Anomalous field dependence of the Hall coefficient in disordered metals

E. Tousson
Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel

Z. Ovadyahu
Racah Institute of Physics, The Hebrew University, Jerusalem, Israel
(Received 19 May 1988; revised manuscript received 23 August 1988)

We report on a comprehensive study of the Hall coefficient, R_H, in disordered three-dimensional In_2O_{3-x} films as a function of the magnetic field strength, temperature, and degree of spatial disorder. Our main result is that, at sufficiently small fields, R_H is virtually temperature, field, and disorder independent, even at the metal-insulator transition itself. On the other hand, at the limit of strong magnetic fields, R_H has an explicit temperature dependence, in apparent agreement with the prediction of Al'tshuler, Aronov, and Lee. For intermediate values of fields, R_H is field and temperature dependent. It is also shown that the behavior of the conductivity as a function of temperature, $\sigma(T)$, at small fields, is qualitatively different than that measured at the limit of strong magnetic fields. The low- and high-field regimes seem to correlate with the respective regimes in terms of the Hall-coefficient behavior. It is suggested that the magnetotransport in the high-field limit is considerably influenced by Coulomb-correlation effects. However, in the low-field regime, where both correlations and weak-localization effects are, presumably, equally important (and where both theories are the more likely to be valid), is problematic; neither R_H nor $\sigma(T)$ gives any unambiguous evidence to the existence of interaction effects. This problem is discussed in light of the experimental results pertaining to the behavior of $R_H(T)$ in two-dimensional In_2O_{3-x} films as well as in other disordered systems. It is argued that, as far as R_H is concerned, the effects of weak localization and Coulomb correlations may not be additive.

I. INTRODUCTION

Our understanding of low-temperature transport properties in disordered metals has progressed considerably over the past decade. Extensive theoretical and experimental investigations[1] led to new concepts regarding two major aspects of charge transport in disordered media; Anderson localization and Coulomb interaction effects. Triggered by the seminal papers of Abrahams *et al.*[2] and Altshuler *et al.*[3] the literature in this field has expanded at a high rate with attempts to elucidate the various specific effects predicted by either approach. It became evident that, in general, two distinct contributions to the low-temperature transport coexist: Those that may be ascribed to the single particle diffusive motion in a random potential and those that reflect the underlying Coulomb interactions in the presence of static disorder. Following the current terminology, we refer to the former as weak localization (WL) and to the latter as Coulomb-correlation (CC) effects. A specific effect due to WL is, e.g., the low-field magnetoresistance associated with suppression of backscattering.[4] Such an effect is found in most disordered metals and semiconductors and in systems with reduced dimensionality it may be unambiguously ascribed to WL due to its anisotropy. A specific CC effect is the single-particle density of states anomaly which is also quite commonly found.[5-7] On the other hand, corrections to the dc conductivity, σ, predicted by these theories, are more difficult to analyze: Both approaches often predict similar behavior. It has been sometimes claimed that by combining magnetoresistance and conductivity versus temperature data, the two "components" may be disentangled. Implicit to such a procedure is the assumption that the two contributions superimpose. A critical test of the theories (and, in particular, the assumption of "superposition"), is the detailed behavior of the Hall coefficient, R_H: According to WL theories[8] there are no specific corrections to R_H which is expected to be temperature and disorder independent. CC theories, on the other hand, predict[9] that there will be corrections to R_H that are, specifically, twice the respective corrections to σ. If the effects due to these two mechanisms do, in fact, simply add up, it should be possible to tell, independently of other measurements, what is the relative contribution of each mechanism to σ by considering $\sigma(T)$ in conjunction with $R_H(T)$ data.

While the relevance of Hall-effect measurements to these issues was immediately recognized, relatively few results have been reported in the literature. Furthermore, in most cases, strong magnetic fields were employed in the measurements such that $L_H < L_{in}$ [$L_H \approx (ch/eH)^{1/2}$ is the magnetic length and $L_{in} \approx (D\tau_{in})^{1/2}$ is the inelastic diffusion length where D and τ_{in} are the diffusion constant and inelastic mean free time, respectively]. In this limit of high field it is usually found[10-14] that R_H is temperature dependent. The low-field limit (namely, $L_H \gg L_{in}$) received much less attention. In 2D (two dimensional) In_2O_{3-x} (Ref. 15) and Pd (Ref. 16) films that were measured using relatively weak fields, R_H was found to be temperature independent. Interestingly, in both systems, prominent density-of-states anomalies were found in tunneling measurements.[6,7] There is no contradiction between the existence of CC effects in the tunneling experiments and a temperature independent R_H (that might be interpreted as no contribution from CC effects to the dc conductivity): The density of states probed by tunneling is the nonequilibrium value, $N(0)$, whereas the conductivity involves the "thermodynamic" density of states. It is nevertheless surprising that the CC contribution to the conductivity may be as small as the Hall measurements would appear to suggest. In theory, CC corrections to conductivity and R_H may be very small if the material relevant parameters happen to be "right." But In_2O_{3-x} and Pd are quite different systems (charge density in Pd is higher by 2 orders of magnitude) which makes it unlikely that the small corrections to R_H in both systems result from a coincidental choice of parameters.

In this paper we report on the behavior of R_H and σ for 3D In_2O_{3-x} samples measured as a function of temperature and disorder. Emphasis is given to $R_H(T)$ measured at extremely weak fields ($H \leq 160$ Oe) where it can be demonstrated that the variation in R_H is much smaller than anticipated by CC theories. We then show that R_H measurements on the same 3D In_2O_{3-x} samples, using higher magnetic fields (up to 7 T), give quite different results. In particular, it is shown that the prediction of CC theories is approached at sufficiently intense fields. We discuss these results as well as those obtained on other systems to question the validity of the "superposition conjecture" alluded to above.

II. EXPERIMENT

The In_2O_{3-x} films used in this study were 2000 Å thick and had carrier density, N, of 2×10^{19} cm^{-3} to 9×10^{19} cm^{-3} (as determined by room-temperature Hall measurements). Full details of sample preparation and characterization techniques are given elsewhere.[17] In the following, different samples are labeled by their room-temperature conductivity, σ_{RT} or by their $K_F l$ values calculated through

$$K_F l = (3\pi^2)^{2/3}\hbar(R_H)^{1/3}\sigma_{RT}/e^{5/3} .$$

The low field R_H and conductivity measurements were made in a standard ^{4}He glass dewar that was mounted within the air gap of a split-coil electromagnet. The latter was energized by an alternating current power supply operating at a fixed frequency of 13 Hz. The maximum intensity of the field used with this arrangement was 160 Oe (rms). A dc current was maintained along the sample (which, typically, was 15 long and 6 mm wide strips with two opposite sets of voltage contacts alongside) and the ac transverse voltage was fed to the differential input of a PAR-124 lock-in amplifier. The latter was phase synchronized to 90° of the voltage induced in a small pick-up coil mounted on a teflon sample holder. This pick-up coil was also used to monitor the field strength *in situ*. The Hall voltage, V_H, was derived by properly summing the final readings obtained for each of the two opposite current directions. Signal averaging was shared between the lock-in amplifier and the statistical features of a HP 3456A digital voltmeter connected to the PAR 124 dc output. With this arrangement, V_H resolution of the order of 2–10 nV was routinely achieved. A typical average value for V_H was ≈ 1 μV giving a basic accuracy figure of the order of 0.2–1 %. All our measurements in this regime include readings at 160 Oe as well as at 100 Oe to ensure that R_H is field independent. It was also ascertained that the longitudinal current used in the measurements is small enough to be in the Ohmic regime by repeating each measurement at a lower current than the nominal value used. The relevance of these steps will become clear below.

For the high-field R_H measurements we employed a standard dc technique. The samples were mounted perpendicularly to the field direction of a 7-T superconducting solenoid and the Hall voltage was obtained as the average of the two perpendicular field directions. This method was considerably less accurate than that employed for the low-field regime discussed above. Typical V_H resolution with the dc method was 5 μV. This resolution resulted in low quality data for H < 0.5 T but it is quite useful for higher fields since V_H is then ≥ 100 μV. Conductivity versus temperature measurements were made by a standard 4-point dc technique. Temperature was determined by means of either gas thermometry or by a calibrated linear resistor.

III. RESULTS AND DISCUSSION

Figure 1 depicts $\sigma(T)$ data for several metallic samples. At the range of temperatures shown it is found that the

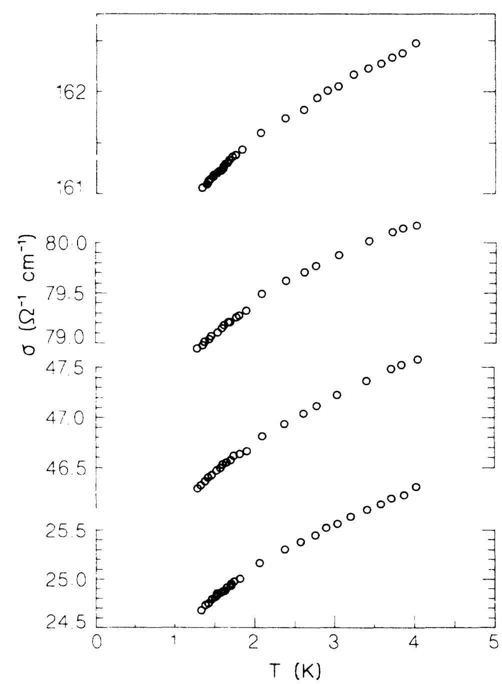

FIG. 1. Conductivity vs temperature for typical "metallic" ($\sigma_0 > 0$) In_2O_{3-x} samples.

data may be fit to $\sigma(T)=\sigma_0+AT^x$ with $x=0.5\pm0.1$. The dependence of the fitting parameters σ_0 and A on disorder is described in Figs. 2 and 3, respectively, for nine conducting ($\sigma_0>0$) samples. Some relevant parameters for these samples are listed in Table I. Though it has no important bearing on the main issue of this paper, we note that σ_0 goes to zero with disorder in agreement with the behavior observed in other systems. Figure 3 deserves more attention: It is noted that A does not show any systematic dependence on disorder over a considerable range. This means that the temperature dependent part of the conductivity is roughly constant, independent of disorder, for conducting In_2O_{3-x} samples. A similar observation was made before on other 3D systems by Dodson *et al.* and by Bishop *et al.*[18] This observation leads to an immediate difficulty in trying to ascribe the observed $\sigma(T)$ to CC effects. Interaction theories[1] predict the following $\sigma(T)$ for a 3D disordered metal:

$$\sigma(T)=\sigma_0+A_I^{(3D)}(k_BT/\hbar D)^{1/2}\ , \qquad (1)$$

where k_B is the Boltzmann constant and A_I is given by

$$A_I^{(3D)}=1.3e^2/4.2^{1/2}\pi^2\hbar(4/3-3F_1^{(3D)}/2)\ , \qquad (1a)$$

and

$$F_1^{(3D)}=-\tfrac{32}{3}[1+3F/4-(1+F/2)^{3/2}]F\ . \qquad (1b)$$

F is the screening parameter that depends on the Fermi wave vector and of the screening length. We estimate that $F\approx0.64$ for In_2O_{3-x} which, using Eq. (1b) gives $F_1^{3D}\approx0.2$. From Eq. (1) and the expression $D=(3\pi^2)\hbar^2/3me^2\sigma_{RT}N^{-1/3}$ one can estimate the value of A expected by CC theories. The calculated A's are plotted in Fig. 3 along with the actual experimental results. The functional temperature dependence (i.e., the value of x), as well as the order of the magnitude of A, seem to be in good agreement with the theory. On the other hand, the dependence of A on disorder is less satisfactory. It may be argued that the observed discrepancy between theory and experiment is not compelling enough to give a reason to be concerned. But the detailed behavior of the Hall effect of these samples does: Interaction theories[9] predict that R_H will exhibit a temperature-dependent correction term similar to that of $\sigma(T)$ but with twice the magnitude of the latter. Namely, it is expected that $\delta R_H/R_H=2\delta\sigma/\sigma$. If, as might be inferred from Fig. 3, most of the temperature dependence of σ is ascribed to CC effects, one must also expect a certain change of R_H with temperature. Our results, however, show that when measured at sufficiently weak fields, these corrections are absent. $R_H(T)$ data for several samples (both, conducting and insulating for comparison) are shown in Fig. 4. In all cases, R_H was also measured at

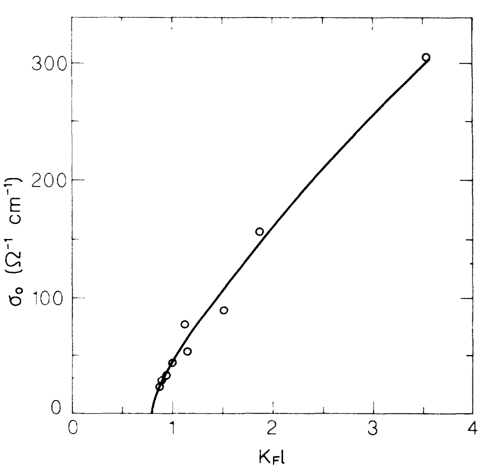

FIG. 2. The dependence of the extrapolated, zero-temperature conductivity, σ_0, on disorder. The solid line is given by $\sigma_0=141(K_Fl-0.80)^{0.75}$.

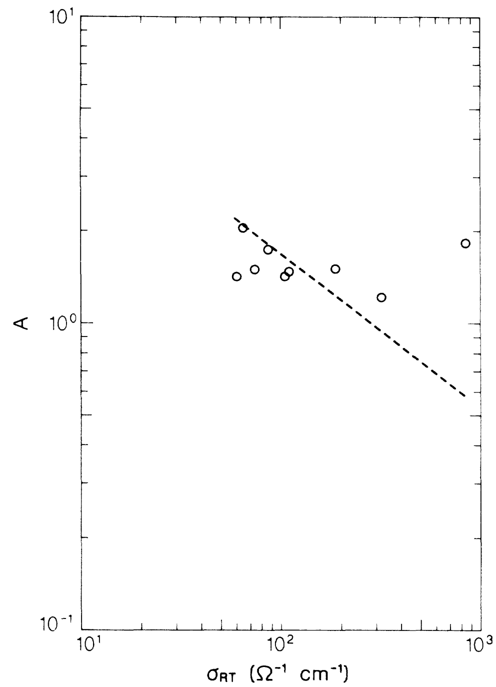

FIG. 3. The dependence of the fitting parameter A on the room-temperature conductivity for the nine samples of Fig. 2. The theoretically expected value for A [based on Eqs. (1), (1a), and (1b) in text] is depicted by the dashed line.

TABLE I. Some relevant parameters for the samples studied in Figs. 2 and 3.

Sample	N (10^{19} cm^{-3})	ϵ_{RT} (Ω^{-1}cm^{-1})	$K_F=1$	σ_0 (Ω^{-1}cm^{-1})
1	5.90	301.2	3.55	303.8
2	9.05	181.2	1.86	158.0
3	4.80	119.4	1.51	89.3
4	4.17	87.3	1.16	53.1
5	7.14	103.7	1.13	77.0
6	4.03	74.8	1.01	44.0
7	4.55	72.5	0.94	32.2
8	3.60	64.8	0.91	29.8
9	2.85	60.3	0.89	23.0

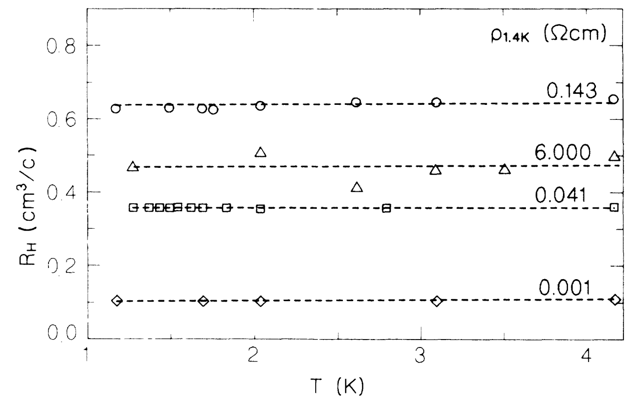

FIG. 4. Characteristic $R_H(T)$ data for two metallic (bottom curves) and two "insulating" (upper curves) samples in the liquid ^{4}He range of temperatures.

room temperature and at 77 K. A list of parameters pertinent to the present discussion is given in Table II. It should be emphasized that the accuracy figures quoted for the various R_H's in Table II reflect the scatter in the data over the measured temperature range. The systematic dependence on temperature expected by the CC theory is not observed in any of these samples (cf. Table II). The behavior of R_H vis-a-vis that of the resistivity of our samples is depicted in Fig. 5. As we pointed out before,[19] these findings are consistent with the scaling theory of Shapiro and Abrahams[20] according to which the temperature dependence of σ is a mobility effect. Here we shall be more concerned with the apparent absence of CC contributions to R_H and restrict the discussion to metallic samples (namely, samples with $K_F l > 1$ for which the theory strictly applies).

Considered in isolation, these data may be interpreted as an indication that there are no CC corrections to the conductivity of 3D In_2O_{3-x} samples. One may consider the possibility that the material-parameters, on which the

TABLE II. Parameters of samples studied in the low-field regime ($H \leq 160$ Oe). $\delta R_H/R_H$ is based on the uncertainty in the data over the entire range of temperature studied (1.14–77 K). These entries should be compared with the respective values of $2\delta\rho/\rho$ in the last column. The latter are based on the prediction of the CC theory (which, according to Fig. 3, is a consistent procedure for samples with $K_F l \leq 3$). $\delta\rho/\rho$ is the measured fractional change of the resistivity in the above temperature range.

Sample	$K_F l$	R_H (cm^3/C)	$\delta R_H/R_H$ (%)	$2\delta\rho/\rho$ (%)
1	8.48	0.103±0.0002	0.20	5.6
2	3.61	0.193±0.0006	0.31	10.4
3	3.37	0.082±0.0005	0.61	12.0
4	2.31	0.122±0.0005	0.41	13.8
5	1.86	0.108±0.0008	0.74	15.6
6	1.51	0.174±0.0010	0.57	18.5
7	1.13	0.139±0.0040	2.90	20.3
8	1.01	0.252±0.0010	0.40	52.0
9	0.89	0.347±0.0050	1.44	78.0

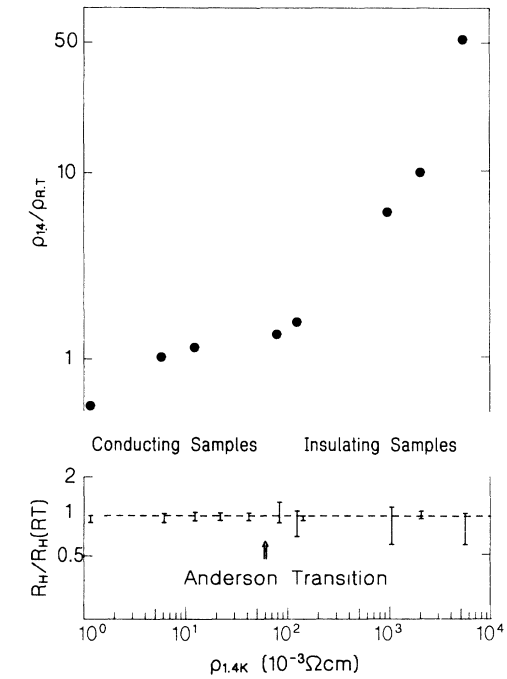

FIG. 5. The variation, with temperature, of the resistivity and the Hall coefficient of In_2O_{3-x} samples as a function of disorder. The upper set of data depicts the resistivity ratio, ρ (at 1.4 K)/ρ (at room temperature). The lower set of data illustrates the behavior of the Hall coefficient in the same range: Each "datum line" signifies the range of values measured for $R_H(T)$, at several temperatures in the 1.4–300 K range, normalized to R_H measured at room temperature.

theoretical estimates for $\delta R_H/R_H$ in Table II are based upon, are off the mark (or that the particular version of the theory we use is inaccurate). There are several ways to see that the problem is more serious than that. In particular, it should be noted that the same problem with $R_H(T)$ discussed here for 3D In_2O_{3-x} samples has been reported before[15] for effectively 2D ones. The Hall-effect measurements on 2D samples were carried out with ~200 Å thick films that in terms of structural aspects and carrier density are identical with the much thicker, effectively 3D samples of the present work. The inverse of the Fermi wave vector and the screening length parameter of In_2O_{3-x} are much smaller than 200 Å (being of the order of 10^{-7} cm). One is then well justified in assuming that the same parameter F must be assigned for both dimensionalities under consideration. We are not aware of any version of a CC theory that gives $\delta R_H = 0$ in 2D and 3D for the same F. To illustrate, let us look at a specific case. Using Eqs. (1), (1a), and (1b) above and our $R_H(T)$ data we solve for $F_1^{(3D)}$ that is consistent with the finding $\delta R_H = 0$. That gives $F_1^{(3D)} \approx 0.88$ which, by (1b), implies $F \approx 0.96$. The 2D variation of Eqs. (1) above reads[1]

$$\delta\sigma(T) = A_I^{(2D)} \ln T \ , \quad (2)$$

$$A_I^{(2D)} = e^2/4\pi^2\hbar(2 - 3F_1^{(2D)}/2) \ , \quad (2a)$$

$$F_1^{(2D)} = 8(1+F/2)\ln(1+F/2)/F - 4 \ . \quad (2b)$$

Inserting $F \approx 0.96$ in (2b) gives $F_1^{(2D)} \approx 0.835$. By Eq. (2), the expected correction to the conductivity of effectively 2D In_2O_{3-x} is given by $\delta\sigma(T) \approx 0.74e^2/(4\pi^2\hbar)\ln T$ which accounts to about one-third of the experimentally observed temperature dependence.[15] That, in turn, means that $R_H(T)$ in Ref. 15 ought to have shown a variation of the order of 20% in the temperature range 1–100 K whereas the actual result was $\delta R_H(T)=0$.

Taking into account the finite accuracy of Hall-effect measurements, ($\sim 1\%$), the finite range of temperature studied (1–100 K), which limits the measured variation in σ, one can claim that the $R_H(T)$ measurements on In_2O_{3-x} samples show that, at least for samples with $K_F l \lesssim 3$, CC theories for the Hall coefficient are inadequate.

To the best of our knowledge, thin Pd (Ref. 16) and Cu:SiO_x films[21] are the only other systems where a temperature independent R_H was observed. A temperature-dependent Hall coefficient, on the other hand, has been reported by a number of researchers: Uren *et al.*[10] and Bishop *et al.*[11] studied Si-inversion layers at subdegree temperatures and fields in the range 0.1–3 T. Both groups reported $R_H(T)$ in fair agreement with the CC theory. A similar observation was made by Drewery and Friend[12] for 3D Cu_xTi_{1-x} films at the temperature range 1.24–25 K (employing 1.3-T field in the Hall measurements). An explicit temperature dependence for R_H has been also reported for granular Al samples[13] measured over the temperature range of 1.5 to 80 K and fields of the order of 9 T and for Ge:Sb samples[14] measured down to 8 mK with magnetic fields of 2.7 kOe.

A temperature-dependent Hall coefficient seems therefore to be the more abundant result. It would appear then, that it is the null result observed for In_2O_{3-x} samples and 2D Pd films that should perhaps be questioned.

We do not believe that Pd and In_2O_{3-x} are the "bad actors" of transport in disordered media. These materials have been studied quite extensively with regards to both the relevant transport properties and structural parameters. In all cases where the interpretation is unambiguous, like, e.g., the magnetoresistance or the tunneling density of states, the results obtained are similar to those observed in other systems. Instead, we propose to question the relevance of the claims that R_H is temperature dependent in other disordered systems.

We want to explore the possibility that the qualitative difference in the $R_H(T)$ results is due to different measuring techniques rather than different materials. In particular, we demonstrate below that the magnitude of the magnetic field used in the Hall-effect measurement may considerably influence the results and thus affect the conclusions drawn from the experiment.

From the theoretical point of view, the limitation on the field strength is quite explicit: H has to be small enough such that $L_H \gg L_{\text{in}}$. Incidentally, this is the range of fields where the magnetoresistance due to WL is quadratic in the field. To our best knowledge, the only 3D system to date, where this caveat has not been ignored, is the present one. Both, WL and CC theories for R_H assume "linear response" and therefore are strictly valid in the $H \to 0$ limit only. That means that the longitudinal resistance and the Hall resistance are supposed to be "Ohmic." At the very least one should verify, then, that R_H is independent of H under the conditions that actually prevail in the experiments before comparison with the theory is attempted. These requirements are very restrictive and place severe limits on the magnetic field as well as on the longitudinal bias current employed in the Hall-effect experiment. Restricting the measurements to very small electric and magnetic fields entails a certain compromise on the ultimate accuracy. But overlooking these restrictions, may cause greater difficulties. Figure 6 describes a typical dependence of the resistance on the applied electric field for one of the samples studied. The limit on the range of Ohmic behavior is clearly observed. As might be expected, this problem becomes more severe with larger disorder or lower temperatures. Biasing the sample deeper into the non-Ohmic regime is usually accompanied by an increasingly poor signal-to-noise ratio in the Hall measurement. We suspect that this problem is due to a current-dependent noise generating mechanism. Since the Hall signal scales linearly with the bias current, the poorer signal-to-noise ratio observed at higher biases might indicate a noise mechanism that has a faster-than-linear dependence on current (e.g., shot noise).

It is more interesting to see what happens when the condition on low magnetic field is not obeyed. This is demonstrated in Figs. 7–11 for a particular sample. The same qualitative features were observed on three other samples that have been measured in the high-field regime described next. Figure 7 depicts the dependence of V_H on the magnetic field at two different temperatures. This curve bears a remarkable similarity to that observed by Field and Rosenbaum[14] on Ge:Sb samples. In particular, both curves are evidently nonlinear above a certain field. In fact, it is easy to overestimate the value of the field below which V_H is linear in H from such $V_H(H)$ plots: As can be seen in Fig. 8, R_H is field dependent at fields much smaller than might be assessed by eye balling Fig. 7. This "non-Ohmicity" is not very strong but it is this

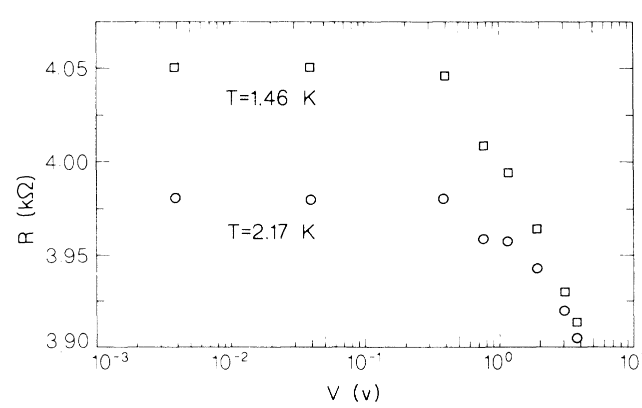

FIG. 6. Longitudinal resistance as a function of the applied electric field for sample 9 in Table I.

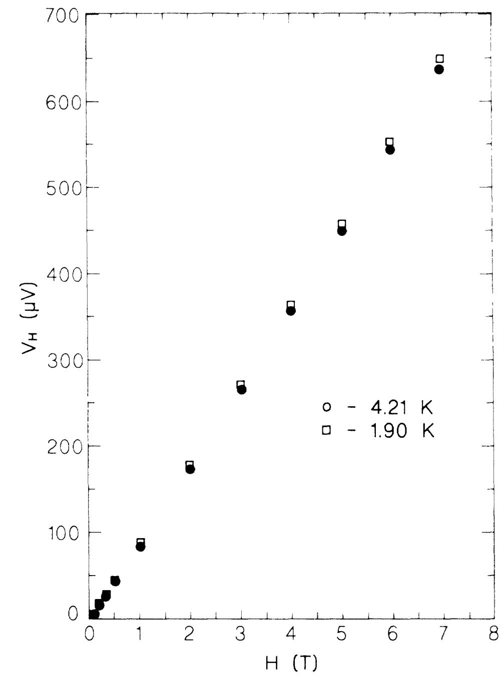

FIG. 7. Hall voltage as a function of the magnetic field for sample 6 in Table II.

small effect that may completely change the qualitative features of the $R_H(T)$ data! That is so because δR_H should be compared with $\delta\rho$ which (in the WL regime) is usually small too. This point can be illustrated by the observed field dependence of the fractional change in R_H normalized to that of the resistance (over the same temperature range). This is shown in Fig. 9. The following features are observed. At small fields, $\delta R_H/\delta\rho$ increases sharply. By extrapolation to zero field it seems conceivable that $\delta R_H/\delta\rho$ will go to zero consistent with the results of our direct measurements in the low-field regime. At higher fields, $\delta R_H/\delta\rho$ tends to saturate at a value close to 2 which appears to be in agreement with CC theories. Ben-Shlomo and Rosenbaum[22] have recently measured $R_H(T)$ in 2D In_2O_{3-x} films similar to those reported by Ovadyahu and Imry.[15] Ben-Shlomo and Rosenbaum employed fields of the order of several tesla

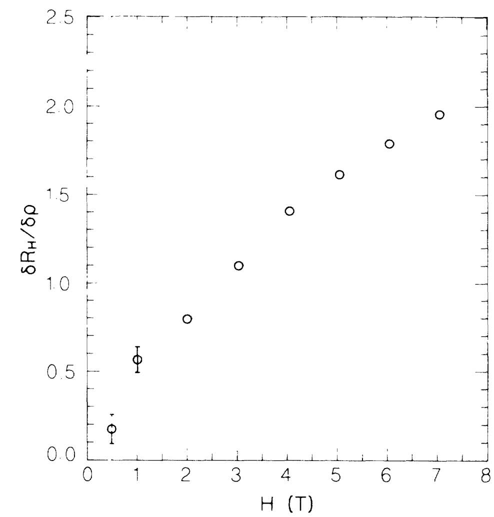

FIG. 9. The fractional change of the Hall coefficient, $\delta R_H \equiv [R_H(T=1.9\text{ K}) - R_H(T=4.2\text{ K})]/R_H(T=4.2\text{ K})$ normalized to the respective fractional change of the resistivity for the sample in Fig. 7.

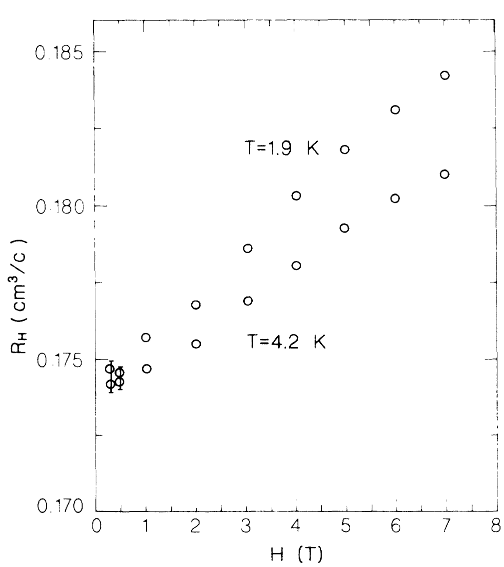

FIG. 8. Hall coefficient as a function of the magnetic field for the sample in Fig. 7.

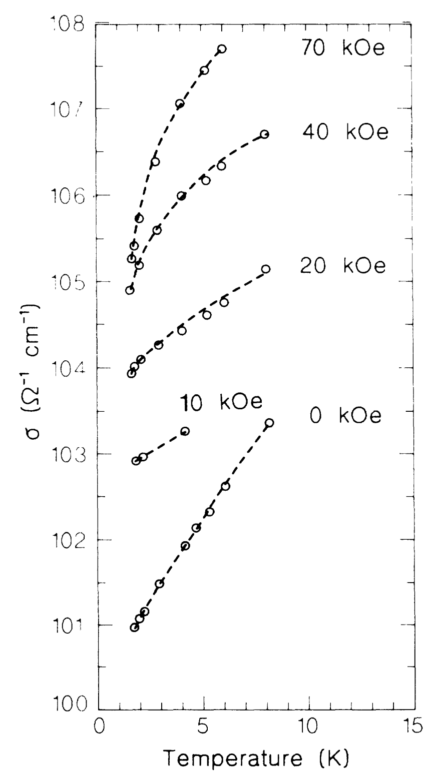

FIG. 10. Conductivity vs temperature at various magnetic fields in the "high-field" regime (same sample as in Fig. 7).

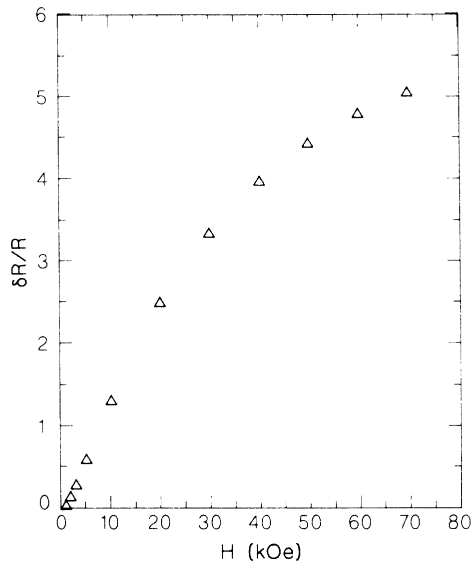

FIG. 11. The negative fractional change of the resistance as a function of magnetic field for the sample shown in Fig. 7 (at $T=4.2$ K).

in their Hall measurements and found $R_H(T)$ in general agreement with CC theories. It is recalled that in the work of Ovadyahu and Imry[15] (where $\delta R_H=0$ was established for the first time), considerably weaker fields (300–1500 Oe) were used.

It is noteworthy that a qualitatively similar field dependence of $\delta R_H/\delta\rho$ (Fig. 9), was reported by Dynes[23] and by Davies and Pepper[24] for Si inversion layers. Dynes, in fact, has deduced that $\delta R_H=0$ for sufficiently small fields on the basis of the decline of $\delta R_H/\delta\rho$ with reduced field.

Figure 10 shows $\sigma(T)$ data for several magnetic fields (within the high-field regime). The application of the field is seen to affect $\sigma(T)$ in a similar way that it does for Si inversion layer:[24] Up to a certain value, the effect of the field is to reduce the temperature dependence of σ. Above $H\approx 2$ T, however, the reverse is observed, namely, $\sigma(T)$ acquires a stronger temperature dependence. Following Davies and Pepper[24] we ascribe this evolution to the delocalizing effect of the field for $H<2$ T and to the growing significance of CC effects (i.e., a positive magnetoresistance component) with increasing H/T for $H>2$ T. From the discussion so far it seems evident that the qualitative features of $R_H(T)$ in other systems may be obtained with In_2O_{3-x} samples (in both, 2D and 3D) by sufficiently increasing the measuring field strength. At the same time, it is observed that there may be more reason to question the relevance of such "finite" field data to the theories that support to account for them. In particular, one should be suspicious of data taken at "small fields" which are not small enough. (By that we refer to data taken in the regime where R_H is manifestly field dependent.) How small should the field be to qualify as small enough? As a partial answer we venture the criterion: "So small that R_H is field independent." But a fuller answer is ultimately connected with our original problem: What is the reason for the apparent absence of CC effects in the low-field regime? The arguments raised above concerning the comparison between data in 2D and 3D of the same material expose a real problem of the CC theory: The $\delta R_H=0$ cannot possibly be explained by an "accidental" choice of parameters.

A possible clue to the origin of the problem may be obtained by comparing the data in Fig. 9 with the magnetoresistance of the same sample shown in Fig. 11. The interesting feature observed in this comparison is that $\delta R_H/\delta\rho$ is small in the same field regime where $\delta R/R$ is. This correlation has been noted before by Dynes[23] at a lower range of fields (cf. Figs. 3 and 4 in Ref. 23). As remarked above, this is consistent with the condition $L_H \gg L_{\text{in}}$ to be in the low-field regime from the point of view of WL. In other words, when the field is so small that WL effects are virtually undisturbed, $R_H(T)$ behaves in agreement with the scaling theory as if CC effects are not there. On the other hand, when WL effects are suppressed, $R_H(T)$ does show significant correlation effects. This observation may suggest that neglecting the maximally crossed diagrams in the CC formal calculation[9] for the Hall coefficient is essential to the validity of the formal result. This, in turn, presumably means that CC and WL effects do not superimpose [at least for the particular case of $R_H(T)$].

Such an interpretation is, admittedly, speculative but it seems to be consistent with the experimental evidence. Note that the condition $L_H > L_{\text{in}}$ implies that the low-field regime is given by $H<H_c$ with $H_c \propto L_{\text{in}}^{-2}$. Much of the confusing state of affairs in this field of research might be related to the violation of this condition. For example, the following additional observations can be qualitatively understood.

(1) For a given range of temperatures and strength of the measuring field, $\delta R_H/\delta\rho$ becomes smaller as the disorder increases (that causes L_{in} to decrease).[11]

(2) 3D In_2O_{3-x} samples were measured before[25] at temperatures $T>12$ K. R_H was reported to be temperature independent up to fields of the order of 1 T whereas in the present work (where lower temperatures were used), the same fields result in R_H being temperature dependent. Further theoretical and experimental work is clearly needed to clarify these issues and to find out whether this problem is unique to the Hall coefficient. At any rate, the anomalous field dependence of R_H discussed above should be borne in mind when analyzing Hall-effect data. One should be aware of the possibility that finite-field measurements may grossly affect the qualitative features of the data and, in particular, may give a temperature and disorder dependent R_H that is not there when the proper, $H\rightarrow 0$ limit is taken.

ACKNOWLEDGMENT

This research has been partially supported by a grant administered by the Israeli Academy for Sciences and Humanities.

[1]P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[2]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[3]B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).

[4]G. Bergmann, Phys. Rep. **101**, 1 (1984).

[5]B. Abeles, P. Sheng, M. D. Coutts, and Y. Arie, Adv. Phys. **23**, 407 (1975); W. L. McMillan and J. M. Mochel, Phys. Rev. Lett. **46**, 556 (1981); R. C. Dynes and J. P. Garno, *ibid.* **46**, 137 (1981).

[6]Y. Imry and Z. Ovadyahu, Phys. Rev. Lett. **49**, 841 (1982).

[7]M. Gijs, Y. Bruinseraede, and A. Gilabert, Solid State Commun. **57**, 141 (1986).

[8]H. Fukuyama, J. Phys. Soc. Jpn. **49**, 644 (1980).

[9]B. L. Altshuler, D. E. Khmelmitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[10]M. J. Uren, R. A. Davies, and M. Pepper, J. Phys. C **13**, L985 (1980).

[11]D. J. Bishop, D. C. Tsui, and R. C. Dynes, Phys. Rev. Lett. **46**, 360 (1981).

[12]J. S. Drewery and R. H. Friend, J. Phys. F **17**, 1739 (1987).

[13]B. Bandyopadhyay, P. Lindenfeld, W. L. McLean, and H. K. Sin, Phys. Rev. B **26**, 3476 (1982).

[14]S. B. Field and T. F. Rosenbaum, Phys. Rev. Lett. **55**, 522 (1985).

[15]Z. Ovadyahu and Y. Imry, Phys. Rev. B **24**, 7439 (1981).

[16]W. C. McGinnis and P. M. Chaikin, Phys. Rev. B **32**, 6319 (1985).

[17]Z. Ovadyahu, J. Phys. C **19**, 5187 (1986).

[18]B. W. Dodson, W. L. McMillan, J. M. Mochel, and R. C. Dynes, Phys. Rev. Lett. **46**, 46 (1981); D. J. Bishop, E. G. Spencer, and R. C. Dynes, Solid State Electron. **28**, 73 (1985).

[19]E. Tousson and Z. Ovadyahu, Solid State Commun. **60**, 407 (1986).

[20]B. Shapiro and E. Abrahams, Phys. Rev. B **24**, 4025 (1981).

[21]N. Savvides, S. P. McAlister, C. M. Hurd, and I. Shiozaki, Solid State Commun. **42**, 143 (1982).

[22]M. Ben-Shlomo and R. Rosenbaum (unpublished).

[23]R. C. Dynes, Surf. Sci. **113**, 510 (1982).

[24]R. A. Davies and M. Pepper, J. Phys. C **16**, 361 (1983).

[25]E. Tousson and Z. Ovadyahu, Phys. Lett. **109A**, 187 (1985).