Quantum-mechanical effects in nonlinear magnetotransport

P. Warmenbol, F. M. Peeters, and J. T. Devreese*
Department of Physics, University of Antwerp, Universiteitsplein 1, B-2610 Wilrijk-Antwerpen, Belgium
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The semiclassical Boltzmann equation and available quantum-magnetotransport models are studied in the crossed electric and magnetic field configuration. The results of these theories are compared with the experimental data of Fujisada, Kataoka, and Beer [Phys. Rev. B **3**, 3249 (1971)] on InSb at 77 K, which are as yet not fully understood. Theoretically it is a very hard problem because one is in the intermediate regime between classical and quantum transport.

I. INTRODUCTION

The addition of a magnetic field causes a rich variety of experimental transport phenomena such as the Hall effect, the magnetophonon resonance effect, the Shubnikov–de Haas oscillations, etc. In the papers of Barker[1] and Mahan[2] much of the theoretical work on this subject is reviewed. The experimental complexity is mirrored in a similar theoretical complexity. Almost all existing nonlinear magnetotransport models are analytically intractable and can be solved only by numerical techniques. In order to obtain analytical solutions it is usually necessary to (i) introduce simplifying or phenomenological assumptions, e.g., the electron effective temperature model[3] is very often applied; its validity has been studied by, e.g., Calecki *et al.*[4] or (ii) derive results for certain limiting cases: Kazarinov and Skobov[5] treated the limiting cases $\hbar\omega_c \ll k_B T_e$ and $\hbar\omega_c \gg k_B T_e$, where T_e is the electron temperature and k_B Boltzmann's constant. Only very few papers go beyond the quasiparticle approximation or even beyond the first Born approximation for the interaction matrix elements, with the exception of Thornber,[6] who worked within the path-integral approach, and Mahan,[2] who applied Green's-function theory.

The aim of the present paper is to investigate the validity range of electric and magnetic fields where the semiclassical Boltzmann equation and some of the available quantum magnetotransport models are valid in the crossed field configuration, i.e., $\mathbf{E}(E,0,0)$ and $\mathbf{B}(0,0,B)$. We will compare the results of these models with the galvanomagnetic transport measurements of Fujisada, Kataoka, and Beer[7] (FKB) on InSb at 77 K. The discussion will be centered around the values of three parameters ζ, η, and $\omega_c\tau$, as discussed below. The parameter $k_B T$ will not explicitly be considered since it is not a large or small parameter here compared to other typical energies. Namely, the magnetic field in the experiments of FKB ranges from $B=0.1$ to 1.5 T, which corresponds to a temperature range of $T=10$–150 K, which compares with the lattice temperature $T=77$ K.

InSb is a polar semiconductor and consequently the interaction of electrons with polar LO phonons will be a very important scattering mechanism. Komiyama *et al.*[8] introduced a classical model which is a generalization of the Shockley model to nonzero magnetic fields. They defined the dimensionless parameter $\zeta = v_{\rm LO}B/E$, where $v_{\rm LO}^2/2m^* = \hbar\omega_{\rm LO}$ is the LO-phonon energy and m^* the electron effective mass. Three different cases can be discriminated: electrons can be (i) *streaming* ($\zeta<1$), (ii) *accumulated* on closed orbits ($\zeta>2$), or (iii) in the transition region ($1<\zeta<2$). Within this model LO-phonon emission is forbidden for $\zeta>2$ (the electron has not enough energy to emit an LO phonon).

The second parameter $\eta=\hbar\omega_c/(eEr_0)$ is a measure for the spatial overlap of the Landau levels. Here ω_c is the cyclotron frequency and $r_0=(\hbar/eB)^{1/2}$ is the radius of the lowest Landau level. For $\eta \ll 1$ the Landau-level overlap is large and the semiclassical Boltzmann equation applies. On the other hand, for $\eta \gg 1$ the Landau levels are well separated and a quantum-mechanical theory has to be used.

The third parameter $\omega_c\tau$ is the product of the cyclotron frequency ω_c and the mean free time τ between collisions. $\omega_c\tau$ gives the average number of cyclotron cycles performed by an electron between two collisions. The semiclassical regime requires $\omega_c\tau \ll 1$, while for $\omega_c\tau \gg 1$ quantum effects[9] are expected to show up, like, e.g., the magnetophonon resonance effect. In extremely high magnetic fields the value of $\omega_c\tau$ is often much larger than 1. In that case Kubo *et al.* derived a rapidly converging series expansions[9] for the linear quantum magnetoconductivity, with $1/\omega_c\tau$ as a small parameter. The generalization of the Kubo formula to the nonlinear regime was first studied rigorously by Budd[10] and later on by, e.g., Barker.[1] Mori *et al.*[11] applied this approach recently to investigate nonlinear magnetic-LO-phonon resonance and derived an analytical expression for the electric field dependence of the oscillatory part of the magnetoconductivity.

Beleznay and Serényi[12] reformulated and approximated the Thornber-Feynman momentum balance approach[13] to the case of an electron in crossed electric and magnetic fields in the limit of weak electron-phonon coupling. They describe the electron in the effective mass approximation and the interaction in the first Born approximation. In principle this approach is valid for arbitrary field strength. On the other hand, the electric field range of validity is restricted by the inherent form of the momentum distribution function, which was shown by Peeters and Devreese[14] to be a drifted Maxwellian. The numerical results in Ref. 12 are for the case of the crossed fields and include scattering by acoustic and polar LO phonons and ionized impurities. A parabolic conduction band was assumed.

The organization of the present paper is as follows. Section II starts with a brief review of the experimental results of Fujisada *et al.*[7] Subsequently these data are compared with the results of different magnetotransport models, of which the validity range of electric and magnetic fields is analyzed in the three parameter space $(\zeta, \eta, \omega_c \tau)$. Our conclusions are presented in Sec. III.

II. ANALYSIS OF THE EXPERIMENTAL DATA OF FUJISADA *et al.*

FKB measured the transverse resistivity as a function of magnetic field and they found the onset of quantum effects for magnetic fields larger than 0.1 T, where $\hbar\omega_c / k_B T$ is of order unity. This value can be derived in the following way. For $B > 0.1$ T the magnetic field dependence of the linear resistivity no longer fits the classical formula $\rho(B) = \rho(0)[1+(\omega_c \tau_0)^2]$ with one single τ_0: quantum effects become important. Also the electric and magnetic field dependence of the Hall mobility were measured by FKB for different samples. They extended the data of Miyazawa and Ikoma[15] to high electric fields and established the magnetic field independence of the mobility at high electric fields. It turned out that the Hall mobility is almost independent of the impurity content (1) for $B > 2$ T and low electric fields and (2) for high electric fields. The latter case is due to the dominance of inelastic scattering processes for high energy electrons. The first case originates from the relatively large spacing between Landau levels, which tends to exclude elastic processes. The magnetic field was slightly tilted in these measurements, but we believe that this will not alter the major trends. The data for the Hall coefficient indicate that the electron density is nearly constant in these measurements, so that it is expected that carrier freezeout and impurity band formation[16] do not play a role here. We choose to analyze the data of the sample that had the largest linear mobility.

The Monte Carlo model, which will be applied below, is identical to the one we used in Ref. 17 for the analysis of the measurements of Alberga *et al.*[18] on InSb at 77 K. It includes scattering by acoustic phonons, polar LO phonons, and ionized impurities for electrons in a nonparabolic conduction band. A value of 30 meV is taken for the acoustic deformation potential[17] and the electron concentration is taken from experiment: 7×10^{13} cm^{-3}. The ionized impurity content is estimated from a fit to the low field Hall mobility: $N_i = 1.2\times10^{14}$ cm^{-3}. These values are subject to discussion, but will not affect the qualitative behavior of the Hall mobility.

Figures 1 and 2 display the electron Hall mobility as a function of applied electric field for n-InSb at 77 K in a fixed magnetic field. In Fig. 1 experimental results of Fujisada *et al.* (FKB) and results from our Monte Carlo calculation are shown for $B = 0.1$ and 1.0 T. Figure 2 displays results for $B = 0.5$ T from experiment and Monte Carlo methods, together with theoretical results from Beleznay and Serényi (BS), for two different impurity contents. The experimental dependence of Hall mobility on electric field for $B > 0.1$ T is characterized by a linear regime for low electric fields, followed by a warm electron

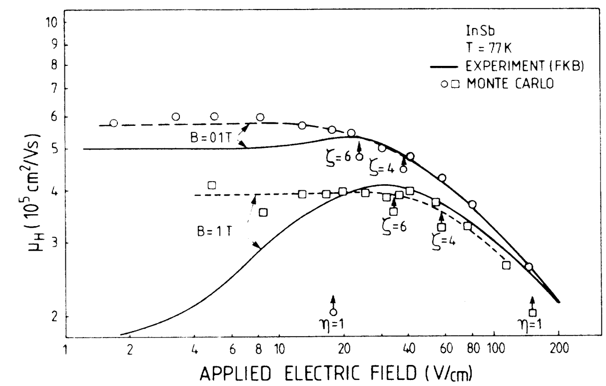

FIG. 1. Electric field dependence of the Hall mobility for InSb at 77 K for different magnetic fields. Results from the experiment of Fujisada *et al.* (solid lines labeled FKB) and from Monte Carlo simulation (the dashed lines labeled MC are a guide to the eye). Characteristic values of the parameters ζ and η are indicated by arrows.

regime where the mobility increases with the electric field; it attains a maximum and subsequently starts decreasing as the hot electron regime sets in. For magnetic fields below 0.1 T the Hall mobility is a monotonically decreasing function of the electric field. Arrows in the figures indicate typical values for the dimensionless field $\zeta = v_{\mathrm{LO}} B / E$ and of the parameter $\eta = \hbar\omega_c / (eEr_0) = 1$ (with ω_c and r_0 the cyclotron frequency and the radius of the first Landau level, respectively). For $B \geq 0.1$ T, ζ is larger than 2 in the whole electric field range displayed in the figure, which implies that streaming motion is forbidden (at least within the model of Komiyama *et al.*[8]). It is apparent that down to electric fields where $\zeta \approx 6$ our Monte Carlo calculation agrees reasonably well with experiment. For lower electric fields where $\zeta > 6$ and $\eta > 1$ the Monte Carlo results do not agree with the experiment, which is due to the fact that magnetic quantum effects are becoming important.

From the Monte Carlo simulation one can extract the collision frequencies ν_i of the different scattering mechanisms. In this way we define $\gamma = \nu_{\mathrm{em}} / \nu_{\mathrm{abs}}$ the ratio of the collision frequencies for emission and absorption of a

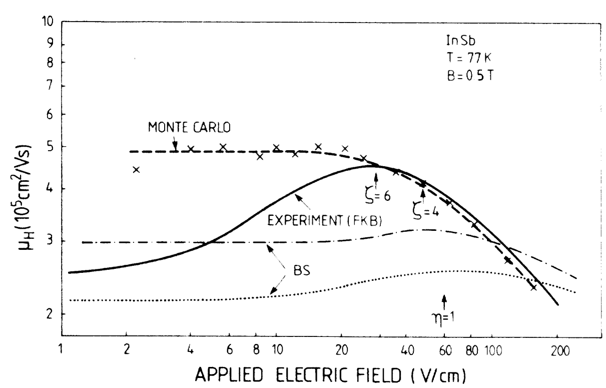

FIG. 2. Electric field dependence of the Hall mobility for InSb at 77 K for a magnetic field of 5 T. Results from FKB and MC simulation (as in Fig. 1) and additionally from the theory of Beleznay *et al.* for two different impurity contents (dash-dotted and dotted lines).

LO phonon. In Fig. 3 this parameter γ is displayed as a function of the applied electric field. The electric field dependence of γ is almost the same for the three different magnetic fields. At lower electric fields γ very slowly deviates from 1. At high electric fields γ increases steeply with the field. The electric field where γ starts to deviate appreciably from unity ($\gamma \approx 1.3$) coincides with the maximum in the experimental Hall mobility and with $\zeta \approx 6$. That is where LO-phonon emission (streaming motion) starts to become important. Since MC simulation is not valid at lower electric fields (or higher ζ) magnetic quantum effects are getting important and thus it is no surprise that our MC simulation deviates from the experimental results.

The experiments of Fujisada *et al.* were analyzed by BS. The magnetophonon oscillations in the linear regime and the maximum in the Hall mobility are described qualitatively by the BS theory. However the agreement with experiment was poor for the nonlinear regime and the BS theory does not show the correct behavior at high electric fields. Moreover the theoretical maximum is exclusively caused by the contribution of impurity scattering, contrary to the experimental results, which do not show any appreciable dependence on the impurity content at these magnetic fields. The failure of the BS theory in the nonlinear regime can be attributed to the joint effect of three approximations: (i) the drifted Maxwell ansatz for the quasimomentum distribution function,[4] (ii) the momentum balance equation is solved without coupling to the energy balance: the electron temperature always equals the lattice temperature and as a consequence this model does not contain any effect of heating, and (iii) the neglect of nonparabolicity of the conduction band.[17]

A measure for the electron mean free time between collisions τ can be obtained from the Monte Carlo method[19] as the total simulation time divided by the number of real interactions (not the self-scattering). $\omega_c\tau$ gives the average number of cyclotron cycles performed by an electron between two collisions. As is shown in Fig. 4, the Monte Carlo estimation for $\omega_c\tau$ at $B=0.5$ T is slightly larger than 1 over the whole range of electric fields: it varies between 1.1 and 1.5. For $B=0.1$ T, $0.25<\omega_c\tau<0.35$, while for $B=1$ T, $2.5<\omega_c\tau<3.0$. $\omega_c\tau$ is almost constant here, because (i) the magnetic field (thus also ω_c) is kept constant and (ii) τ is approximately constant, since for $\zeta \gg 1$ the contribution of electron LO-phonon scattering is very small. The slight increase of $\omega_c\tau$ with E_a is due to the diminishing contribution of the elastic scattering processes as the electric field and the average electron energy increase. The Monte Carlo estimation for the mean free time τ is much smaller than the relaxation time τ_μ obtained from a fit to the linear mobility at zero magnetic field ($\mu = e\tau_\mu/m^*$). This difference originates from the fact that τ and τ_μ are different quantities with a different definition. In the Monte Carlo estimation for τ every collision is treated on the same footing, while the relaxation time involves a weighted average over **k** space with the wave-vector distribution function as a weight.

Application of Eq. (15) of Mori *et al.*[7] to InSb for $B=1$ T and for $0<E_a<30$ V/cm and for realistic values of the Landau-level broadening parameter ($\Gamma=2$ meV) leads to a $\Delta\sigma_{xx}/\sigma_0$ of 20%, while the experimental data show a $\Delta\mu_H/\mu_H$ of 200%. This discrepancy reflects the unimportance of the magnetophonon effect in the measurements of Fujisada *et al.* Moreover the $1/\omega_c\tau$ expansion approach of Kubo *et al.* cannot be applied here because $\omega_c\tau$ is of order unity.

III. DISCUSSION AND CONCLUSION

In conclusion one can state that up to now there are no theoretical results which can quantitatively describe the field dependence of electron mobility (or resistivity) in the whole range of electric and magnetic fields where η and $\omega_c\tau$ vary from larger than 1 to smaller than 1. The Monte Carlo method and the Boltzmann equation are invalid for $\eta \gg 1$. The BS theory is valid only for $\eta \gg 1$ and the generalized Kubo formalism requires $\omega_c\tau \gg 1$. In the experiment of Fujisada *et al.* one encounters a transition from a quantum regime for low electric fields

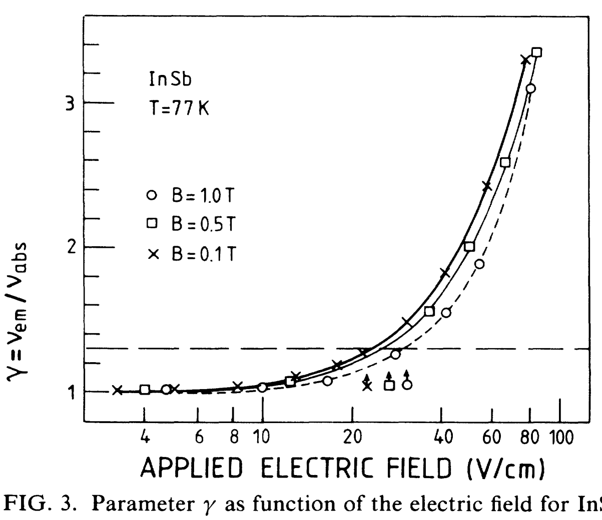

FIG. 3. Parameter γ as function of the electric field for InSb at 77 K as obtained from Monte Carlo simulation for different values of the magnetic field. γ is defined as the ratio of the collision frequency for LO-phonon emission to the collision frequency for LO-phonon absorption. The fields for which the straight line with $\gamma=1.3$ crosses the curves $\gamma(E_a)$ coincide with the position of the maxima in the experimental Hall mobility (indicated by arrows).

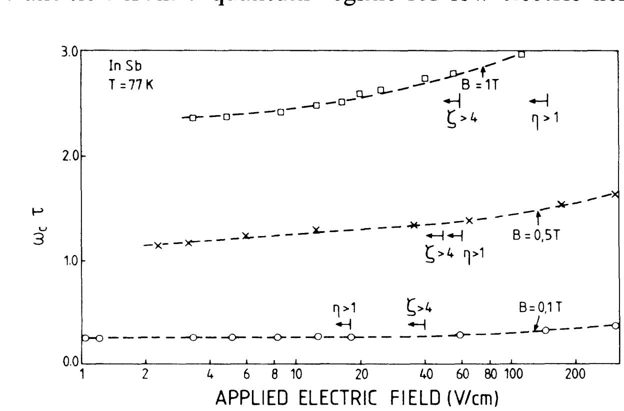

FIG. 4. Dimensionless parameter $\omega_c\tau$ (see text for definition) as function of the electric field InSb at 77 K as obtained from Monte Carlo simulation for different values of the magnetic field.

and high magnetic fields to a semiclassical regime for high electric fields.

(i) $\hbar\omega_c \approx k_B T$ for the considered magnetic field range.

(ii) For $\omega_c \tau \approx 1$ expansions to $\omega_c \tau$, or to $1/\omega_c \tau$, are clearly not applicable. One is in the intermediate regime, the regime between classical and quantum transport.

(iii) For $\eta < 1$ the Landau-level overlap, induced by the electric field, is considerable and the electron can be described by a plane wave, while for $\eta > 1$ the Landau-level separation is larger than eEr_0 and the plane-wave approximation should break down.

In the regime characterized by $\omega_c \tau \approx 1$, $\eta \approx 1$, and $\hbar\omega_c \approx k_B T$, the four typical energies $\hbar\omega_{\mathrm{LO}}$, $\hbar\omega_c$, eEr_0, and $k_B T$ are of the same order of magnitude. Consequently a larger number of Landau levels contribute to the conductivity and the calculation of the transport parameters forms a nontrivial numerical problem which is unsolved up to now. Whereas MC describes the correct high electric field behavior, but not the linear quantum regime, the theory of Beleznay and Serényi is qualitatively correct in the linear quantum regime, but does not correctly model the nonlinear regime.

In a previous paper we analyzed hot electron magnetotransport in AgCl and AgBr at $T = 4$ K. The main differences with the case of InSb considered here are (i) the parabolicity of the conduction band, (ii) the relatively large electron-LO-phonon coupling constant, which makes the LO-phonon scattering rate an order of magnitude larger than in InSb, and (iii) the lower temperature. As a consequence the transition from the quantum regime to the semiclassical regime in AgCl and AgBr is rather abrupt: around $\zeta = v_{\mathrm{LO}} B/E = 1$, $\omega_c \tau$ changes rapidly from $\omega_c \tau \ll 1$ to $\omega_c \tau \gg 1$. For electric and magnetic fields such that $\eta > 1$ the Hall mobility is an increasing function of the electric field with a maximum at $\eta \approx 1$. From the data on InSb it appears that at $\zeta \approx 1$ there is no step in $\omega_c \tau$ and that the maximum in the electric field dependence is not located at $\eta \approx 1$. From our Monte Carlo simulation we find that the maximum coincides with the onset of streaming motion. We suggest that this is caused by the strong nonparabolicity of the conduction band and by the very small electron-LO-phonon coupling constant in InSb. However, up to now there is no theory that can explain in detail the increase of the Hall mobility with increasing electric field and the location of the subsequent maximum. Equivalently a quantitative model for the field dependence of the warm electron coefficient is lacking: the experimental values are an order of magnitude larger than the theoretical estimates (see, e.g., Refs. 12, 20, and 21).

Kazarino and Skobov[5] have analyzed the dependence of the resistivity on the electric current density for the two limiting cases $k_B T_e \gg \hbar\omega_c$ and $k_B T \ll k_B T_e \ll \hbar\omega_c$ for a semiconductor system where the electrons interact with acoustic phonons and neutral impurities. They found that the electron temperature T_e depends on the electric and magnetic fields and on an energy average of the ratio of collision frequencies of electrons with acoustic phonons and impurities. Their results suggest that both heating and quantization are important and that all relevant scattering mechanisms should be taken into account accurately in order to obtain a quantitative description of the field dependence of the Hall mobility in the warm electron regime. One possibility is to extend the BS model by including the strong nonparabolicity of the conduction band of InSb and by coupling their momentum balance to the energy balance, thereby introducing an electron temperature. Another possibility is to generalize the Boltzmann equation to include a quantizing magnetic field.

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*Also at University of Antwerp (Rijksuniversitair Centrum Antwerpen) and Eindhoven University of Technology, NL-5600 MB Eindhoven, Belgium.

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