Generalized master equations and other theories of the phonon-assisted hopping conduction

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A new method of solving exactly the time-convolution generalized master equations linearized in the external field but exact to all powers in the electron-phonon coupling is reported for a standard dc phonon-assisted hopping problem. This method allows one to classify existing theories of this type of transport, using an exact identity derived and according to the degree of approximation used, into three categories. It is clarified why theories of the worst category comply with the exact solution but are appreciably distorted when formally amended so as to fall into the second category of the lowest-order Markovian kinetic theories.

I. INTRODUCTION

Recently, generalized master equations (GME) for the standard model describing phonon-assisted hopping conduction in amorphous semiconductors have been formulated and solved exactly in the linear approximation in the external electric field.[1] For the dc conductivity, this result yields surprisingly the old Kasuya-Koide[2] explicit result

$$\sigma=\frac{\beta e^2}{2\Omega}\sum_{p,r}\sum_{s=\pm 1/2}\Gamma_{pr}(x_p-x_r)^2+o(g^2)\ . \tag{1}$$

Here β, e, Ω, x_r, and g are the reciprocal temperature in energy units, electronic charge, normalizing volume, mean coordinate of the localized single-electron state $|r\rangle$ in the mobility gap in the direction of the field, and the electron-phonon coupling constant. $\Gamma_{pr}=\Gamma_{rp}$ is the mean statistical transfer rate $r\leftrightarrow p$; for the Hamiltonian

$$H=H_e+H_{\rm ph}+H_{e\text{-ph}}=H_0+H_{e\text{-ph}}=\sum_{r,s=\pm 1/2}\varepsilon_r a_{rs}^\dagger a_{rs}+\sum_k \hbar\omega_k b_k^\dagger b_k+\frac{g}{\sqrt{2\Omega}}\sum_{p,r}\sum_{s=\pm 1/2}\sum_k U_{pr}^k a_{ps}^\dagger a_{rs}(b_k+b_k^\dagger)\ , \tag{2}$$

it is

$$\Gamma_{pr}=W_{pr}^{\rm eq}n_F(\varepsilon_r)[1-n_F(\varepsilon_p)]\ , \tag{3a}$$

$$W_{pr}^{\rm eq}=\frac{\pi g^2}{\hbar\Omega}\sum_k |U_{pr}^k|^2\{\delta(\varepsilon_r-\varepsilon_p+\hbar\omega_k)n_B(\hbar\omega_k)+\delta(\varepsilon_r-\varepsilon_p-\hbar\omega_k)\times[1+n_B(\hbar\omega_k)]\}\ , \tag{3b}$$

$$n_F(z)=(e^{\beta(z-\mu)}+1)^{-1},\quad n_B(z)=(e^{\beta z}-1)^{-1}\ . \tag{3c}$$

On the other hand, converting the only correct order of limits ($\Omega\to+\infty$, frequency $\omega+i\delta\to 0$, and then $g\to 0$), i.e., expanding as usual in powers of g before performing the dc limit, we recover (up to kinematic corrections properly included in GME) at low but finite frequencies $\omega+i\delta$[1] the Miller and Abrahams equations[3]

$$-i(\omega+i\delta)\delta f_r^{\omega+i\delta}=\sum_{p\,(\neq r)}\beta\Gamma_{rp}[e\mathcal{E}^{\omega+i\delta}(x_r-x_p)-\delta\mu_r^{\omega+i\delta}+\delta\mu_p^{\omega+i\delta}]\ , \tag{4a}$$

$$\delta\mu_r^{\omega+i\delta}=\{\beta n_F(\varepsilon_r)[1-n_F(\varepsilon_r)]\}^{-1}\delta f_r^{\omega+i\delta}\ , \tag{4b}$$

$$\delta f_r^{\omega+i\delta}=\int dt\,\delta f_r(t)e^{i(\omega+i\delta)t}\ . \tag{4c}$$

Here, the frequency picture has been used. Furthermore,

$$\delta f_r(t)={\rm Tr}[\delta\rho(t)a_{rs}^\dagger a_{rs}] \tag{5}$$

is the linear field-induced change of the mean number of electrons at site r; $\delta\mu_r$ is the corresponding linear local field-induced change of the chemical potential. Finally,

$$\delta\rho(t)=\rho(t)-\rho_{\rm eq}\equiv\rho(t)-\frac{e^{-\beta H}}{{\rm Tr}(e^{-\beta H})}\to 0 \text{ as } t\to-\infty \tag{6}$$

is the linear change of the electron-phonon density matrix due to the electric field

$$\mathcal{E}(t)=\int\frac{d\omega}{2\pi}\mathcal{E}^{\omega+i\delta}e^{-i(\omega+i\delta)t}\to 0 \text{ as } t\to-\infty\ . \tag{7}$$

Equations (4a) with (4b) form a complicated set of equations to be solved either approximately or using computers. They result (upon linearization with respect to $\mathcal{E}$) from standard lowest-order Markovian rate equations:

$$\frac{\partial}{\partial t}f_r(t)=\sum_{p\,(\neq r)}\{W_{rp}f_p(t)[1-f_r(t)]-W_{pr}f_r(t)[1-f_p(t)]\}\ , \tag{8a}$$

$$f_r(t)={\rm Tr}[\rho(t)a_{rs}^\dagger a_{rs}]\ . \tag{8b}$$

For the dc conductivity, (4a) yields the Miller and Abrahams formula[3]

$$\sigma=\frac{\beta e^2}{2\Omega}\sum_{p,r}\sum_{s=\pm 1/2}\Gamma_{pr}(x_p-x_r)\times\left[x_p-x_r-\frac{\delta\mu_p}{e\mathcal{E}}+\frac{\delta\mu_r}{e\mathcal{E}}\right]+o(g^2)\ , \tag{9}$$

where $\delta\mu_r/\mathcal{E}$ is the dc limit of $\delta\mu_r^{\omega+i\delta}/\mathcal{E}^{\omega+i\delta}$. Relations (4a) and (9) may be converted into the classical random resistance network problem which serves then as a starting point for most of the contemporary dc phonon-assisted hopping theories.

Equation (9) clearly contradicts (1) due to the presence of the local shifts $\delta\mu_q$. It is of course known to be just approximate, owing to the lowest-order (in g) expansion used to derive (4a) [or (8a)] before performing the dc limit. On the other hand, result (1) was originally derived under *two* important approximations—the first one being mentioned just now as inherent to (4a) [or (8a)] and the second one being the full neglect of the shifts $\delta\mu_q$ in (4a) and (9). It is therefore surprising that (1) results from a rigorous theory[1,4,5] (see also Ref. 6 and 7 though this older form of a rigorous theory has never been given a physical interpretation).

A physical explanation of the contradication between (9) and rigorous result (1) was given[5] where terms exactly canceling those leading to $\delta\mu_q$ in (9) have been identified among higher-order terms in GME. Here, another and yet more transparent form of these arguments is given which is based on a new method of solving the exact linearized GME. In addition to physical transparency, the present theory clearly shows why the two above-mentioned crude approximations used by Kasuya-Koide[2] to derive a closed formula for the dc conductivity effectively cancel each other, i.e., why this crude approximate theory complies with the exact result (1). Standard theories leading to (9), due to the finite-order expansion performed before taking the dc limit, are then shown to disturb one exact relation derived here. As this is the only approximation used, there is nothing in these theories which might compensate the corresponding error. This is then the reason for their contradiction with the exact formula (1).

II. dc CONDUCTIVITY CALCULATION

As we must go to the infinite order in g if we want to construct a rigorous theory, the formal structure of our approach is more complicated than in standard theories. First, we introduce the projection superoperator D $(=D^2)$, i.e., operator in the Liouville space, with matrix elements

$$D_{l\lambda,m\mu,n\nu,p\pi}=\rho^R_{\lambda\mu}\delta_{\nu\pi}\delta_{lm}\delta_{ln}\delta_{mp},\quad \sum_{\lambda}\rho^R_{\lambda\lambda}=1\ , \tag{10a}$$

i.e.,

$$(DA)_{l\lambda,m\mu}=\sum_{n\nu,p\pi}D_{l\lambda,m\mu,n\nu,p\pi}A_{n\nu,p\pi}=\rho^R_{\lambda\mu}\delta_{lm}(\mathrm{Tr}_{\mathrm{ph}}A)_{mm}\ , \tag{10b}$$

and the Liouville superoperators

$$LA=\frac{1}{\hbar}[H,A],\ L_0A=\frac{1}{\hbar}[H_0,A],\ \mathcal{L}A=\frac{1}{\hbar}[H_{e\text{-ph}},A]\ , \tag{11a}$$

i.e.,

$$L_{l\lambda,m\mu,n\nu,p\pi}=\frac{1}{\hbar}(H_{l\lambda,n\nu}\delta_{m\mu,p\pi}-H_{p\pi,m\mu}\delta_{l\lambda,n\nu}) \tag{11b}$$

and similarly for L_0 and $\mathcal{L}$. Here A is an arbitrary operator; the Latin (Greek) indices designate the set of the electron site (phonon mode) occupation numbers, i.e., $m\mu$ designates the whole many-body eigenstate of H_0.

Upon linearization with respect to $\mathcal{E}(t)$, the Liouville equation

$$i\frac{\partial}{\partial t}\rho(t)=\frac{1}{\hbar}[H-d\cdot\mathcal{E}(t),\rho(t)]\equiv L\rho(t)-\frac{1}{\hbar}\mathcal{E}(t)\cdot[d,\rho(t)] \tag{12}$$

yields the linearized GME (Ref. 1)

$$\frac{\partial}{\partial t}[D\,\delta\rho(t)]=-iDLD\,\delta\rho(t)-\int_{-\infty}^{t}DLe^{-i(1-D)L(t-\tau)}(1-D)\left\{-\frac{1}{\hbar}\mathcal{E}(\tau)\cdot[d,\rho_{\mathrm{eq}}]+LD\delta\rho(\tau)\right\}d\tau-\frac{1}{\hbar}\mathcal{E}(t)\cdot D[d,\rho_{\mathrm{eq}}]\ . \tag{13}$$

Here $d=\sum_r\sum_{s=\pm1/2}e\,x_r a^{\dagger}_{rs}a_{rs}$ is the dipole moment in the direction parallel to $\mathcal{E}(t)$. As a consequence of (10)–(11),

$$DLD=0\ , \tag{14a}$$

$$DL_0=L_0D=0\ , \tag{14b}$$

and, due to the choice of d,

$$D[d,\rho_{\mathrm{eq}}]=0\ . \tag{14c}$$

Consequently, the first and the last terms on the right-hand side of (13) disappear. Expanding formally in powers of g, the first (second) term in curly brackets on the right-hand side of (13) would reduce to the first term $\sim x_r-x_p$ (second term $\sim-\delta\mu_r^{\omega+i\delta}+\delta\mu_p^{\omega+i\delta}$) on the right-hand side of (4a) as far as the kinematic corrections (due to a correlation in occupation of neighboring sites, included in GME) are neglected. So the structure of (4a) and (13) is formally the same.

Before going further, let us mention that (4a) may be converted to the matrix form

$$[(\omega+i\delta)\mathbb{1}-\mathcal{A}^{(2)}]\delta f^{\omega+i\delta}=\mathcal{B}^{(2)}\ , \tag{15}$$

where the operator matrices $\mathcal{A}^{(2)}/g^2$ and $\mathcal{B}^{(2)}/g^2$ are g independent. Here, $\mathbb{1}$ designates the unit operator matrix. The necessity of performing the inversion $[(\omega+i\delta)\mathbb{1}-\mathcal{A}^{(2)}]^{-1}$, i.e., solving the complicated set of dynamic equations (15), is the reason for enormous technical problems of standard lowest-order Markovian (percolation, rate equation, random resistance network, etc.) theories. Now, with (13), we shall perform an analogous algebra but exactly to all powers of g and on our superoperator level.

Performing the Fourier transformation of (13) and taking (14a)–(14c) into account, we obtain exact dynamic equations in the superoperator form

$$[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]D\,\delta\rho^{\omega+i\delta}=B^{\omega+i\delta} \tag{16}$$

from which the solution $D\,\delta\rho^{\omega+i\delta}$ must (and, as we are going to show, also can easily) be found. Here, $\mathbb{1}$ is the superoperator unit matrix. The superoperator

$$A^{\omega+i\delta}=D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}\mathcal{L}D \tag{17a}$$

as well as the operator

$$B^{\omega+i\delta}=-\frac{1}{\hbar}D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}] \tag{17b}$$

are both of the formal order $\sim g^2$ but contain arbitrarily high powers of g. Multiplying (16) from the right-hand side by $a_{rs}^{\dagger}a_{rs}$, taking the trace and neglecting the kinematic corrections would turn (16) to (15) in the lowest order in g. Instead, we devote our attention to the exact form of $B^{\omega+i\delta}$.

Because of (14b),

$$B^{\omega+i\delta}=-\frac{1}{\hbar}DL\left[\frac{1}{(\omega+i\delta)\mathbb{1}-L}-\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}DL\frac{1}{(\omega+i\delta)\mathbb{1}-L}\right]\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}]$$

$$=-\frac{1}{\hbar}\left[\mathbb{1}-DL\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}D\right]DL\frac{1}{(\omega+i\delta)\mathbb{1}-L}\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}]\ . \tag{18}$$

Here, we have used the idempotency property $D=D^2$. As it is

$$L\frac{1}{(\omega+i\delta)\mathbb{1}-L}=-\mathbb{1}+(\omega+i\delta)\frac{1}{(\omega+i\delta)\mathbb{1}-L}\ , \tag{19}$$

(18) gives using (14c)

$$B^{\omega+i\delta}=-\frac{1}{\hbar}\left\{(\omega+i\delta)\mathbb{1}-DL\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}[(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L+(\mathbb{1}-D)L]D\right\}$$

$$\times D\frac{1}{(\omega+i\delta)\mathbb{1}-L}\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}]\ . \tag{20}$$

Because of (14a)–(14c), (20) finally yields

$$B^{\omega+i\delta}=-\frac{1}{\hbar}\left[(\omega+i\delta)\mathbb{1}-D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-(\mathbb{1}-D)L}\mathcal{L}D\right]D\left[\frac{1}{(\omega+i\delta)\mathbb{1}-L_0}+\frac{1}{(\omega+i\delta)\mathbb{1}-L_0}\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-L}\right]$$

$$\times\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}]=[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]\left[-\frac{1}{\hbar(\omega+i\delta)}\right]D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-L}\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\rm eq}]$$

$$\equiv[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]C^{\omega+i\delta}\ . \tag{21}$$

This is our chief result showing that in the exact (to all powers of g) theory, the right-hand side $B^{\omega+i\delta}$ of our dynamic equations (16) is [in contrast with the standard treatment (15)] a product of two factors; the left-hand side factor in $B^{\omega+i\delta}$ is *exactly* the same superoperator which appears on the left-hand side of (16) while the right-hand side factor $C^{\omega+i\delta}$ is proportional to $g^2/(\omega+i\delta)$. This relevant observation has several important implications.

(a) The exact solution of (16) is [due to (21) but in con-

trast to the standard theory] absolutely elementary. Namely, multiplying (16) by $[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]^{-1}$, we obtain

$$D\delta\rho^{\omega+i\delta}=-\frac{1}{\hbar(\omega+i\delta)}D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-L}\mathcal{E}^{\omega+i\delta}\cdot[d,\rho_{\mathrm{eq}}]\ . \tag{22}$$

This is singular in the dc limit and yields exactly the Kasuya-Koide formula for the dc conductivity (1).[1,4,5]

(b) Proceeding as in the standard lowest-order Markovian (rate equation, percolation, etc.) theory [i.e., like in (15)] means to replace (16) by the second-order equation

$$[(\omega+i\delta)\mathbb{1}-A^{(2)\omega+i\delta}]D\delta\rho^{\omega+i\delta}\approx B^{(2)\omega+i\delta} \tag{23}$$

equivalent in the above-mentioned sense to (15). In (23),

$$A^{(2)\omega+i\delta}=D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-L_0}\mathcal{L}D\ , \tag{24a}$$

but according to (17b) or (21),

$$B^{(2)\omega+i\delta}=-\frac{1}{\hbar}D\mathcal{L}\frac{1}{(\omega+i\delta)\mathbb{1}-L_0}\mathcal{E}^{\omega+i\delta}\cdot\left[d,-\int_0^\beta ds\,\frac{e^{-sH_0}H_{e\text{-ph}}e^{(s-\beta)H_0}}{\mathrm{Tr}(e^{-\beta H_0})}\right]\ . \tag{24b}$$

Here $A^{(2)\omega+i\delta}/g^2$ and $B^{(2)\omega+i\delta}/g^2$ are g independent. Clearly, this means to approximate $[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]$ as $[(\omega+i\delta)\mathbb{1}-A^{(2)\omega+i\delta}]$ on the left-hand side of (16) but as only $(\omega+i\delta)\mathbb{1}$ on the right-hand side of (16) because $A^{\omega+i\delta}\sim g^2$. This is of course inconsistent and is the very source of technical, but unphysical as our approach shows, difficulties in the standard treatment. These are due to roughly approximate converting elementary and exact problem (16) with (21) with the exact solution (22) to a completely nonelementary problem (23) or (15) without any explicit solution, motivated by naive attempts to make the standard theory "simpler." After the full neglect of $A^{\omega+i\delta}$ in (21) on the right-hand side of (16), it is

$$\frac{1}{\omega+i\delta}[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]\to\frac{1}{\omega+i\delta}(\omega+i\delta)\mathbb{1}=\mathbb{1}\ . \tag{25}$$

Consequently, there is no wonder that for $D\,\delta\rho^{\omega+i\delta}/\mathcal{E}^{\omega+i\delta}$, i.e., also $\delta\mu_q^{\omega+i\delta}/\mathcal{E}^{\omega+i\delta}$, one obtains finite results when $\omega+i\delta\to 0$ in the standard theory. The fact that $(\omega+i\delta)\mathbb{1}-A^{(2)\omega+i\delta}$ is singular in the dc limit on the left-hand side of (23) does not change the situation as this is due to the fact that

$$\sum_{l,\lambda}A^{(2)\omega+i\delta}_{l\lambda,l\lambda,m\mu,n\nu}=0 \tag{26a}$$

while simultaneously,

$$\sum_{l,\lambda}B^{(2)\omega+i\delta}_{l\lambda,l\lambda}=0\ , \tag{26b}$$

as it may be easily verified. This also contradicts the exact result (22) but as it is seen from (25), this contradiction is just due to the above mentioned inconsistency. Finally, writing $[(\omega+i\delta)\mathbb{1}-A^{\omega+i\delta}]$ as $(\omega+i\delta)[\mathbb{1}-A^{\omega+i\delta}/(\omega+i\delta)]$ in (21) on the right-hand side of (16), it is also seen that theories of this kind are based on an unjustified neglect of higher-order terms $\sim g^2/(\omega+i\delta)$.

(c) One might try to be "consistently crude" in the sense of being formally yet more crude than in (b) but in a consistent manner: If the approximation $B^{\omega+i\delta}\to B^{(2)\omega+i\delta}$ means to neglect fully $A^{\omega+i\delta}$ as a part of the prefactor in $B^{\omega+i\delta}$ on the right-hand side of (16) [see (21)], one should neglect also $A^{\omega+i\delta}$ on the left-hand side of (16). It is amusing to observe that this yields

$$D\,\delta\rho^{\omega+i\delta}\approx\frac{1}{\omega+i\delta}[B^{(2)\omega+i\delta}+o(g^2)] \tag{27}$$

which is nothing but (22) up to higher-order terms $\sim g^2/(\omega+i\delta)$. Consequently, as the conductivity is given by the current $\sim(\omega+i\delta)d^{\omega+i\delta}$, this approach reproduces the exact result for the dc conductivity σ (1). It is worth mentioning that neglecting $A^{\omega+i\delta}$ on the left-hand side of (16) means to neglect the field-induced changes of $D\delta\rho^{\omega+i\delta}$ on the right-hand side of (13), i.e., $\delta\mu_q^{\omega+i\delta}$ in (4a). Exactly this type of consistently crude approximation was used by Kasuya-Koide in their original work.[2] Finally, writing $[(\omega+i\delta)\mathbb{1}-A^{(2)\omega+i\delta}]^{-1}$ as

$$(\omega+i\delta)^{-1}[\mathbb{1}-A^{(2)\omega+i\delta}/(\omega+i\delta)]\approx\frac{1}{\omega+i\delta}\left[1+\frac{O(g^2)}{\omega+i\delta}\right],$$

it is seen that from the point of view of the standard theories, theories of the present group are (formally) due to unjustified neglect of higher-order terms.[8,9] In fact, however, as we have seen, it is successively necessary to neglect the terms proportional to $g^2/(\omega+i\delta)$ twice when going from (16) via (23) to (27). As shown above, it is then not fully fortuitous that the net effect of these two formal approximations effectively cancels in the sense that it leads to just the term $o(g^2)$ in (1).

III. CLASSIFICATION OF THE PHONON-ASSISTED HOPPING THEORIES

As we have seen, the most characteristic feature of any theory of the dc phonon-assisted hopping is whether it preserves (21) as an exact identity connecting the left- and right-hand side of the dynamic equation (16) or not. We have also seen that preserving this identity is a necessary condition for obtaining the exact result (1) for the dc con-

ductivity. Therefore, we shall classify the existing theories of the dc phonon-assisted hopping according to the degree to which the identity (21) is preserved.

Historically the first theory of the dc phonon-assisted hopping in amorphous or doped crystalline semiconductors was due to Kasuya-Koide;[2] it was based on the consistently crude approximation neglecting $A^{\omega+i\delta}$ on the left-hand side of dynamic equation (16) (i.e., neglecting local shifts $\delta\mu_q$) as well as on the right-hand side of (16) (as a part of taking just the lowest order in g). Theories presented in Refs. 10–12 belong to the same group and all of them legitimately reproduce the exact result (1) which has been, however, found later.[1,6]

The second group of theories is formed by those which use arguments equivalent to retaining $A^{\omega+i\delta}$ (as $A^{(2)\omega+i\delta}$) on the left-hand side of (16), i.e., keeping $\delta\mu_q^{\omega+i\delta}$ in (4a) but neglecting it fully on the right-hand side of (16) as a part of taking just the lowest-order contribution in the driving term of relevant dynamic equations. Hence, the identity (21) is not preserved. Most of the contemporary theories belong to this class.[3,8,9,13–20] In this list, we do not include papers on the rate equations, percolation, or computer modeling of random resistence networks, which are based on results of works quoted here. Also the works[9,21] belong to this group in the sense of disturbing relation (21); nevertheless, in this case, the situation is more complicated since an incomplete Hamiltonian is used and technically, the theory is formulated in terms of electronic functions which are not eigenfunctions of H_0 [Eq. (2)].

The third group of theories is formed by those approaches which retain $A^{\omega+i\delta}$ on both sides of (16). To this group, theories[1,4,5] belong; historically, also the works[6,7] belong here. They all yield the same exact result (1) irrespective of whether they use the kinetic equations[1,4,5] or not.[6,7] The fact that the theories of the first group yield the same result for $\sigma(1)$ means (irrespective of their roughly approximate nature) their rehabilitation in practice. It should be mentioned here that in contrast to the usual opinion,[22] result (1) for the dc phonon-assisted hopping conductivity may be shown to agree well with experiment[6] so that it forms a sound basis for further discussion.

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