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Magnetoplasmons in thin films in the perpendicular configuration

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We have derived the dispersion relation for magnetoplasma polaritons guided by a thin semiconducting film subjected to an applied, perpendicular magnetic field. The film is bounded by two different (in general) dielectric media. The nonradiative modes are classified as surface, bulk (or "waveguide"), hybrid surface-bulk, and "complex," depending on the nature of the decay constants in the film. The general dispersion relation is studied in the nonretarded limit and for very thin films. In the case of a very thin film on a surface-wave active substrate there may exist resonances between the magnetoplasmons and the substrate surface polaritons; as a result splittings occur at the cyclotron and screened-plasma frequencies. Similar splittings are also found for a very thin, unsupported film.

I. INTRODUCTION

This paper is the third in a series dealing with electromagnetic (polariton) modes propagating in a thin semiconducting film, bounded, in general, by two dissimilar dielectric media, in the presence of an applied magnetic field $\mathbf{B}_0$. There are three main configurations, namely, $\mathbf{B}_0$ parallel to the interfaces and to the propagation vector $\mathbf{Req}$ (Faraday configuration), $\mathbf{B}_0$ parallel to the interfaces and perpendicular to $\mathbf{Req}$ (Voigt configuration), and $\mathbf{B}_0$ perpendicular to the interfaces and to $\mathbf{Req}$ (perpendicular configuration). In the previous work, we have presented a detailed theoretical investigation of the propagation characteristics of magnetoplasma polaritons in the Faraday and the Voigt geometries.[1,2] Most of our results are reviewed in Ref. 3. Our goal in this paper is to present the theoretical foundation for the magnetoplasmons propagating in a thin semiconducting film subjected to an applied magnetic field in the perpendicular configuration. We are interested in electromagnetic (EM) modes whose fields decay exponentially away from the interfaces.

In the past, the effect of an applied magnetic field on *surface* plasmons in metals and semiconductors in the perpendicular configuration has been investigated by several authors.[4–8] The propagation of polariton modes in a transparent dielectric film bounded by two identical semi-infinite semiconductors in this configuration was studied by Kanada *et al.*[9] They concluded that the interaction of the magnetoplasma polaritons localized at the two interfaces produces shifts in the dispersion curves. It is noteworthy that, although the perpendicular configuration is of wide current interest, particularly in systems of lower dimensionality, we are not aware of any study of magnetoplasmons in thin semiconducting films in this configuration.

In this paper we will study the propagation characteristics of the magnetoplasma modes propagating in a thin semiconducting film subjected to a perpendicular magnetic field. The exact dispersion relation is derived for a local magnetoplasma tensor characterizing the semiconductor film and scalar dielectric functions representing the two bounding dielectric media. While we defer the exact numerical calculations to a future publication, a number of interesting special cases have been studied in the thin-film approximation invoked upon the exact dispersion relation. In particular, we have shown that if the substrate is a polar semiconductor where the dispersion is entirely due to phonons, then the magnetoplasma dispersion curve exhibits splittings at certain characteristic frequencies due to resonance between the thin-film magnetoplasmons and the substrate phonons. In the other two geometries, we have found such a splitting occurring at the hybrid cyclotron-plasmon frequency.[10] In the case of an unsupported film we find two magnetoplasma modes terminating at resonant frequencies in the short-wavelength limit.

The rest of the paper is organized as follows: In Sec. II we present the derivation of the exact dispersion relation for the magnetoplasma polaritons in the geometry depicted in Fig. 1. In Sec. III we analyze the dispersion rela-

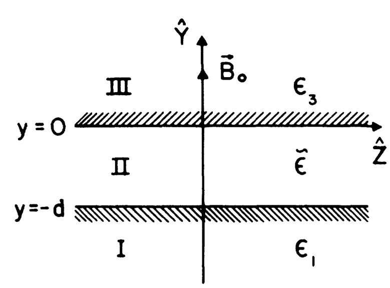

FIG. 1. Bounded region II of thickness d is occupied by a semiconducting film subjected to an applied magnetic field $\mathbf{B}_0$ that is perpendicular to the interfaces ("perpendicular configuration"). This medium is characterized by a magnetoplasma tensor $\tilde{\epsilon}\,(\omega,\omega_p,\omega_c)$. The bonding media I and III have scalar dielectric constants ϵ_1 and ϵ_3, respectively. The guided modes studied in the paper propagate in a direction ($\hat{\mathbf{z}}$) that is parallel to the interfaces; they decay exponentially away from the interfaces.

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tion in the nonretarded limit ($c \to \infty$). In Sec. IV we use an approximation for very thin films to simplify the general dispersion relation and study two cases of interest: (a) surface phonon polaritons modified by a magnetoplasmon thin layer, and (b) a magnetized film bounded by two identical dielectric media. We discuss our numerical results in the respective sections. It is worth mentioning that most of the analytic results in the present paper are independent of any specific model. The magnetoplasma model for the dielectric tensor ($\tilde{\epsilon}$) used in the numerical calculations is relegated to the Appendix.

II. GENERAL DISPERSION RELATION

We consider a semiconducting film (II) of finite thickness (d) characterized by a dielectric tensor $\tilde{\epsilon}$ which is independent of the propagation (wave) vector $\mathbf{q}$. The film is assumed to be bounded by two dissimilar, semi-infinite dielectric media I and III characterized, respectively, by the dielectric constants ϵ_1 and ϵ_3. The magnetostatic field ($\mathbf{B}_0$) is assumed to be oriented along the $\hat{\mathbf{y}}$ axis which is perpendicular to the interfaces. The wave is taken to propagate along the $\hat{\mathbf{z}}$ axis (thus $q_x=0$), i.e., parallel to the interfaces. We are therefore concerned with the perpendicular configuration: It is to be noted that in the absence of $\mathbf{B}_0$ all the three media are isotropic. The geometry under consideration is shown in Fig. 1.

After eliminating the magnetic field variable ($\mathbf{B}$) in Maxwell's curl field equations, we obtain the following wave equation for the macroscopic electric field $\mathbf{E}$:

$$\nabla\times(\nabla\times\mathbf{E})-q_0^2\tilde{\epsilon}\cdot\mathbf{E}=0 , \tag{1}$$

where $q_0=\omega/c$ is the vacuum wave vector, ω being the angular wave frequency and c is the velocity of light in vacuum. We assume that the spatial and temporal dependence of the fields is of the form $\sim e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$. In the configuration at hand ($\mathbf{B}_0\|\hat{\mathbf{y}}$) the dielectric tensor $\tilde{\epsilon}$ is simplified by the symmetry requirements such that $\epsilon_{xx}=\epsilon_{zz}$, $\epsilon_{xz}=-\epsilon_{zx}$, and $\epsilon_{xy}=\epsilon_{yx}=\epsilon_{yz}=\epsilon_{zy}=0$. As a result, Eq. (1) may be rewritten as

$$\begin{bmatrix} q_0^2\epsilon_{xx}-q_y^2-q_z^2 & 0 & q_0^2\epsilon_{xz} \\ 0 & q_0^2\epsilon_{yy}-q_z^2 & q_yq_z \\ -q_0^2\epsilon_{xz} & q_yq_z & q_0^2\epsilon_{xx}-q_y^2 \end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} . \tag{2}$$

Equation (2) is a set of three linear, homogeneous equations satisfied by the electric field in the dispersive, anisotropic semiconducting film (medium II in Fig. 1). The same set of three equations also give valid solutions of Maxwell's equations in the isotropic media I and III, provided that we set $\epsilon_{xz}=0$ and $\epsilon_{xx}=\epsilon_{yy}=\epsilon_i$, where $i\equiv 1$ and 3, respectively, for media I and III. The condition of the nontrivial solutions applied to Eq. (2) leaves us with the following relation:

$$-q_y^2=\beta_\pm^2=\frac{1}{2\epsilon_{yy}}\{[q_z^2(\epsilon_{xx}+\epsilon_{yy})-2q_0^2\epsilon_{xx}\epsilon_{yy}] \pm[q_z^4(\epsilon_{xx}-\epsilon_{yy})^2+4\lambda^2q_0^2\epsilon_{xz}^2\epsilon_{yy}]^{1/2}\} , \tag{3}$$

where

$$\lambda^2=q_z^2-q_0^2\epsilon_{yy} \tag{4}$$

in the semiconducting medium (II) and

$$-q_y^2=\alpha_i^2=q_z^2-q_0^2\epsilon_i, \quad i\equiv 1,3 \tag{5}$$

in the bounding dielectric media. In Eqs. (3) and (5) $\beta_\pm$, α_1, and α_3 refer to the decay constants in media II, I, and III, respectively.

We write the field distributions in the three media in the form (see Fig. 1):

$$\mathbf{E}(r,t)=\mathbf{E}(y)e^{i(q_zz-\omega t)} \tag{6}$$

where $\mathbf{E}(y)$ for regions I ($y\le -d$), II ($-d\le y\le 0$), and III ($y\ge 0$) is expressed as follows:

$$E^{\text{I}}(y)=E^{\text{I}}e^{\alpha_1 y} , \tag{7}$$

$$E^{\text{II}}(y)=E_1e^{-\beta_+y}+E_2e^{\beta_+y}+E_3e^{-\beta_-y}+E_4e^{\beta_-y} , \tag{8}$$

$$E^{\text{III}}(y)=E^{\text{III}}e^{-\alpha_3 y} . \tag{9}$$

Analogous solutions can be written for the magnetic field variables ($\mathbf{B}$) in the three regions. The determination of the dispersion relation requires the matching of certain EM boundary conditions at both the $y=0$ and the $y=-d$ interfaces. The boundary conditions are the continuity of the tangential components of the electric and magnetic fields: E_x, E_z, B_x, and B_z. Making use of Maxwell's curl field equations and Eq. (2) leads one to express E_z, B_x, and B_z in terms of E_x in the magnetoplasma (region II). Similarly, B_x and B_z in the dielectric media (regions I and III) are expressible in terms of E_z and E_x, respectively. This greatly reduces the number of unknowns involved. Employing the boundary conditions at the two interfaces yields the following relations.

For $y=0$, we have

$$E_x^{\text{III}}=E_{1x}+E_{2x}+E_{3x}+E_{4x} , \tag{10}$$

$$q_0^2\epsilon_{xz}E_z^{\text{III}}=A_+E_{1x}+A_+E_{2x}+A_-E_{3x}+A_-E_{4x} , \tag{11}$$

$$\left[-\frac{q_0^2\lambda^2\epsilon_{xz}\epsilon_3}{\epsilon_{yy}\alpha_3}\right]E_z^{\text{III}}=-\beta_+A_+E_{1x}+\beta_+A_+E_{2x} -\beta_-A_-E_{3x}+\beta_-A_-E_{4x} , \tag{12}$$

$$-\alpha_3E_x^{\text{III}}=-\beta_+E_{1x}+\beta_+E_{2x}-\beta_-E_{3x}+\beta_-E_{4x} . \tag{13}$$

For $y=-d$, we have

$$E_x^{\text{I}}e^{-\alpha_1d}=E_{1x}e^{\beta_+d}+E_{2x}e^{-\beta_+d}+E_{3x}e^{\beta_-d}+E_{4x}e^{-\beta_-d} , \tag{14}$$

$$q_0^2\epsilon_{xz}E_z^{I}e^{-\alpha_1d}=A_+E_{1x}e^{\beta_+d}+A_+E_{2x}e^{-\beta_+d} +A_-E_{3x}e^{\beta_-d} +A_-E_{4x}e^{-\beta_-d} , \tag{15}$$

$$\left[\frac{q_0^2\lambda^2\epsilon_{xz}\epsilon_1}{\epsilon_{yy}\alpha_1}\right]E_z^I e^{-\alpha_1 d} = -\beta_+ A_+ E_{1x} e^{\beta_+ d} + \beta_+ A_+ E_{2x} e^{-\beta_+ d} - \beta_- A_- E_{3x} e^{\beta_- d} + \beta_- A_- E_{4x} e^{-\beta_- d} , \quad (16)$$

$$\alpha_1 E_x^I e^{-\alpha_1 d} = -\beta_+ E_{1x} e^{\beta_+ d} + \beta_+ E_{2x} e^{-\beta_+ d} - \beta_- E_{3x} e^{\beta_- d} + \beta_- E_{4x} e^{-\beta_- d} , \quad (17)$$

where

$$A_\pm = k^2 - \beta_\pm^2 \quad (18)$$

and

$$k^2 = q_z^2 - q_0^2\epsilon_{xx} . \quad (19)$$

Equations (10)–(17) are eight homogeneous equations in terms of eight unknowns; E_x^{I}, E_z^{I}, E_x^{III}, E_z^{III}, E_{1x}, E_{2x}, E_{3x}, and E_{4x}. Setting up these equations in compact (matrix) form and employing the condition of nontrivial solutions gives us the required dispersion relation for the magnetoplasma polaritons. After laborious algebra we arrive at the following form of the general dispersion relation:

$$\begin{aligned} &A_+^2[(\beta_-^2+\alpha_1\alpha_3)T_- + \beta_-(\alpha_1+\alpha_3)][(\beta_+^2\alpha_1\alpha_3\epsilon_{yy}^2+\lambda^4\epsilon_1\epsilon_3)T_+ + \beta_+(\alpha_1\epsilon_3+\alpha_3\epsilon_1)\lambda^2\epsilon_{yy}] \\ &\quad + A_-^2[(\beta_+^2+\alpha_1\alpha_3)T_+ + \beta_+(\alpha_1+\alpha_3)][(\beta_-^2\alpha_1\alpha_3\epsilon_{yy}^2+\lambda^4\epsilon_1\epsilon_3)T_- + \beta_-(\alpha_1\epsilon_3+\alpha_3\epsilon_1)\lambda^2\epsilon_{yy}] \\ &\quad - A_+A_-\{[(\beta_-^2\alpha_1\epsilon_{yy}+\lambda^2\alpha_3\epsilon_1)T_- + \beta_-(\alpha_1\alpha_3\epsilon_{yy}+\lambda^2\epsilon_1)][(\beta_+^2\alpha_3\epsilon_{yy}+\lambda^2\alpha_1\epsilon_3)T_+ + \beta_+(\alpha_1\alpha_3\epsilon_{yy}+\lambda^2\epsilon_3)] \\ &\qquad + [(\beta_-^2\alpha_3\epsilon_{yy}+\lambda^2\alpha_1\epsilon_3)T_- + \beta_-(\alpha_1\alpha_3\epsilon_{yy}+\lambda^2\epsilon_3)][(\beta_+^2\alpha_1\epsilon_{yy}+\lambda^2\alpha_3\epsilon_1)T_+ + \beta_+(\alpha_1\alpha_3\epsilon_{yy}+\lambda^2\epsilon_1)]\} \\ &\quad + 2A_+A_-\beta_+\beta_-(\lambda^2\epsilon_1-\alpha_1^2\epsilon_{yy})(\lambda^2\epsilon_3-\alpha_3^2\epsilon_{yy})(1-T_+^2)^{1/2}(1-T_-^2)^{1/2} = 0 , \quad (20) \end{aligned}$$

where

$$T_\pm = \tanh(\beta_\pm d) . \quad (21)$$

We have examined Eq. (20) by subjecting it to various special limits, viz., $d=0$, $d\to\infty$, and $\mathbf{B}_0=0$. It can readily be shown that the general dispersion relation, Eq. (20), for these limits reproduces exactly the proper results previously reported in the literature for a surface ($\mathbf{B}_0\neq 0$) and for a thin film in the absence of an applied magnetic field.[4,6,11,12] A careful inspection reveals a close analogy between Eqs. (10)–(17) in the present paper and the corresponding equations of Ref. 1 in the Faraday configuration. This leads us to conclude that the present dispersion relation, Eq. (20), can be obtained directly from the general dispersion relation, Eq. (19), derived in Ref. 1. It is found that redefining $A_\pm$ in Eq. (18) of Ref. 1 such that it is given by Eq. (18) in the present paper and replacing $\alpha_i\epsilon_{zz}/\epsilon_i$ [in Eq. (19) of Ref. 1] by $\lambda^2\epsilon_i/\alpha_i\epsilon_{yy}$ yields Eq. (20).

It is worth pointing out that we are interested in the situation where q_z is real when the absorption is neglected. Then α_1 and α_3, given by Eq. (5), are either real or pure imaginary. The latter case is of substantial interest in waveguide theory.[11] In the present work we will confine our attention to the solution for which the EM fields decay exponentially away from the interfaces. Such solutions are characterized by both α_1 and α_3 being real and positive; see Eqs. (7) and (9). In the presence of damping the conditions $\mathrm{Re}\alpha_i>0$ ($i\equiv 1,3$) must be imposed and $\mathrm{Re}\beta_\pm>0$ may be imposed with no loss of generality.

The magnetoplasma modes with real q_z, α_1, and α_3 may be further classified according to the nature of $\beta_\pm$. Depending upon the spectral range in the ω-q plane the following possibilities may arise: (i) β_+ and β_- are both real and positive, (ii) β_+ and β_- are both pure imaginary, (iii) β_+ is real and β_- is pure imaginary, or vice versa, and (iv) β_+ and β_- are complex conjugates of each other. We have classified the magnetoplasma modes corresponding to the aforementioned possibilities as surface modes, bulk (waveguide) modes, hybrid surface-bulk modes, and generalized (or complex) modes, respectively.[1] It is evident that with regard to the classification of the modes, the situation in the perpendicular configuration is identical to that in the Faraday configuration.[1]

In general, the three components of the electric field in the media I, II, and III, Eqs. (7)–(9), are all nonvanishing. Hence the electric field is elliptically polarized in a tilted plane (neither parallel, nor perpendicular to the surface). Similarly, the magnetic field of the wave is polarized in another tilted plane. Our dispersion relation, Eq. (20), is even in the propagation constant q_z, that is, $\omega(-q_z) = \omega(q_z)$ for a given solution. For such "reciprocal" propagation we may limit the discussion, without loss of generality, to the case $q_z\geq 0$.

In what follows, we will quote a number of equations directly from Ref. 1 (hereinafter referred to as I) and specify them as (I.n), where n stands for the number of the equation in I.

III. NONRETARDED LIMIT

In the nonretarded (NR) limit we assume that $q_z \gg q_0$ which is mathematically equivalent to taking $c\to\infty$. Then

$$\alpha_1=\alpha_3=k=\beta_-=q_z$$

and

$$\beta_+=(\epsilon_{xx}/\epsilon_{yy})^{1/2}q_z .$$

As such, Eq. (20) reduces to

$$(\epsilon_{xx}\epsilon_{yy}+\epsilon_1\epsilon_3)\tanh[(\epsilon_{xx}/\epsilon_{yy})^{1/2}q_z d] + \epsilon_{yy}(\epsilon_{xx}/\epsilon_{yy})^{1/2}(\epsilon_1+\epsilon_3)=0 \ . \quad (22)$$

This is the dispersion relation for the magnetoplasma polaritons in the NR limit for an arbitrary thickness of the semiconducting film. Note that the off-diagonal element ϵ_{xz} has dropped out of the calculation. This leads us to conclude that, in the NR limit, the transverse (Hall) field is absent just as in the Faraday configuration.[1] Thus there is no dynamical Hall effect and we do not expect heliconlike modes for $q_z \gg q_0$. This, however, does not preclude the long-range propagation under suitable conditions. We now analyze Eq. (22) in two cases.

The case $q_z \to \infty$. First we consider the case $q_{zz} \gg 1/d$; taken together with $q_z \gg q_0$, this implies that $q_z \to \infty$. The hyperbolic tangent in Eq. (22) reaches a limiting value (one) only when its argument is real; that is, $\beta_\pm$ must be real. Therefore $\epsilon_{xx}(\omega)$ and $\epsilon_{yy}(\omega)$ must have the same sign. Moreover, because of our conventions that $q_z>0$ and $\beta_\pm>0$ we must choose the positive root: $(\epsilon_{xx}/\epsilon_{yy})^{1/2}>0$. Then Eq. (22) reduces to

$$[(\epsilon_{xx}/\epsilon_{yy})^{1/2}\epsilon_{yy}+\epsilon_1][(\epsilon_{xx}/\epsilon_{yy})^{1/2}\epsilon_{yy}+\epsilon_3]=0 \ . \quad (23)$$

Equation (23) may be written in the form

$$(\epsilon_{xx}/\epsilon_{yy})^{1/2}\epsilon_{yy}=-\epsilon_i, \quad i\equiv 1,3 \ . \quad (24)$$

In the case $\mathbf{B}_0=0$ and hence $\epsilon_{xx}=\epsilon_{yy}=\epsilon_2(\omega)$, say, Eq. (24) reduces to $\epsilon_2(\omega)=-\epsilon_i$, as it should be.[13] Equation (24), for $\epsilon_i=1$, gives the solutions for the asymptotic modes at the semiconductor-vacuum interface as specified by Eqs. (28) and (46) in Ref. 12.

In this section ϵ_1 and ϵ_3 are taken to be positive. Then it follows from Eq. (24) that $\epsilon_{yy}(\omega)$ must be negative. Thus the correct behavior is given by

$$\epsilon_{xx}<0, \quad \epsilon_{yy}<0 \quad (q_z\to\infty) \ . \quad (25)$$

In view of this, the asymptotic solutions predicted by Eq. (24) must lie in the frequency window specified by $\omega_c<\omega<\omega_p/\sqrt{\epsilon_L}$; therefore propagation is not possible for $\omega_c>\omega_p/\sqrt{\epsilon_L}$. For the magnetoplasma model, specified in the Appendix, Eq. (24) may be solved to obtain the same result as in the Faraday configuration.[1]

The case $q_0 \ll q_z \ll 1/d$. Assuming that $|\epsilon_{xx}/\epsilon_{yy}|^{1/2}q_z d \ll 1$, Eq. (22) yields an explicit solution for q_z,

$$q_z=-\frac{1}{d}\frac{\epsilon_{yy}(\epsilon_1+\epsilon_3)}{(\epsilon_{xx}\epsilon_{yy}+\epsilon_1\epsilon_3)} \ . \quad (26)$$

Because the NR limit requires that

$$q_0 \ll q_z \ll (\epsilon_{yy}/\epsilon_{xx})^{1/2}/d \ ,$$

this result holds for very thin films, namely, $q_0 d \ll 1$. Equation (26) when solved using ϵ_{xx} and ϵ_{yy}, as given in the Appendix, yields a biquadratic equation in ω, which predicts two branches $\omega(q_z)$ and can be studied numerically for the asymmetric ($\epsilon_1\neq\epsilon_3$) and the symmetric ($\epsilon_1=\epsilon_3$) cases.

It is useful to look at the limiting cases $q_z=0$ and $q_z\to\infty$, although these are really outside the range of validity of the present approximation. For $q_z=0$, we must have either $\epsilon_{yy}=0$ or $\epsilon_{xx}\to\infty$, as may be seen from Eq. (26). This gives, respectively, for the higher and the lower branches

$$\omega=\omega_p/\sqrt{\epsilon_L}, \quad \omega=\omega_c \quad \text{for } q_z\to 0 \ . \quad (27)$$

where ω_c and ω_p are, respectively, the cyclotron frequency and the *unscreened* plasma frequency, defined in the Appendix. It should be pointed out that the asymptotic frequencies, as predicted by Eq. (26), are given by $\epsilon_{xx}\epsilon_{yy}+\epsilon_1\epsilon_3=0$. This is clearly wrong, since we know that the asymptotic frequencies are correctly given by Eq. (24). This discrepancy is hardly surprising because the present approximation is limited to $q_z \ll 1/d$.

In the NR limit, the decay constants α_1, α_3, and β_- all have the value q_z, so they are real and positive. However,

$$\beta_+=(\epsilon_{xx}/\epsilon_{yy})^{1/2}q_z$$

is real (and positive) or pure imaginary depending upon whether the quantity $(\epsilon_{xx}/\epsilon_{yy})$ is positive or negative. This quantity is positive for $\omega_c<\omega<\omega_p/\sqrt{\epsilon_L}$. [In the case $\omega_p/\sqrt{\epsilon_L}<\omega<\omega_c$ we should have $\epsilon_{xx}>0$ and $\epsilon_{yy}>0$, and then Eq. (22) cannot be satisfied.] As such the two modes, as predicted by Eq. (26), propagating in this window correspond to surface polaritons. Neglecting the damping and using Eq. (26), our approximation may be expressed as follows:

$$q_0 d \ll \frac{|\epsilon_{yy}|(\epsilon_1+\epsilon_3)}{|\epsilon_{xx}\epsilon_{yy}+\epsilon_1\epsilon_3|} \ll (\epsilon_{yy}/\epsilon_{xx})^{1/2} \ . \quad (28)$$

The first inequality always fails for $\omega=\omega_p/\sqrt{\epsilon_L}$ and $\omega=\omega_c$. This is reasonably expected because, for $q_z\to 0$, the phase velocities for both the modes are enormous, whereas the NR limit requires that $q_z \gg \omega/c$. The second inequality is satisfied for $\omega=0$, ω_c, $\omega_p/\sqrt{\epsilon_L}$, and ω_H. With this understanding, we may conclude that this approximation works well provided that we do not approach too closely one of the asymptotic frequencies.

IV. APPROXIMATE DISPERSION RELATION FOR VERY THIN FILMS

In this section we invoke the thin-film approximation (TFA) $\tanh(\beta_\pm d)\simeq\beta_\pm d$. With this approximation, the general dispersion relation, Eq. (20), reduces to

$$\epsilon_{yy}(\alpha_1+\alpha_3)(\alpha_1\epsilon_3+\alpha_3\epsilon_1)+d[(\alpha_1+\alpha_3)(\alpha_1\alpha_3\epsilon_{xx}\epsilon_{yy}+\lambda^2\epsilon_1\epsilon_3)+\epsilon_{yy}(\alpha_1\alpha_3+k^2)(\alpha_1\epsilon_3+\alpha_3\epsilon_1)] + d^2[(\alpha_1\alpha_3+k^2)(\alpha_1\alpha_3\epsilon_{xx}\epsilon_{yy}+\lambda^2\epsilon_1\epsilon_3)-\alpha_1\alpha_3 q_0^2\epsilon_{xz}^2\epsilon_{yy}]=0 \ . \quad (29)$$

Note that in writing Eq. (29) we have omitted a prefactor (P) given by

$$P=\lambda^2(\beta_+^2-\beta_-^2)\beta_+\beta_- \tag{30}$$

and treated it as a nonvanishing quantity. We will analyze Eq. (29) in two different cases of interest.

A. Surface phonon polaritons modified by magnetized overlayer

In this case we assume that medium III is air ($\epsilon_3=1.0$) and that medium I is surface-wave active ($\epsilon_1<0$). We use an ansatz specified by Eq. (I.36) for a film of small thickness ($q_0d \ll 1$). Substituting in Eq. (29) and following the procedure stated in I leads to the following formula:

$$q_z \simeq q_0 \left\{ \left[\frac{\epsilon_1}{1+\epsilon_1} \right]^{1/2} - i\frac{(q_0 d)\epsilon_1^{3/2}}{(1+\epsilon_1)^2(1-\epsilon_1)} \times \left[\epsilon_1 \left[\frac{1}{\epsilon_{yy}} - 1 \right] - 1 + \epsilon_{xx} \right] \right\} . \tag{31}$$

Note that $-i\epsilon_1^{3/2}=-i(-|\epsilon_1|)(-i|\epsilon_1|^{1/2})=|\epsilon_1|^{3/2}$; and hence the right-hand side of Eq. (31) is a real quantity. Thus the propagation constant q_z is linear in the film thickness, to the lowest order in d. Equation (31) is a good approximation provided that $q_0 d \ll 1$. For the special case of $\mathbf{B}_0=0$, Eq. (31) reduces to Eq. (I.39) which is an exact analogue of Eq. (3) of Lopez-Rios,[14] provided that ϵ_0, the dielectric constant of medium III in his notation, is equal to 1. This is a justification of our TFA.

It should be pointed out that the requirement of the validity of Eq. (31) (i.e., $q_0 d \ll 1$) implies that the first term in the curly bracket predominates over most of the spectral range of interest. An exception, however, occurs at ω_c and $\omega_p/\sqrt{\epsilon_L}$. This is because ϵ_{yy} vanishes at $\omega=\omega_p/\sqrt{\epsilon_L}$ and $\epsilon_{xx}\to\infty$ at $\omega=\omega_c$. Consequently, by Eq. (31), $q_z\to\infty$. It is thus understood that although our perturbational approach breaks down, the dispersion relation of the surface magnetoplasmon polariton must exhibit splittings at these frequencies provided that they are within the surface-polariton region of the substrate. Therefore there may be two, or one, or zero splittings, depending on whether ω_c and $\omega_p/\sqrt{\epsilon_L}$ both fall inside the surface-polariton pass band (ω_T,Ω_s), or only one of these frequencies is within the pass band, or both frequencies are outside this band. The splittings correspond to a resonance between the magnetoplasmons of the thin film and the surface polaritons of the bare substrate.[15] This will be shown in the numerical examples discussed later.

We consider the substrate (region I in Fig. 1) to be an undoped polar semiconductor wherein the dispersion is entirely due to phonons. Neglecting the damping, we have

$$\epsilon_1(\omega)=\epsilon_\infty\frac{\omega^2-\omega_L^2}{\omega^2-\omega_T^2} , \tag{32}$$

where ϵ_∞ is the high-frequency dielectric constant and ω_T and ω_L are, respectively, the transverse and longitudinal optical phonon frequencies at the center of the first Brillouin zone. The thin film is assumed to be strongly doped and we neglect in it the phonons. This is a good approximation provided that $\omega^2 \gg \omega_{Lf}^2$, ω_{Lf} being the longitudinal phonon frequency of the film semiconductor.

In what follows, we discuss the two cases. (i) $\omega_c<\omega_T$: This leads to one resonance at $\omega=\omega_p/\sqrt{\epsilon_L}$. (ii) $\omega_c>\omega_T$: This is the situation where one finds two resonances, one at $\omega=\omega_c$ and the other at $\omega=\omega_p/\sqrt{\epsilon_L}$. We choose the following parameters in our calculation: $\epsilon_L=15.7$, $\epsilon_\infty=9.65$, $\omega_T=12$ THz, $\omega_L=21.6$ THz, $\omega_p d/c=2\pi\times10^{-2}$, ω_c ($<\omega_T$)$=0.5\omega_p/\sqrt{\epsilon_L}$, and ω_c ($>\omega_T$)$=0.8\,\omega_p/\sqrt{\epsilon_L}$. This corresponds to a strongly doped InSb film on an undoped MgO substrate.

(i) $\omega_c<\omega_T<\omega_p/\sqrt{\epsilon_L}<\Omega_s$, where Ω_s is the limiting frequency of the surface-phonon polariton: In this case the numerical results in terms of rationalized variables are shown in Fig. 2. The dotted line is the light line in the vacuum (medium III), and the dashed curve is the surface phonon-polariton mode (in the absence of the film). The solid curve, representing the magnetoplasma polariton mode, starts at ω_T, rises to the right of the light line (and to the right of the surface phonon-polariton mode), and approaches resonance at $\omega=\omega_p/\sqrt{\epsilon_L}$, implied by $\epsilon_{yy}=0$ in Eq. (31). Although our perturbational approach breaks down [at $\omega=\omega_p/\sqrt{\epsilon_L}$ the correction term in Eq. (31) diverges], it is clear that a splitting occurs due to the resonance with the thin magnetoplasma transition layer. Thus we have a lower branch $\omega_-(q)$ and an upper branch $\omega_+(q)$. The upper branch starts on the

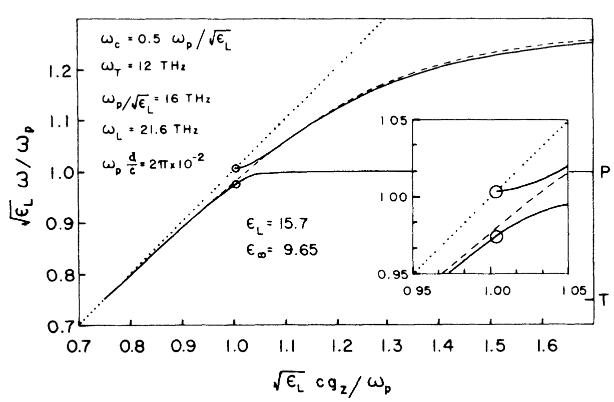

FIG. 2. Normalized frequency vs normalized propagation constant for a *very* thin, strongly doped semiconducting film (InSb) on an undoped, polar semiconductor substrate (MgO). The dashed line is the substrate phonon polariton that starts at the transverse phonon frequency (T). The magnetoplasma polariton of the film (continuous line) closely follows the dashed line, except for $\omega\sim\omega_p/\sqrt{\epsilon_L}$. In this region a splitting occurs due to a resonance between the thin-film plasmon and the substrate phonon polaritons. In this case ($\omega_c<\omega_T$) the applied magnetic field does not play an important role. By definition, we define the "splitting" as the vertical distance between the two circles (amplified in the inset). The dotted line is the vacuum light line. The plasma frequency $\omega_p/\sqrt{\epsilon_L}$ has been chosen within the surface-phonon polariton range; therefore a splitting in the magnetoplasma-polariton dispersion occurs at this frequency.

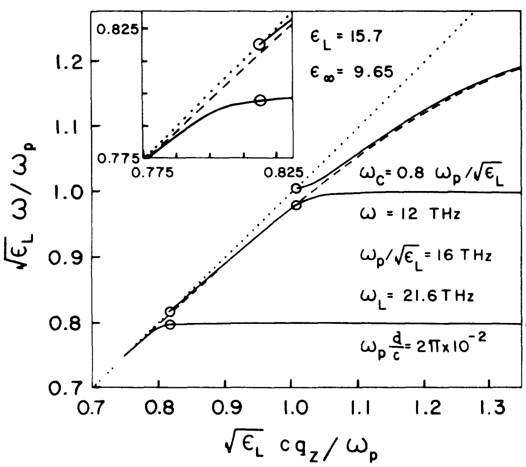

FIG. 3. As in Fig. 2, however, in addition to the plasma frequency $\omega_p/\sqrt{\epsilon_L}$ the cyclotron frequency ω_c now also falls inside the propagation window of the substrate surface polariton. As a result we observe two splittings at the corresponding frequencies.

light line; say, at the wave vector q_+. We define the splitting between the two branches as

$$\Delta\omega=\omega_+(q_+)-\omega_-(q_+) . \tag{33}$$

Note that this is just the vertical distance between the two branches evaluated at the initial point ($\omega=cq$) of the upper branch. It is noteworthy that we have retained our definition of splitting, used in the Faraday and the Voigt geometries,[10] which is quite different from that of Agranovich;[15] the reason being that, in the Voigt geometry, the definition of Ref. 15 is not applicable (see Fig. 3 in Ref. 10).

(ii) $\omega_T<\omega_c$, $\omega_p/\sqrt{\epsilon_L}<\Omega_s$. In this case we encounter two resonances: one at $\omega\simeq\omega_c$ and another at $\omega\simeq\omega_p/\sqrt{\epsilon_L}$. The former is implied by $\epsilon_{xx}\to\infty$ and the latter by $\epsilon_{yy}\to 0$. The numerical results in terms of rationalized variables are displayed in Fig. 3. In general, the repulsion in the two branches at both resonance frequencies is a consequence of the resonance between the thin-film magnetoplasmons and the substrate phonons. It is worthwhile mentioning that we have disregarded, both in Figs. 2 and 3, the solutions with $q_z<q_0$, the reason being that, by Eq. (5), these would give an imaginary decay constant α_3. This would result in radiative polariton modes in the medium III.

In Fig. 4 we compare the splitting $\Delta\omega$ as a function of the normalized ω_c in the present geometry with that in the Faraday and Voigt geometries.[10] The curves designated as F, $V_\pm$, and P_t refer, respectively, to the Faraday, Voigt, and perpendicular configurations. It is evident that $\Delta\omega$ in the Faraday and Voigt geometries depends strongly on the intensity of the applied magnetic field $\mathbf{B}_0$. The situation in the perpendicular geometry is rather different. For $\omega_c<\omega_T$, where one finds only one splitting at $\omega\simeq\omega_p/\sqrt{\epsilon_L}$, $\Delta\omega$ is a very slowly varying function of $\mathbf{B}_0$. This behavior may be understood from the fact that in the vicinity of the resonance $\omega\simeq\omega_p/\sqrt{\epsilon_l}$ the last term in the square brackets of Eq. (31) is negligible, resulting in a field-independent expression. This expression is essentially the same as the one obtained for $\mathbf{B}_0=0$.[14] For $\omega_c>\omega_T$ (with both ω_c and $\omega_p/\sqrt{\epsilon_L}$ now being inside the surface-phonon polariton range) there are two resonances, as we have seen in the example of Fig. 3. The curves labeled as P_p and P_c correspond to the splittings at $\omega_p/\sqrt{\epsilon_L}$ and ω_c, respectively. In the special case $\omega_c=\omega_p/\sqrt{\epsilon_L}$ there is only one resonance; the corresponding value of $\Delta\omega$ has been marked by a dot. This causes a discontinuity in the curves P_p and P_c at $\omega_c=\omega_p/\sqrt{\epsilon_L}$. For $\omega_c>\omega_p/\sqrt{\epsilon_L}$ the curves P_p and P_c "exchange roles." It is interesting to note that the two curves exhibit opposite behaviors: when $P_p(\omega_c)$ increases then $P_c(\omega_c)$ decreases, and vice versa. Generally speaking, Fig. 4 shows that the splitting depends in a qualitative way on the direction and magnitude of the applied magnetic field.

B. Magnetized film bounded by identical media

Here we have $\epsilon_1=\epsilon_3=\epsilon_0$ and hence $\alpha_1=\alpha_3=\alpha_0$. As a result Eq. (29) assumes the form

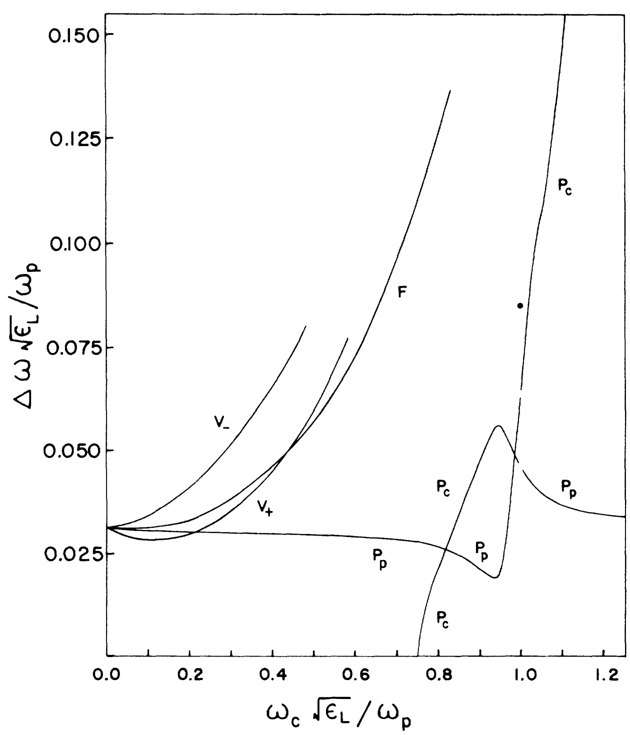

FIG. 4. The splitting $\Delta\omega$, as defined by Eq. (33), as a function of the normalized cyclotron frequency. We compare results for the perpendicular geometry (curves labeled P_p or P_c) with former calculations[10] in the Faraday (F) and Voigt ($V_\pm$) configurations. The splittings arise as a result of a resonance between a magnetoplasmon of the thin film (ω_p, or ω_c, or ω_H) and the surface-phonon polariton of the bare surface. In the Voigt configuration different results obtain for propagation in the positive and negative directions (nonreciprocity). In the perpendicular configuration there is only one resonance at $\omega_p/\sqrt{\epsilon_L}$ for $\omega_c<\omega_T$ (curves P_p). For $\omega_c>\omega_T$ an additional resonance is obtained at ω_c (curves P_c). At $\omega_c=\omega_p/\sqrt{\epsilon_L}$ there is only one resonance, and the corresponding value of $\Delta\omega$ is marked by a dot. Notice the strong dependence of $\Delta\omega$ on both direction and magnitude of $\mathbf{B}_0$.

$$4\epsilon_0\,\epsilon_{yy}\alpha_0^2+2\alpha_0 d\,[\alpha_0^2\epsilon_{xx}\epsilon_{yy}+\lambda^2\epsilon_0^2+(\alpha_0^2+k^2)\epsilon_0\epsilon_{yy}]+d^2[(\alpha_0^2+k^2)(\alpha_0^2\epsilon_{xx}\epsilon_{yy}+\lambda^2\epsilon_0^2)-q_0^2\alpha_0^2\epsilon_{xz}^2\epsilon_{yy}]=0\ . \tag{34}$$

We now use an ansatz specified by Eq. (I.43) for a film of small thickness. Taking into account the discussion after Eqs. (I.43) and (I.44), we derive the following dispersion relations to be satisfied simultaneously:

$$q_z^{(p)}\simeq q_0\left[\epsilon_0^{1/2}+\frac{(q_0 d)^2}{8}\epsilon_0^{3/2}\left[1-\frac{\epsilon_0}{\epsilon_{yy}}\right]^2\right] \tag{35a}$$

and

$$q_z^{(s)}\simeq q_0\left[\epsilon_0^{1/2}+\frac{(q_0 d)^2}{8\epsilon_0^{1/2}}(\epsilon_{xx}-\epsilon_0)^2\right]\ . \tag{35b}$$

Equation (35a) is the same as the formula derived[16] for p-polarized surface polaritons in the limit of a very thin film. In the absence of an applied magnetic field $\epsilon_{xx}=\epsilon_{yy}\equiv\epsilon$, where ϵ is the dielectric function of the thin film. Then Eq. (35b) reduces to the corresponding expression for s-polarized waveguide modes.[1]

It is possible to prove that the above-mentioned polarization properties are preserved even in the presence of an external magnetic field. In other words, Eq. (35a) describes p-polarized modes, while Eq. (35b) corresponds to s-polarized modes. The former (for our very thin film) are independent of $\mathbf{B}_0$; the contrary is true for the latter solutions. Another important distinction: the p-polarized solutions exhibit a resonance at $\omega_p/\sqrt{\epsilon_L}$, on the other hand the s-polarized modes resonate at ω_c (the pole of ϵ_{xx}). Due to these simple polarization properties for a very thin film it should not be difficult to excite these modes optically.

We have calculated the dispersion curves using Eqs. (35), for a given value of $\mathbf{B}_0$. For this purpose, we employ the following material parameters: $\epsilon_L=15.7$, $\epsilon_0=1.0$, $\omega_p d/c=2\pi\times10^{-2}$, and $\omega_c=0.5\omega_p/\sqrt{\epsilon_L}$. The parameters describe an unsupported InSb film. The numerical results in terms of dimensionless variables are depicted in Fig. 5.

Corresponding to Eqs. (35a) and (35b) we obtain two values of q_z for each value of ω. It is observed that one of the two values effectively coincides with the light line ($\omega=cq_z$). The other solution deviates from the light line and gives resonances at the cyclotron and screened-plasma frequencies.

We find that just above the higher resonance (at $\omega\simeq\omega_p/\sqrt{\epsilon_L}$) neither Eq. (35a) nor Eq. (35b) give bona fide solutions. This situation corresponds to the fact that, in this region, the decay constant α_0 attains negative values for both modes. Because of the very small thickness of the film α_0 is a perturbational quantity, given by

$$\alpha_0^{(p)}=\tfrac{1}{2}(q_0^2 d)\epsilon_0\left[1-\frac{\epsilon_0}{\epsilon_{yy}}\right] \tag{36a}$$

and

$$\alpha_0^{(s)}=\tfrac{1}{2}(q_0^2 d)(\epsilon_{xx}-\epsilon_0)\ , \tag{36b}$$

corresponding, respectively, to Eqs. (35a) and (35b). Analysis reveals that $\alpha_0^{(s)}<0$ in the range specified by

$$\omega_c<\omega<[\omega_c^2+\omega_p^2/(\epsilon_L-\epsilon_0)]^{1/2}\ .$$

Similarly, $\alpha_0^{(p)}<0$ in the interval defined by

$$\omega_p/\epsilon_L^{1/2}<\omega<\omega_p/(\epsilon_L-\epsilon_0)^{1/2}\ .$$

Since according to the prescribed form of the fields, Eqs. (7) and (9), α_0 has to be real and positive, we disregard the solutions of Eqs. (36) in the range where $\alpha_0<0$. The result is that a gap opens up in the spectrum in the range specified by $\omega_p/\sqrt{\epsilon_L}<\omega<\omega_p/\sqrt{\epsilon_L-\epsilon_0}$; where $\alpha_0^{(s)}$ and $\alpha_0^{(p)}$ are *both* negative.[17] As mentioned above, while our perturbational approach breaks down at both resonances ($\epsilon_{yy}=0$ and $\epsilon_{xx}\to\infty$), it is clear that splittings occur in the dispersion curves of the magnetoplasmons propagating in an unsupported film.

For sufficiently high frequencies a "fork" is seen in Fig. 5; a similar behavior was found in the Faraday configuration.[1] Of course, for $\omega\gg\omega_c$, $\omega_p/\sqrt{\epsilon_L}$ we have $\epsilon_{xx}\simeq\epsilon_{yy}\simeq\epsilon_L$ and $\epsilon_{xz}\simeq0$, that is, simply a very thin dielectric slab in vacuum. Then it is clear that the two high-frequency solutions correspond to the p- and s-

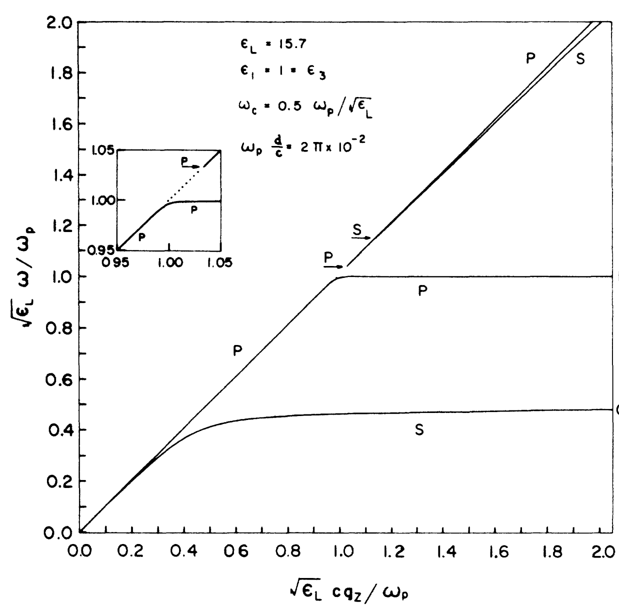

FIG. 5. Dispersion relation for magnetoplasma polaritons guided by a very thin, unsupported film corresponding to highly doped InSb. One solution practically coincides with the vacuum light line, except for being interrupted by a gap just above the screened plasma frequency; see inset. The arrows labeled p and s indicate the points at which modes of the corresponding polarization resume. The second solution exhibits resonances at the cyclotron and screened plasma frequencies. Note that, in the limit of a very thin film the modes are either p- or s-polarized, as marked.

polarized modes of a "single-mode," planar dielectric waveguide.

We conclude with the remark that although we have presented some numerical results in special cases (e.g., in the TFA), we have not classified the magnetoplasma modes as surface, waveguide, etc. In a future publication we intend to report detailed (exact) numerical results for the dispersion relation given by Eq. (20) and specify there the nature of the magnetoplasma modes.

APPENDIX

The dielectric tensor elements relevant to the present geometry (Fig. 1) are

$$\epsilon_{xx}=\epsilon_{zz}=\epsilon_L-\frac{\omega_p^2(\omega+i\nu)}{[(\omega+i\nu)^2-\omega_c^2]}\ ,$$

$$\epsilon_{xz}=i\frac{\omega_p^2\omega_c}{\omega[(\omega+i\nu)^2-\omega_c^2]}\ , \tag{A1}$$

$$\epsilon_{yy}=\epsilon_L-\frac{\omega_p^2}{\omega(\omega+i\nu)}\ .$$

Here ϵ_L is the background dielectric constant of the film medium, ν is free-carrier collision frequency, and ω_c and ω_p are, respectively the cyclotron frequency and unscreened plasma frequency, defined as follows:

$$\omega_c=\frac{e|\mathbf{B}_0|}{m_e c}\ ,\quad \omega_p^2=\frac{4\pi n e^2}{m_e}\ . \tag{A2}$$

Here e, m_e, and n are, respectively, the electronic charge, effective mass, and free-carrier concentration in the semiconducting film (region II in Fig. 1). In Eqs. (A1), if we also consider the effect of phonons, which, in a way, allows the coupling of the magnetoplasma polaritons to optical phonons, then the background dielectric constant ϵ_L has to be replaced by a frequency-dependent expression, given by (I.A3).

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[17]Just above ω_c, $\alpha_0^{(p)}>0$, therefore the p-polarized mode is allowed to propagate, while there is a gap for s-polarized waves.