Properties of strong-coupled superconductors

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By use of Eliashberg equations the system of electrons and phonons with the Einstein spectrum $\alpha^2(\omega)F(\omega)=(\lambda\Omega/2)\delta(\omega-\Omega)$ is studied. The orbital and paramagnetic upper critical magnetic fields are obtained in the case of strong electron-phonon coupling λ (including $\lambda \gg 1$). For $\lambda \gg 1$, superconducting parameters at very low temperatures differ remarkably from those near T_c; the crossover takes place at a temperature corresponding to the phonon frequency Ω. For $\lambda \gtrsim 4$, at temperatures below about $\Omega/3$ the superconducting correlation length decreases, giving the positive curvature in the temperature dependence of the upper critical field. The coefficients of the Ginzburg-Landau functional are calculated. The absolute value of the specific-heat jump grows with λ while its relative value drops. For $\lambda \gtrsim 4$, the resistivity above T_c increases linearly with temperature.

I. INTRODUCTION

The Bardeen-Cooper-Schrieffer (BCS) model is known to describe the behavior of superconductors with weak electron-phonon interaction, $\lambda \ll 1$, well. The superconducting properties of the most real compounds with $\lambda \approx 1$ deviate slightly from the BCS behavior (see, for example, Ref. 1). Recently a number of superconductors were studied whose properties differ remarkably from the standard BCS results.

For example, in organic layered superconductors bis[(ethylenedithio)tetrathiafulvalene] tri-iodide [β_H-(BEDT-TTF)$_2$I$_3$] with $T_c=8$ K and β-(BEDT-TTF)$_2$AuI$_2$ with $T_c=5$ K the anomalous temperature dependence of the upper critical fields H_{c2} was observed. In the former the dependence $H_{c2}(T)$ for the magnetic field perpendicular to the layers has a positive curvature and the reduced upper critical field $h_{c2}(T)=H_{c2}(T)/(-dH_{c2}/dT)_{T=T_c}T_c$ reaches the value of 1.4 as $T\rightarrow 0$,[2] while in the BCS model $h_{c2}(0)\simeq 0.7$. It should be noted that for the perpendicular field the orbital effect alone is significant. For the parallel direction both the effects (orbital and paramagnetic) determine the upper critical field. Measurements of the parallel upper critical field[3] show that $h_{c2}(0)$ obviously exceeds the BCS result which corresponds to the combined action of the orbital and paramagnetic effects. In β-(BEDT-TTF)$_2$AuI$_2$ the dependence $H_{c2}(T)$ is linear down to very low temperatures and the ratio $\Delta(0)/T_c$ as high as 7 was observed at $T \ll T_c$ (Ref. 4) instead of 1.76 in the BCS model. Here $\Delta(0)$ is the superconducting gap at $T=0$ (a more detailed discussion of organic superconductors is given in a review[5]).

A similar behavior of $H_{c2}(T)$ was observed also in the system Ba(Rb$_{1-x}$Bi$_x$)O$_3$ and in the new high-temperature superconductors of La-Sr-Cu-O or Y-Ba-Cu-O type. It is impossible to explain the above-mentioned strong deviations from the BCS behavior in the framework of the electron-phonon model with intermediate coupling $\lambda \approx 1$ though the theoretical calculations based on Eliashberg equations show the tendency of the values of $h_{c2}(0)$ and $\Delta(0)/T_c$ to grow with λ.[6-8]

II. ELECTRON-PHONON MODEL WITH EINSTEIN SPECTRUM

Henceforth we consider a system of electrons coupled with a one-phonon mode, whose spectral density of interaction (Eliashberg function) is $\alpha^2(\omega)F(\omega)=(\lambda\Omega/2)\delta(\omega-\Omega)$. In the limit $\lambda \gg 1$ the asymptotic behavior of T_c is known, $T_c=0.18\Omega\sqrt{\lambda}$ (see Ref. 9). We study here the dependence of the upper critical field $H_{c2}(T)$ on λ. Part of the results obtained for $H_{c2}(T)$ and some thermodynamical properties of the model under consideration were discussed in Ref. 10.

Some remarks concerning the model with a one-phonon mode will be useful. First, all the results for this model are valid in the case of the excitonic mechanism of superconductivity also, i.e., in the case of electron pairing due to exchange by any excitations with a frequency Ω such that $\Omega \ll E_F/\lambda$. Second, the deviations of the real extended phonon spectrum from the Einstein one can be taken into account in our final results if we change the value Ω by the corresponding moment of the function $\alpha^2(\omega)F(\omega)$. For example, in the asymptotic expression of T_c the value $\Omega\sqrt{\lambda}$ should be changed by the value

$$(\lambda\langle\omega^2\rangle)^{1/2}=[M(1)]^{1/2}$$

where

$$M(1)=2\int_0^\infty d\omega\,\omega\alpha^2(\omega)F(\omega)\ .$$

More important is the question about the possibility of very large λ values in real compounds. Values as high as

2.5 were observed up to now in alloys such as Pb-Bi. Theoretical estimations give similar values in La-Sr-Cu-O compounds.[11] In principle, larger λ's can be realized in crystals near structural instabilities.

In the general case λ and phonon frequencies are determined by the expressions

$$\lambda = N(0)\langle I^2\rangle\langle 1/M\omega^2\rangle \;, \qquad \langle\omega^2\rangle = \langle\omega_0^2\rangle = (4n_0/5)N(0)\langle I^2\rangle/M \;, \tag{1}$$

where $\langle I^2\rangle$ is the mean value of the electron-phonon matrix elements over the Fermi surface, M the ion mass, n_0 the number of conducting electrons per atom, $\langle\omega^2\rangle$ the mean value of the squared frequency of phonons, and ω_0 the pseudoatomic bare frequency (the bare frequency in the Fröhlich electron-phonon model); see Refs. 12–14. As the value $(4n_0/5)N(0)\langle I^2\rangle/M$ grows, the phonon frequencies fall and the system approaches structural instability. This fall is maximal for some mode i, and the parameter λ_i which describes the coupling of this soft mode with electrons increases and can reach a very high value. What is important is that this growth gives an increase of T_c limited to $T_c^{\max}=0.18[N(0)\langle I^2\rangle/M]^{1/2}$. According to (1) the stability condition $\omega_i^2>0$ restricts the maximal values of $N(0)\langle I^2\rangle/M$, and as result T_c is limited to a value of about $0.2\langle\omega_0\rangle$. Thus, very high values of λ may occur near a structural instability, and that mode is important for the pairing which softens here. In such a case the electron-phonon model under consideration may be good enough.

III. THE UPPER-CRITICAL MAGNETIC FIELDS

Now we write the equations for the upper-critical field H_{c2} which take into account the orbital effect of a magnetic field as well as a paramagnetic one. (We neglect the Coulomb pseudopotential μ^* in comparison with large λ.) They have the form (see Ref. 7).

$$\tilde\Delta(i\omega_n) = \pi T\sum_m \lambda(\omega_n-\omega_m)[\chi^{-1}(\tilde\omega_n)-(2\tau_{\rm imp})^{-1}]^{-1} \times\tilde\Delta(i\omega_m)\;,$$
$$\tilde\omega_n=\omega_n+\pi T\sum_m\lambda(\omega_n-\omega_m)\,{\rm sgn}\omega_m+(2\tau_{\rm imp})^{-1}\,{\rm sgn}\omega_n\;,$$
$$\lambda(\omega_n-\omega_m)=\lambda\Omega^2/[\Omega^2+(\omega_n-\omega_m)^2],\quad \omega_n=\pi T(2n+1)\;, \tag{2}$$

where

$$\chi(\tilde\omega_n)=\frac{2}{\sqrt{\alpha}}\int_0^\infty dq\,\exp(-q^2)\tan^{-1}\times\left[\frac{q\sqrt{\alpha}}{|\tilde\omega_n|+i\mu H_{c2}\,{\rm sgn}\tilde\omega_n^{-1}}\right], \tag{3}$$
$$\alpha=\pi H_{c2}v_F^2/2\phi_0\;,$$

for an isotropic three-dimensional system. Here ϕ_0 is the quantum flux, μ the Bohr magneton, and $\tau_{\rm imp}$ the scattering time due to the nonmagnetic impurities.

We first consider the orbital upper-critical field H_{c2}^* without paramagnetic effect [i.e., setting $\mu=0$ in (3)]. This approach is good enough if the obtained value of H_{c2}^* is small as compared with the paramagnetic upper-critical field $H_{c2}^{(p)}$ which will be studied hereafter. For a dirty superconductor with $\tau_{\rm imp}^{-1}\gg\lambda\Omega$ Eqs. (2) take a more simple form,

$$\tilde\Delta(i\omega_n)=\pi T\sum_m\lambda(\omega_n-\omega_m)\frac{\tilde\Delta(i\omega_n)}{|\tilde\omega_m|+\rho}\;,\qquad \tilde\omega_n=\omega_n+\pi T\sum_m\lambda(\omega_n-\omega_m)\,{\rm sgn}\omega_m\;, \tag{4}$$

where $\rho=eH_{c2}^*D$, and $D=v_F^2\tau_{\rm imp}/3$ is the diffusion coefficient. So the calculation of H_{c2} is reduced to the problem of finding the maximal eigenvalue of the linear finite-difference equation.

In the limiting case $\lambda\gg1$, the inequality $T_c\gg\Omega$ is fulfilled, and near T_c the eigenfunction may be approximated as $\tilde\Delta(\pi T)=\tilde\Delta(-\pi T)\neq0$, all other $\tilde\Delta(\omega_n)$ being zero. Such an approximation gives a rather good value of T_c for $\lambda\gg1$ (see Ref. 9). The result for $H_{c2}^*(T)$ is

$$H_{c2}^*(T)\simeq\frac{T\phi_0}{D}[\lambda(\pi T)-1]=\frac{T\phi_0}{D}\left[\frac{T_c^2}{T^2}-1\right],\quad T\gg\Omega\;. \tag{5}$$

When $T\to T_c$ we get $H_{c2}^*(T\to T_c)=2\phi_0T_ct/D$, where $t=(T_c-T)/T_c$, while the extrapolation to temperatures $T\approx\Omega$ gives $H_{c2}^*\approx\phi_0T_c\sqrt{\lambda}/D$, i.e., it is larger by the factor $\sqrt{\lambda}$ than the corresponding BCS result. The correct results obtained numerically are

$$H_{c2}^*(T)=\frac{2.20\phi_0T_c}{D}t,\quad t\ll1\;,\qquad H_{c2}^*(0)=\frac{1.08\phi_0T_c\sqrt{\lambda}}{D}\;. \tag{6}$$

By use of Eq. (6) the reduced orbital critical field may be found, $h_{c2}^*(0)=0.45\sqrt{\lambda}$, for $\lambda\gg1$. This asymptotic behavior is correct for $\lambda\gtrsim6$. For the intermediate values of λ the numerical results for $h_{c2}^*(T/T_c)$ are presented in Fig. 1. For $\lambda\gg1$ the value of $h_{c2}^*(0)$ greatly exceeds the BCS value of 0.7. At $\lambda\gtrsim4$ the dependence $H_{c2}^*(T)$ has a positive curvature.

Now the correlation length $\xi(T)$ can be obtained by means of the relation $H_{c2}^*(T)\sim\phi_0/\xi^2(T)$. Near T_c we get the usual BCS expression for a dirty superconductor, $\xi(T)\sim(\xi_sl_{\rm imp})^{1/2}t^{-1/2}$, where $l_{\rm imp}$ is the electron mean free path, $l_{\rm imp}=v_F\tau_{\rm imp}$, and the actual superconducting length $\xi_s(T_c)\simeq v_F/T_c$. However, when $T\to0$ and $\lambda\gg1$, the value of ξ_s is decreased by the factor $\sqrt{\lambda}$, i.e., $\xi(0)=[\xi_s(0)l_{\rm imp}]^{1/2}$ with $\xi_s(0)\approx\xi_s(T_c)/\lambda$.

If we introduce the order parameter $\Delta(0)$ at $T=0$ by means of the relation $\Delta(0)=v_F/\xi_s(0)$, we obtain $\Delta(0)=C\lambda\Omega$, where C is some numerical factor. We note that the order parameter may differ from the energy gap $\Delta_g(0)$ because in the case of strong coupling there is no reason to identify these quantities. Moreover, they actually differ according to the result $\Delta_g(0)\sim\Omega\sqrt{\lambda}$ at large λ obtained in Refs. 15 and 16.

We now consider a clean superconductor, i.e., $\tau_{\rm imp}^{-1}\ll\lambda\Omega$, and $\lambda\gg1$. The equation for $\Delta(i\omega_n)$ can be

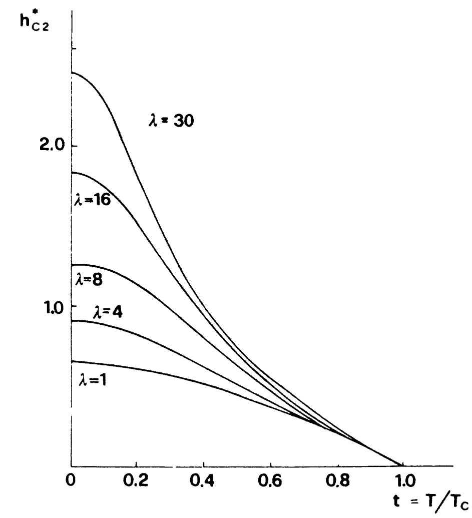

FIG. 1. The temperature dependence of the reduced orbital upper critical field $h_{c2}^{*}(T/T_c)$ in the dirty superconductor with Einstein spectrum for some values of λ. The curve $\lambda=1$ represents the BCS result well.

reduced to the standard eigenvalue problem in the case $T \rightarrow T_c$ only. Then

$$\chi(\tilde{\omega}_n) = |\tilde{\omega}_n|^{-1}[1-(\alpha/3)|\tilde{\omega}_n|^{-2}]$$

and as a result,

$$H_{c2}^{*}(T \rightarrow T_c) = 12\pi\lambda T_c^2 \phi_0 t / v_F^2 \ .$$

This expression coincides with the corresponding one in a dirty system if we introduce the diffusion coefficient $D_{\mathrm{ph}} = v_F^2 \tau_{\mathrm{ph}}/3$ with $\tau_{\mathrm{ph}}^{-1} = 2\pi\lambda T_c$, which describes the scattering of electrons on thermal phonons. In the following, we will discuss this relation in more detail. The numerical calculations for $\lambda \gg 1$ yield

$$H_{c2}^{*}(T) = \frac{12.5\pi\lambda T_c^2 \phi_0}{v_F^2} t, \quad t \ll 1 \ ,$$

$$H_{c2}^{*}(0) = \frac{18.8\pi\lambda T_c^2 \phi_0}{v_F^2} \ . \tag{7}$$

The behavior of $h_{c2}^{*}(T/T_c)$ for the intermediate values of λ in clean superconductors is shown in Fig. 2. In Fig. 3 we present the value 2α, which is $(-dH_{c2}/dT)_{T=T_c}$ normalized to the corresponding BCS value as a function of λ, as well as $h_{c2}^{*}(0)$ in clean and dirty superconductors.

Making use of Eq. (7) we see that $h_{c2}^{*}(0)$ is saturated as $\lambda \rightarrow \infty$ for clean superconductors, while in dirty ones h_{c2}^{*} grows as $\sqrt{\lambda}$. This distinction is connected with different behavior of $H_{c2}^{*}(T)$ near T_c and not near $T=0$. Actually, $\xi_s(T_c) \simeq v_F/T_c$ and $\xi_s(0) \approx v_F/T_c\sqrt{\lambda}$ irrespective of the system purity. By use of these values of $\xi_s(T)$ we get the expected values of $H_{c2}^{*}(0) \approx \phi_0/l_{\mathrm{imp}}\xi_s(0)$ in dirty and $\phi_0/\xi_s^2(0)$ in clean superconductors. However, near T_c we obtain the expected value $H_{c2}^{*} \approx \phi_0 t/l_{\mathrm{imp}}\xi_s(T_c)$ in a dirty system, while the actual H_{c2}^{*} differs from the expected

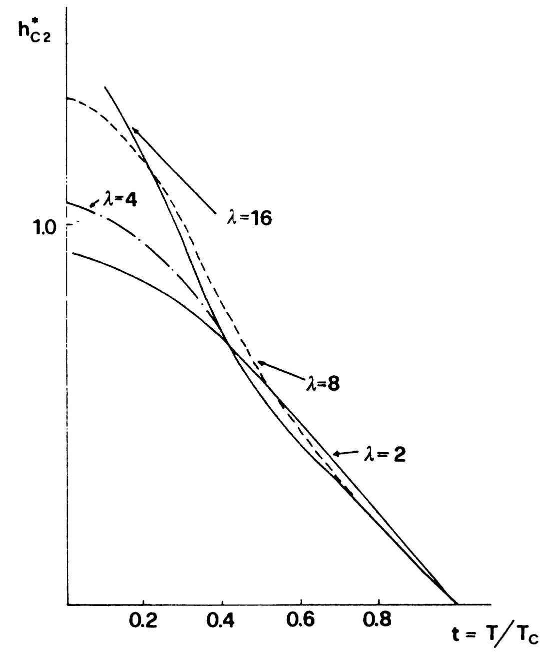

FIG. 2. The temperature dependence of the reduced orbital critical field $h_{c2}^{*}(T/T_c)$ in an isotropic clean superconductor for some values of λ.

value $\phi_0 t/\xi_s^2(T_c)$ by the factor λ in a clean superconductor.

We now explain, first, why the value $\Delta(0)$ is much larger than T_c (by the factor $\sqrt{\lambda}$) in systems with $\lambda \gg 1$ and, second, the reason of the above-mentioned differences between the behavior of H_{c2}^{*} in dirty and clean

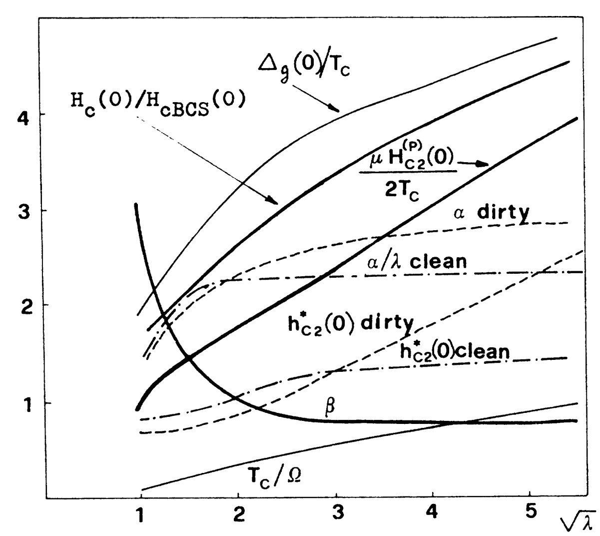

FIG. 3. The dependence of superconducting parameters on λ for the electron-phonon model with Einstein spectrum. The data for $\Delta(0)/T_c$ are taken from Ref. 15, the ratio T_c/Ω from Ref. 9. The value 2α is $(-dH_{c2}/dT)_{T=T_c}$ normalized to the corresponding BCS value. The $H_{c2}^{(p)}$ is the upper critical paramagnetic field (without orbital effect) for the appearance of infinitely small superconducting nuclei as the magnetic field is decreasing. The dependence of parameter β in GL functional and $H_c(0)/H_{c\mathrm{BCS}}(0)$ on λ is also shown.

systems.

Qualitatively the first result may be explained by depression of the Cooper pairing near T_c due to the presence of thermal phonons ($\Omega \ll T_c$ at large λ) and the corresponding disappearance of this depression factor at $T \ll \Omega$ where the thermal phonons are frozen out. At $T = T_c$ the attraction of electrons via virtual phonons would give $T_c \sim \lambda\Omega$ which should be diminished by the small factor $N_{\rm ph}^{-1} = (e^{\Omega/T} - 1) \simeq \Omega/T_c$ because thermal phonons destroy the coherence of the interaction via virtual phonons. Thus we get $T_c \sim \lambda\Omega(\Omega/T_c)$ from which the result $T_c \sim \Omega\sqrt{\lambda}$ follows. At low temperatures $T \ll \Omega$ the depression factor disappears and the relation $\Delta(0) \sim \lambda\Omega$ ought to be valid. Thus the superconducting length ξ_s is V_F/T_c near T_c and less by the factor $\lambda^{-1/2}$ at $T \ll \Omega$. For this reason the rapid growth of $h_{c2}^*(T/T_c)$ below the temperature $T \approx \Omega$ occurs in dirty superconductors, and in this case $h_{c2}^*(0) \sim \sqrt{\lambda}$.

In clean superconductors the effect of impurities is negligible. However, the thermal phonons present near T_c here determine the electron scattering time $\tau_{\rm ph} = (2\pi\lambda T_c)^{-1}$ and $\tau_{\rm ph}^{-1} \gg T_c$ for $\lambda \gg 1$. So with respect to the electron scattering on phonons, the clean superconductors with $\lambda \gg 1$ near T_c occur actually as dirty ones because

$$l_{\rm ph} \simeq v_F \tau_{\rm ph} \simeq \zeta_s(T_c)/\lambda \ll \xi(T_c) \ .$$

Thus using the standard theory for H_{c2}^* in dirty systems we get

$$T_c(-dH_{c2}^*/dT)_{T_c} \simeq \phi_0/l_{\rm ph}\xi_s(T_c) \simeq \phi_0\lambda/\xi_s^2(T_c) \ ,$$

i.e., $\alpha_{\rm clean} \sim \lambda$. On cooling below $T \simeq \Omega$ the value $l_{\rm ph}$ grows. However the increase of $l_{\rm ph}$ is compensated here by the decrease of $\zeta_s(T)$. So the rapid growth of H_{c2}^* below $T \approx \Omega$ is absent.

By use of (6) we find the relation connecting the value $(dH_{c2}/dT)_{T=T_c}$ the density of states $N(0)$, and the resistivity $\rho(T_c)$ at $T = T_c$ for dirty superconductors (the Gor'kov relation)

$$-\left[\frac{dH_{c2}}{dT}\right]_{T=T_c} = 16\alpha_{\rm dirty}(\lambda) e\rho(T_c) N(0)/\pi \ , \qquad (8)$$

where $\alpha_{\rm dirty} \to 0.5$ as $\lambda \to 0$. The same relation is valid in clean superconductors with $\lambda \gg 1$ if their resistivity is determined primarily by electron-phonon scattering near T_c.

By means of Eqs. (2) and (3) we can calculate the upper critical field due to the paramagnetic effect alone ($\mu \neq 0$, $\alpha \to 0$). We then get the linear difference equation with a complex kernel. It gives the critical field $H_{c2}^{(p)}$ at which for the first time infinitely small superconducting nuclei appear as the magnetic field is decreasing. [Actually the first-order transition from the normal state to the superconducting one can occur at $H = H_p$ where the magnetic field H_p is thermodynamical critical field which is determined by the free-energy consideration. For the BCS model $\mu H_{c2}^{(p)} = \Delta(0)/2$, while $\mu H_p = \Delta(0)/\sqrt{2}$ at $T = 0$.] We are interested here in the behavior of $H_{c2}^{(p)}$ as a function of λ at $T = 0$. The numerical results for the ratio $\mu H_{c2}^{(p)}/2T_c$ are shown in Fig. 3. The asymptotic behavior of this ratio is $\sqrt{\lambda}$ for large λ. The BCS value ($\lambda \ll 1$) is 0.44. Again, the value $\mu H_{c2}^{(p)}(0)$ should be determined by the value $\Delta(0)$ (with some numerical coefficient) and we obtain additional evidence in favor of the relations $\Delta(0) \sim \lambda\Omega$ and $\Delta(0)/T_c \sim \sqrt{\lambda}$ for large λ.

The numerical calculations of $\Delta_g(0)/T_c$ for values of λ up to 28 were done in Ref. 15 any they are presented in Fig. 3. Here $\Delta_g(0)$ is the energy gap for tunneling of electrons. Based on the analytic consideration the authors of Ref. 15 came to the conclusion that the value $\Delta_g(0)/T_c$ is saturated at very large λ. We give here strong evidence as well as a clear physical picture in favor of the ratio for order parameter $\Delta(0)/T_c$ to be proportional to $\sqrt{\lambda}$ for $\lambda \gtrsim 4$. This discrepancy shows that the order parameter and the gap may be quite different at large λ.[17]

IV. DERIVATION OF THE GINZBURG-LANDAU FUNCTIONAL

The free-energy functional in the case of the space-homogeneous parameter $\Delta(i\omega_n)$ was obtained in Ref. 18,

$$(F_s - F_n)/N(0) = \pi T \sum_n \omega_n(\mathrm{sgn}\omega_n - \sin\psi_n) + \pi^2 T^2 \sum_{n,m} [\mathrm{sgn}(\omega_n\omega_m) - \cos(\varphi_n - \varphi_m)]\lambda(\omega_n - \omega_m), \quad \varphi_n \equiv \varphi(i\omega_n) \ . \qquad (9)$$

By use of (9) we calculate the thermodynamical critical field $H_c(0)$ at $T = 0$. The asymptotic behavior for large λ is $H_c(0)/H_{c_{\rm BCS}}(0) = 1.16\sqrt{\lambda}$, where $H_{c\rm BCS}(0)$ is the value $H_c(0)$ which corresponds to T_c by the BCS relation. The dependence $H_c(0)/H_{c\rm BCS}(0)$ on λ for intermediate λ is shown in Fig. 3. Thus $H_c(0)$ is proportional to λ at large λ like $H_{c2}^{(p)}(0)$ and $\xi_s^{-1}(0)$.

Minimization over φ_n gives equilibrium free energy near T_c

$$(F_s - F_n) = N(0)(2\pi T_c)^2 t^2/\beta(\lambda) \ . \qquad (10)$$

The function $\beta(\lambda)$ is shown in Fig. 3. The BCS value of β is $7\zeta(3) \simeq 8.4$ [for $\lambda \gg 1$ in the approximation $\tilde{\Delta}(\pi T) = \tilde{\Delta}(-\pi T) \neq 0$ and others, $\tilde{\Delta}(\omega_n)$ being zero, we get $\beta = 1$]. Now we can write Ginzburg-Landau (GL) functional. It has the form

$$(F_s - F_n)/N(0)(2\pi T_c)^2 = \xi^2 \left| \left[\nabla - \frac{2ie}{c}A\right]\psi \right|^2 - t\psi^2 + \tfrac{1}{4}\beta(\lambda)\psi^4 \ . \qquad (11)$$

The value of ξ^2 is determined by H_{c2}^* near T_c according to the relation

$$(-dH_{c2}^*/dT)_{T_c}=\phi_0/2\pi\xi^2 T_c \ .$$

Thus

$$\xi^2=\pi D/16T_c\alpha_{\rm dirty}(\lambda)t \tag{12}$$

for dirty superconductors, and for clean ones

$$\xi^2=0.009v_F^2/T_c^2\alpha_{\rm clean}(\lambda)t \ . \tag{13}$$

We can now calculate the London penetration depth near T_c. For clean superconductors

$$\frac{\lambda_L^{-2}(T)}{\lambda_L^{-2}(0)}=8.4\frac{t}{\alpha_{\rm clean}(\lambda)\beta(\lambda)} \ , \quad \lambda_L^{-2}(0)=\frac{8\pi e^2}{3c^2}N(0)v_F^2 \ , \tag{14}$$

and for dirty superconductors

$$\frac{\lambda_L^{-2}(T)}{\lambda_L^{-2}(0)}=\frac{2\pi^2\tau_{\rm imp}T_c t}{\alpha_{\rm dirty}(\lambda)\beta(\lambda)} \ . \tag{15}$$

By use of (10) we find the specific heat jump

$$\Delta C=8\pi^2 N(0)T_c/\beta(\lambda) \ . \tag{16}$$

From the GL functional, the following relation can be obtained for isotropic superconductor:

$$N(0)=\frac{H'_{c1}H'_{c2}\beta(\lambda)}{(\ln\kappa+0.497)32\pi^2}=\frac{\Delta C\beta(\lambda)}{8\pi^2 T_c} \ , \quad \frac{\kappa^2}{\ln\kappa+0.497}=\frac{H'_{c2}}{H'_{c1}} \ , \tag{17}$$

where $H'_{c1}=(-dH_{c1}/dT)_{T=T_c}$ and κ is the GL parameter. According to (16) and the dependence $\beta(\lambda)$ the absolute value of the specific-heat jump grows with λ. However, its relation to the full specific heat (C_e+C_{ph}) at $T=T_c$ falls because (C_e+C_{ph}) increases more rapidly than ΔC with λ due to the phonon contribution.[19]

V. DISCUSSION

We obtain the upper-critical magnetic fields (orbital and paramagentic) and the coefficients of GL equations as functions of λ for $\lambda \geq I$ in the electron-phonon system with an Einstein spectrum. The asymptotic behavior peculiar to large λ starts at $\lambda \gtrsim 4-6$; see Fig. 3. In the regime of large λ the thermal phonons weaken Cooper pairing near T_c, giving $T_c \sim \sqrt{\lambda}$, and they freeze out at $T \lesssim \Omega$, giving $\Delta(0) \sim \lambda$ and the ratio $\Delta(0)/T_c \sim \sqrt{\lambda}$. Thus the superconductors with $\lambda \gtrsim 4-6$ may be called strong-coupling superconductors. They are characterized by the depairing effect of thermal phonons near T_c and by the corresponding anomalous enhancement of superconductivity at low temperatures.

The properties of strong-coupling superconductors in the normal state are characterized by the linear dependence of resistivity above T_c. According to our result for H_{c2}^* in clean systems the scattering of electrons on the thermal phonons gives the temperature-dependent part

$$\rho(T)=8\pi^2\lambda T/\omega_p^2 \ , \tag{18}$$

which allow us to estimate λ if the plasma frequency ω_p is known.

Now we consider briefly the possibility to explain the high T_c values in oxides in the framework of electron-phonon mechanism of coupling. Three points are essential in this discussion.

(1) The limiting value of T_c may be estimated from the dependence of T_c on λ and λ on Ω for $\lambda \gtrsim 2$. Thus

$$T_c=0.18[N(0)\langle I^2\rangle/M]^{1/2} \approx (300-400\ {\rm K})(m_p/M)^{1/2} \ , \tag{19}$$

where m_p is the proton mass. So, in principle, values as high as 100 K (for $M=16m_p$) may be obtained in oxides if the conducting electrons interact strongly with oxygen ion vibrations. The singularities of the density of states near the Fermi surface additionally enhance the values of T_c.

(2) The small value of the isotope effect in the Y-Ba-Cu-0 system does not exclude in principle the photon mechanism. The Debye-Waller factor in electron-phonon coupling which takes into account the effect of nonlinearity of ionic displacements [it is dropped in Eq. (19)] can compensate for the dependence of T_c on M given by Eq. (19). For example, in palladium hydrides PdH and PdD, the isotope effect has opposite sign.

(3) The linear dependence ρ on T was observed in oxide superconductors Y-Ba-Cu-O giving $\lambda \approx 2.5$ for $d\rho/dT \approx 2\ \mu\Omega$ cm/K (see Ref. 18 and references therein).

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[1] D. J. Scalapino, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969).

[2] F. Creuzet, G. Creuzet, D. Jerome, D. Schweitzer, and H. J. Keller, J. Phys. Lett. **46**, L1079 (1985).

[3] V. N. Lauchin, S. I. Pesotskii, and E. B. Yagubskii, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 394 (1987) [JETP Lett. **45**, 394 (1987)].

[4] M. E. Hawley, K. E. Gray, B. D. Terris, H. H. Wang, K. D. Carlson, and J. M. Williams, Phys. Rev. Lett. **57**, 629 (1986).

[5] L. N. Bulaevskii, Adv. Phys. (to be published).

[6] B. T. Geilikman, V. Z. Kreskin, and N. F. Macharov, J. Low Temp. Phys. **18**, 241 (1975).

[7] M. Schossmann and E. Schachinger, Phys. Rev. B **33**, 6123 (1986).

[8] F. Marsiglio, M. Schossmann, E. Schachinger, and J. P. Carbotte, Phys. Rev. B **35**, 3226 (1987).

[9] P. B. Allen and R. C. Dynes, Phys. Rev. B **11**, 1895 (1975).

[10] L. N. Bulaevskii and O. V. Dolgov, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 413 (1987) [JETP Lett. **45**, 526 (1987)].

[11]W. Weber, Phys. Rev. Lett. **58**, 1371 (1987).
[12]J. R. Gomersall and B. L. Gyorffy, Phys. Rev. Lett. **33**, 1286 (1974).
[13]P. B. Allen, in *Dynamical Properties of Solids,* edited by G. K. Horton and A. A. Maradudin (North-Holland, Amsterdam, 1980), Vol. 3.
[14]O. V. Dolgov and E. G. Maksimov, Usp. Fiz. Nauk **138**, 95 (1982) [Sov. Phys. Usp. **25**, 688 (1982)].
[15]J. P. Carbotte, F. Marsiglio, and B. Mitrović, Phys. Rev. B **33**, 6135 (1986).
[16]V. Z. Kresin, Solid State Commun. **63**, 725 (1987).
[17]Now the gap $\Delta_g(0)$ at large λ was calculated numerically by use of the Eliashberg equations in real frequencies representation, see O. V. Dolgov and A. A. Golubov, in Proceedings of the Adriatico Research Conference, Trieste, 1988 (unpublished). Such an approach gives the gap $\Delta_g(0)$ which is proportional to λ. The ratio $\Delta_g(0)/T_c$ grows as $1.14\lambda^{1/2}$ with λ. The different results of Refs. 15 and 16 for $\Delta_g(0)$ were obtained by the use of the Matsubara frequency representation and the following analytical continuation to the real frequencies. The accuracy of the latter procedure at large λ is in question.
[18]D. Rainer and G. Bergmann, J. Low Temp. Phys. **14**, 501 (1974).
[19]The GL functional (11) for the Eliashberg model gives the mean-field description only. This functional does not describe correctly the fluctuations. This conclusion follows from the fact that the fluctuation contribution to the specific heat C_{fl} obtained by use of the GL functional (11) and the result of the direct microscopic calculation of C_{fl} in the framework of the Eliashberg equations coincide at $\lambda \rightarrow 0$ only and differ significantly at large λ; see L. N. Bulaevskii and O. V. Dolgov, Solid State Commun. **67**, 63 (1988).
[20]P. A. Lee and N. Read, Phys. Rev. Lett. **58**, 2691 (1987).