_c)|$, where $y_L(T)\equiv 1/\lambda_L^2(T)$. [$y_L(T)$ is directly proportional to the depolarization muon rate in muon-spin-relaxation measurements.] Results for this quantity are presented in Fig. 8. We see a very rapid drop from 0.50 (which is the BCS value) as T_c/ω_{ln} increases, with a minimum reached around 0.25 and then the curve starts increasing again recovering a value near BCS for $T_c/\omega_{ln}=1.2$. All three cases considered $\mu^*=0.1$, $\mu^*=0.6$ with the La-Sr-Cu-O spectrum, and $\mu^*=0.136$ with Pb give about the same results. It is remarkable that in this limit, results close to BCS are again obtained. As T_c/ω_{ln} continues to increase ($\lambda\to\infty$), it can, in fact, be shown that $y_L(0)/[T_c|y_L'(T_c)|]\propto\sqrt{\lambda}$ and so can increase indefinitely.

The relationship between $y_L(T)$ and the $\eta_{\lambda_L}(T)$ is of interest as are their BCS limits. Near $t=1$ we have

$$\lambda_L^{\mathrm{BCS}}(t) \cong \lambda_L^{\mathrm{BCS}}(0)\frac{1}{\sqrt{2(1-t)}} , \tag{10}$$

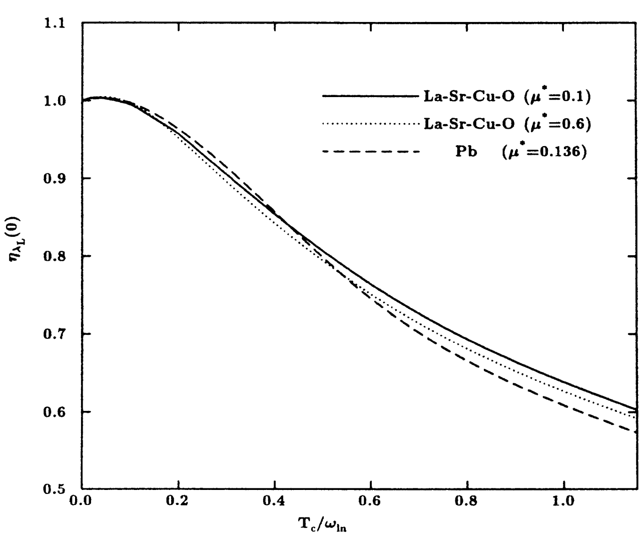

FIG. 6. The variation with T_c/ω_{ln} of the strong-coupling correction $\eta_{\lambda_L}(T)$ at zero temperature which measures departure from BCS. Note that two different spectral densities (Pb and La-Sr-Cu-O), and in one case (La-Sr-Cu-O), two values of Coulomb pseudopotential μ^* are used. No qualitative differences result.

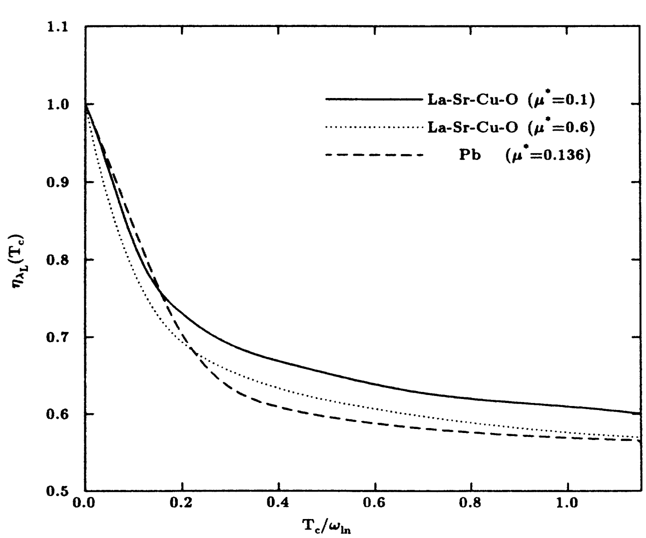

FIG. 7. Same as for Fig. 5, but now we show $\eta_{\lambda_L}(T)$ at $T=T_c$.

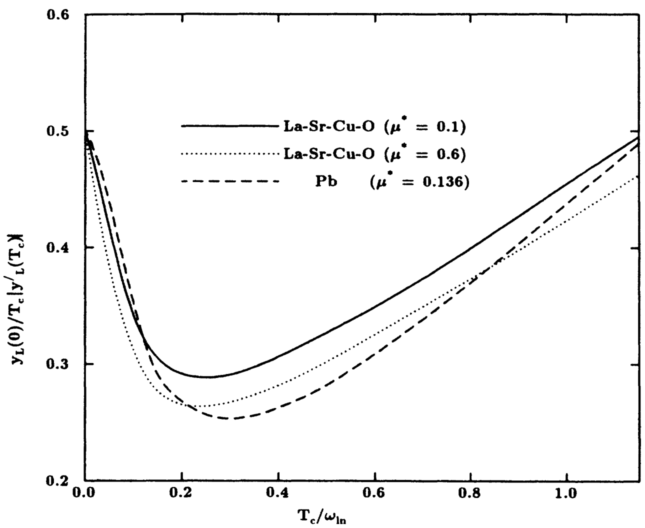

FIG. 8. The dimensionless ratio $y_L(0)/[T_c|y'_L(T_c)|]$ as a function of T_c/ω_{ln}. Two different densities (Pb and La-Sr-Cu-O), and in one case (La-Sr-Cu-O), two values of Coulomb pseudopotential μ^* are used. No qualitative differences result.

so that

$$T_c y'_L(1)^{\rm BCS} = \left[\frac{1}{\lambda_L^{\rm BCS}(0)}\right]^2 (-2) \tag{11}$$

and $[y_L(0)/T_c|y'_L(1)|]^{\rm BCS}$ becomes equal to 0.5. The strong-coupling parameter η_{λ_L} is related to this last quantity through

$$\frac{y_L(0)}{T_c|y'_L(T_c)|} = \frac{1}{2}\frac{\eta^2_{\lambda_L}(T_c)}{\eta^2_{\lambda_L}(0)} \tag{12}$$

so that Figs. 6–8 are related.

IV. CONCLUSIONS

We have calculated the temperature dependence of the London penetration depth for several values of the strong-coupling ratio T_c/ω_{ln} with ω_{ln} a characteristic boson energy associated with the electron-boson exchange spectral density $\alpha^2F(\omega)$ which appears as a kernel in the Eliashberg equations. While these equations were first derived and used for phonon superconductors, inasmuch as they can be used to describe, as a first approximation, some other boson exchange mechanism, our results will also apply in those cases. Corrections due to the failure of Migdal's theorem go beyond the scope of the present work.

As the coupling T_c/ω_{ln} is increased from the BCS limit ($T_c/\omega_{ln} \sim 0$) towards the outer edge of the conventional strong-coupling region which is $T_c/\omega_{ln} \lesssim 0.25$, the temperature dependence of

$$D_L(t) \equiv \left[\frac{\lambda_L(0)}{\lambda_L(T)}\right]^2 - \left[1 - \left[\frac{T}{T_c}\right]^4\right]$$

is gradually reduced in magnitude and can even stop being negative definite with a deep minimum around $t \simeq 0.75$. These results are similar to those found by Rammer although we have not attempted a detailed comparison. More interestingly, we find that as we go towards the very strong coupling limit with T_c/ω_{ln} of order 1.0, the trend in $D_L(t)$ reverses and it can now display a negative minimum which falls below the BCS value. This was quite unexpected and implies that a BCS temperature variation of $D(t)$ does not necessarily imply weak coupling.

The impurity dependence of $\lambda_L(T)$, while similar qualitatively, is very different quantitatively in the $T_c/\omega_{ln} \sim 1.0$ limit than it is for $T_c/\omega_{ln} \sim 0.0$.

The strong-coupling parameter $\eta_{\lambda_L}(T) \equiv \lambda_L(T)/\lambda_L^{\rm BCS}(T)$ also shows interesting behavior. Both $\eta_{\lambda_L}(0)$ and $\eta_{\lambda_L}(T_c)$ start from one at $T_c/\omega_{ln} = 0$ but vary very differently with increasing value of this same parameter. $\eta_{\lambda_L}(0)$ is at first only slightly affected with a gradual drop towards ~ 0.6 at $T_c/\omega_{ln} \sim 1.0$. On the other hand, $\eta_{\lambda_L}(T_c)$ first drops very rapidly and then its variation moderates as the very strong coupling asymptotic limit is reached. The behavior of the dimensionless ratio $y_L(0)/T_c|y'_L(T_c)|$ with $y_L(T) \equiv 1/\lambda_L^2(T)$ was particularly unexpected. In the BCS limit it has value 0.5 and drops to a minimum value of about 0.25 around $T_c/\omega_{ln} \sim 0.2$–0.3 and then begins to rise again to take on a value near BCS in the extreme case $T_c/\omega_{ln} \cong 1.2$. These results do not depend strongly on the value of Coulomb pseudopotential used or on the shape assumed for the electron-boson exchange spectral density.

ACKNOWLEDGMENTS

The research was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). We thank R. Kiefl *et al.* for giving us permission to display their data in our Fig. 5.

*Present address: MPB Technologies, Inc., 1725 North Service Road, Trans-Canada Highway, Dorval, Quebec, Canada H9P 1J1.

[1] Many papers in *Novel Superconductivity*, edited by S. A. Wolf and V. Z. Kresin (Plenum, New York, 1987).

[2] J. Ruvalds, Phys. Rev. B **35**, 8869 (1987).

[3] V. Kresin, Phys. Rev. B **35**, 8716 (1987).

[4] C. M. Varma, S. Schmitt-Rink, and E. Abrahams, Solid State Commun. **62**, 681 (1987).

[5] D. Allender, J. Bray, and J. Bardeen, Phys. Rev. B **7**, 1020 (1973).

[6] F. Marsiglio and J. P. Carbotte, Phys. Rev. B **36**, 3937 (1987).

[7] F. Marsiglio, R. Akis, and J. P. Carbotte, Solid State Commun. **64**, 905 (1987).

[8] V. J. Emery, Phys. Rev. Lett. **59**, 2794 (1987).

[9] D. Vaknin, S. K. Sinha, D. E. Moncton, D. C. Johnston, J. M. Newsam, C. R. Safinya, and H. E. King, Jr., Phys. Rev. Lett. **58**, 2802 (1987).

[10] P. W. Anderson, Science **235**, 1196 (1987).

[11] S. Kivelson, D. Rokhar, and J. Sethna, Phys. Rev. B **35**, 8865 (1987).

[12] P. W. Anderson, G. Baskaran, Z. Zou, and T. Hsu, Phys. Rev. Lett. **58**, 2790 (1987).

[13] J. M. Daams and J. P. Carbotte, J. Low Temp. Phys. **43**, 263 (1981).

[14] D. Rainer and G. Bergmann, J. Low Temp. Phys. **14**, 501 (1974).

[15] J. Rammer, Europhys. Lett. **5**, 77 (1988).

[16] F. N. Gygax, B. Hitti, E. Lippelt, A. Schenck, D. Cattani, J. Cors, M. Decroux, O. Fischer, and S. Barth, Europhys. Lett. **4**, 473 (1987).

[17] G. Aeppli, R. J. Cava, E. J. Ansaldo, J. H. Brewer, S. R. Kreitzman, G. M. Luke, D. R. Noakes, and R. F. Kiefl, Phys. Rev. B **35**, 7129 (1987).

[18] W. J. Kossler, J. R. Kempton, X. H. Yu, H. E. Schone, Y. J. Vemura, A. R. Moodenbaugh, and M. Suenaga, Phys. Rev. B **35**, 7133 (1987).

[19] D. R. Hanshman, G. Aeppli, E. J. Ansaldo, B. Batlogg, J. H. Brewer, J. F. Carolan, R. J. Cara, M. Celio, A. C. D. Chaklader, W. N. Hardy, S. R. Kreitzman, G. M. Luke, D. R. Noakes, and M. Senba, Phys. Rev. B **36**, 2386 (1987).

[20] T. R. Lemberger, D. M. Ginsberg, and G. Rickayzen, Phys. Rev. B **18**, 6057 (1978).

[21] S. B. Nam, Phys. Rev. **156**, 470 (1967); **156**, 487 (1967).

[22] P. D. Scholten, J. D. Legeune, W. M. Saslow, and D. G. Maugle, Phys. Rev. B **16**, 1068 (1977).

[23] H. R. Kerchner and D. M. Ginsberg, Phys. Rev. B **8**, 3190 (1973).

[24] J. Blezius and J. P. Carbotte, Phys. Rev. B **33**, 3509 (1986).

[25] W. Weber, Phys. Rev. Lett. **58**, 1371 (1987).

[26] P. B. Allen and R. C. Dynes, Phys. Rev. B **12**, 905 (1975).

[27] F. Marsiglio and J. P. Carbotte, Phys. Rev. B **33**, 6141 (1986).

[28] B. Mitrovic, H. G. Zarate, and J. P. Carbotte, Phys. Rev. B **29**, 184 (1984).

[29] F. Marsiglio, R. Akis, and J. P. Carbotte, Phys. Rev. B **36**, 5245 (1987).

[30] R. F. Kiefl *et al.*, in *Proceedings of the International Conference on High-T_c Superconductors: Materials and Mechanisms of Superconductivity, Interlaken, Switzerland, 1988,* edited by J. Muller and J. L. Olsen [Physica C (to be published)].

[31] W. L. McMillan and J. M. Rowell, in *Superconductivity,* edited by R. D. Parks (Dekker, New York, 1969), Vol. 1, p. 562.