Polaritons in an *n-i-p-i* semiconductor superlattice: Bulk and surface modes

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We consider the propagation of bulk and surface polaritons of a superlattice consisting of *n*- and *p*-doped semiconductors separated by an intrinsic semiconductor (*n-i-p-i* structure). Using a transfer-matrix method we obtain the bulk modes in an infinite superlattice and the surface modes in a semi-infinite superlattice. The effects of the charge of carriers at the interfaces of the doped semiconductors and the thickness of the layers are analyzed. Our results can be specialized in order to obtain the bulk and surface polaritons of a two-component (binary) dielectric superlattice.

I. INTRODUCTION

In recent years considerable effort has been devoted to studying systems composed of alternating layers of different materials, the so-called superlattices.[1,2] Collective excitations that can propagate in these superlattices, such as phonons,[3] magnons,[4] and polaritons,[5] have been studied and it was observed that their properties depend not only on the materials used as the constituents of the superlattices, but also on the ratio of the thickness of the alternating materials.[6]

Particularly the existence of electromagnetic collective excitations on a superlattice, such as bulk and surface polaritons, can be understood in the following way. An excitation of a polariton within a material layer produces electric fields that extend outside its boundaries, and these fields can couple with elementary excitations of the other layers. Through the use of Bloch's theorem, one sees that this coupling creates a set of collective excitations of the entire superlattice. This collective mode is characterized by a wave normal to the interfaces and can transmit energy normal to the layers of the superlattice structure. Taking Q as the wave vector normal to the interfaces and L as the unit-cell length of the superlattice, the bulk polariton can propagate when $0 \leq QL \leq \pi$. Surface-polariton modes exist in the regions above, below, and in between the bulk bands. Among other important parameters in the dispersion relation of bulk and surface polaritons on superlattices, the existence of two-dimensional electron-gas layers at the interfaces of the different materials on the superlattice plays an important role. Recently Constantinou and Cottam[7] obtained a dispersion relation of bulk and surface polaritons on a superlattice composed by a doped semiconductor and an intrinsic semiconductor. In this case, the superlattice is composed of electron-gas layers separated alternately by two media with different thicknesses and dielectric constants.

The *n-i-p-i* crystal is a different superlattice composed of doped semiconductors that presents many interesting properties. This system, which is formed by a periodic array of *n*- and *p*-doped semiconductor layers, separated by an intrinsic semiconductor, was proposed by Döhler[8,9] some years ago. A certain fraction of the donors and acceptors will be ionized, producing doped layers with positive charge in each n layer and negative charge in each p layer.

In this paper we obtain the dispersion relation for bulk and surface polaritons on an *n-i-p-i* semiconductor superlattice structure by using the transfer-matrix technique. The results of Camley[6] and Constantinou and Cottam[7] are obtained as particular cases of our results.

The remainder of this paper is organized as follows. In Sec. II we present the general theory for both bulk and surface polaritons propagating on an *n-i-p-i* superlattice. In Sec. III we present special cases that can be obtained from our results. Section IV is devoted to a discussion of numerical results and a summary of our principal conclusions.

II. GENERAL THEORY

A. Bulk modes

The *n-i-p-i* structure that we consider in the present paper is shown in Fig. 1. Materials A and C are *n*- and *p*-doped semiconductors with dielectric constants $\epsilon_a(\omega)$ and $\epsilon_c(\omega)$, with thickness a and c, respectively. Materials B and D are intrinsic semiconductors with frequency-independent dielectric constant ϵ_b and ϵ_d, and thickness b and d, respectively. The unit-cell length is $L = a + b + c + d$, and is designated by the index n, as illustrated in Fig. 1. In the nth unit cell, at the interface defined by $z = nL$ and $z = nL + a$, we have a two-dimensional hole gas (2D HG), and at $z = nL + a + b$ and $z = nL + a + b + c$ we have a two-dimensional electron gas (2D EG), respectively.

Considering an isolated slab, we assume a p polarization for the electromagnetic mode, characterized by a wave vector $\mathbf{k}$ parallel to the xy plane. Since we assume

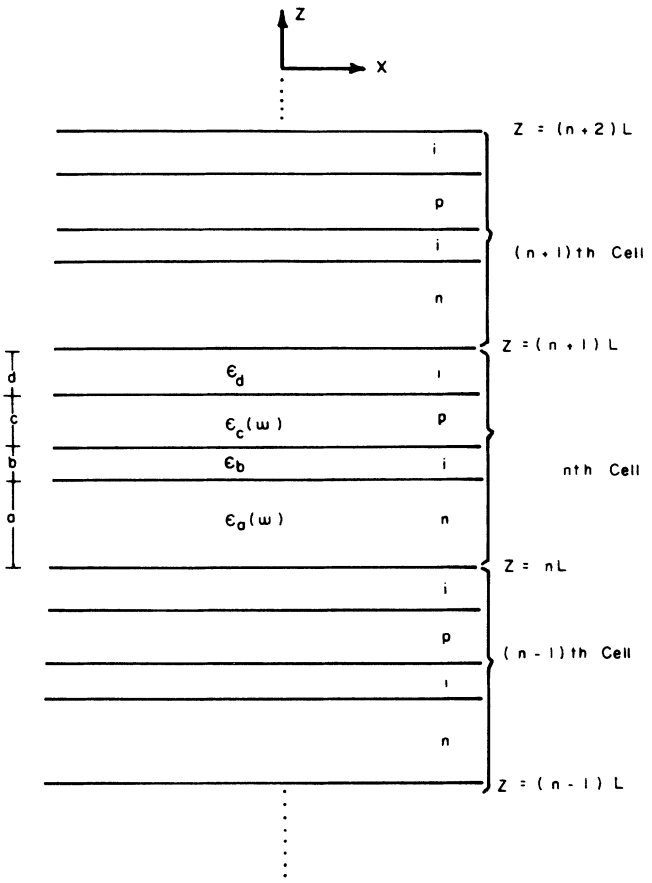

FIG. 1. A schematic illustration of an *n-i-p-i* superlattice of period L. The n and p layers contain 2D HG and 2D EG gas layers, respectively, separated by media C and D of different thickness and dielectric constants. The unit cells of the structure are indexed by n, as illustrated.

isotropic materials we can take $\mathbf{k}$ parallel to the x axis without loss of generality. Thus the electric and magnetic fields can be written as

$$\mathbf{E}^{(n)}(\mathbf{x},t)=(E_x^{(n)}(z \mid k\omega),0,E_z^{(n)}(z \mid k\omega))e^{i(k_x x-\omega t)} , \tag{2.1}$$

and

$$\mathbf{H}^{(n)}(\mathbf{x},t)=(0,H_y^{(n)}(z \mid k\omega),0)e^{i(k_x x-\omega t)} . \tag{2.2}$$

Inside each layer these fields satisfy Maxwell's equations

$$\nabla\times\nabla\times\mathbf{E}^{(n)}(\mathbf{x},t)=-\varepsilon_0\epsilon_j(\omega)\frac{\partial^2}{\partial t^2}\mathbf{E}^{(n)}(\mathbf{x},t) , \tag{2.3}$$

and

$$\nabla\times\mathbf{H}^{(n)}(\mathbf{x},t)=\varepsilon_0\epsilon_j(\omega)\frac{\partial}{\partial t}\mathbf{E}^{(n)}(\mathbf{x},t) , \tag{2.4}$$

where ε_0 is the vacuum permittivity, ϵ_j is the dielectric constant, and $j=a$, b, c, or d.

From Eqs. (2.1)–(2.4), we have that the x component of the electric field and the y component of the magnetic field in each layer of the nth cell is given by

$$E_{xj}^{(n)}(z \mid k\omega)=A_{1j}^{(n)}e^{-\alpha_j z}+A_{2j}^{(n)}e^{\alpha_j z} , \tag{2.5a}$$

and

$$H_{yj}^{(n)}(z \mid k\omega)=-i\frac{\omega\varepsilon_0\epsilon_j}{\alpha_j}(A_{1j}^{(n)}e^{-\alpha_j z}-A_{2j}^{(n)}e^{\alpha_j z}) , \tag{2.5b}$$

where $\epsilon_j=\epsilon_j(\omega)$ and

$$\alpha_j=\begin{cases}(k_x^2-\epsilon_j\omega^2/c^2)^{1/2}, & k_x>\epsilon_j\omega/c\\ i(\epsilon_j\omega^2/c^2-k_x^2)^{1/2}, & k_x<\epsilon_j\omega/c\end{cases} . \tag{2.6}$$

The boundary conditions for the electromagnetic field at the interfaces are that the component $E_{xj}^{(n)}(z \mid k\omega)$ is continuous across an interface and the magnetic field $H_{yj}^{(n)}$ is discontinuous across an interface, due to the presence of a current density at the interface, given by

$$J_{xy}^{(n)}(z \mid k\omega)=i\frac{n_p e^2}{m_p^*\omega}E_{xj}^{(n)}(z \mid k\omega) , \tag{2.7}$$

where the subscript p reads e for electrons or h for holes, n_p is the carrier concentration per unit area, and m_p^* is the effective mass of the charge carriers. Therefore, using the boundary conditions for the electric and magnetic fields given by Eq. (2.5) at the interfaces $z=nL+a$, $nL+a+b$, $nL+a+b+c$, and $z=(n+1)L$, we obtain the following equations:

$$A_{1a}^{(n)}f_a+A_{2a}^{(n)}\bar{f}_a=A_{1b}^{(n)}+A_{2b}^{(n)} , \tag{2.8a}$$

$$\epsilon_a'(A_{1a}^{(n)}f_a-A_{2a}^{(n)}\bar{f}_a)=(\epsilon_b'+\sigma_h)A_{1b}^{(n)}-(\epsilon_b'+\sigma_h)A_{2b}^{(n)} , \tag{2.8b}$$

$$A_{1b}^{(n)}f_b+A_{2b}^{(n)}\bar{f}_b=A_{1c}^{(n)}+A_{2c}^{(n)} , \tag{2.9a}$$

$$\epsilon_b'(A_{1b}^{(n)}f_b-A_{2b}^{(n)}\bar{f}_b)=(\epsilon_c'-\sigma_e)A_{1c}^{(n)}-(\epsilon_c'+\sigma_e)A_{2c}^{(n)} , \tag{2.9b}$$

$$A_{1c}^{(n)}f_c+A_{2c}^{(n)}\bar{f}_c=A_{1d}^{(n)}+A_{2d}^{(n)} , \tag{2.10a}$$

$$\epsilon_c'(A_{1c}^{(n)}f_c-A_{2c}^{(n)}\bar{f}_c)=(\epsilon_d'-\sigma_e)A_{1d}^{(n)}=(\epsilon_d'+\sigma_e)A_{2d}^{(n)} , \tag{2.10b}$$

$$A_{1d}^{(n)}f_d+A_{2d}^{(n)}\bar{f}_d=A_{1a}^{(n+1)}+A_{2a}^{(n+1)} , \tag{2.11a}$$

$$\epsilon_d'(A_{1d}^{(n)}f_d-A_{2d}^{(n)}\bar{f}_d)=(\epsilon_a'-\sigma_h)A_{1a}^{(n+1)}-(\epsilon_a'+\sigma_h)A_{2a}^{(n+1)} . \tag{2.11b}$$

In Eqs. (2.8)–(2.11) we have redefined $A_{ij}^{(n)}$ and $A_{2j}^{(n)}$ as

$$A_{ma}^{(n)}=A_{ma}^{(n)}\exp[(-1)^m\alpha_a nL] , \tag{2.12a}$$

$$A_{mb}^{(n)}=A_{mb}^{(n)}\exp[(-1)^m\alpha_b(nL+a)] , \tag{2.12b}$$

$$A_{mc}^{(n)}=A_{mc}^{(n)}\exp[(-1)^m\alpha_c(nL+a+b)] , \tag{2.12c}$$

$$A_{md}^{(n)}=A_{md}^{(n)}\exp[(-1)^m\alpha_d(nL+a+b+c)] , \tag{2.12d}$$

where $m=1,2$, and

$$\epsilon_j'=\epsilon_j/\alpha_j, \quad j=a,b,c,d \tag{2.13}$$

$$\sigma_p = \frac{n_p e^2}{m_p^* \omega^2 \epsilon_0}, \quad p = e,h \ . \tag{2.14}$$

The functions f_j are defined by

$$f_j = e^{-\alpha_j j} \tag{2.15}$$

and

$$\bar{f}_j = f_j^{-1} \ . \tag{2.16}$$

Defining for each medium, the two-column vector

$$| A_j^{(n)} \rangle = \begin{bmatrix} A_{1j}^{(n)} \\ A_{2j}^{(n)} \end{bmatrix} , \tag{2.17}$$

Eqs. (2.8)–(2.11) can be written in a matrix form as

$$\begin{aligned} M_a | A_a^{(n)} \rangle &= N_b | A_b^{(n)} \rangle \ , \\ M_b | A_b^{(n)} \rangle &= N_c | A_c^{(n)} \rangle \ , \\ M_c | A_c^{(n)} \rangle &= N_d | A_d^{(n)} \rangle \ , \\ M_d | A_d^{(n)} \rangle &= N_a | A_a^{(n+1)} \rangle \ , \end{aligned} \tag{2.18}$$

where we have defined the matrices

$$\overleftrightarrow{M}_j = \begin{bmatrix} f_j & \bar{f}_j \\ \epsilon_j' f_j & -\epsilon_j' \bar{f}_j \end{bmatrix} , \tag{2.19}$$

and

$$\overleftrightarrow{N}_j = \begin{bmatrix} 1 & 1 \\ \epsilon_j' - \sigma_p & -\epsilon_j' - \sigma_P \end{bmatrix} , \tag{2.20}$$

with $p = h$ for $j = a,b$ and $p = e$ for $j = c,d$.

From Eqs. (2.18)–(2.20) it is easy to see that

$$| A_j^{(n+1)} \rangle = \overleftrightarrow{T} | A_j^{(n)} \rangle \ , \tag{2.21}$$

where the matrix $\overleftrightarrow{T}$ is given by

$$\overleftrightarrow{T} = \overleftrightarrow{N}_a^{-1} \overleftrightarrow{M}_d \overleftrightarrow{N}_d^{-1} \overleftrightarrow{M}_c \overleftrightarrow{N}_c^{-1} \overleftrightarrow{M}_b \overleftrightarrow{N}_b^{-1} \overleftrightarrow{M}_a \ . \tag{2.22}$$

The matrix $\overleftrightarrow{T}$ in Eq. (2.22) is a transfer matrix because it relates the coefficients of the electric field in one cell to those in the preceding cell. Taking into account the translational symmetry of the problem, we can use Bloch's ansatz, that is

$$| A_j^{(n+1)} \rangle = e^{iQL} | A_j^{(n)} \rangle \ . \tag{2.23}$$

Using Eqs. (2.21) and (2.23), we have that

$$\overleftrightarrow{T} | A_j^{(n)} \rangle = e^{iQL} | A_j^{(n)} \rangle \ , \tag{2.24a}$$

$$\overleftrightarrow{T}^{-1} | A_j^{(n)} \rangle = e^{-iQL} | A_j^{(n)} \rangle \ , \tag{2.24b}$$

and consequently

$$[\cos(QL)\overleftrightarrow{I} - \tfrac{1}{2}(\overleftrightarrow{T} + \overleftrightarrow{T}^{-1})] | A_j^{(n)} \rangle = 0 \ . \tag{2.25}$$

Since $| A_j^{(n)} \rangle$ is a general vector of the structure considered, the dispersion relation of the bulk polaritons on the *n-i-p-i* superlattice will be given by

$$\cos(QL)\overleftrightarrow{I} = \tfrac{1}{2}(\overleftrightarrow{T} + \overleftrightarrow{T}^{-1}) \ . \tag{2.26}$$

From the definition of the transfer matrix in Eq. (2.22) and using Eqs. (2.19) and (2.20) we can show that $\det\overleftrightarrow{T} = 1$; therefore

$$\overleftrightarrow{T}^{-1} = \begin{bmatrix} T_{22} & -T_{12} \\ -T_{21} & T_{11} \end{bmatrix} , \tag{2.27}$$

and hence, from Eqs. (2.26) and (2.27) our dispersion relation for the bulk modes is simply given by

$$\cos(QL) = \tfrac{1}{2}\mathrm{Tr}\overleftrightarrow{T} \ . \tag{2.28}$$

B. Surface modes

In order to study the surface modes on an *n-i-p-i* superlattice we consider the geometry illustrated in Fig. 2. In this case our structure is terminated at the plane $z=0$, with the half-space $z<0$ filled with a material that has a frequency-independent dielectric constant ϵ_s. Then the periodicity in the z direction is destroyed and we can no longer assume Bloch's ansatz as in Eq. (2.23). Therefore, we have to consider electromagnetic modes that have their excitations localized in the near vicinity of the interface between the material with dielectric constant ϵ_s and the superlattice. For these modes, instead of Eq. (2.23)

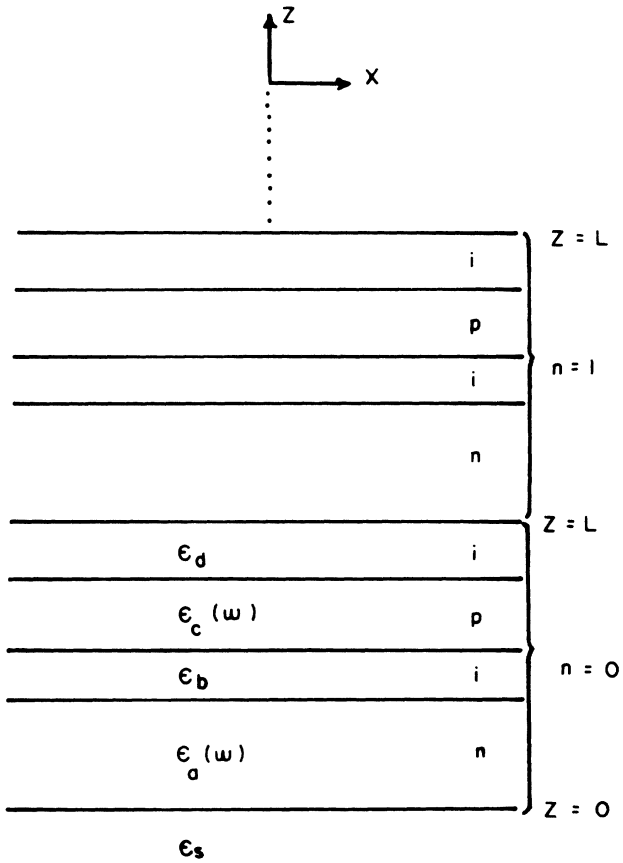

FIG. 2. Semi-infinite *n-i-p-i* superlattice. The structure presents the same periodicity as in Fig. 1, but is terminated with the half-space $z<0$ filled with material of dielectric constant ϵ_s.

we consider that

$$| A_j^{(n+1)}\rangle = e^{-\beta L} | A_j^{(n)}\rangle \ , \tag{2.29}$$

with $\mathrm{Re}(\beta)>0$, in order to obtain a localized mode. Therefore, Eq. (2.28) still holds, provided we replace Q by $i\beta$, i.e.,

$$\cosh(\beta L)=\tfrac{1}{2}\mathrm{Tr}\overleftrightarrow{\mathrm{T}} \ . \tag{2.30}$$

Since we now have to consider a boundary condition at the plane $z=0$, this implies a further constraint in Eq. (2.30). In fact, the solution of Eq. (2.2) for the x component of the electric field and the y component of the magnetic field, in the region $z<0$, is given by

$$E_x^{(s)}(z \mid k\omega)=E^{(s)}(k \mid \omega)e^{\alpha_s z} \ , \tag{2.31a}$$

and

$$H_y^{(s)}(z \mid k\omega)=\frac{i\omega\varepsilon_0\epsilon_s}{\alpha_s}E^{(s)}(k \mid \omega)e^{\alpha_s z} \ , \tag{2.31b}$$

where

$$\alpha_s=\left[k_x^2-\epsilon_s\frac{\omega^2}{c^2}\right]^{1/2} , \quad k_x>\epsilon_s\frac{\omega}{c} \ . \tag{2.32}$$

At the interface $z=0$, the x component of the electric field and particularly the y component of the magnetic field are continuous. Consequently, using Eqs. (2.5) and (2.31) we have that

$$E^{(s)}(k \mid \omega)=A_{1a}^{(0)}+A_{2a}^{(0)} \tag{2.33}$$

and

$$\epsilon_s' E^{(s)}(k \mid \omega)=-\epsilon_a'(A_{1a}^{(0)}-A_{2a}^{(0)}) \ , \tag{2.34}$$

respectively, where we have defined $\epsilon_s'=\epsilon_s/\alpha_s$.

Since the surface mode can be associated to the bulk mode, provided we replace Q by $i\beta$, Eq. (2.24a) can be written for the vector $| A_a^{(0)}\rangle$ in the form

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} A_{1a}^{(0)} \\ A_{2a}^{(0)} \end{bmatrix}=e^{-\beta L}\begin{bmatrix} A_{1a}^{(0)} \\ A_{2a}^{(0)} \end{bmatrix} \ . \tag{2.35}$$

From Eqs. (2.33)–(2.35) the coefficients $E^{(s)}(k \mid \omega)$, $A_{1a}^{(0)}$, and $A_{2a}^{(0)}$ can be eliminated, and then we obtain

$$(T_{11}+T_{12}\lambda)\lambda=T_{21}+T_{22}\lambda \ , \tag{2.36}$$

with

$$\lambda=\frac{\epsilon_a'+\epsilon_s'}{\epsilon_a'-\epsilon_s'} \ . \tag{2.37}$$

Equation (2.36) represents an implicit dispersion relation for the surface polaritons on the *n-i-p-i* structure. In fact, once Eq. (2.36) is solved we must obtain a value of β which satisfies Eq. (2.30), provided that $\mathrm{Re}(\beta)>0$.

III. SPECIAL CASES

A. Superlattice of alternating layered electron gases

The dispersion relation of polaritons in a superlattice of alternating layered electron gases calculated by Constantinou and Cottam[7] can be obtained from our results as a particular case. For this we have to consider in our calculations, $a=c$, $b=d$, and $\sigma_h=\sigma_e$. Therefore the periodicity of the superlattice is $L=a+b$, and to obtain the transfer matrix it is necessary to consider only the boundary conditions given by Eqs. (2.8) and (2.9). Thus the transfer matrix will be given by

$$\overleftrightarrow{\mathrm{T}}_1=\overleftrightarrow{\mathrm{N}}_a^{-1}\overleftrightarrow{\mathrm{M}}_b\overleftrightarrow{\mathrm{N}}_b^{-1}\overleftrightarrow{\mathrm{M}}_a \ , \tag{3.1}$$

where $\overleftrightarrow{\mathrm{M}}_{a,b}$ and $\overleftrightarrow{\mathrm{N}}_{a,b}$ have been defined in Eqs. (2.19) and (2.20), respectively. With Eqs. (2.28) and (3.1) the dispersion relation of bulk polaritons in this superlattice is given by

$$\cos[Q(a+b)]=\tfrac{1}{2}\mathrm{Tr}\overleftrightarrow{\mathrm{T}}_1 \ . \tag{3.2}$$

Using the definitions of the matrices $\overleftrightarrow{\mathrm{M}}_{a,b}$, Eq. (2.19), and $\overleftrightarrow{\mathrm{N}}_{a,b}$, Eq. (2.20), the trace of $\overleftrightarrow{\mathrm{T}}_1$ is equal to

$$\mathrm{Tr}\overleftrightarrow{\mathrm{T}}_1=\frac{\alpha_a\alpha_b}{4\epsilon_a\epsilon_b}[\sigma_\epsilon^2(f_af_b-f_a\bar{f}_b-\bar{f}_af_b+\bar{f}_a\bar{f}_b)+2\sigma_e\epsilon_{ab}^{+}(f_af_b-\bar{f}_a\bar{f}_b)$$
$$+2\sigma_e\epsilon_{ab}^{-}(\bar{f}_af_b-f_a\bar{f}_b)+(\epsilon_{ab}^{+})^2(f_af_b+\bar{f}_a\bar{f}_b)-(\epsilon_{ab}^{-})^2(f_a\bar{f}_b+\bar{f}_af_b)] \ , \tag{3.3}$$

with $\epsilon_{ab}^{\pm}=\epsilon_a'\pm\epsilon_b'$.

Substituting Eqs. (2.13), (2.15), and (2.16) in Eq. (3.3), the dispersion relation in Eq. (3.2) can be written in the form

$$\cos[Q(a+b)]=\frac{\alpha_a\alpha_b}{2\epsilon_a\epsilon_b}\left[\left[\sigma_e^2+\frac{\epsilon_a^2}{\alpha_a^2}+\frac{\epsilon_b^2}{\alpha_b^2}\right]\sinh(\alpha_a a)\sinh(\alpha_b b)\right.$$
$$\left.-2\sigma_e\left[\frac{\epsilon a}{\alpha_a}\cosh(\alpha_a a)\sinh(\alpha_b b)+\frac{\epsilon_b}{\alpha_b}\sinh(\alpha_a a)\cosh(\alpha_b b)\right]+\frac{2\epsilon_a\epsilon_b}{\alpha_a\sigma_b}\cosh(\alpha_a a)\cosh(\alpha_b b)\right] \ . \tag{3.4}$$

Particularly, if we consider the limit where retardation effects can be neglected, we have that

$$\alpha_a=\alpha_b=k_x \ , \tag{3.5}$$

and consequently Eq. (3.4) is simplified and we obtain

$$\left[\frac{\omega}{\Omega}\right]^2=\frac{K_x\{\cosh[(1+s)K_x]-\cosh[(1-s)K_x]\}}{(r+1)\sinh[(1+s)K_x]-(r-1)\sinh[(1-s)K_x]\pm F^{1/2}}\ , \tag{3.6}$$

with

$$F=2(r^2+1)\{\cosh[(1+s)K_x]\cosh[(1-s)K_x]-1\}-2(r^2-1)\sinh[(1+s)K_x]\sinh[(1-s)K_x]$$
$$+4r\cosh[(1+s)Q_z]\{\cosh[(1+s)K_x]\cosh[(1-s)K_x]\}\ , \tag{3.7}$$

where we have introduced in Eqs. (3.6) and (3.7) the same notation used by Constatinou and Cottam,[7] that is, $K_x=k_xa$, $Q_z=Qa$, $r=\epsilon_a/\epsilon_b$, $S=b/a$, and $\Omega=(n_e e^2/am_e^*\varepsilon_0\epsilon_b)^{1/2}$.

B. Plasmons in superlattices

Another special case of our results is the dispersion relation of bulk plasmons obtained by Camley and Mills.[6] In this case the superlattice is constituted by two dielectric materials A and B with dielectric constants $\epsilon_a(\omega)$ and ϵ_b (frequency independent), respectively, on which the retardation effect is ignored. The dispersion relation of bulk plasmons in this superlattice can be obtained from Eq. (3.4), by considering $\alpha_a=\alpha_b=k$ and $\sigma_e=0$. Thus, using these restrictions in Eq. (3.4) we obtain

$$\cos[Q(a+b)]=\frac{1}{2\epsilon_a\epsilon_b}[(\epsilon_a^2+\epsilon_b^2)\sinh(ka)\sinh(kb)$$
$$+2\epsilon_a\epsilon_b\cosh(ka)\cosh(kb)]\ , \tag{3.8}$$

which corresponds to the result obtained by Camley and Mills.[6]

A different way to obtain Eq. (3.8) is to consider in the *n-i-p-i* superlattice σ_e, $\sigma_h=0$, $a=c$, and $b=d$. Thus, the transfer matrix in Eq. (2.22) is given by

$$\overleftrightarrow{T}=(\overleftrightarrow{T}_1)^2\ , \tag{3.9}$$

with $\overleftrightarrow{T}_1$ given by Eq. (3.1). Therefore Eq. (2.28) can be written as

$$\cos[2Q(a+b)]=\tfrac{1}{2}\mathrm{Tr}(\overleftrightarrow{T}_1^2)\ . \tag{3.10}$$

Using the matrices $\overleftrightarrow{M}_{a,b}$ and $\overleftrightarrow{N}_{a,b}$ defined in Eqs. (2.19) and (2.20), with $\sigma_e=\sigma_h=0$, we can show that Eq. (3.10) is equivalent to

$$\cos^2[Q(a+b)]=\tfrac{1}{4}(\mathrm{Tr}\overleftrightarrow{T}_1)^2\ , \tag{3.11}$$

or

$$\pm\cos[Q(a+b)]=\tfrac{1}{2}\mathrm{Tr}\overleftrightarrow{T}_1\ . \tag{3.12}$$

The two roots in Eq. (3.12) mean that we need to consider only $0\le Q<\pi/2$, instead of $0\le Q\le\pi$ as used in Eq. (3.8).

IV. RESULTS AND DISCUSSION

In order to obtain numerical results we consider the dielectric materials A and C as Si doped with n and p impurities. Since we do not use highly-doped semiconductors, we assume that $\epsilon_a(\omega)=\epsilon_c(\omega)$, and the dielectric constant of the Si can be taken as[5]

$$\epsilon(\omega)=\epsilon_L(1-\omega^2/\omega_p^2)\ . \tag{4.1}$$

where $\epsilon_L=11.7$ is the background dielectric constant of the material, and $\omega_p=7.65\times10^{13}$ s^{-1} is the electronic plasma frequency and we consider $\epsilon(\omega)$ independent of the impurity density. The effective mass of the electrons and holes are related to the electron mass m_0 by $m_e^*=0.2m_0$ and $m_h^*=0.4m_0$, respectively.[10] We also assumed that the dielectrics b and d consist of SiO_2 with dielectric constant $\epsilon_b=\epsilon_d=3.7$.

In Figs. 3(a) and 3(b) we plot the dispersion relation for bulk polaritons by considering the semiconductor layers (n and p) with 400 Å of thickness and the insulators with 200 Å of thickness. We also consider that the density of electrons and holes at the interfaces are equal to 2×10^{16} carriers/m^2. We observed the existence of four bands separated by gaps that tend to crowd together when k increases.

In Figs. 4(a) and 4(b) we present the dispersion curve for bulk polaritons on which the layers have the same thickness as shown in Figs. 3(a) and 3(b), but with a density of carriers (electrons and holes) at the interfaces equal to 4×10^{16} carriers/m^2. Comparing Figs. 3 and 4 we see that the increase in the density of charges at the interfaces changes the values of frequency on which the modes tends to crowd together. We also observe for the two upper bands, Figs. 3(b) and 4(b), that the larger density of charges corresponds to larger bulk bands for small values of k. The effects observed for the bulk bands with large density of charges at the interfaces can also be found in systems on which the layers have a small thickness.

In order to analyze the dispersion relation of surface polaritons on the *n-i-p-i* structure, we consider in Figs. 5(a) and 5(b) a superlattice with all layers with the same thickness and equal to 400 Å. We also assume that the density of charges (n and p) at the interfaces are the same absolute value and equal to 2×10^{16} carriers/m^2.

In Fig. 5(a) we have one surface mode localized below the lower bulk band, corresponding to βL purely real and positive, and another surface mode inside the gap between the bulk bands, which is associated with $\beta L=i\pi+\chi_1$, where χ_1 is positive. In Fig. 5(b) we see two surface modes, above and below the bulk bands, corresponding to βL purely real. The other mode between the gap of the bulk bands corresponding to $\beta L=i\pi+\chi_2$,

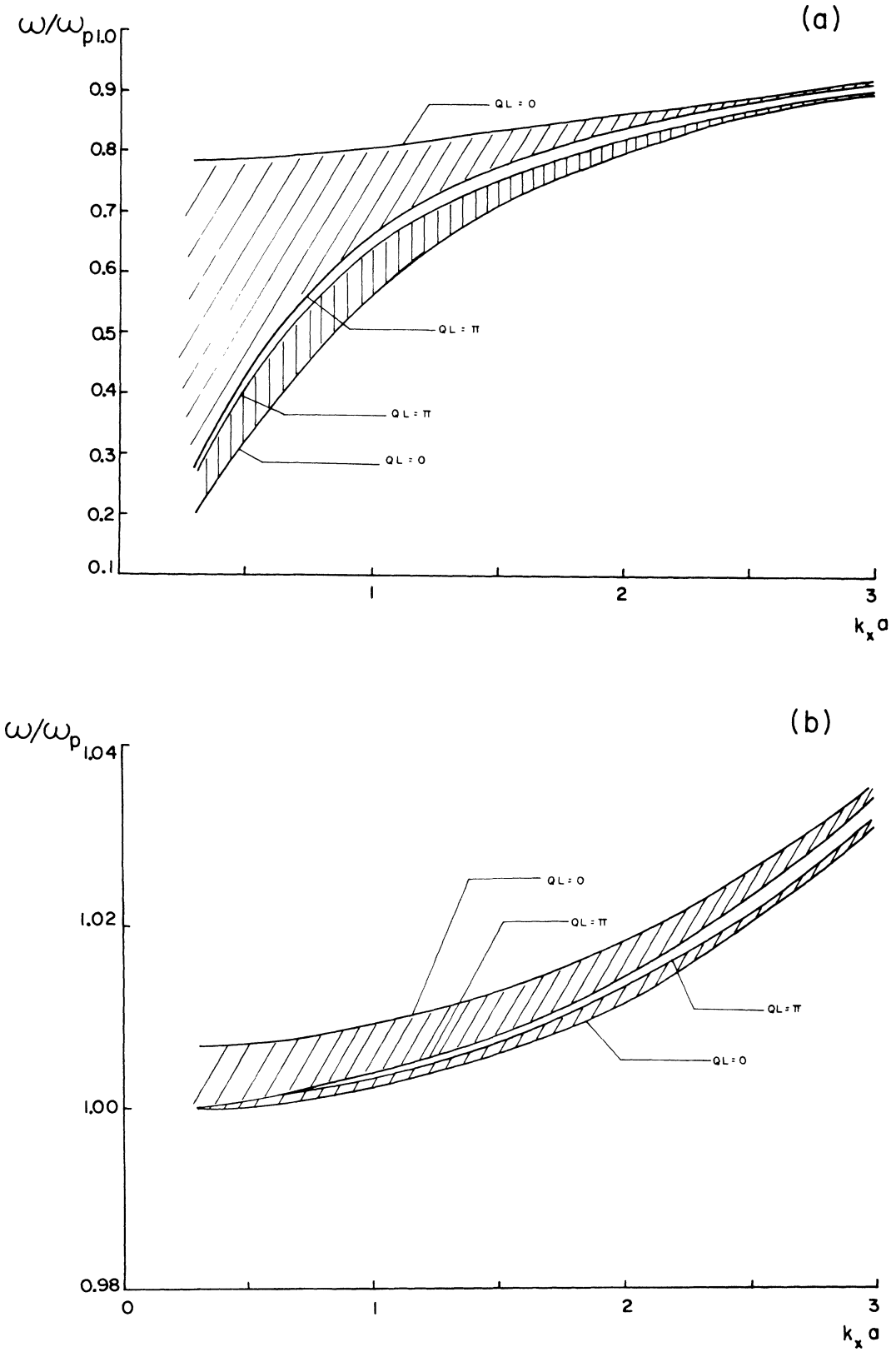

FIG. 3. (a) The frequency (ω/ω_p) of the two lower bands of the bulk polaritons as a function of $k_x a$, Eq. (2.28), for an *n-i-p-i* superlattice with $a=c=400$ Å, $b=d=200$ Å, and $|\sigma_e| = |\sigma_n| = 2\times 10^{16}$ carriers/m^2. (b) The frequency (ω/ω_p) of the two upper bands of the bulk polaritons as a function of $k_x a$, Eq. (2.28), for an *n-i-p-i* superlattice with $a=c=400$ Å, $b=d=200$ Å, and $|\sigma_e| = |\sigma_n| = 2\times 10^{16}$ carriers/m^2.

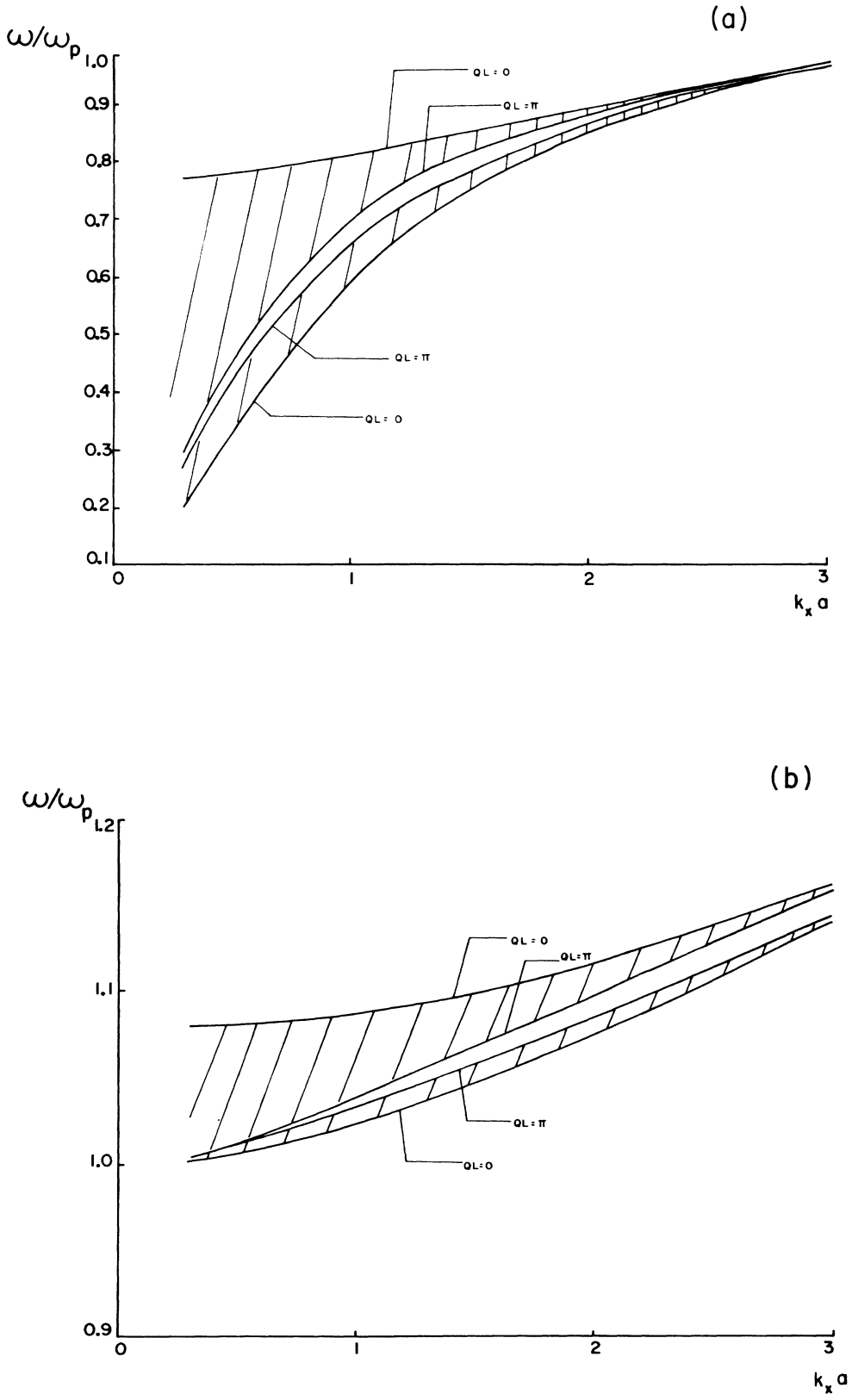

FIG. 4. (a) The frequency (ω/ω_p) of the two lower bands and (b) the two upper bands of the bulk polaritons as a function of $k_x a$, Eq. (2.28), for an *n-i-p-i* superlattice with $a=c=400$ Å, $b=d=200$ Å, and $|\sigma_e| = |\sigma_n| = 4\times 10^{16}$ carriers/m^2.

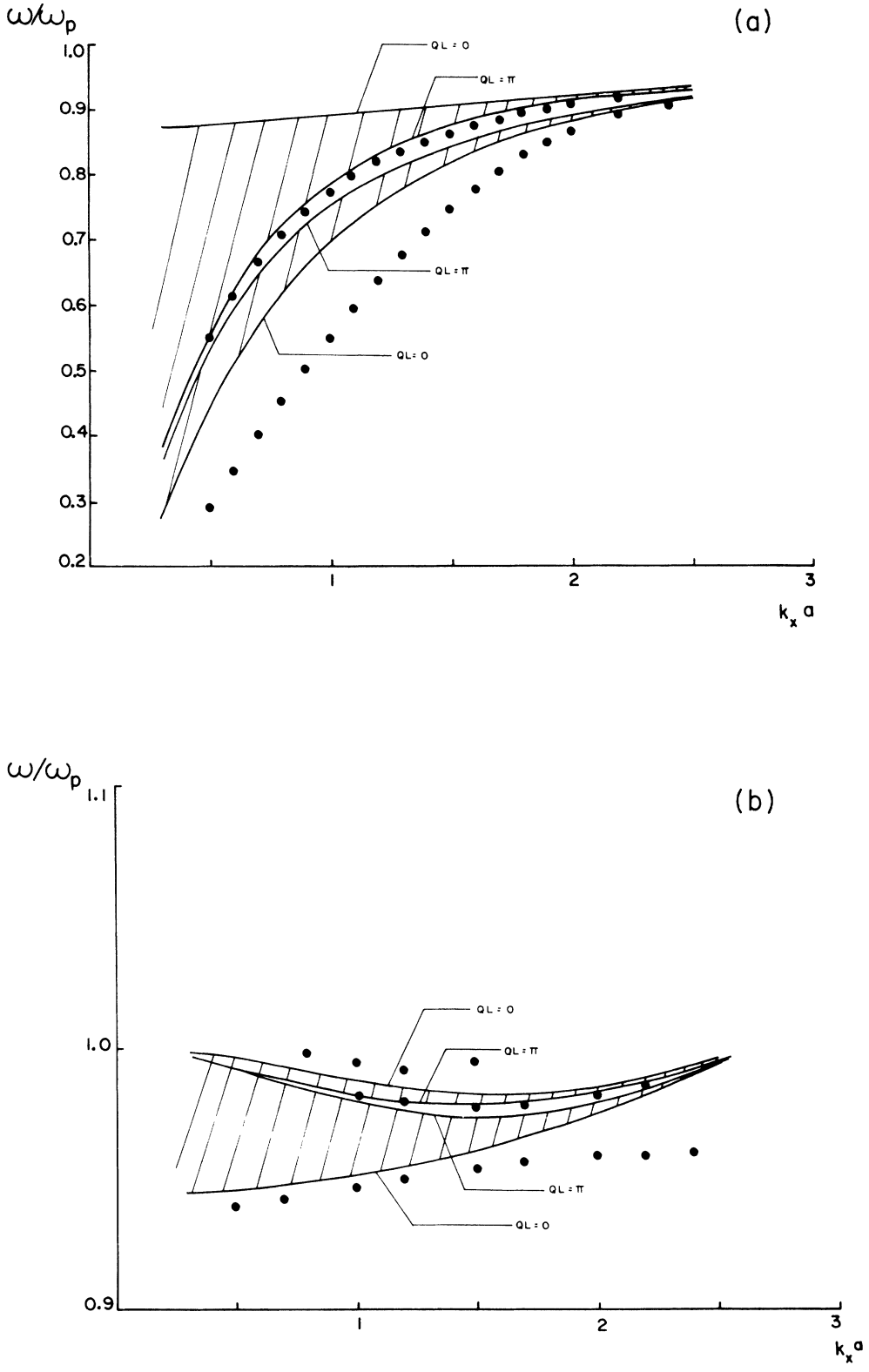

FIG. 5. (a) The frequency (ω/ω_p) of the two lower bands and (b) two upper bands of the bulk polaritons as a function of $k_x a$, Eq. (2.28), for an *n-i-p-i* superlattice with $a=b=c=\mathrm{d}=400$ Å and $|\sigma_e| = |\sigma_n| = 2\times 10^{16}$ carriers/m^2. The dotted curves of surfaces polaritons, Eq. (2.36), correspond to a semi-infinite *n-i-p-i* superlattice with the same parameters.

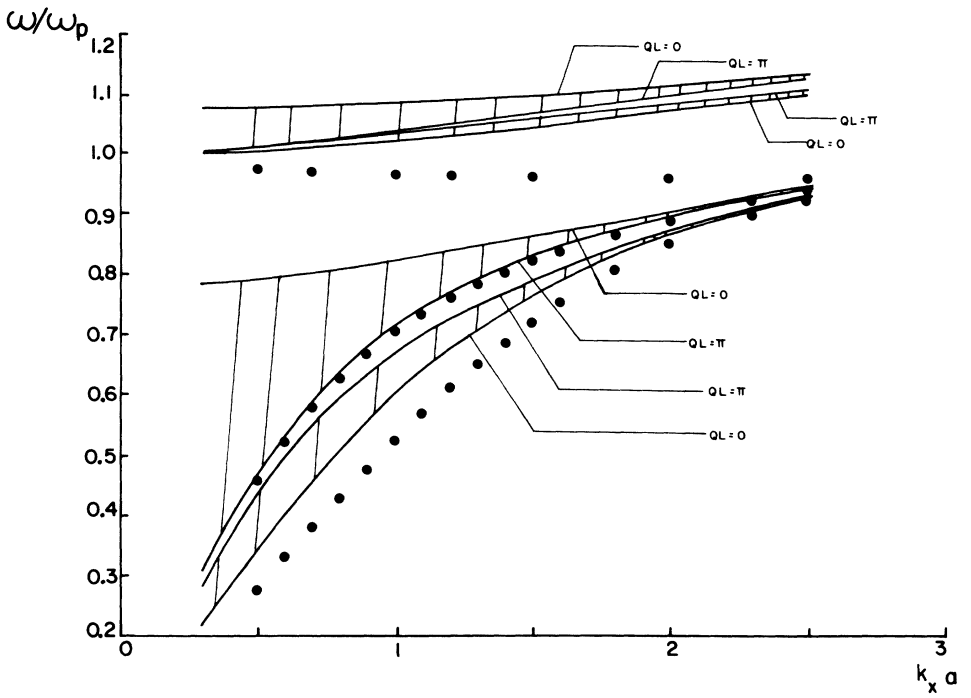

FIG. 6. The frequency (ω/ω_p) of the bands of bulk polaritons as a function of $k_x a$, Eq. (2.28), for an *n-i-p-i* superlattice with $a=b=c=d=200$ Å and $|\sigma_e|=|\sigma_n|=2\times10^{16}$ carriers/m^2. The three curves of surface polaritons (dotted curves), Eq. (2.36), correspond to a semi-infinite *n-i-p-i* superlattice with the same parameters.

with χ_2 positive, merges with the upper bulk band for small values of k. In Fig. 5(b) the numerical results for the upper surface mode are very difficult to be obtained for values of k smaller than 0.5 and larger than 1.5, due to the fact that in this region of k the frequency of the surface mode tends to the plasma frequency ω_p.

In Fig. 6 we show the bulk bands and the surface modes of the polaritons for a superlattice with the same density of charges as shown in Fig. 5, but with layers equal to 200 Å. Since the surface modes of the polariton are limited to frequencies lower than ω_p we found only three surface modes, the one between the bulk bands corresponds to $\beta L=i\pi+X$, with X positive. For large values of k we can see that all surface modes tend to merge with the bulk bands.

In all cases considered here we observed that the retardation effects are important only in the region of small values of k.

In summary, we have derived the general dispersion relations for bulk and surface plasmon modes in a superlattice of an *n-i-p-i* type. Although our treatment generalizes previous theories, it cannot be considered as a mere mathematical and complex extension of these theories, since new physical results are presented and discussed here. Our model, which is based on the transfer-matrix treatment, simplifies enormously the algebra, which is otherwise quite involved. It was also used previously to discuss the propagations of other excitations in infinite semiconductor superlattices[11,12] and we envisage that it is of sufficient generality and should prove useful for the study of the kinematic properties of other superlattice excitations. We believe that, although there are so far no experimental results, the predictions of our theory could be tested by scattering experiments, as is described by Olego *et al.*[13]

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