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Critical exponents of percolating wire networks

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Measurements of electronic and magnetic properties of percolating, aluminum wire networks are presented and compared with numerical and analytic results. As opposed to vapor-deposited films, these networks are well suited for comparison to theory of two-dimensional percolation since they are exact replicas of models used to study percolation. We find that the ratio of the exponents for the conductivity and the percolation length, $t/\nu = 0.98 \pm 0.03$, is consistent with recent computer studies but not with the Alexander-Orbach conjecture. The critical current exponent, within error, is given by $v = \nu = \frac{4}{3}$. Finally, the critical field exponent is given by $k = 1.11 \pm 0.04$, consistent with our prediction of $k = \nu\Theta \sim 1.15$ as well as with numerical studies.

I. INTRODUCTION

Much of the current work on disordered systems uses percolation as a model for the sample geometry. The effects of this geometry on both normal and superconducting behavior are of interest. For example, the percolation model may help to describe the metal (superconducting) -insulator transition in certain metal-nonmetal composites.[1] Also of interest are the properties of fractals which may offer a systematic method for studying disorder.[2,3] Percolating networks are expected to reflect an underlying fractal geometry common to many disordered systems.[1,3]

To date, studies of percolating systems have been dominated by analytic and numerical approaches. Experimental work has been carried out on thin, single-component films and alloys, each of which is only an approximate example of the regular lattices explored theoretically. Critical exponents for normal-state properties have been studied numerically and include t, the conductivity exponent,[4] ν, the correlation length exponent,[1] β, the mass exponent, and f, the elasticity exponent.[5] More recently, analytical and numerical work has been used to determine critical exponents for superconducting properties such as k, the critical field exponent[6-8] and v, the critical current exponent.[9] In addition, measurements have been performed on thin-film materials such as Al-Ge systems which are expected to have a percolative geometry.[1,6]

There are inconsistencies between theory and the results of experiments on real, disordered materials. These include measurements of k, where theory ranges from 0.7 to 1.16 and experiment has found $k \simeq 0.6$, and analytic determinations of v. For the latter exponent, the only measurements are those on quasi-two-dimensional Pb-Ge films where the value of t may be in question.[10] Inconsistencies may be due to the differences between the underlying models which involve regular lattices in the theory and random lattices in experiments.

Advances in lithography have allowed us to fabricate percolating wire networks on a square lattice, precisely the model upon which calculations are made. In this paper, we present measurements of three critical exponents [t,v (or ν) and k] that are relevant to two-dimensional bond percolation on a square lattice. Because our samples exactly replicate the models used by numerical and analytical studies, and are actually very large on the scale of objects studied numerically, we expect our results to not be size limited and to accurately reflect theoretical work. Deviations between critical exponents characteristic of our networks and those of thin films suggest important differences in their respective geometries.

II. EXPERIMENTAL DETAILS

Samples were prepared by depositing pure aluminum films onto oxidized silicon substrates. The films, 50 nm thick, were evaporated through a lift-off mask written in two-layer electron-beam resist by a Cambridge model EBMF-2-150 electron beam microfabricator. These networks are identical to those reported upon earlier.[3] They are formed on an 800×800 square lattice of wire bonds, present with probability p. The wire widths were approximately 0.3 μm and the mesh size was $a = 1.7$ μm. Normal-state resistances varied from 1 to 100 Ω and the superconducting coherence length in all samples was $\xi_s(0) = 0.22$ μm.

III. RESULTS

First we consider a normal state property of the lattice, its resistance, $R = 1/G$ (G is the conductance). The critical exponent for the resistance of the network is defined by

$$R \propto (p - p_c)^{-t}, \tag{1a}$$

or equivalently,

$$G \propto (p - p_c)^{t}, \tag{1b}$$

where p_c is the percolation threshold. We can estimate t using the "nodes, links, and blobs" picture of percolation.[11] In this picture, the original lattice with mesh size a is replaced by a larger lattice with a mesh size equal to the percolation correlation length, $\xi_p \simeq a(p-p_c)^{-\nu} > a$, where $\nu = \frac{4}{3}$. The resistance of any square lattice scales with the mesh size and thus

$$R \propto \xi \propto (p-p_c)^{-\nu}, \tag{2}$$

or, $t = \nu$.

Based on the assumption of a universal spectral dimension, $\bar{\bar{d}} = \frac{4}{3}$, for homogeneous fractals, Alexander and Orbach[12] (AO) have conjectured that

$$t/\nu = (3D/2 - 2 - \beta/2\nu) = 91/96 = 0.947\ldots, \tag{3}$$

where $\nu = \frac{4}{3}$ and $\beta = \frac{5}{36}$ and D is the Euclidean dimensionality of the system taken to be 2. Computer calculations[4] have shown that while (3) is close, it is probably not exact. Numerical results agree to high precision and give $t/\nu = 0.973 \pm 0.004$, in disagreement with the AO conjecture. From measurements on thin Au films, Palevski *et al.*[13] found, making the assumption $d - d_c \propto p - p_c$, that $t = 1.25 \pm 0.05$ and $t/\nu = 0.94$. Here $d - d_c$ is the difference between the nominal thickness and thickness at the percolation threshold. Octavio, Gutierrez, and Aponte[14] determined t for sputtered thin films using the same assumption and found $t/\nu = 0.94 \pm 0.05$.

In Fig. 1 we show the resistance as a function of $(p-p_c)$ for various values of p. In our samples, the percolation parameters p and p_c are determined from the geometrical design and no assumptions regarding their relation to d is needed. The solid line reflects a best fit to the data yielding $t = 1.31 \pm 0.03$, which together with $\nu = \frac{4}{3}$ gives

$$t/\nu = 0.98 \pm 0.03. \tag{4}$$

This is consistent with both simulations and thin film work, but, within the calculated error, not with the AO conjecture (although the margin is quite small). It is interesting to note that a single exponent t describes data at all values of p, not just in the critical region, $(p-p_c) \ll 1$, a result also found in the work of Refs. 13 and 14.

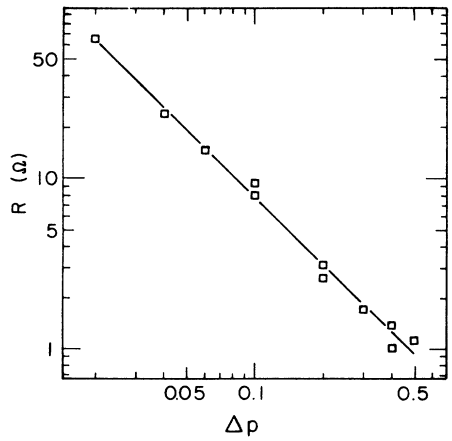

FIG. 1. Log-log plot of resistance vs $\Delta p = (p-p_c)$. Squares are experimental data and the line is a best fit with slope $t = 1.31 \pm 0.03$.

Next, we discuss the critical current in the superconducting state as a function of $p-p_c$. In Fig. 2 we show the temperature dependence of the critical current in two networks with different values of $\Delta p = (p-p_c)$. First, we notice that, at least for small values of $\Delta T = T - T_c$ ($T_c = 1.2485$ K), there is a linear relationship between I_c and ΔT. Theoretically, we might expect the critical current to depend upon $\Delta T^{3/2}$ in the homogeneous case or $\Delta T^{4/3}$ for inhomogeneous samples.[9] Previous measurements, however, exhibited linear behavior, characteristic of short, weak links.[15] This is consistent with the idea of phase slips in the strandlike superconducting links near p_c. I-V curves for our sample are shown in the inset to Fig. 2. Here, the characteristic length is likely the inelastic diffusion length which may be several micrometers in aluminum wires at T_c, $L_i \gg L_{\text{bond}}$. Jumps in the voltage are reminiscent of discrete normal regions forming on the links.[16]

Clearly, the critical current is expected to decrease as p approaches p_c. Again, we can use the "nodes, links, and blobs" model[11] to estimate the behavior of I_c as a function of Δp. We envision a hypercube, one face of which cuts $(L/\xi_p)^{D-1}$ current-carrying channels giving us a critical current of the form

$$I_c = I_{c1}(L/\xi_p)^{D-1}, \tag{5}$$

where I_{c1} is the critical current of a single link. It immediately follows that the dependence of I_c on Δp is

$$I_c \propto \xi_p^{1-d} \propto \Delta p^{\nu(D-1)} \tag{6}$$

as discussed in Ref. 9. For $D = 2$, this gives $v = \nu = 1.3$, a result supported by analytic work in Ref. 5.

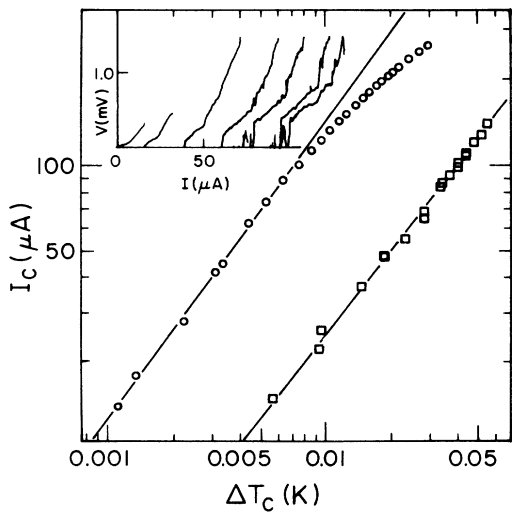

FIG. 2. Log-log plot of critical current vs $\Delta T_c = (T_c - T)$. The circles are for a sample with $p = 0.60$ and the squares for one with $p = 0.52$. The straight lines identify the region linear in ΔT_c. The inset shows typical I-V characteristics for $p = 0.52$ at several temperatures ($T = 1.244$, 1.239, 1.230, 1.220, 1.214, 1.205, and 1.208 K from left to right).

Equation (6) does not consider phase coherence between the various links and is, therefore, accurate only in the inhomogeneous region where $\Delta p \ll 1$ and $\xi_p \gg \bar{\xi}_s$, where $\bar{\xi}_s$ is the superconducting coherence length on the network. Entin-Wohlman *et al.*[9] have considered both the inhomogeneous and homogeneous limits and in the latter they find

$$I_c \propto \Delta p^{(t+\beta)/2} . \tag{7}$$

Using well-known two-dimensional results $t \simeq 1.3$ and $\beta \simeq 0.14$, we expect that $v = (t+\beta)/2 \simeq 0.72$.

In Fig. 3 we have plotted dI_c/dT as a function of Δp. The derivative is measured in the linear, small ΔT, regime of Fig. 2. One can see a change of behavior as one passes from the, presumably, inhomogeneous region ($\Delta p \ll 1$) to the homogeneous region ($\Delta p \gg 0$). Unfortunately our networks do not lie close enough to $p = p_c$, and at sufficiently dense values of p, to completely enter the inhomogeneous regime. However, v is clearly larger at small Δp than large and is close to $v = \frac{4}{3}$ (see solid line of Fig. 3). At larger Δp, the exponent decreases and is consistent with the homogeneous prediction, $v = 0.72$. The crossover is broad and occurs somewhere in the range $0.02 < \Delta p < 0.08$. Below we shall see that this is similar to the crossover in the behavior of the critical field $H_{c2}(\Delta p)$.

Finally, we consider the behavior of $H_{c2}(\Delta p)$. In Ref. 3 we described the method used to make these measurements. There we presented results for $0.04 \leq \Delta p \leq 0.10$ in which $H_{c2}(T)$ was always linear in $\Delta T = (T_c - T)$ and we defined the critical-field strength as dH_{c2}/dT. For values of $\Delta p > 0.2$, reported here, the temperature dependence of H_{c2} is no longer linear ($\Delta T \propto H^{\alpha}$, with $0.85 < \alpha < 1.0$) and we adopt the convention that the critical-field derivative is represented by $dH_{c2}/dT \equiv 0.1G/\Delta T(0.1G)$. The nature of this sublinear temperature dependence is unclear.

In Fig. 4, we plot the critical field strength as a function of Δp for $0.02 \leq \Delta p \leq 0.5$. Again, we might expect a transition between inhomogeneous ($\Delta p \ll 1$) and homogeneous ($\Delta p \gg 1$) regimes. This appears to be the case, with the power-law behavior, $dH_{c2}/dT \propto \Delta p^{-k}$, characteristic of the homogeneous regime, saturating as Δp approaches zero. The broad nature of this transition is suggested by Alexander[17] and is possibly seen in the data of Ref. 6.

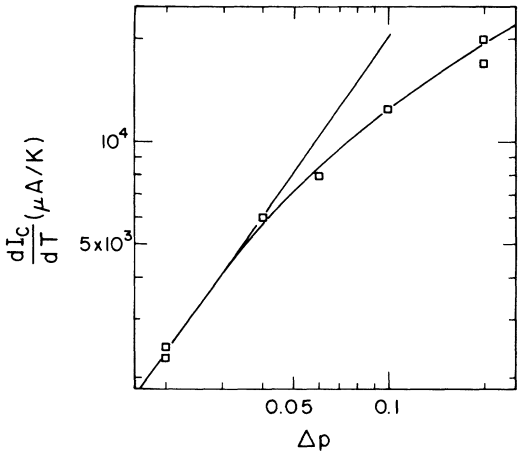

FIG. 3. Log-log plot of dI_c/dT vs $\Delta p = (p - p_c)$. Squares are experimental data and the curved line is a guide. The straight line is a fit to the points with $p \leq 0.54$ and has a slope of $v = 1.33$.

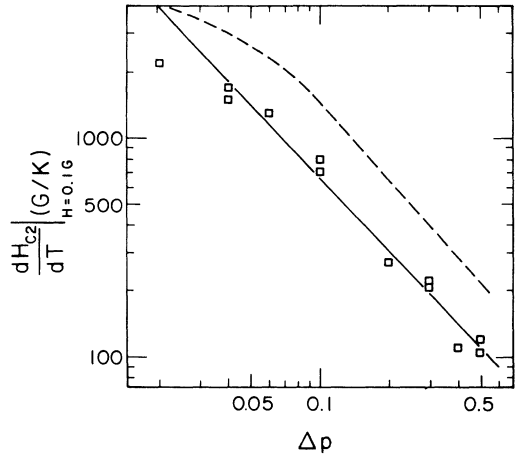

FIG. 4. Log-log of $dH_{c2}/dT \propto A$ vs $\Delta p = (p - p_c)$. The straight line is a fit to $p \geq 0.54$ and has a slope of $k = 1.11 \pm 0.04$. The dashed line is the result of the simulation of Ref. 7.

Analysis of our data for $\Delta p > 0.04$ yields $k = 1.1 \pm 0.05$. Using our earlier result,[3] $k = \nu\Theta$, we calculate $k \simeq 1.15$, where we use the two-dimensional values $\nu = \frac{4}{3}$ and $\Theta = (t - \beta)/\nu \simeq 0.87$. This is in good agreement with both our data and the simulation of Simonen and Lopes[7] (the dashed line in Fig. 4). They actually plotted the related quantity,

$$A = dH_{c2}/dT\{[\xi(0)/L]^2(T_c/H_0)\} .$$

The dependence of dH_{c2}/dT on Δp is almost identical in the simulation of Ref. 7 and our data, each showing a "saturation" in the limit of small Δp. The crossover is again quite broad and over about the same range as that in Fig. 3. The measurements of $\Delta T(H)$ were generally taken in the range $0.05 \ll H \ll 0.5$, or, equivalently, where the magnetic length has the values $6\ \mu\text{m} \ll L_H = (\phi_0/H)^{1/2} \ll 20\ \mu\text{m}$. If we assume that $\xi_p \simeq L_H$ at the crossover we would expect it somewhere in the range $0.03 < \Delta p < 0.08$. Soukoulis *et al.*[8] have predicted two different definitions for k, one describing the band-edge states and one describing the onset of extended states. They find $k_{\text{band edge}} = 0.84 < k = 1.15 < k_{\text{extended}} = 2.50$ and suggest this as evidence that our measurements lie somewhere in between, most likely due to the absence of nonlinear terms in the theory.

In summary, we have fabricated percolating networks which reproduce theoretical models. The critical exponents measured for conductivity (t), critical current (v), and the critical-field exponent (k) all agree with theoretical values. This suggests that results of measurements on thin films which deviate from theoretical expectations are probably due to deviations from true percolating geometry.

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